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Analysis of cooperative stability for reputation evaluation rules in spatial prisoner's dilemma game

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ABSTRACT

Research on reputation-based indirect reciprocity has found profound achievements, elucidating its role in promoting cooperation over selfish actions. However, some evaluation methodologies have limitations, such as the image scoring model, a classic first-order paradigm. Several studies have suggested that higher-order rules with more individual information can enhance the stability of cooperation. In this study, we introduce a reputation incentive mechanism to explore the cooperative differences among various evaluation rules. Specifically, players evaluate their opponents' actions following first-order and second-order evaluation rules, respectively. Given that players possess varying degrees of social influence, the evaluative intensity is influenced by the neighbor environment and updated in each round. This resultant fluctuations in reputation exhibit heterogeneity and dynamism. Numerical simulations based on the spatial prisoner's dilemma game demonstrate that under stringent conditions, the first-order rule can sustain cooperation, while the second-order rule may fail, leading to complete group defection. Under more relaxed conditions, the second-order rule proves more effective in promoting full cooperation than the first-order rule. Our research contributes to understanding the guidance and influence of reputation on collective behavior.

1. Introduction

Cooperation among self-interested individuals remains a significant research focus, prompting extensive interdisciplinary studies [1–4]. Evolutionary game theory [5,6] provides a robust mathematical framework for understanding complex altruistic behaviors [7]. This theory enables the simulation and prediction of decision-making processes in scenarios involving cooperation versus competition [8,9]. The integration of complex network theory with game theory has further enhanced our understanding of cooperative phenomena [10]. Various network topologies, such as square-lattice [10–12], small-world [13,14], and scale-free networks [15,16], influence the evolution of group cooperation. Furthermore, subsequent studies have examined multi-layer networks [17,18], temporal networks [19], and higher-order networks [20], offering deeper insights into real-life group dynamics.

Several mechanisms have been proposed to explain cooperation [21], such as direct reciprocity [22,23], indirect reciprocity [24], group selection [25], kin selection [26], and network reciprocity [27]. In these studies, the reputation-based indirect reciprocity mechanism stands out as particularly prevalent. Within a group, a player A may forgo personal benefits to help another player B.

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When such cooperative behavior is observed by a third party, it enhances the reputation of player A, thereby fostering trust as a compensation for its contribution. Therefore, establishing a feasible and reliable criterion for evaluating reputation is crucial to maintaining cooperative behavior [28].

The simplest and most classic model is the image scoring proposed by Nowak and Sigmund [29], which has inspired numerous intriguing investigations. In [30], a feedback mechanism with evolutionary stability was introduced, building upon this pioneering work, and can influence cooperation in relatively relaxed games. Berger and Grüne [31] observed that the general image scoring model remained stable even in the presence of errors. Moreover, image scoring represents a first-order rule that adjusts reputation based on player's behavior, wherein cooperation increases scores while defection decreases scores [32]. However, the rule also encounters inherent dilemmas [24,33]: when confronted with opponents of bad reputation, players may exhibit non-cooperative behavior, thereby lowering their image. To address this drawback and stabilize cooperation, researchers have adopted higher-order information to assess reputation [28,34–36].

Based on the first-order rule, the second-order rule takes both the player's action and the opponent's reputation into account. Sasaki et al. [37] explored public and private information, considering evaluation errors among the main second-order rules. Their study demonstrated that the simple standing mechanism is the most powerful in maintaining full cooperation. Further exploration of additional reputation models, such as shunning [38] and stern judging [39,40], has demonstrated their reciprocity in numerous studies. Moreover, when player's reputation is factored into second-order rules, the evaluation criterion becomes third-order. Ohtsuki and Iwasa [41] extensively examined all evaluation combinations, identifying eight evaluation criteria that stabilize cooperative behavior, thus proposing the "leading eight." However, despite the more sophisticated evaluation system, the question arises: does the stability of cooperation necessarily increase as evaluation rules encompass more information?

From the perspective of reputation formation, this study introduces a reputation incentive mechanism. Various reputation evaluation rules, including first-order and second-order rules, are considered to analyze the stability of cooperation. Previous studies have often assumed that reputation changes are uniform and constant [42,43]. However, in real life, reputation changes are neither homogeneous nor linear. Given the diverse clusters of players within networks, we assume that players exhibit varying degrees of social influence across different contexts. In our model, each player evaluates the action of their neighbors in the previous round, with the evaluation intensity influenced by the neighbor environment, thus demonstrating heterogeneity and dynamism. Subsequently, we explore differences in cooperation under first-order and second-order rules in both square-lattices and small-world networks, respectively. Numerical simulation results show that even under strict conditions, the first-order rule can sustain cooperation, while the second-order rule may require more lenient conditions to achieve cooperation, potentially result in cooperation levels higher than first-order rules. Furthermore, the accurate evolution of reputation scores promptly reflects player behavior, thereby fostering positive incentives for cooperative stability.

