COSC343: Artificial Intelligence

Lecture 11 : Recurrent neural networks

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In today's lecture

- Examples of neural networks in action
- Softmax function – network output as a probability distribution
- Simple Recurrent Network (SRN)
- Teaching SRN to talk

Recap: Notation

weight

layer

neuron # in the layer

$w_{ij}^{[l]}$

neuron # in previous layer

#of layers

L

neuron # in the layer

$b_j^{[l]}$

bias

neuron # in the layer

$v_j^{[l]}$

activity

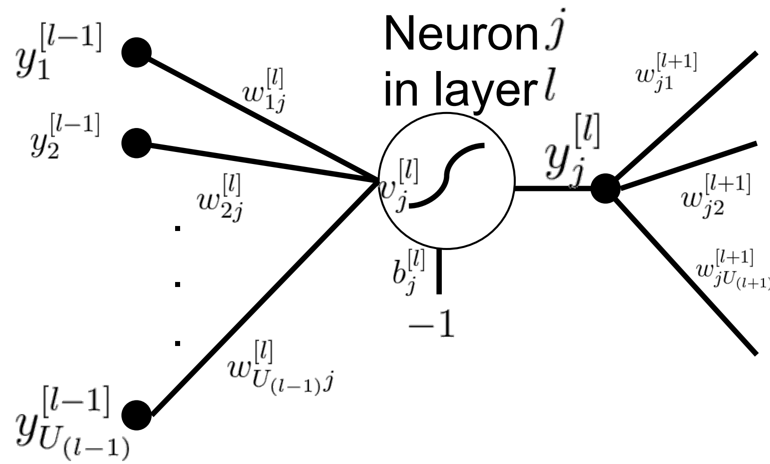
neuron # in the layer

$y_j^{[l]}$

output

U_l

number of neurons in layer l



- Activity is the weighted sum of inputs from the previous layer minus the bias

$$v_j^{[l]} = \sum_{i=1}^{U^{[l-1]}} w_{ij}^{[l]} y_i^{[l-1]} - b_j^{[l]}$$

- Output is a function of activity

$$y_j^{[l]} = f_{\text{logsig}}(v_j^{[l]}) = \frac{1}{1+\exp^{-v_j^{[l]}}}$$

where $y_i^{[0]} = x_i$ and $U_0 = M$

Recap: Backpropagation

Let's assume that we are minimising some cost function that depends on the network output and the desired output

$$J\left(y_j^{[L]}, \tilde{y}_j\right)$$

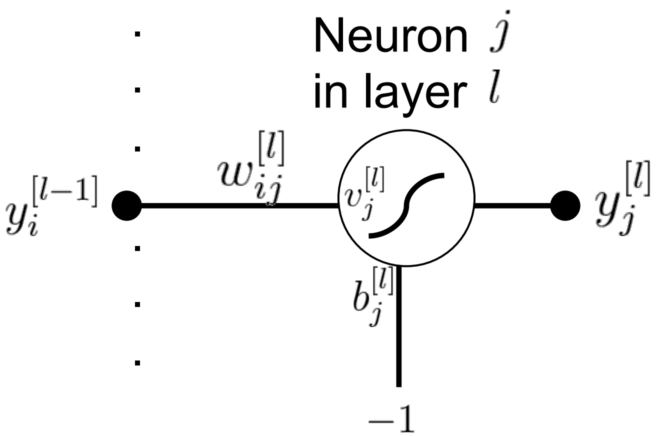
Steepest gradient descent update for weight connecting input i with neuron j in layer l is :

$$w_{ij}^{[l]} = w_{ij}^{[l]} - \alpha \Delta w_{ij}^{[l]}$$

Steepest gradient descent update for bias on neuron j in layer l is :

$$b_j^{[l]} = b_j^{[l]} - \alpha \Delta b_j^{[l]}$$

where...



The change in the weight connecting input i with neuron j in layer l is:

$$\Delta w_{ij}^{[l]} = y_i^{[l-1]} \frac{dy_j^{[l]}}{dv_j^{[l]}} \delta J_j^{[l]}$$

The change in the bias for neuron j in layer l is:

$$\Delta b_j^{[l]} = -\frac{dy_j^{[l]}}{dv_j^{[l]}} \delta J_j^{[l]}$$

Sigmoid function has a derivative

$$y_j^{[l]} = f_{\text{logsig}}(v_j^{[l]}) = \frac{1}{1+\exp^{-v_j^{[l]}}}$$

$$\frac{\partial y_j^{[l]}}{\partial v_j^{[l]}} = y_j^{[l]}(1 - y_j^{[l]})$$

where...

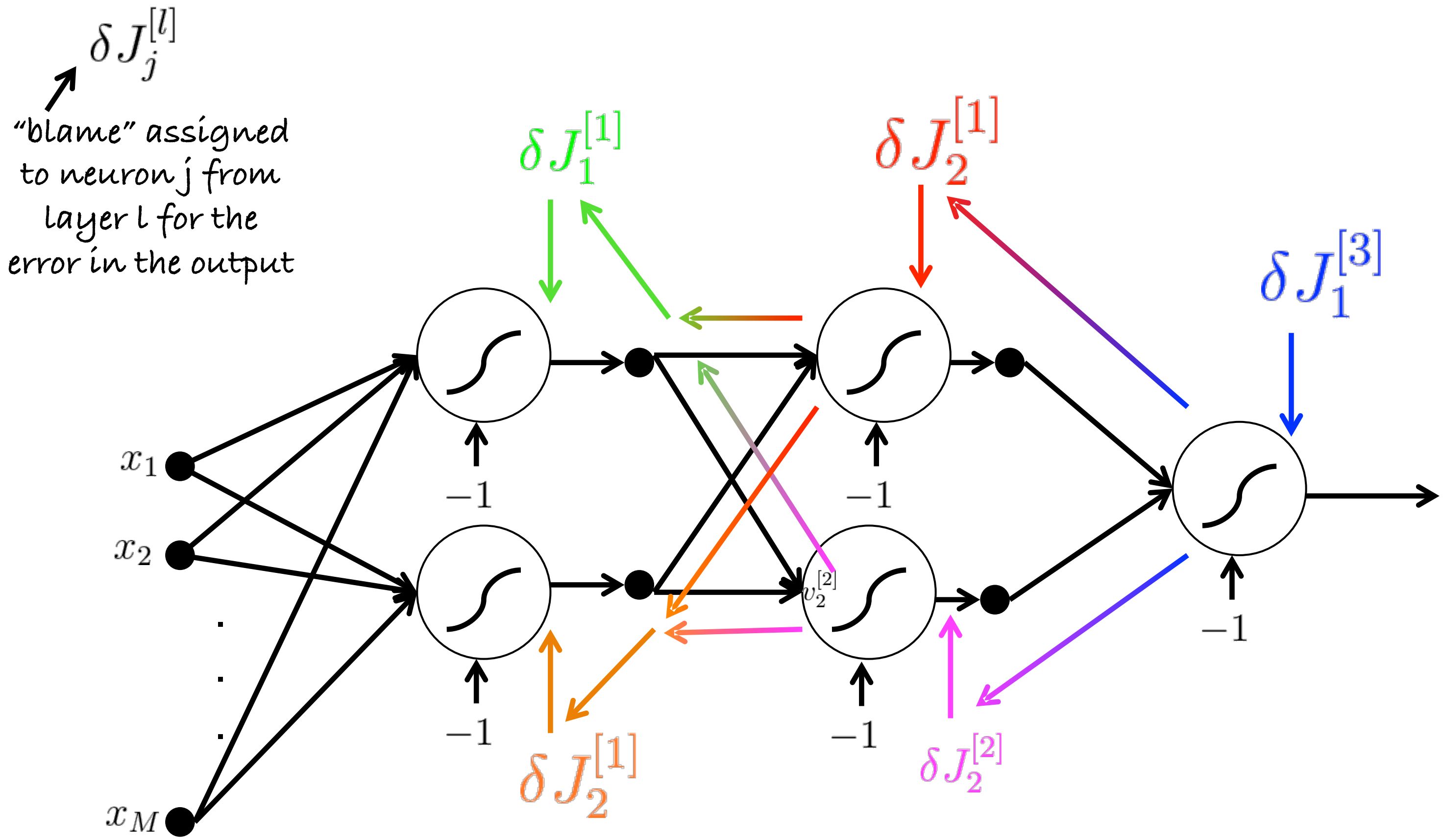
The cost blame for neuron i in layer l-1 is:

$$\delta J_i^{[l-1]} = \sum_{j=1}^{U_l} w_{ij}^{[l]} \frac{dy_j^{[l]}}{dv_j^{[l]}} \delta J_j^{[l]}$$

The cost blame for neuron j in the output layer is a derivative of the overall cost with respect to the neuron's output:

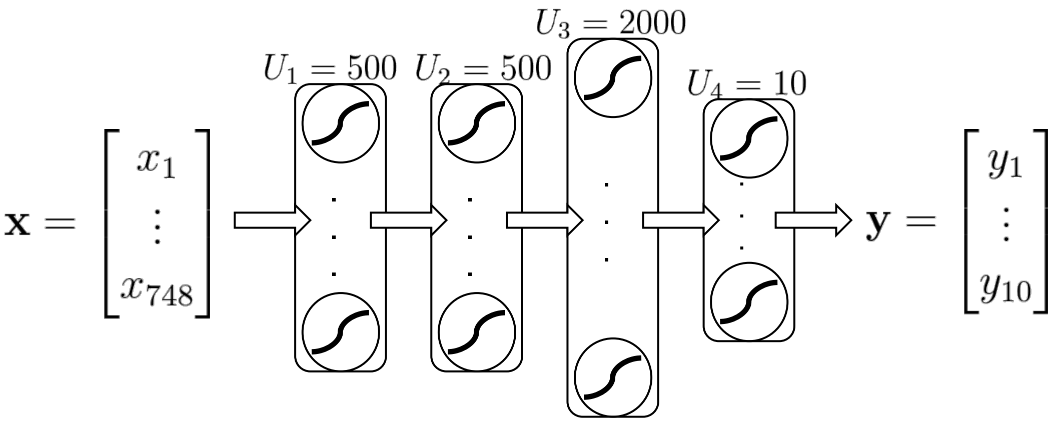
$$\delta J_j^{[L]} = \frac{dJ\left(y_j^{[L]}, \tilde{y}_j\right)}{dy_j^{[L]}}$$

Recap: Backpropagation



An example: an artificial neural network for classification

748-500-500-2000-10 neural network:



$$\text{class label} = \arg\max_i y_i$$

Input:
- Each image is 28x28 pixels
- Digits are normalised for size, position and orientation.

Output:
- One-zero vector code for digit identified in the image

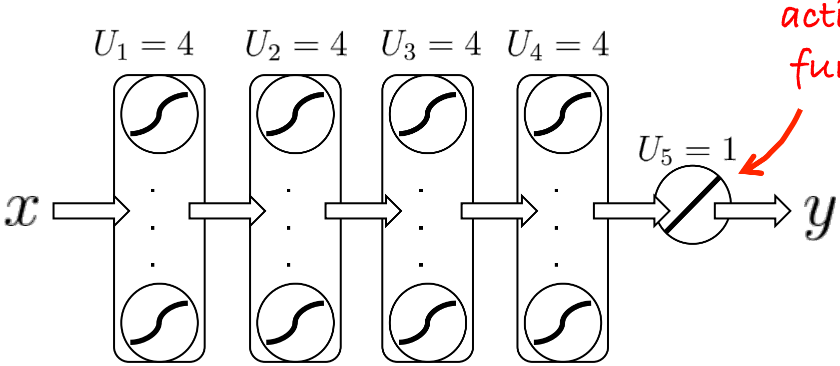
Mean squared error

Training with backpropagation of MSE:
- 60000 images
- 2.5% classification error

Testing:
- 10000 images
- 3% training error

An example: an artificial neural network for regression

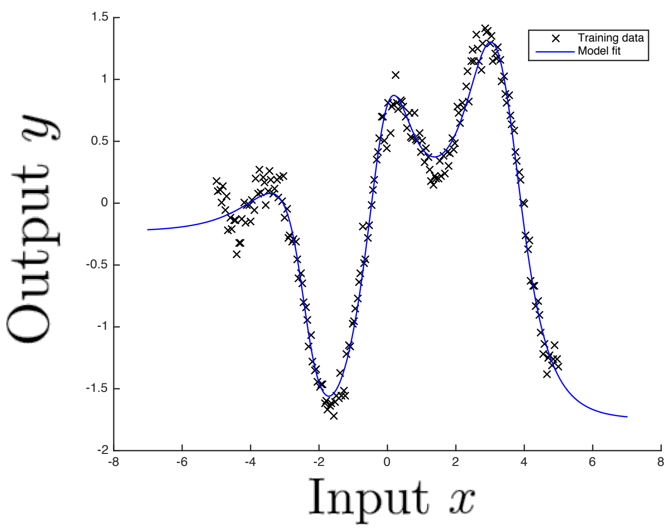
1-4-4-4-4-1 neural network:



Linear activation function

Input:
- Value between -5 and 5

Output:
- Inferred function of input x



Training with backpropagation of MSE:
- 100 samples
- RMSE=0.72

Root mean squared error

Testing:
- 100 samples
- RMSE=0.77

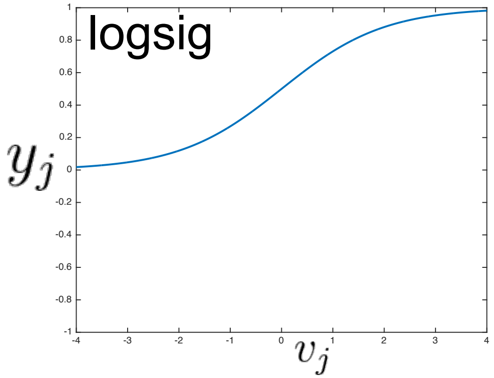
Selection of activation functions

- Logistic sigmoid

j indexes an individual neuron

$$y_j = \frac{1}{1+e^{-v_j}} \quad , \quad \frac{dy_j}{dv_j} = y_j(1-y_j)$$

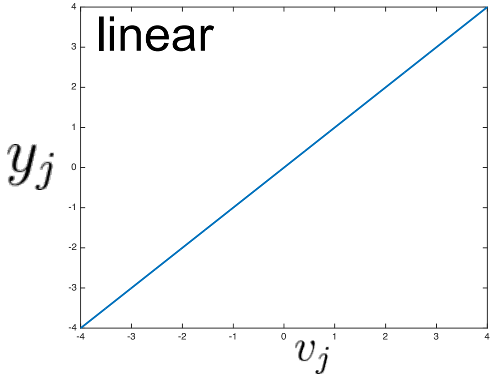
  - Output bounded between 0 and 1
  - Useful for classification, hidden units, and interpreting output as a probability



