**Lab 1: Calculations for the Saturn V and Apollo 11**

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## I. Introduction

The Saturn V rocket is set to launch with the aim of landing humans on the Moon. During this mission, the spacecraft will be influenced by the gravitational potential and forces exerted by both the Earth and the Moon. It is crucial to determine when the rocket will deplete its fuel and how high above the Earth's surface it will be at that moment. This report aims to summarize the calculations made to provide a comprehensive view of what the Saturn V will experience at launch and what the Apollo 11 will then experience in space.

## II. The gravitational potential of the Earth-Moon system

The first calculations were made to determine and visualize the gravitational potential of Earth the rocket will experience as it gets further and further from the planet. Gravitational potential tells you the amount of energy per unit mass needed to move an object (Saturn V) a distance away from another object (Earth), which is exerting a gravitational force on its surroundings. It is calculated with the following formula:

$$\Phi(\mathrm{r}) = -\frac{GM}{r}$$

where G is the gravitational constant, M is mass, and r is distance from the mass.

Being able to visualize how this value changes as distance from Earth increases can help us make conclusions about how much energy will be needed for the Saturn V to escape Earth's grasp.

Once calculations had been made for the Earth, we could easily add the influence of the Moon to this system. While the Moon is much smaller than Earth, its gravitational force will still play a role in the gravitational potential of the overall system. Gravitational potential was visualized in the following way:

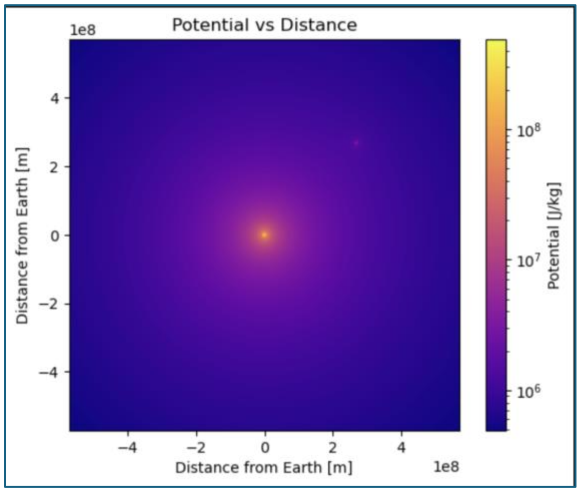


*Figure 1: Mesh plot of Gravitational Potential vs Distance from Earth*

## III. The gravitational force of the Earth-Moon system

The next step was to map the gravitational force that the Earth-Moon system will exert on the Apollo 11 command module. Having a visualization to show how force changes with position informs us of what direction and with how much strength the module will be pulled at any given location in this system. The gravitational force is calculated as follows:

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

where $M_1$ and $m_2$ are the masses of the two objects, $\vec{r}_{21}$ is the magnitude vector of the force and $\hat{r}_{21}$is the direction vector of the force.

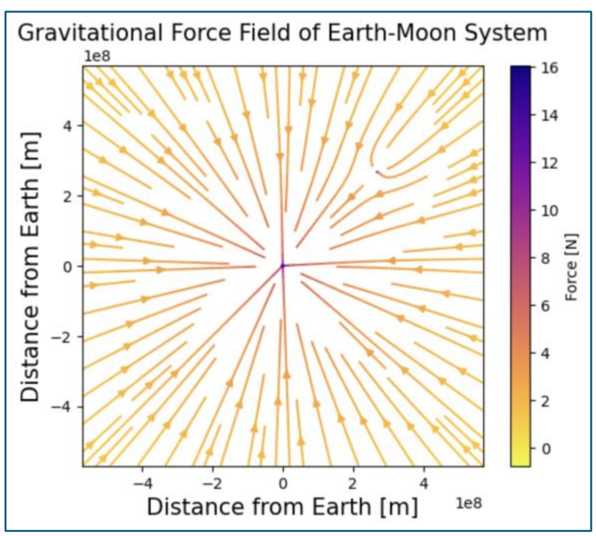


*Figure 2: Stream plot of Gravitational Force vs Distance from Earth*

This plot shows us that the module will experience a fairly uniform force at most positions within a $5.7 \times 10^8$m radius of the Earth except for right near the Earth or Moon's surface.

## IV. Projected performance of the Saturn V Stage 1

The final step was to calculate the total burn time of the first stage of the Saturn V and then use that value to predict the altitude of the rocket at the end of that time. The formula for burn time is

$$T = \frac{m_0 - m_f}{\dot{m}}$$

where $m_0$ is the wet mass, $m_f$ is the dry mass, and $\dot{m}$ is the fuel burn rate of the rocket. This formula yields a burn time of 157.69 seconds.

With the burn time, we can calculate the altitude $h$ of the rocket at burnout with this formula:

$$h = \int_0^T (v_e \ln\left(\frac{m_0}{m(t)}\right) - gt)dt$$

where $v_e$ is the fuel exhaust velocity, $m(t) = m_0 - \dot{m}t$, and g is the gravitational acceleration. This formula yields an altitude of $74{,}093.98 \pm 5.849 \times 10^{-8}$m.

## V. Discussion and Future Work

The calculations summarized above relied on a few key assumptions. We assumed that the Earth and Moon are point masses which mean that our calculations for the gravitational potential and force fields of the Earth and Moon are not exact. We could make more accurate calculations by treating these two bodies as the non-uniform density spheres that they are.

We also assumed that Saturn V maintains a constant fuel burn rate throughout its entire launch. This means that predictions for values such as the burn time and burnout altitude will not be exactly what is measured during the actual launch. In fact, the first prototype of Saturn V burned for about 160 seconds and ended up at an altitude of about 70,000m. This differs from our calculated values of 157.69 seconds and 74,093.98m. Another explanation for these differences could be that we neglected the impact of drag. This force would slightly slow the rocket and make it waste some fuel on overcoming the force rather than increasing in altitude. In the future, we could edit these calculations to take all of these extra factors into account to get an even more realistic view of this mission.