The rest of this paper is organized as follows. In Section 2, we introduce the detailed reputation incentive mechanism, encompassing both first-order and second-order rules. And the following Section 3 presents relevant simulation processes, evolutionary results, and deep analysis. Finally, a comprehensive summary of the entire work will be presented in Section 4.

2. Model

In this paper, the reputation mechanism considers the classical prisoner's dilemma game (PDG), with foundational backgrounds and detailed models introduced in this section. In the two-player PDG, each player has two alternative strategies: cooperation (C) or defection (D). If both players cooperate, they receive the same reward (R); if they both defect, they receive the same punishment (P). However, if one cooperates while the other defects, the cooperator receives the sucker's payoff (S), and the defector receives the temptation (T). Additionally, all payoffs satisfy the conditions $T > R > P > S$ and $2R > S + T$. For simplification of the model, we assume $R = 1$, $P = S = 0$, and $T = b$, where the temptation to defect can be adjusted within the range $1 < b < 2$. The cumulative payoff Π_i is obtained by player i playing with each neighbor in turn, and the payoff matrix for this interaction is as follows:

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \mathrm{C} \\ \mathrm{D} \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array} \tag{1}$$

Building upon aforementioned background, the model transitions into the evaluation phase. We assume a structured group wherein players possess varied reputation scores, which may be cooperators or defectors. Meanwhile, we introduce the reputation threshold parameter γ to regulate the reputation range R_i, denoted as $R_i \in [0, 2\gamma]$ with $\gamma \geq 1$. In each game round, the player sequentially evaluates the behavioral performances of its neighbors.

Taking the second-order reputation rule as an example, the updated evaluation scores are presented in Table 1. Specifically, when player's reputation score $R_i \geq \gamma$, they are recognized as possessing "good-image" (G), encouraging cooperative behavior and upholding social norms. When G players face a cooperative opponent, they provide a positive evaluation, whereas when faced with a defector, they offer a negative evaluation to criticize the behavior. Conversely, players with lower reputation scores, $R_i < \gamma$, are perceived as having "bad-image" (B) within the group, disrupting public order by giving positive evaluations to defectors and negative evaluations to cooperators.

It is worth noting that players with low-reputation seem to be untrustworthy. Therefore, we define the probability, denoted by the ratio of cooperators in the group $N_c(t)/N$, which represents the likelihood that the evaluation provided by a bad player is disregarded. When an evaluation is recognized as invalid, it is recorded as zero. Furthermore, due to the diverse neighborhood environments among players, they possess varying levels of social influence, resulting in heterogeneous evaluation intensity. For

Table 1
Evaluation score updates on the second-order rule.

		Type of i	
		Good-image (G)	Bad-image (B)
Strategy of j	Cooperation (C)	ω_G	$-\omega_B$
	Defection (D)	$-\omega_G$	ω_B

Table 2
Evaluation score updates on the first-order rule.

		Type of i
		Good-image (G) / Bad-image (B)
Strategy of j	Cooperation (C)	ω_G
	Defection (D)	$-\omega_B$

example, a player with high-reputation surrounded by all cooperators will have more persuasive evaluations, indicating stronger evaluation intensity. Conversely, a player with low-reputation surrounded by all defectors is more prone to social retaliation, also resulting in stronger evaluation intensity. The evaluation intensity of player depends on the strategies of its neighbors, and is updated accordingly each round. The specific function for this is expressed as follows:

$$\begin{cases} \omega_G = \dfrac{\sum_{j\in\Omega_i, S_j=\mathbf{C}}(1)}{n_i+1}, & \text{if player } i \text{ has “good-image”}, \\ \omega_B = \dfrac{\sum_{j\in\Omega_i, S_j=\mathbf{D}}(1)}{n_i+1}, & \text{if player } i \text{ has “bad-image”}, \end{cases} \tag{2}$$

where Ω_i denotes the set of neighbors including focal player i, and n_i denotes the total number of neighbors for player i. ω_G represents the evaluation intensity from G player, determined by the proportion of cooperators within the Ω_i set. ω_B represents the evaluation intensity from B player, determined by the proportion of defectors within the Ω_i set.

Table 2 illustrates the updated evaluation scores given by i to j under the first-order evaluation rule. In contrast to the second-order rule, which considers both strategy of player and type of its neighbor, the evaluation scores under the first-order rule are determined solely by changes in strategies of each player.

After players evaluate each other, reputations are formed based on these publicly available evaluations. We define the reputation increment as the average of evaluations received by players in each round. The range of reputation scores is constrained to the interval $[0, 2\gamma]$, with γ to adjust this range. In this model, although evaluation scores consider the neighbor set including the focal player, due to the subjectivity of self-evaluation, reputation changes are solely based on neighbors' evaluations. The calculation for updating reputation scores can be expressed as:

$$R_i(t+1) = R_i(t) + \frac{\sum_{j\in\Phi_i} E_{ji}(t+1)}{n_i}, \tag{3}$$

where Φ_i is the set of neighbors of player i, and E_{ji} represents the evaluation score given by j to i.