- Linear

$$y_j = v_j \quad , \quad \frac{dy_j}{dv_j} = 1$$

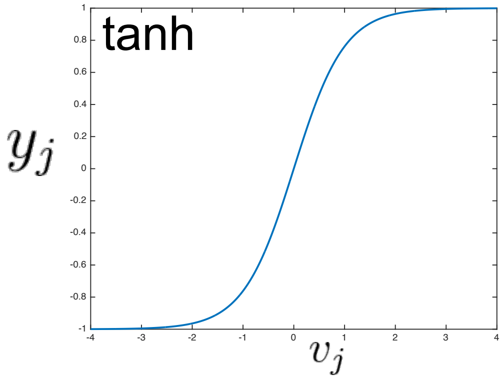
  - Output not bounded
  - Useful for output neurons in regression tasks



- Hyperbolic tangent

$$y_j = \tanh(v_j) \quad , \quad \frac{dy_j}{dv_j} = (1+y_j)(1-y_j)$$

  - Output bounded between -1 and 1
  - Sometimes gives richer hidden layer representation than logistic sigmoid



Network output as a probability distribution

A neural network with *K* outputs can represent probability distribution of a discrete random variable with *K* possible outcomes

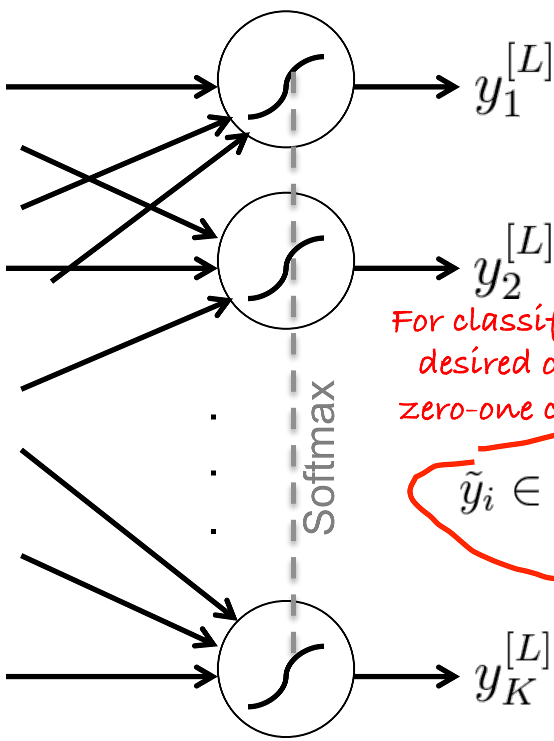
- Most commonly used for classification, where each output represents probability of classifying the input a belonging to one of *K* possible labels: $y_j^{[L]} = p(\text{label} = c_j)$, where $\text{label} \in \{c_1, \ldots, c_K\}$.
- Probability distribution at the output is computed with the softmax function

$$y_j^{[L]} = \frac{e^{v_j}}{\sum_{k=1}^{K} e^{v_k}}$$ for which the derivative

is $$\frac{dy_j^{[L]}}{dv_j^{[L]}} = y_j^{[L]}(1 - y_j^{[L]})$$

This is all you need to backpropagate the softmax function

Output layer:



For classification, the desired output is a zero-one coded vector

$$\tilde{y}_i \in \{0, 1\}$$

- The training error at the output is $\delta J_j^{[L]} = \tilde{y}_j(y_j^{[L]} - 1) + (1 - \tilde{y}_j)y_j^{[L]}$, which

corresponds to minimisation of the following cost: $J = -\sum_j \left(\tilde{y}_j \ln y_j^{[L]} + (1 - \tilde{y}_j)\ln(1 - y_j^{[L]})\right)$

An example: Classification with softmax

A 784-500-500-2000-10 neural network trained with a the softmax function to recognise digit in an image.

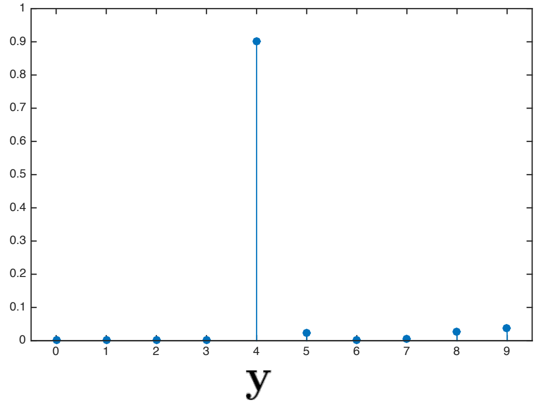
For the following test inputs,

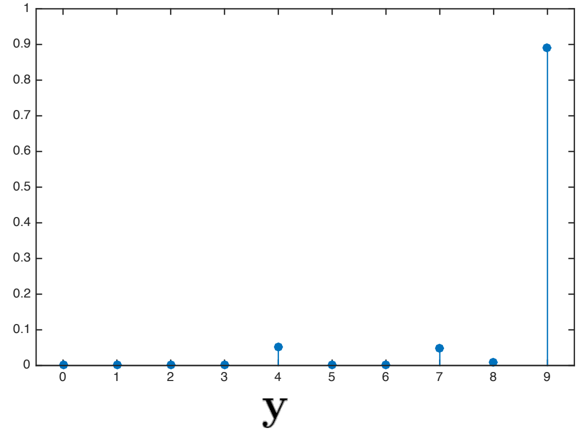
this input is misclassified

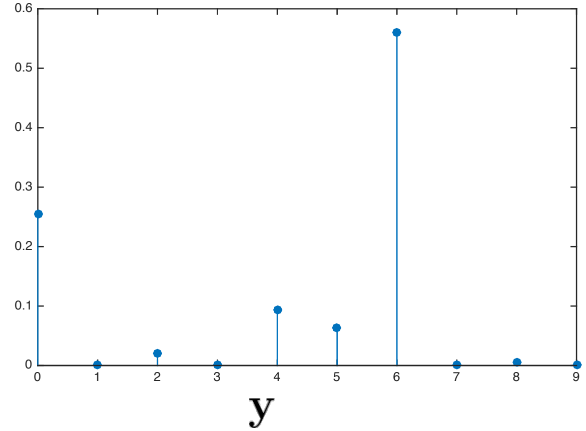
the network produces the following probability distributions over 10 possible labels:

target label is "4"

$P(\mathbf{y}|\mathbf{x})$







What about temporal patterns?

So far we have only considered learning tasks where the order of inputs (during training and testing) was of no consequence

- E.g. In digit recognition, input  should be recognised as a "2" regardless whether the previously processed input was a , , or .

What about a language related task?

- E.g. If we wanted to train a neural network to predict the next word in a sentence....the prediction depends on the context:

  "Dogs are ..."

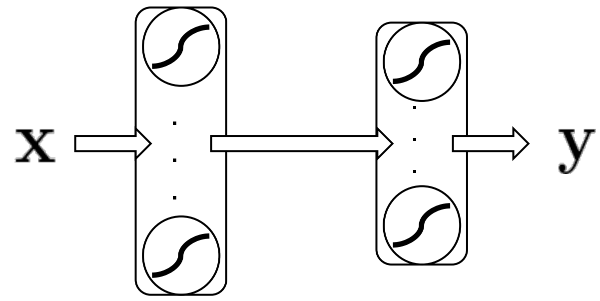
  "Cars are ..."

