1 01. AMORTIZED ANALYSIS

- $\sum$ amortized cost $\geq \sum$ actual cost

1.1 Types of amortized analysis

1.1.1 Aggregate method

- Add up the cost of all operations and divide by number of operations
- amortized cost per op $= \frac{1}{k} \sum_i \text{cost}(i)$
- Simple but unable to handle more complex algorithms

1.1.2 Accounting method

- Allow an op to charge extra cost to store credit for future use, and allow an op to withdraw credit to pay for its cost
- Charge only if amortized cost $\geq$ actual cost
- Withdraw only if amortized cost $<$ actual cost

1.1.3 Charging method

- Allow an op to charge cost retroactively to past ops
- amortized cost of an op = actual cost of this op - total cost charged to past ops + total cost charged by future ops

1.1.4 Potential method

- Define a potential function $\phi : \text{DS} \to \text{value}$
- ϕ denotes the total unused credits stored by all past ops
- **Invariant**: $\phi(n) \geq \phi(0)$ for all $n \geq 0$
- amortized cost of an op = actual cost of this op + $\Delta\phi$
- $\sum$ amortized cost = $\sum$ actual cost + ϕ(final DS) − ϕ(initial DS)
- Fundamentally equivalent to the accounting method