Finally, all players update their strategies through imitation. We normalize the reputation scores, considering their impact on player payoffs during strategy transformation, and introduce α to control the influence of reputation scores on player payoffs. The payoff adjustment of player i is as follows:

$$\Pi_i^*(t) = \Pi_i(t) \times \left(\frac{R_i(t) - R_{min}}{R_{max} - R_{min}}\right)^{\alpha}, \tag{4}$$

where $\alpha \geq 0$. When $\alpha = 0$ it reverts to the traditional PDG model. A higher α indicates that reputation is more valued in the learning of strategies.

Here, we employed the Fermi function synchronously to update individuals' strategies based on adjusted payoffs. Player i, who is randomly selected in network, randomly selects neighbor j in its set of neighbors. The probability that player i imitates player j's strategy is given by

$$w(S_i \leftarrow S_j) = \frac{1}{1 + e^{(\Pi_i^* - \Pi_j^*)/k}}, \tag{5}$$

where k denotes the noise effect, with higher k indicating less rational the player's choice. For simplicity, this study sets $k = 0.1$.

The complete reputation incentive model is described in this section, primarily consisting of three components: mutual evaluation, reputation update, and imitation of strategy. To provide a clearer illustration of our model, Fig. 1 depicts the mutual evaluation process based on the second-order rule. In Section 3, we will further analyze cooperative stability under different reputation evaluation rules across various scenarios.

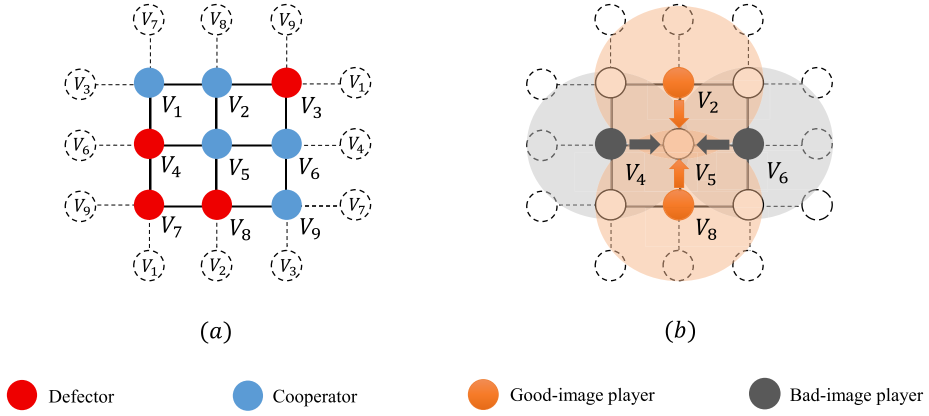


Fig. 1. The evolution of reputation incentive mechanism based on second-order rule on a square-lattice network. This network consists of 3×3 nodes with periodic boundary conditions. Nodes colored in red, blue, orange, and gray represent defectors, cooperators, good-image players, and bad-image players, respectively. The shaded area indicates the neighbors set of the focal player, with arrows denoting evaluations provided by the focal player. The evolution of player V_5's reputation score unfolds as follows: during interaction with player V_6, a bad-image player, V_6 provides a negative evaluation $\omega_B^{V_6}$ for V_5's cooperative behavior. Given that V_6 has two defectors (V_3 and V_4) in its neighbor set, its evaluation intensity is $\omega_B^{V_6} = 2/5$. In the same round, V_5 receives evaluations from its other three neighbors: $\omega_G^{V_2} = 3/5$, $\omega_B^{V_4} = 2/5$, and $\omega_G^{V_8} = 3/5$. Additionally, with five cooperators in the group, there is a 5/9 probability that evaluation of a bad-image players are invalid. Assuming all evaluations are accepted, the increment of V_5's reputation score is calculated as $\Delta R_{V_5} = (\omega_G^{V_2} - \omega_B^{V_4} + \omega_G^{V_6} - \omega_B^{V_8})/4 = 0.1$.