Feed forward and recurrent neural networks

Networks we were looking at up to this point

A **feed forward neural network** is a directed acyclic graph:

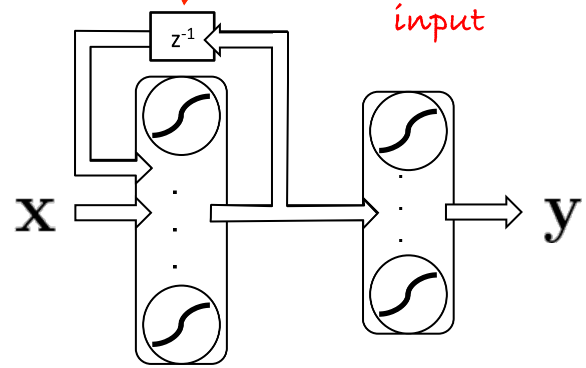
- The internal state of the network depends only on its current input, which allows the model to perform static input-output mapping.



State of the output is saved and used to supplement the next input

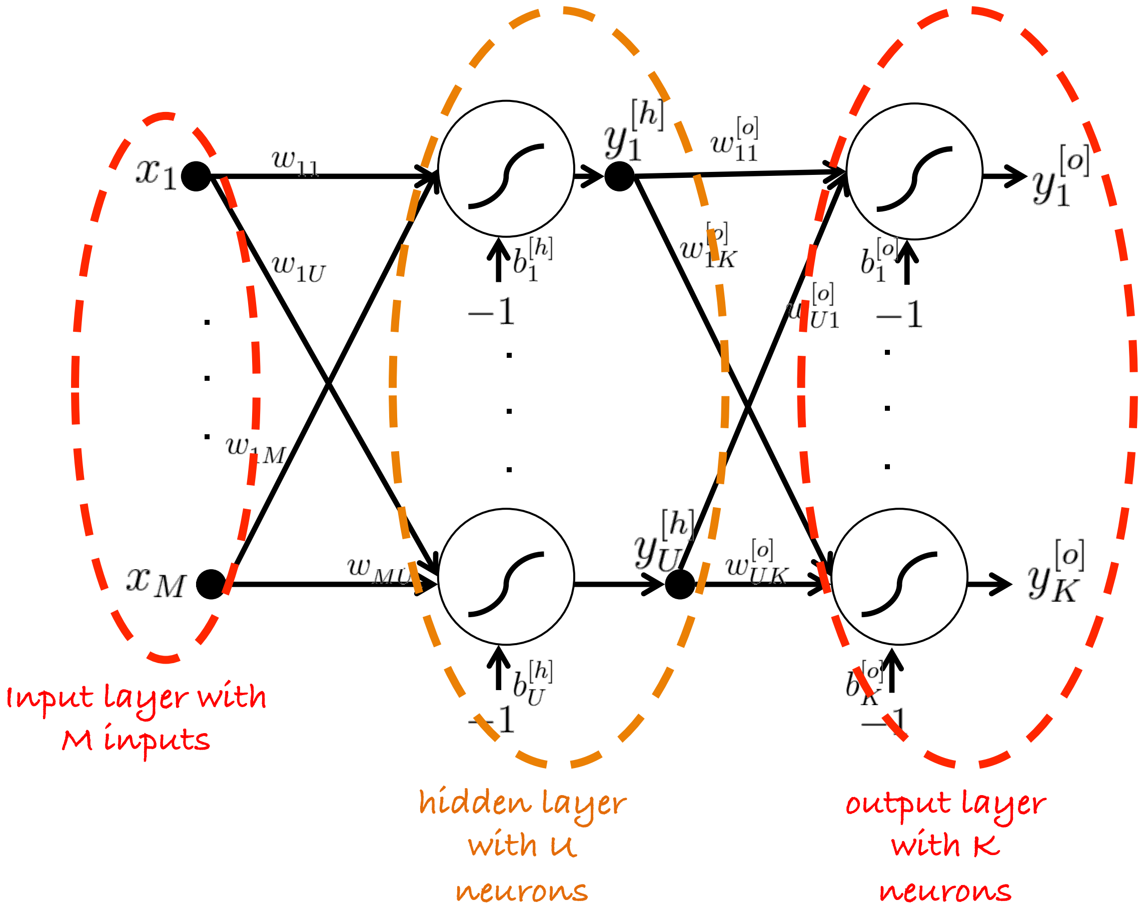
A **recurrent neural network** is a directed graph with cycles (loops):

- The internal state of the network depends on its previous state, which allows the model to exhibit a dynamic temporal behaviour.



Simple Recurrent Network (SRN)

- Also referred to as *Elman network* for its inventor Jeff Elman



w_{ij} - weight connecting input i to neuron j in the hidden layer

$b_j^{[h]}$ - bias of neuron j in the hidden layer

$w_{ij}^{[o]}$ - weight connecting neuron i from the hidden layer to neuron j in the output layer

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x_i - input i to the hidden layer

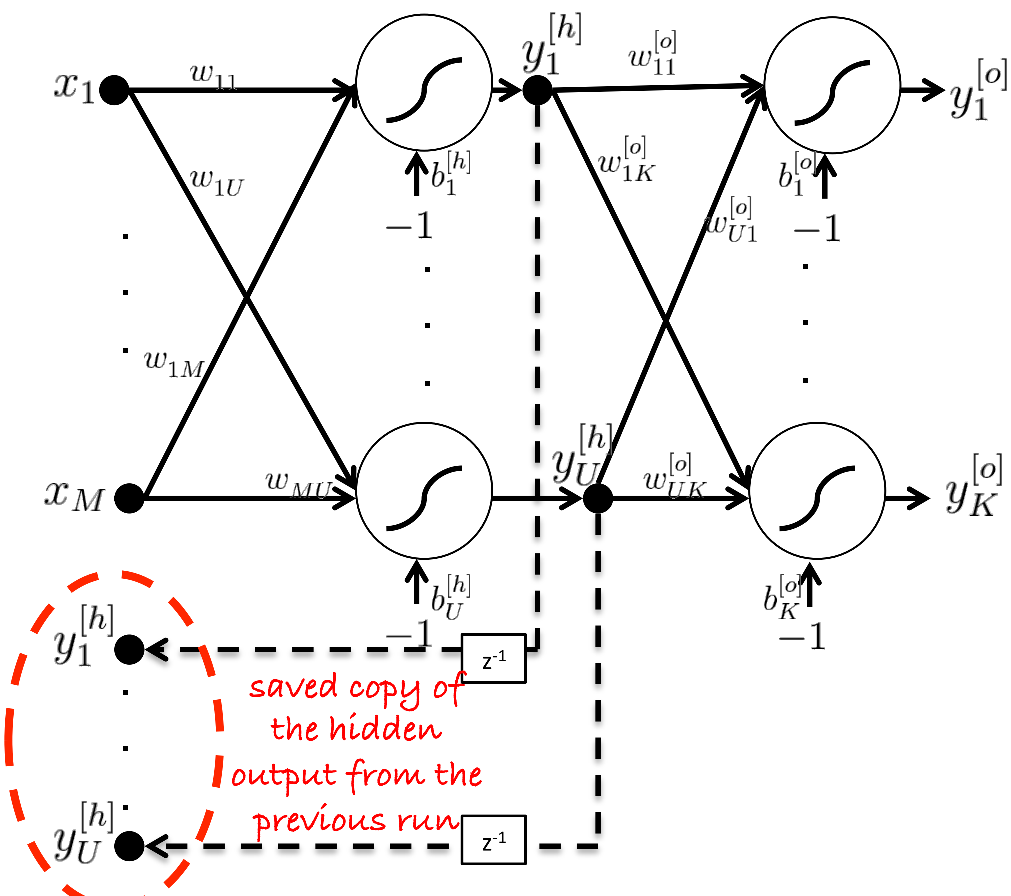
$y_i^{[h]}$ - output of neruon i in the hidden layer

$y_i^{[o]}$ - output of neruon i in the output layer

M - # of inputs U - # of hidden neurons K - # of outputs

Simple Recurrent Network (SRN)

- Output of the hidden layer feeds into the output layer as well as the hidden layer.



