PHYS265 Lab 2: Mine Crafting

Zya Woodfork

I. Introduction

The purpose of this report is to summarize the calculations and visuals made to determine a more accurate number for the depth of our company's 4 4-kilometer mineshaft. Predictions were made in several ways, each of which made their own assumptions about the nature of the Earth-mass system. The following sections will detail each of these calculations and their results, as well as a discussion on whether this depth measurement technique is a good method to proceed with.

II. Calculation of fall time

The first calculations were done to compute how long it would take a 1kg mass object to reach the bottom of a mineshaft that was exactly 4 kilometers deep. The purpose of these calculations was to determine how much of an influence drag force would have on the fall time. To calculate fall time without drag, we first had to find the acceleration the object would undergo due to Earth's gravity. Since this object is in free fall, we can then use a kinematic equation to solve for fall time. Simplifying one of the kinematic equations and solving for time gives us

$$t = \sqrt{\frac{2y}{a}} \quad (1)$$

where y is the object's distance from the surface of the Earth and $a = \frac{GM}{R^2}$ where G is the gravitational constant, M is the mass of Earth, and R is Earth's radius. This method yields a fall time of 28.6 seconds. We then numerically integrated the following differential equation for this system as this is another way to calculate fall time without drag.

$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma} \quad (2)$$

where α is the drag coefficient (set to 0 in this case) and γ is the speed dependence of drag. This calculation yields a very similar fall time which also rounds to 28.6 seconds. Using this differential equation to create a system of coupled first-order differential equations, we then plot the object's position and velocity as a function of time.

In reality, the amount of Earth's mass below causing the mass to accelerate decreases as a function of time, and the mass is also influenced by drag which points opposite the mass's direction of motion. These two factors add to the fall time of the object and should therefore be accounted for to get a more realistic value. The formula for time-dependent gravitational acceleration is

$$g(r) = g_0\left(\frac{r}{R_E}\right) \qquad (3)$$

where g_0 is the acceleration of gravity at the planet's surface, r is the mass's distance from the Earth's center, and R_E is the radius of Earth. While a time-dependent g does not increase the fall time by a very noticeable amount, yielding a fall time that also rounds to 28.6 seconds, drag drastically changes the time. To account for drag in equation 3, we set γ equal to 2 and calibrated α using a terminal speed of 50 m/s. With drag, the mass takes 83.5 seconds to fall, almost three times larger than the fall speed calculated by neglecting drag. The position and velocity graph versus time for this new fall time is shown in Figure 1 below.

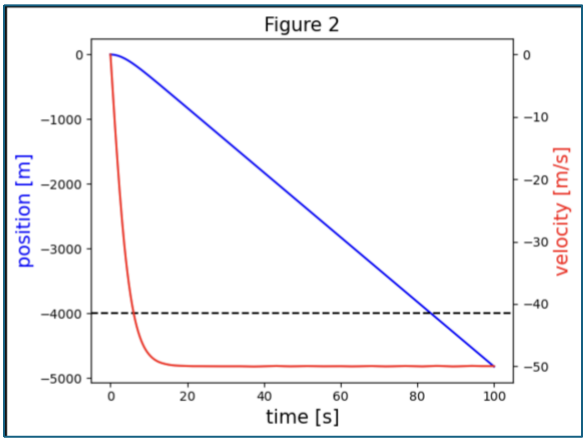

Figure 1: Position and Velocity vs Time of a 1kg mass with drag force and time-dependent g

III. Feasibility of depth measurement approach

The next step is to add a little more complexity to the simulation to see how another real-life factor will affect fall time. As the mass falls down the mineshaft, the Earth continues in its rotation, and if the object falls for long enough, this rotation can cause the mass to bump into the side of the shaft before it hits the bottom. This is called the Coriolis force and is defined by the following equation.

$$\vec{F_c} = -2m(\vec{\Omega} \times \vec{v}) \quad (4)$$

where $\vec{\Omega}$ is the Earth's rotation rate and m is the mass of the test object. When we include this force in the equations of motion derived from equation 2 and plot the transverse position of the mass versus its depth, we get the following plot.