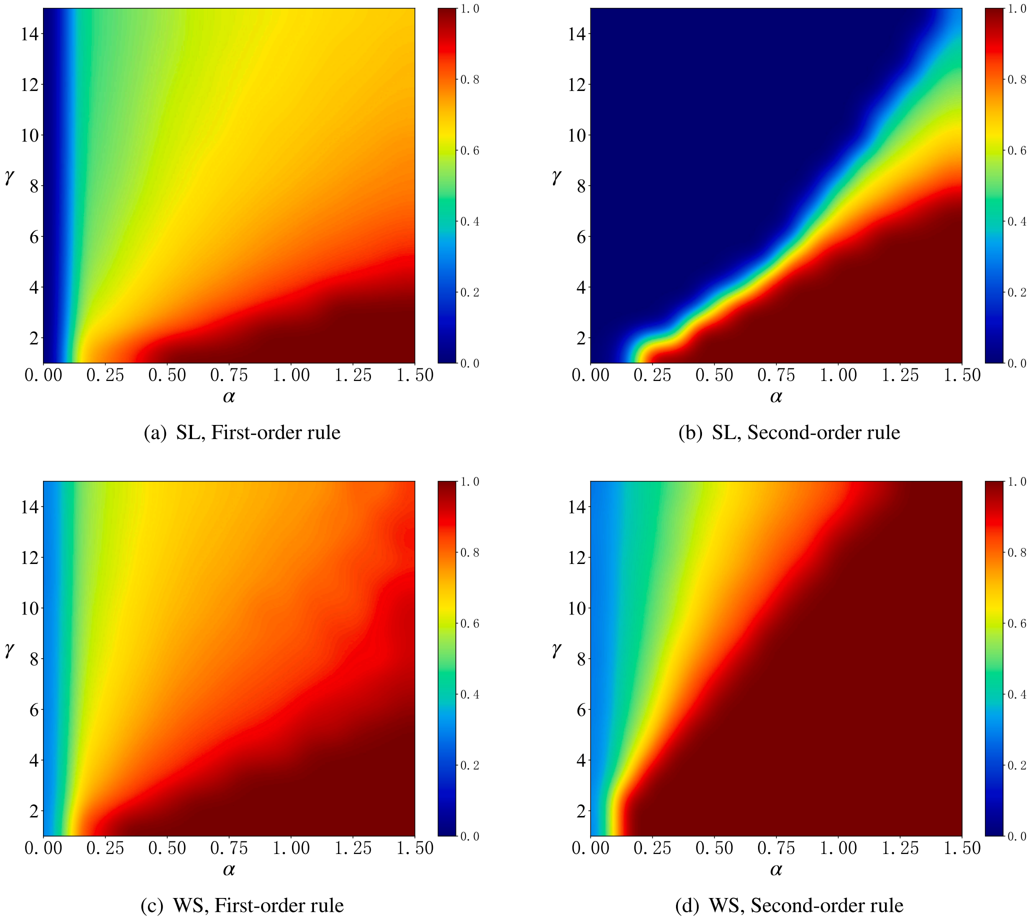
3. Simulation and analysis

In this section, we conducted numerical simulation experiments using both a square lattice (SL) and a small-world (WS) network, each consisting of $N = 6400$ nodes. The SL network employed periodic boundary conditions, where each player selects their nearest four neighbors as opponents. In the WS network, each player connects to n_i nodes based on a specific probability distribution. In both types of networks, the strategies of all players were randomly initialized, with half being cooperators and half defectors. Initial reputation scores for all players followed a uniform distribution within the range $[0, 2\gamma]$. The results of numerical simulations are obtained after 10^4 Monte Carlo Steps, and the cooperation level F_c being computed as the average over the last 10^3 time steps. Furthermore, to ensure the accuracy of the conclusions, the final experimental results are statistically averaged over 20 independent experiments.

Firstly, to illustrate the impact of various variables on F_c based on different evaluation rules (first-order and second-order rules), Fig. 2 presents heat maps for $b = 1.2$ in both SL and WS networks. The reputation ratio α and reputation threshold γ are respectively represented as the horizontal and vertical axes of the images. The color bar represents different cooperation levels within group, where blue indicates complete defection, red indicates complete cooperation, and other colors signify coexistence of cooperation and defection within the group. In all subplots of Fig. 2, conspicuous red areas appear in the lower-right region, indicating the effectiveness of our mechanism in maintaining full cooperation under these conditions.

We proceed to analyze the evolution of cooperation in the SL network, i.e., Fig. 2(a) and 2(b). At smaller α, the reputation mechanism has not yet acted. However, as α increases and in a certain range of γ, cooperation experiences significant promotion and sustenance. Fig. 2(a) demonstrates that under the first-order rule, when α exceeds 0.1, the image rapidly transitions from blue to light green, and as α grows the image shifts towards large areas of yellow and orange. When α exceeds 0.4 and γ decreases to a certain value, the image color transitions to red, indicating full cooperation within the entire group. However, it is evident that the red and blue areas in Fig. 2(b) are larger and broader compared to Fig. 2(a). The image in Fig. 2(b) exhibits a wide range of blue, transitioning rapidly into a substantial expanse of red when α exceeds 0.2 and γ decreases, with only a relatively small intermediate color area. The above phenomenon illustrates that under constraints of smaller α and larger γ, the first-order rule effectively maintains cooperation evenly. Conversely, the second-order rule requires favorable conditions of larger α and smaller γ to generate cooperation, despite it can achieve a higher cooperation level than the first-order rule. For the WS network depicted in Fig. 2(c) and 2(d), similar results are obtained with slight discrepancies. The steady-state cooperation level in the WS network is generally higher than in the SL network, especially characterized by a larger red area. In Fig. 2(d), under the second-order rule, while not displaying large-scale defection behavior, the cooperation level in the upper-left region is lower compared to Fig. 2(c). This similarly implies that the first-order rule effectively incentivizes cooperative behavior even under adverse conditions, while the second-order rule requires favorable conditions to fully motivate cooperative behavior within the group.

