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Letter

The impact of environmental disorder-induced constraints on spatial public goods games with social exclusion

Xingping Sun [1], Zhiyuan Huang [1], Hongwei Kang *, Zhekang Li, Yong Shen, Qingyi Chen

School of Software, Yunnan University, Kunming, 650000, China

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ABSTRACT

Social exclusion is a powerful strategy in spatial public goods games for fostering cooperation, but it comes with costs and limitations. Inspired by the Broken Windows theory, connecting disorderly environments to community breakdown, we present two environmentally constrained cases. One case limits exclusion by dynamic exclusion probability, while the other faces constraints based on the dynamic unit cost of exclusion. Our analysis studies the evolutionarily stable state and phase transition processes under various parameter spaces for both cases. Results show that cooperation is promoted widely in both scenarios. Interestingly, when the unit cost of exclusion is dynamic, there is a notable expansion in the parameter space, leading to complete cooperation. Despite disorderly environments, both exclusion strategies effectively promote cooperation, with the dynamic cost strategy outperforming the dynamic exclusion probability strategy. These findings enhance our understanding of how environmental constraints and social exclusion shape prosocial behaviors.

1. Introduction

Cooperative behavior is a very common phenomenon in both biological systems and human societies [1–4]. However, it proposes a conceptual challenge within the framework of Darwinian evolution [5,6]. Cooperation often hinges on choices driven by individual self-interest [7], yet myopia and selfishness can undermine the very foundation on which cooperation relies. This dilemma has given rise to the “tragedy of the commons,” exemplified by the clash between individual interests and the common good [8]. Consequently, the study of the evolution of cooperative behavior and strategies for sustaining it in social dilemmas has become a focal point in the realms of social and biological sciences [9]. The Public Goods Game (PGG) and evolutionary game theory offer theoretical models and practical methodologies for investigating these issues. In the context of human society, specific structural characteristics define social networks and interpersonal interactions [10,11]. The Spatial Public Goods Game (SPGG) has emerged as a direct outcome of scholarly inquiries into these structural attributes.

Within the context of the SPGG theoretical framework, extensive research efforts have been dedicated to the exploration of social dilemmas [12–16]. Researchers have identified various mechanisms that can promote the evolution of cooperation, offering potential solutions to a range of social dilemmas. These solutions encompass the rewarding of cooperators [17–20], the punishment of defectors [21–28], and the amalgamation of both approaches [29]. Additionally, social diversity [30,31], heterogeneous investments [32–37], tax mechanisms [38,39], reputation [40,41], voluntary participation [42,43], any multiplayer game [44], corrupt enforcers [45] and institutional incentives [46] have been studied. Notably, punishment has emerged as a central theme in this body of research. Social exclusion, as a special form of punitive action, has garnered significant attention, particularly after its examination from an evolutionary perspective by Sasaki and Uchida [47].

In comparison to punishment, exclusion stands out as a more potent strategy [48,49] due to its capacity to remove defectors from a group with a certain degree of probability. This expulsion results in defectors gaining nothing, while the remaining group participants can collectively reap the benefits of cooperation. Extensive research efforts have been devoted to the study of the exclusion strategy [50–55]. For instance, by considering the scale effect, Quan et al. [56] introduced the concept of dynamic costs into social exclusion. Lv et al. [57] integrated heterogeneous investment with exclusion. The comparison of exclusion and punishment is a common approach, as exemplified by Liu et al. [49], who conducted a comparative analysis of social exclusion and punishment strategies, ultimately highlighting the superior efficacy

* Corresponding author.
E-mail address: hwkang@ynu.edu.cn (H. Kang).
[1] These authors contributed equally to this work.

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of exclusion in resolving social dilemmas. To address the second-order free-riding problem, Sun et al. [58] combine antisocial punishment with exclusion.

The existing body of literature has primarily concentrated on the examination of exclusion as a standalone strategy or its integration with other mechanisms to foster its efficacy. However, the exclusion strategy, while demonstrating exceptional success in deterring defection when compared to conventional strategies, is not without its limitations. In reality, the idealized conditions that would allow the exclusion strategy to operate optimally are seldom encountered. Consequently, this paper incorporates the Broken Windows Theory to elucidate the environmental constraints that the exclusion strategy may face.

Some researchers have studied the Broken Windows Theory. For example, D'Orsogna et al. [59] wrote a review for statistical physics of crime and Perc et al. [60] integrated the Broken Windows Theory with heterogeneous punishment. This theory is typically regarded as a community-level theory of crime, illustrating how a disordered environment can influence individual behavior [61]. Wilson and Kelling [62] provided a comprehensive exposition of this theory, positing that in areas where criminal behavior goes unpunished, two potential paths leading to crime emerge. The first path delineates a direct correlation between disordered environments and criminal conduct. Acts such as breaking windows or other forms of disorder send a signal that uncivil or unlawful behavior will not face retribution, thereby fostering a climate conducive to unlawful actions. This can be analogously viewed as akin to defection in the context of social dilemmas. Consequently, this theory finds resonance in the update of strategies within public goods games: when defectors can evade exclusion, neighbors are more inclined to embrace this unpunished, higher-yield strategy.