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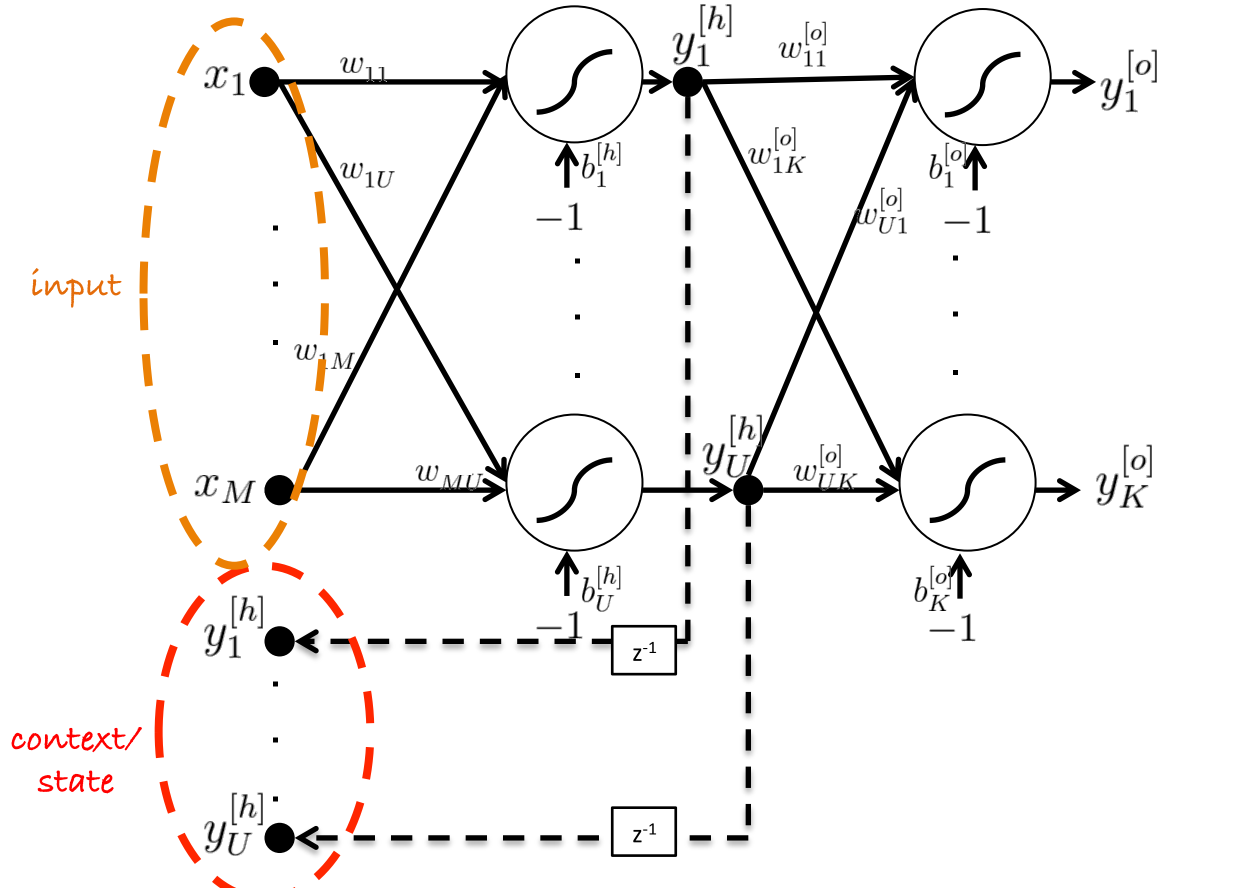

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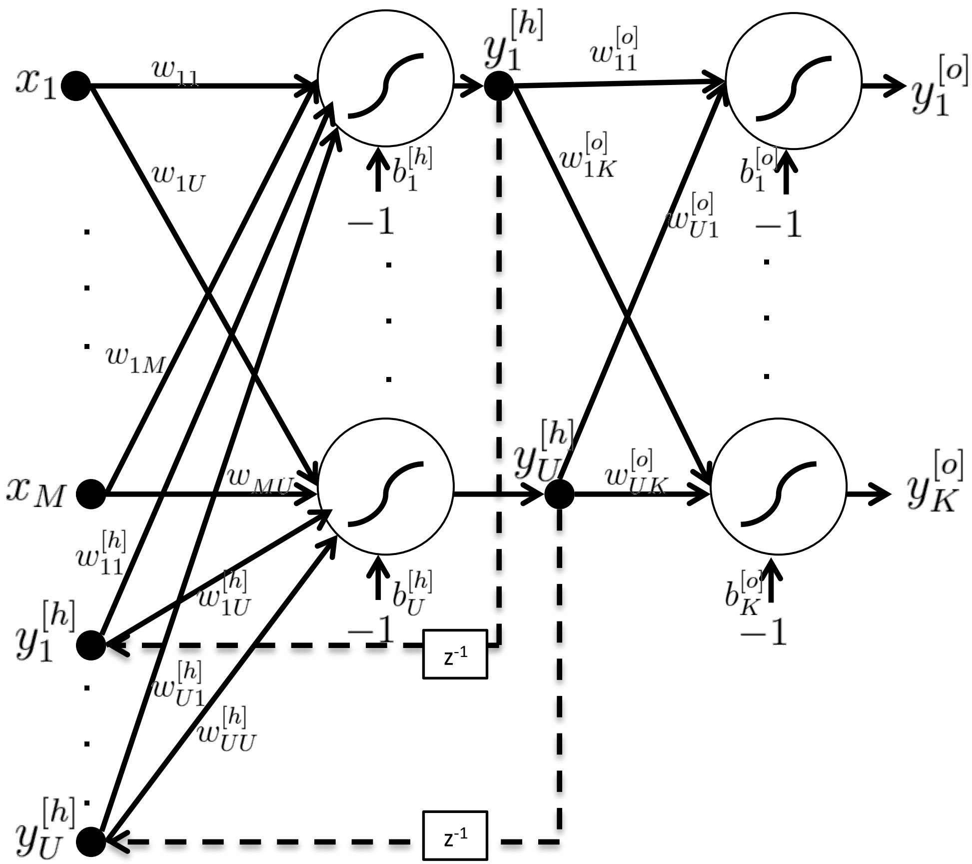