In Fig. 2, we manipulated various parameter variations while maintaining a constant b. Subsequently, in Fig. 3, we illustrate the evolution of F_c across a wide range of b to explore more information. Evidently, it can be visually observed that for a certain γ, F_c raises as α increases, which aligns with the findings in Fig. 2. In both Fig. 3(a) and 3(b), under the same conditions of γ and α, the critical parameters for achieving higher F_c in the first-order rule compared to the second-order rule are as follows: when $\alpha = 0.1$, $b = 1.02$; when $\alpha = 0.5$, $b = 1.08$; when $\alpha = 1.0$, $b = 1.16$; when $\alpha = 1.5$, $b = 1.24$; and when $\alpha = 2.0$, $b = 1.36$. For instance, for $\alpha = 1.0$ and $\gamma = 8$, F_c is higher in the second-order rule when $b < 1.16$, while the first-order rule exhibits stronger cooperation stability when $b \geq 1.16$. In comparison to the SL network, the second-order rule proves to be more effective in sustaining cooperation



(a) SL, First-order rule

(b) SL, Second-order rule

(c) WS, First-order rule

(d) WS, Second-order rule

Fig. 2. The cooperation level F_c at the equilibrium state as a function of the reputation threshold variable γ and reputation ratio α. The color bar from blue to red corresponds to the stable group cooperation ratio F_c ranging from 0 to 1. The subplots horizontally correspond to different reputation evaluation rules and vertically different network models. Under the same rule, the cooperation ratio is higher on WS network compared to SL network. Additionally, in all four subplots, the temptation to defect b is set to 1.2. For the WS network (subplots (c) and (d)), the rewiring probability is set to 0.3 and the average degree of 4.

within the WS network. Similarly, in Fig. 3(c) and 3(d), the critical parameters for the differences in cooperation among different evaluation rules are: when $\alpha = 0.1$, $b = 1.08$; when $\alpha = 0.5$, $b = 1.24$; when $\alpha = 1.0$, $b = 1.34$; when $\alpha = 1.5$, $b = 1.40$; and when $\alpha = 2.0$, $b = 1.46$. Based on observations from both networks, these results indicate that the second-order rule is more effective in scenarios with low temptation to defect. However, in situations with high temptation to defect, the second-order rule falters, leading to the dominance of defectors within the network. Conversely, the first-order rule consistently maintains higher F_c regardless of the value of b. Additionally, in Fig. 3(d), the transition of F_c from 1 to 0 occurs more rapidly as α increases. In other words, at high α values, even a slight increase in b leads the group to abruptly transition from full cooperators to full defectors.

The following graph, Fig. 4, illustrates the time evolution of F_c with α fixed at 0.5, and reputation thresholds γ set at 4.0, 6.0, 8.0, 12.0, and 16.0, respectively. We proceed to horizontally compare the disparity among different evaluation rules. Initially, during the early stages of evolution, F_c exhibits varying degrees of decline. Notably, F_c reaches its lowest point within shorter time intervals in Fig. 4(a) and 4(c). This observation suggests that the reputation incentive mechanism operates more promptly under the first-order rule. As time progresses, F_c begins a steady ascent, indicating the influence of network reciprocity in the evolutionary process [44,45]. Under the first-order rule, F_c peaks before descending and ultimately stabilizing, except for the case of $\gamma = 4.0$ in Fig. 4(c). In contrast, under the second-order rule, F_c continues to rise and stabilizes across most parameter configurations. For $\gamma = 12.0, 16.0$ in Fig. 4(b) and $\gamma = 16.0$ in Fig. 4(d), F_c initially increases before declining. Across all subplots in Fig. 4, it is evident that F_c increases as γ decreases. Particularly for $\gamma = 4.0$, in Fig. 4(a), Fig. 4(b) and 4(c), the growth trend of F_c significantly surpasses other γ conditions, indicating that a smaller γ can notably enhance cooperation stability under the reputation incentive mechanism.

Finally, to further validate the results discussed above, we analyze the underlying micro-evolutionary processes. Fig. 5 and 6 illustrate the distribution of individual strategy and reputation scores evolution for $\gamma = 5.0, 10.0$ at several typical time steps. The first and third rows present snapshots of evolution under the first-order rule, while the second and fourth rows present snapshots under the second-order rule. In Fig. 5, light blue and dark blue represent bad-image cooperators (BC) and good-image cooperators (GC) respectively, while pink and red represent good-image defectors (GD) and bad-image defectors (BD) respectively. Initially, the four