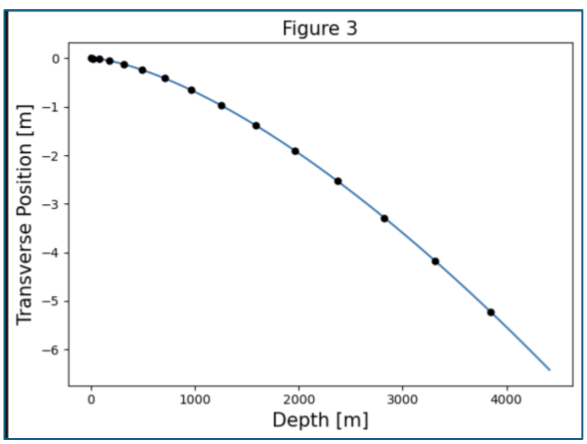

Figure 2: The Transverse position vs Depth of a 1kg mass without drag

We can calculate when the mass will hit the side of the shaft by seeing at what time the mass's x-position is equal to the width of the shaft, which is 5m. This event happens 21.9 seconds after the mass is dropped, but the mass still hits the bottom of the shaft at 28.6 seconds.

Based on these results, this method would not be best for measuring the depth of the shaft. As stated in the previous section, drag almost triples the fall time of the mass, and this would lead to the mass hitting the side of the shaft higher up from the bottom. The mass hitting the side of the shaft before getting to the bottom risks it breaking into multiple pieces or even getting lodged in the wall, which would add inaccuracy to the depth calculations, especially if the mass collides with the wall early on in its descent.

IV. Calculation of crossing times for homogeneous and non-homogeneous Earth

The final factor to account for is the Earth's non-uniform density which is defined as

$$\rho(r) = \rho_n \left(1 - \frac{r^2}{{R_E}^2}\right)^n \qquad (5)$$

where ρ_n is the normalizing constant. With this density distribution, we could solve for the time it would take the mass to fall to the center of the Earth. Calculating this over an array of n values, we found that it would take the mass 1267.3 seconds to reach the center when n=0 and 943.8 seconds when n=9. The plots below show the position and velocity of the mass versus time if it could fall through the center of the Earth to the other side with n values of 0, 1, 2, and 9.

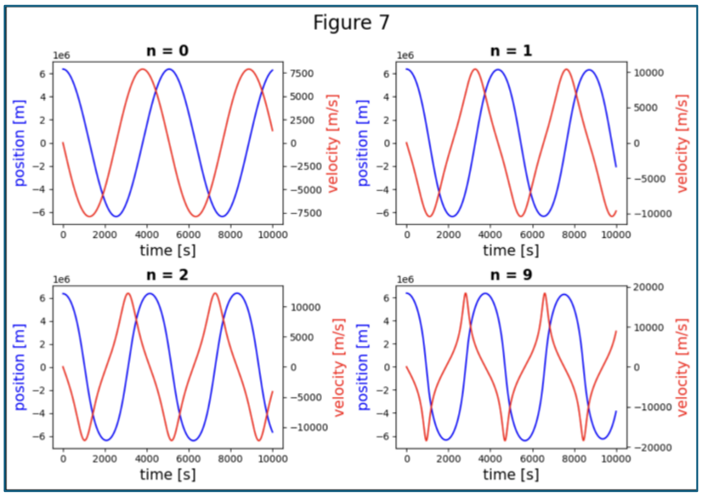

Figure 3: Position and Velocity vs Time of a 1kg mass at n-values of 0, 1, 2, and 9

Based on these results, it seems that the larger the n-value, meaning the more dense the core of the planet is, the faster the object falls and the faster it travels at the center of the planet.

V. Discussion and Future Work

In summary, the calculations for this report started by assuming no drag or Coriolis force and that the Earth has a constant density concentration. This system would be ideal for measuring the depth of the shaft, but in reality, things would not go as smoothly. Drag will greatly increase the object's fall time, and the Coriolis force will cause the mass to eventually hit the side of the shaft which could unintentionally end the experiment early or at the very least make our depth calculations less accurate if the mass is still able to reach the bottom of the mineshaft. In addition to the outside factors we *did* take into account, there are more that have been neglected. To have even more realistic calculations, a good next step would be to assume that the Earth is not a perfect sphere. While this would help us to make even better predictions about how a test mass would behave as if falls down the shaft, the results of this report conclude that the company should consider other methods of measuring the depth of our shaft.