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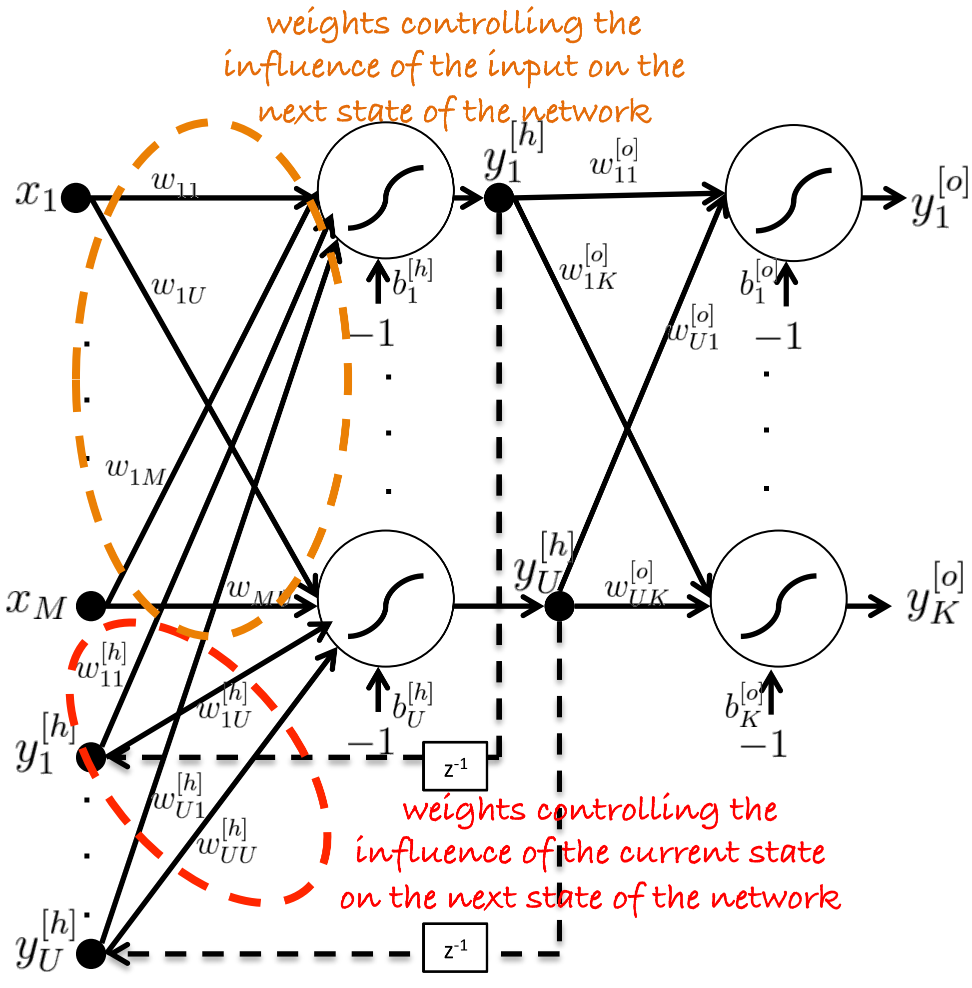

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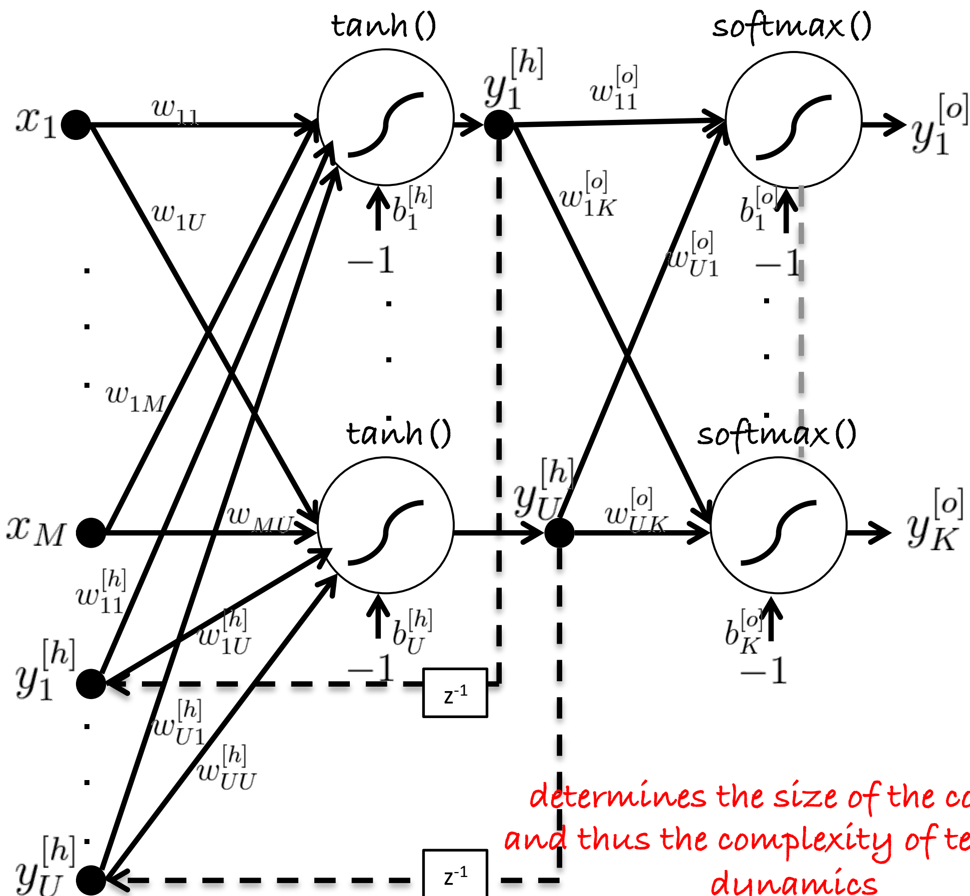

determines the size of the context and thus the complexity of temporal dynamics

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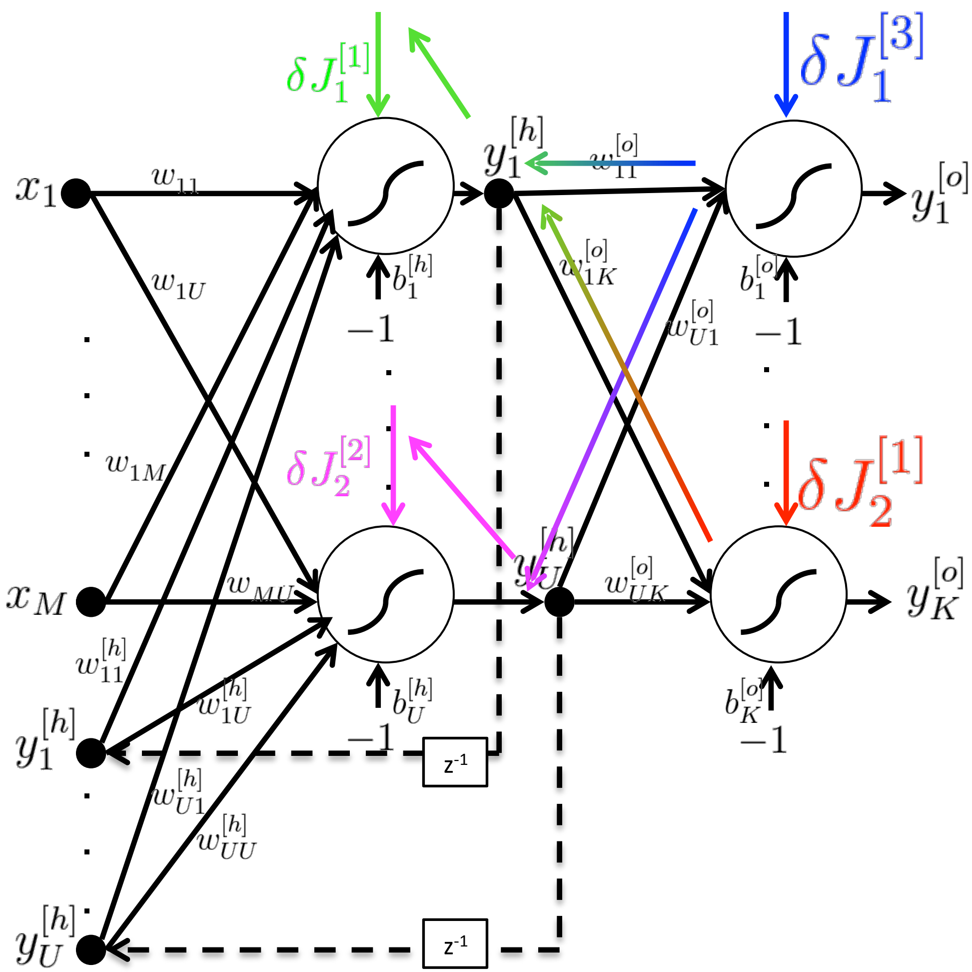
Simple Recurrent Network (SRN)

- Backpropagation works in SNR.