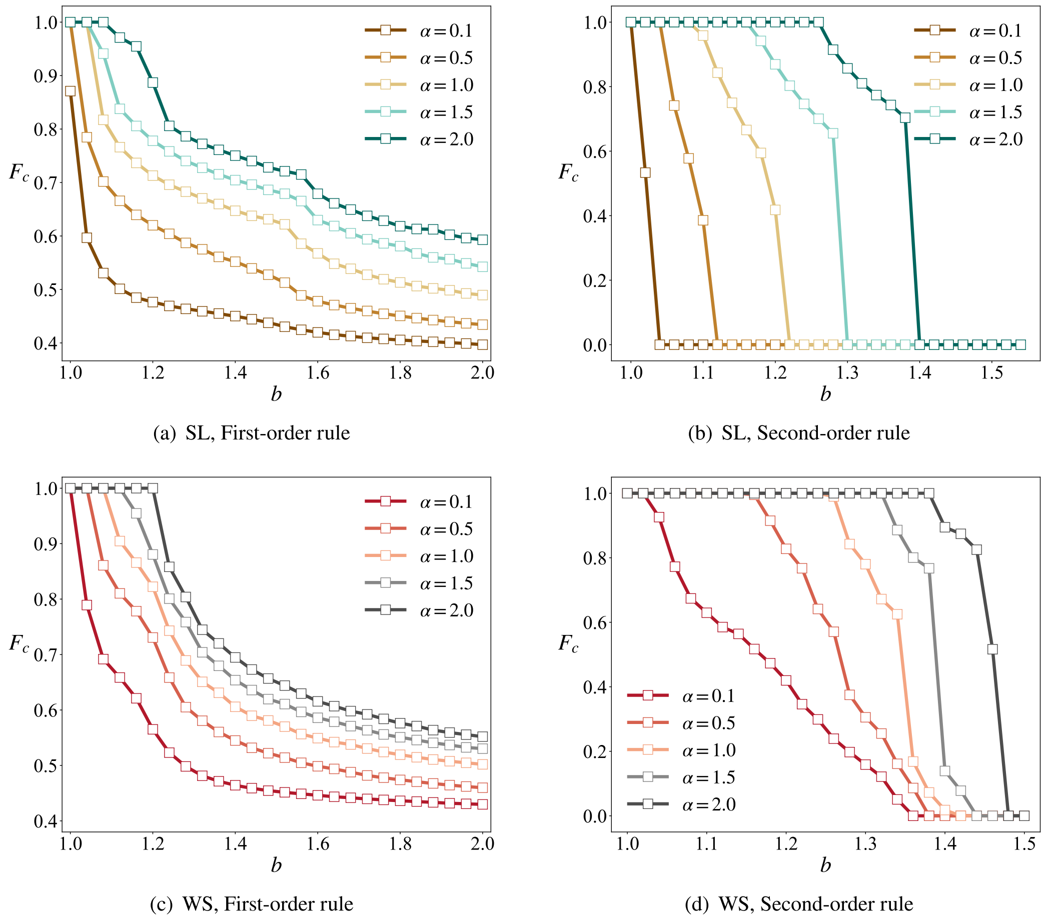


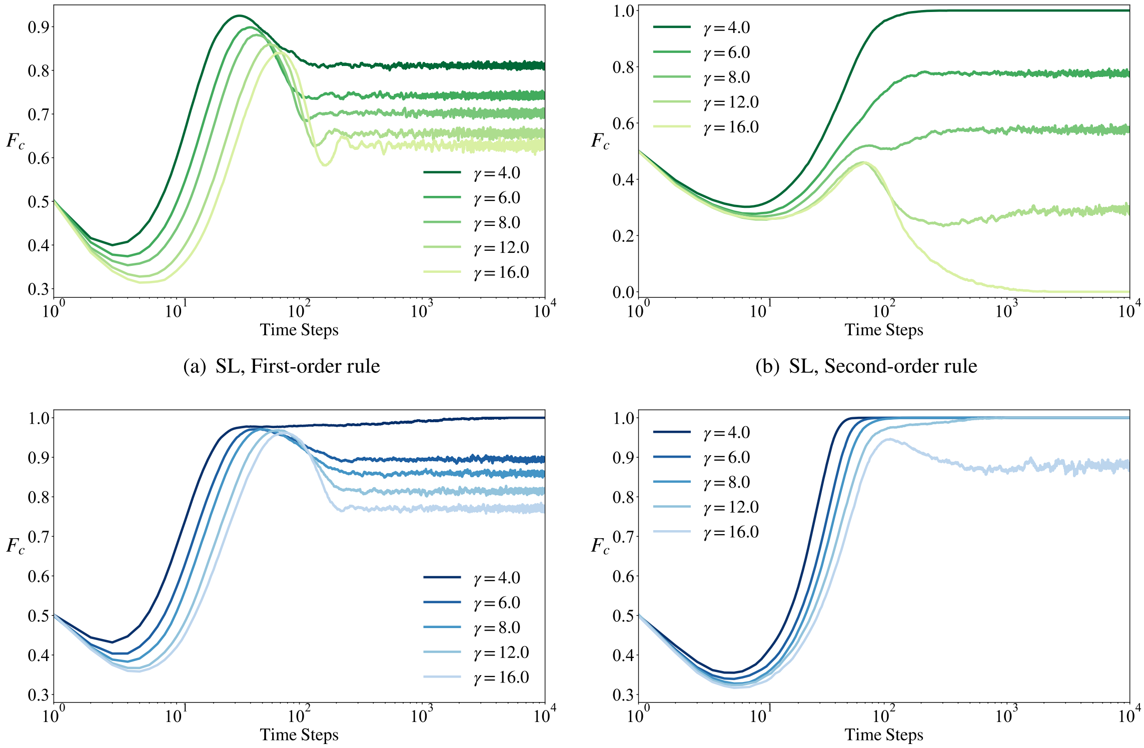
Fig. 3. The final stable cooperation level F_c as a function of the temptation to defect b for different reputation ratios α. The different colored lines in the subplots represent various parameter values, with α set to 0.1, 0.5, 1.0, 1.5, and 2.0, respectively. A consistent trend observed across all four subplots is that a higher reputation influence ratio α is associated with increased group cooperation levels. The remaining parameters are fixed as follows: reputation ranges from 0 to 16, with a reputation threshold γ set at 8. For the WS network, the random rewiring probability is 0.3, and the average degree is 4.

types of players are randomly and evenly distributed across the network, forming low-density and dispersed clusters of cooperation. However, after a short period, defectors begin to dominate, particularly evident in the second and fourth rows of images. Over time, it becomes apparent that the blue areas in the first two rows of images rapidly expand, with cooperators growing faster under the second-order rules. This suggests that in an environment with $\gamma = 5.0$, the second-order rule better supports the survival of cooperators compared to the first-order rule. Conversely, when $\gamma = 10.0$, cooperators can only establish survival patches within a limited range under the first-order rule, while cooperation clusters fail to form extensively under the second-order rule.

Fig. 6 depicts the evolution of reputation scores corresponding to Fig. 5, providing additional detail. During the early stages of evolution, reputation scores evolve more rapidly under the first-order rule, with small blue areas observed in the images. Subsequently, in the images of the first three rows, the appearance of large red areas alongside some blue areas indicates that the reputation scores of most players have reached the critical threshold. Compared to the results under the same parameter conditions in Fig. 5, it is observed that cooperators are typically those players with critical reputation values. Additionally, in the fourth-row images, when $\gamma = 10$, a large number of players in the group have reputation scores concentrated between $[15, 20]$, with GD players dominating the network, rendering the reputation mechanism ineffective. We find that the reputation mechanism proves effective only when reputation scores evolve accurately towards the critical value. In other words, the rapid and efficient evolution of reputation scores enhances cooperative stability.

4. Conclusion

Our investigation focused on the impact of different evaluation rules on cooperative stability, utilizing an improved reputation mechanism. The reputation incentive model comprises three key components: mutual evaluation, reputation updating, and strategy transformation. Initially, players assess their opponents' actions following interactions, employing both first-order and second-order evaluation methods. Furthermore, the evaluation intensity varies for each player and is influenced by the surrounding environment, undergoing updates in each round. Following mutual evaluations, reputation increments of player are determined based on the



(a) SL, First-order rule

(b) SL, Second-order rule

(c) WS, First-order rule

(d) WS, Second-order rule

Fig. 4. The time evolution of the cooperation level F_c for varying reputation threshold variables, with the reputation ratio α fixed at 0.5. The different colored lines represent the results for each γ, which are set to 4.0, 6.0, 8.0, 12.0, and 16.0. The model parameters are as follows: for both networks, the temptation to defect b is set to 1.08, and the Monte Carlo steps = 10,000. For WS networks, the rewiring probability is 0.3 and the average degree is 4.

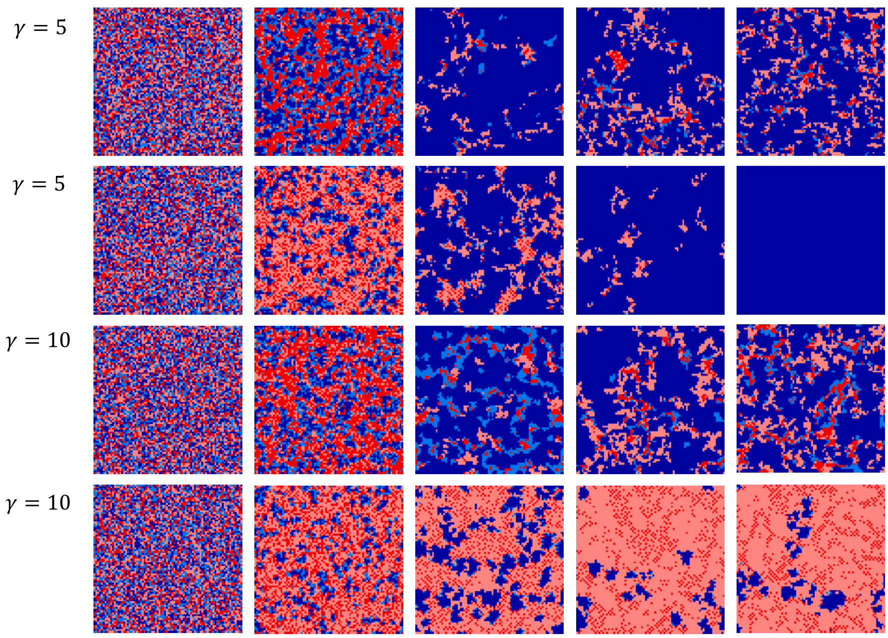


Fig. 5. Snapshots of individual strategy distribution on the square-lattice network with reputation thresholds $\gamma = 5.0, 10.0$. The first and third rows depict snapshots under the first-order rule, with time steps progressing from left to right: 0, 10, 50, 100, 1000. Similarly, the second and fourth rows depict snapshots under the second-order rule, with time steps progressing from left to right: 0, 10, 100, 1000, 10000. In these images, light blue, dark blue, pink, and red regions respectively represent player types: BC, GC, GD, BD. The parameters are set as follows: $L = 80$, $b = 1.15$, $\alpha = 0.8$.