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SNR training

Backpropagation works in SNR:

error "blame" on output neurons $$\delta J^{[o]} = \tilde{y}_j(y^{[o]}_{jt} - 1) + (1 - \tilde{y}_j)y^{[o]}_{jt}$$

error "blame" on hidden neurons $$\delta J^{[h]}_i = \sum_{j=1}^{U} w^o_{ij}\left(y^{[o]}_{jt}(1 - y^{[o]}_{jt})\right)\delta J^{[o]}_j$$

updates to hidden-output weights $$\Delta w^{[o]}_{ij} = y^{[h]}_{it}\left(y^{[o]}_{jt}(1 - y^{[o]}_{jt})\right)\delta J^{[o]}_j$$

updates to output layer biases $$\Delta b^{[o]}_j = -\left(y^{[o]}_{jt}(1 - y^{[o]}_{jt})\right)\delta J^{[o]}_j$$

updates to hidden-hidden weights $$\Delta w^{[h]}_{ij} = y^{[h]}_{i(t-1)}(1 + y^{[h]}_{jt})(1 - y^{[h]}_{jt})\delta J^{[h]}_j$$

updates to hidden layer biases $$\Delta b^{[h]}_j = -(1 + y^{[h]}_{jt})(1 - y^{[h]}_{jt})\delta J^{[h]}_j$$

updates to input-hidden weights $$\Delta w_{ij} = x_i(1 + y^{[h]}_{jt})(1 - y^{[h]}_{jt})\delta J^{[h]}_j$$

SNR training

Backpropagation works in SNR:

derivative of softmax with respect to network output

error "blame" on output neurons $\delta J^{[o]} = \tilde{y}_j(y_{jt}^{[o]} - 1) + (1 - \tilde{y}_j)y_{jt}^{[o]}$

error "blame" on hidden neurons $\delta J_i^{[h]} = \sum_{j=1}^{U} w_{ij}^{o} \left(y_{jt}^{[o]}(1 - y_{jt}^{[o]})\right) \delta J_j^{[o]}$

derivative of softmax with respect to output neuron's activity

output of the hidden neuron i due to previous input (indexed t-1)

output of the hidden neuron j due to current input (indexed t)

updates to hidden-output weights $\Delta w_{ij}^{[o]} = y_{it}^{[h]} \left(y_{jt}^{[o]}(1 - y_{jt}^{[o]})\right) \delta J_j^{[o]}$

updates to output layer biases $\Delta b_j^{[o]} = -\left(y_{jt}^{[o]}(1 - y_{jt}^{[o]})\right) \delta J_j^{[o]}$

updates to hidden-hidden weights $\Delta w_{ij}^{[h]} = y_{i(t-1)}^{[h]} (1 + y_{jt}^{[h]})(1 - y_{jt}^{[h]}) \delta J_j^{[h]}$

updates to hidden layer biases $\Delta b_j^{[h]} = -(1 + y_{jt}^{[h]})(1 - y_{jt}^{[h]}) \delta J_j^{[h]}$

updates to input-hidden weights $\Delta w_{ij} = x_i (1 + y_{jt}^{[h]})(1 - y_{jt}^{[h]}) \delta J_j^{[h]}$

derivative of tanh with respect to hidden neuron's activity

An example: sentence generation

- A dictionary of 6 strings encoded as a 6-dim zero-one vectors (M=6)

$$\mathbf{x} \in \left\{ \begin{array}{cccccc} . & \text{Cars} & \text{Dogs} & \text{are} & \text{furry} & \text{red} \\ \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} \end{array} \right\}$$

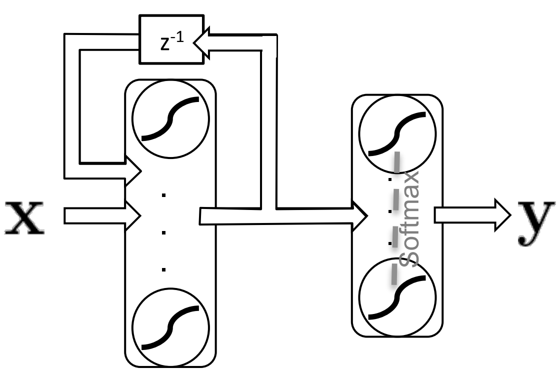
- Output is a 6-value probability distribution over all possible strings (K=6)

$$p(\text{string})$$

$$\mathbf{y} = \begin{bmatrix} y_1^{[o]} \\ y_2^{[o]} \\ y_3^{[o]} \\ y_4^{[o]} \\ y_5^{[o]} \\ y_6^{[o]} \end{bmatrix} \begin{array}{l} P(\text{string} = .) \\ P(\text{string} = \text{Cars}) \\ P(\text{string} = \text{Dogs}) \\ P(\text{string} = \text{are}) \\ P(\text{string} = \text{furry}) \\ P(\text{string} = \text{red}) \end{array}$$

SRN:

- 16 hidden neurons (U=16)
- Tanh activity function in the hidden layer
- Softmax function on the output



Training:

- Initial state $\mathbf{y}_h = \begin{bmatrix} y_1^{[h]} & \ldots & y_U^{[h]} \end{bmatrix}^T$ set to zeros.