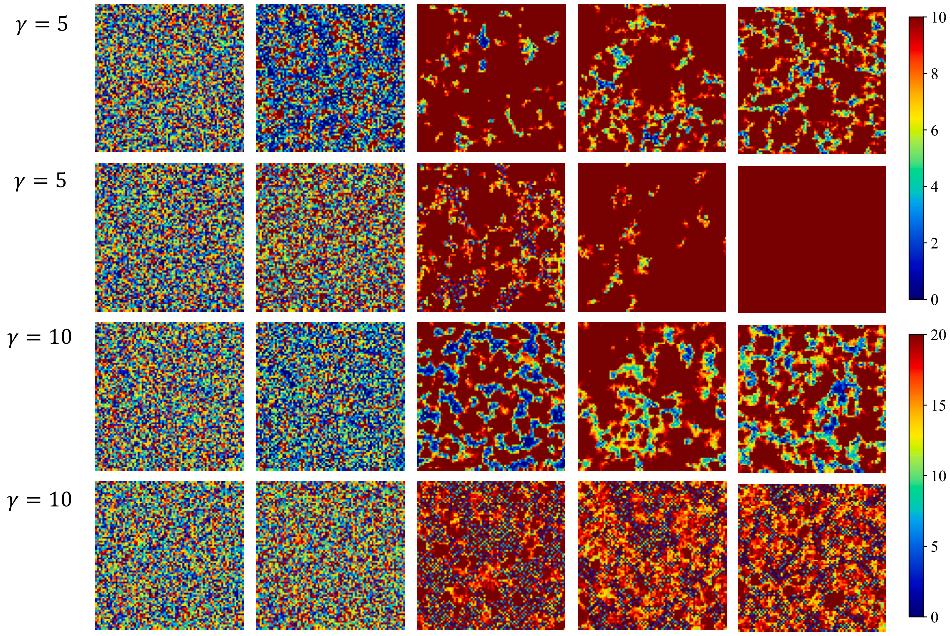


Fig. 6. Snapshots of individual reputation score distribution on the square-lattice network with reputation thresholds $\gamma = 5.0, 10.0$. The color bar associates different colors with corresponding reputation scores. The first and third rows depict snapshots under the first-order rule, with time steps progressing from left to right: 0, 10, 50, 100, 1000. The second and fourth rows represent snapshots under the second-order rule, with time steps progressing from left to right: 0, 10, 100, 1000, 10000. The parameters are set as follows: $L = 80$, $b = 1.15$, $\alpha = 0.8$.

evaluations they receive. Ultimately, these normalized reputation scores are integrated with players' payoffs, forming a comprehensive consideration during the strategy learning process. Through numerical simulations, we explored the effects of the reputation ratio α and reputation threshold variables γ on cooperation, with the goal of identifying factors that promote cooperative behavior. The experimental results from SL and WS networks revealed that, under stringent constraints with larger γ and smaller α, cooperation remains stable with the first-order rule, while complete defection occurs within groups under the second-order rule. Conversely, under lenient conditions with smaller γ and larger α, the second-order rule can incentivize fully cooperative behavior within groups, surpassing F_c achieved with the first-order rule under specific parameters. Moreover, it is evident that the reputation mechanism effectively fosters cooperation, but only when the reputation scores of the majority of players in the group reach a critical threshold. Smaller temptation to defect b positively influences such phenomenon.

In this work, we found that the reputation mechanism fails with stringent parametric conditions under the second-order rule. This phenomenon could be attributed to players' restricted capacity to process extensive information, thereby impacting their decision-making efficiency. This finding aligns with research on memory mechanism [46], and is corroborated by previous social experiments [47], indicating that memory is easily constrained in reputation-based interactions. Additionally, extending our model to the third-order reputation rule [41] or introducing gossip [48,49] and misinformation [50] could lead to different results. Meanwhile, it is also meaningful to use more methods, especially the semi-tensor product of matrices [51,52] to analyze the stability of networked evolutionary games. All these works may be further explored in future studies. In the end, we expect that our work can provide a positive insight into the study of social group behavior.

CRediT authorship contribution statement

Qi Hu: Conceptualization, Data curation, Methodology, Software, Writing – original draft. **Mengyu Zhou:** Data curation, Validation, Writing – original draft. **Yulian Jiang:** Supervision, Validation, Writing – review & editing. **Xingwen Liu:** Supervision, Validation, Writing – review & editing.

Data availability

Data will be made available on request.

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