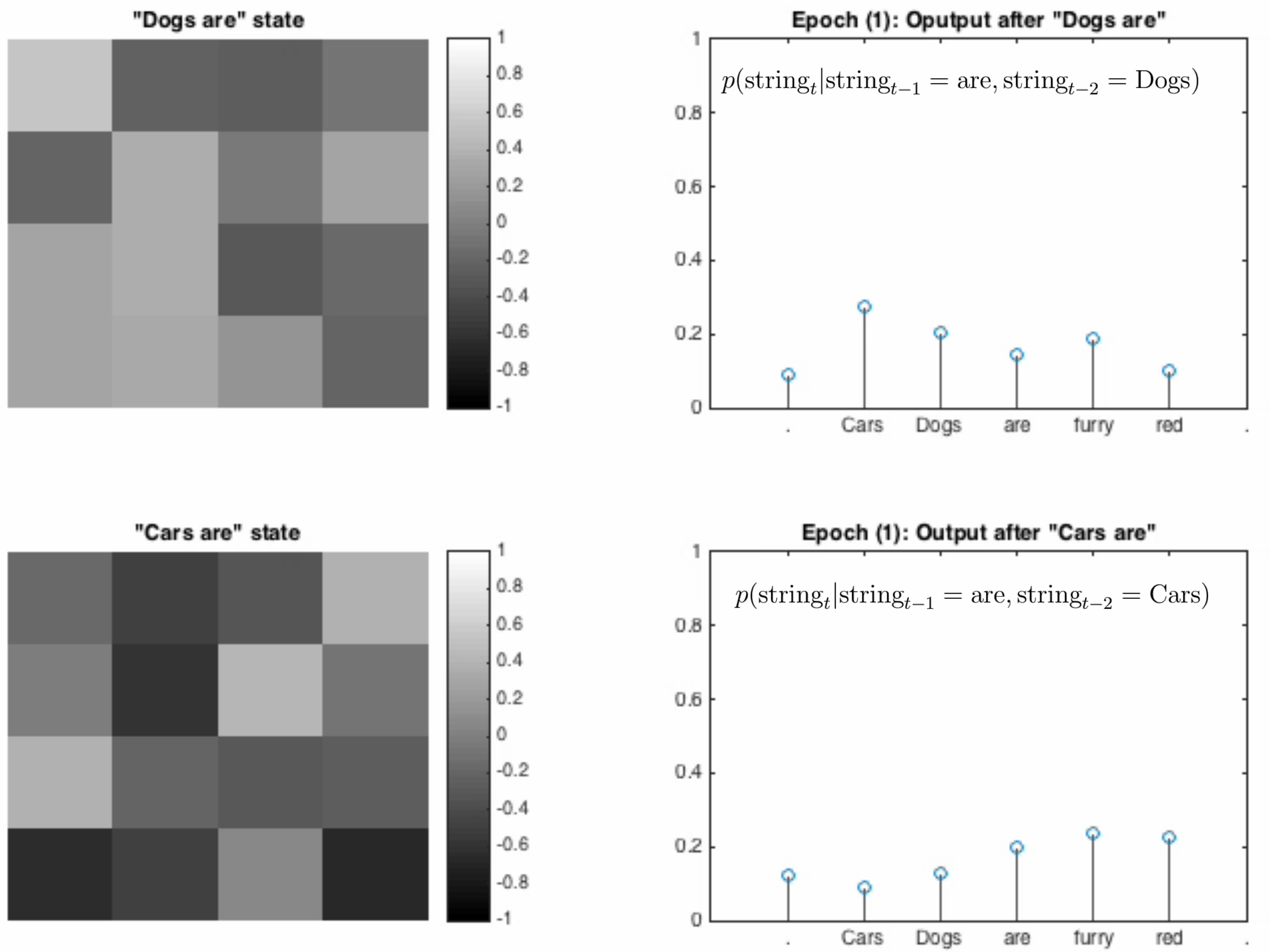
Input sequence 1:
$$\left\{ \begin{array}{ccc} \text{Dogs} & \text{are} & \text{furry} \\ \begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix} \end{array} \right\}$$

Desired output 1:
$$\left\{ \begin{array}{ccc} \text{are} & \text{furry} & . \\ \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}, & \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix} \end{array} \right\}$$

Input sequence 2:
$$\left\{ \begin{array}{ccc} \text{Cars} & \text{are} & \text{red} \\ \begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} \end{array} \right\}$$

Desired output 2:
$$\left\{ \begin{array}{ccc} \text{are} & \text{red} & . \\ \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, & \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}, & \begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix} \end{array} \right\}$$

An example: sentence generation



An example: more complicated text generation

- A dictionary of 1038 words, zero-one coded into a 1038-dimensional input
- SRN with 64 hidden units and 1038-dimensional output
- Trained with softmax on the first 5,000 words of "Pride and Prejudice" to predict the next word in a sentence.
- Used to generate text by priming the network with a sentence fragment...
- ...then taking the predicted most likely word to follow and feeding it as the next input...repeating this over and over.

An example: more complicated text generation

- Sample of the training data:

“It is a truth universally acknowledged, that a single man in possession of a good fortune must be in want of a wife. However little known the feelings or views of such a man may be on his first entering a neighbourhood, this truth is so well fixed in the minds of the surrounding families, that he is considered as the rightful property of some one or other of their daughters.

- Sentences generated after the starting sequence “**It is**”:

    - After 1 epoch, J=4.331e+04

    *“**It is** rode seminaries no respectable enduring one little your does fortune bad everywhere please sensible information your views too”*

    - After 300 epochs, J=2.742e+04

    *“**It is** barefaced truth the I. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley. Bingley”*

    - After 3500 epochs, J=7.595e+03

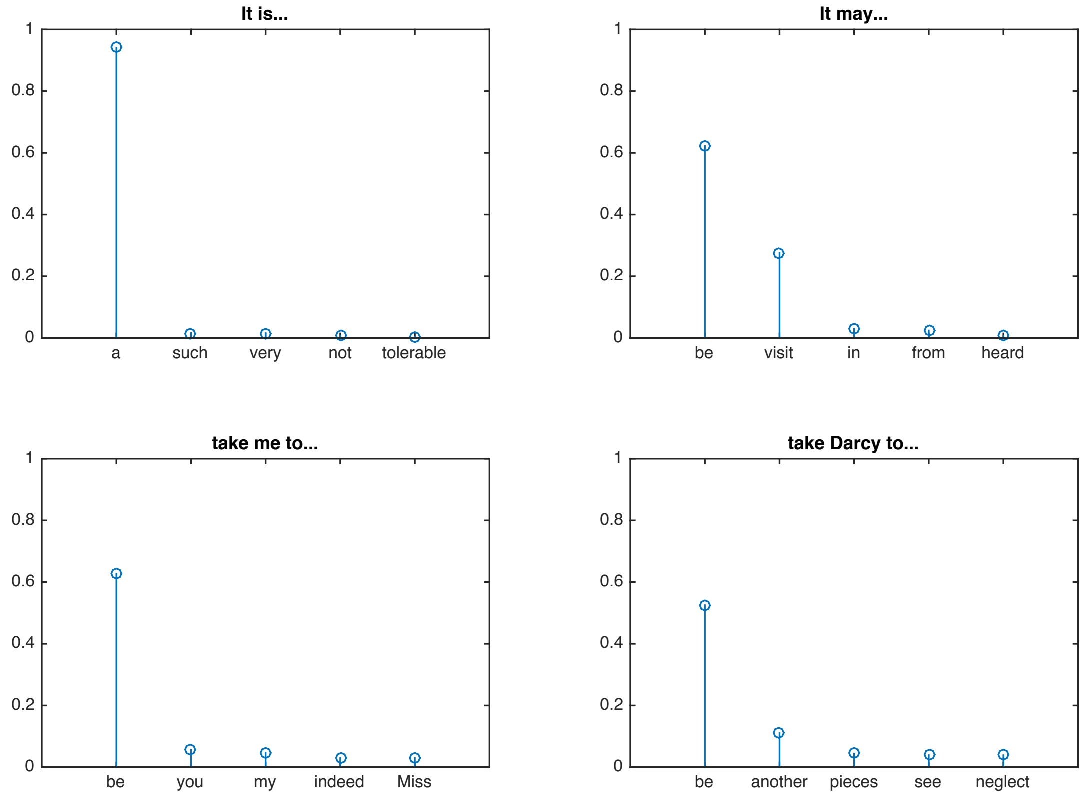
    *“**It is** a truth universally acknowledged, and no two for your husband day, and the two sixth of joy was a lively and it till the room”*

    - After 9900 epochs, J=2.310e+03

    *“**It is** a truth universally acknowledged, that a single man ought to be above his company, and above his father, my dear. I have a high respect for your nerves.”*

An example: more complicated text generation

- Probabilities of the top 5 most likely words after a certain word sequence, as given by the “Pride and Prejudice”-trained SRN after 10000 training epochs:



Summary

- Feed forward neural networks can do classification and regression
- Softmax function gives network output as a probability distribution
- Recurrent neural networks find temporal patterns in the sequences of input data
    - Simple Recurrent Network can be trained using the backpropagation algorithm

Reading for the lecture: Andrej Karpahty, "**The Unreasonable Effectiveness of Recurrent Neural Networks**", http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Reading for next lecture: AIMA Chapter 18.9