The second theory, which is predominantly invoked in this paper, explains an indirect relationship between a disordered environment and crime (defection). The proliferation of criminals within a community triggers a heightened perception of danger among the residents, leading them to avoid this perceived threat. As a consequence, their willingness to confront and deter such threats diminishes. This reduction in community engagement also results in a decline in the community's capacity for self-governance (a concept also reflected in the studies of social disintegration theory [63,64]). Drawing from this theory, we introduce the concept of constrained exclusion strategies. The first case of limitation arises when an increase in the number of defectors in the community leads to a decrease in the willingness and ability of enforcers (excluders), manifested as a reduction in the success rate of exclusion. In the second case of limitation, we assume that enforcers can maintain a consistent rate of exclusion success but must bear the burden of increased risks as the number of defectors in the community escalates, leading to an elevated unit exclusion cost. Under the stipulated conditions in both cases, the exclusion probability or unit exclusion cost within each group experiences continual fluctuations in response to the dynamic evolution of the system.

Our work studies the repercussions of environmental constraints within communities on the exclusion strategy, and we undertake a comparative analysis of two distinct constraint cases. This comparative assessment seeks to identify the shared attributes and distinctions between these two cases. Parameters that can modulate the rate of exclusion success, the unit cost of exclusion, and the overall dilemma strength of the game have been introduced in both cases. We analyze the evolutionarily stable states within the system across different parameter spaces. Simulation results demonstrate that both cases still promote cooperation, which can occur under strong dilemma strength. Furthermore, it becomes evident that the exclusion strategy in the second constraint case exerts a more favorable influence in fostering cooperation when contrasted with that in the first case.

The remaining parts of this article are arranged as follows: Section 2 introduces the SPGG models with two cases of constraints. The simulation results are presented in Section 3. In Section 4, the conclusions of this study and future work directions are presented.

2. Model

We study the evolutionary process on an $L \times L$ square lattice network. Assuming the presence of N players, each player is located at a specific node within the network. The network is regular, with each node having 4 neighbors. In the conventional Spatial Public Goods Game (SPGG), every node forms a group in conjunction with its immediate 4 neighbors, and each node partakes in 5 rounds of such group games, thereby accruing payoffs. Within this model, each node is presented with a choice between adopting either a cooperative or a defection strategy. Cooperators contribute a fixed amount c ($c = 1$) to the common pool, while defectors make no contributions whatsoever. The aggregate contribution is subsequently multiplied by a synergy factor r, with the resulting sum being evenly distributed among all group members. Consequently, defectors can attain the highest short-term gains, although cooperation represents the superior long-term choice. Nevertheless, a pervasive inclination toward self-interest engenders a scenario wherein eventually all participants opt for defection, culminating in a collective dilemma characterized by an overall payoff of zero.

We consider game groups as communities, while the overall participants constitute the system. The environmental quality within the system, as determined by synergy factor r, in conjunction with the parameters governing exclusion, notably the exclusion probability and unit exclusion cost, plays an important role in shaping the quality of the system's environment. Within a single group, the degree of disorder within the community environment is contingent upon the number of defectors present in that group, denoted as N_D. In the context of exclusion strategies, we introduce the parameters of exclusion probability β, and unit cost of exclusion θ, which are intimately linked to the community's disordering factor N_D. The exclusion probability β comprises the maximum exclusion probability $\beta_{\max}$, and the minimum exclusion probability $\beta_{\min}$, thereby delineating the upper and lower bounds that govern the success rate of exclusion within the groups. Likewise, the unit cost of exclusion θ encompasses the maximum unit cost of exclusion $\theta_{\max}$, and the minimum unit cost of exclusion $\theta_{\min}$, thereby specifying the range of unit exclusion cost variations within each group.

In this paper, we introduce two prosocial exclusion strategies based on the Broken Windows Theory in the context of the traditional Spatial Public Goods Game. The first strategy referred to as Case I: EP strategy (Exclusion with dynamic Probability), is designed for situations in which a constrained environment facilitates defectors' evasion from exclusion. The second strategy, Case II: EC strategy (Exclusion with dynamic Cost), is employed when the exclusion cost is significantly higher. These two cases serve to illustrate the influence of "Broken Windows". It assumes that the management efficacy of enforcers within a group will dynamically diminish or the enforcement cost will dynamically increase in response to the growing disorder in the group's environment. Both of these exclusion strategies need to donate c to the common pool, as same as the pure cooperation strategy does. Specifically, the EP strategy utilizes a dynamic probability parameter α, and a static unit cost δ, to exclude defectors. In contrast, the EC strategy relies on a static probability parameter γ, and a dynamic unit cost ϕ, for the execution of exclusion. The formulas calculating these different probabilities and unit costs are provided as follows:

$$\alpha = \beta_{\max} - (\beta_{\max} - \beta_{\min})\frac{N_D}{G-1}, \tag{1}$$

$$\delta = \theta_{\min} + (\theta_{\max} - \theta_{\min})\frac{1}{G-1}, \tag{2}$$

$$\gamma = \beta_{\max} - (\beta_{\max} - \beta_{\min})\frac{1}{G-1}, \tag{3}$$

$$\phi = \theta_{\min} + (\theta_{\max} - \theta_{\min})\frac{N_D}{G-1}, \tag{4}$$

where G denotes the number of participants in a game group, N_D signifies the count of defectors in the same group. When N_D increases, the "Broken Windows" cast its effect through the following consequences:

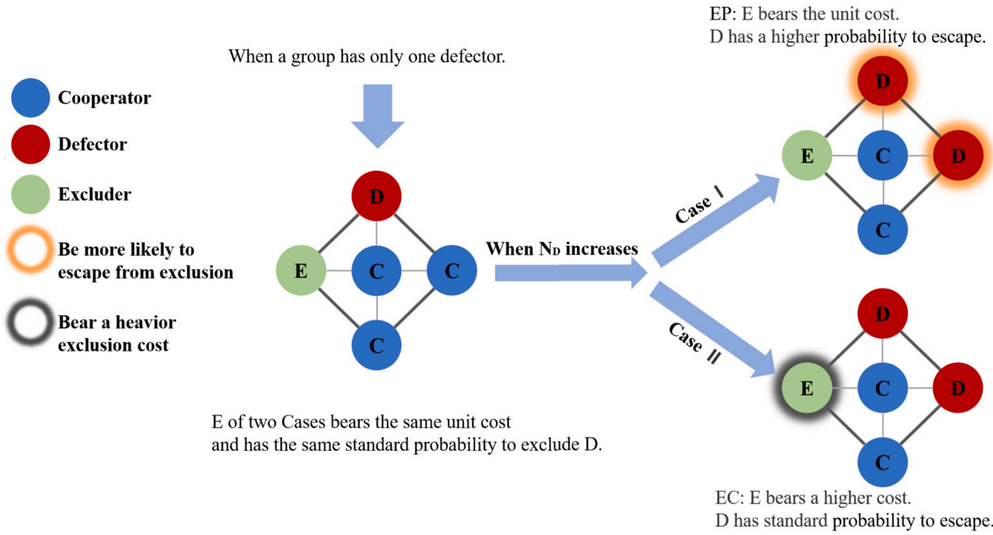

Fig. 1. An example to illustrate the specifics of the two exclusion strategies. Let's consider a game group with three cooperators (blue, C), one defector (red, D), and one excluder (green, E). The dynamic exclusion probability in Case I matches the static exclusion probability in Case II and the unit cost of exclusion is also the same in both cases. As the number of defectors increases, the group environment exhibits greater disorder. In Case I, the probability of defectors escaping from exclusion increases (highlighted in orange), while in Case II, the unit exclusion cost for excluders increases (shaded in black).

(1) a decrease in α, leading to a reduced successful rate of exclusion in Case I; (2) an increase in ϕ, resulting in an increased unit cost of exclusion in Case II. δ and γ keep constant and do not change with the disordered environment of the group.

To assess the impact of the introduced exclusion strategies within the conventional SPGG framework under both Case I and Case II, dedicated models were constructed. When the community (group) contains only one defector, the exclusion probability and unit cost of exclusion are identical for both Case I and Case II. It is only with a rise in the number of defectors within the group that distinctions between the two cases begin to emerge, as shown in Fig. 1. Subsequently, we calculate the benefits of the three strategy types of exclusion, cooperation, and defection in the group for each of these cases.

In Case I, the exclusion strategy employs a dynamic probability of exclusion. Pure cooperators act as second-order free-riders, benefiting from the exclusion carried out by the excluders against defectors. Notably, when there are no defectors present within a group, the payoffs for excluders are equivalent to those of cooperators. Consequently, in a group ($G = 5$), the payoffs for the three strategies are as follows:

$$\Pi_C = \frac{r(N_C + N_E + 1)}{G - N_K} - 1, \tag{5}$$

$$\Pi_D = \begin{cases} 0 & \text{expelled with probability } \alpha, \\ \frac{r(N_C+N_E)}{G-N_K} & \text{otherwise,} \end{cases} \tag{6}$$

$$\Pi_E = \frac{r(N_C + N_E + 1)}{G - N_K} - 1 - \delta \frac{N_D}{G-1}, \tag{7}$$

where N_C, N_D, and N_E denote the numbers of cooperators, defectors, and excluders, respectively, excluding itself. And N_K represents the number of defectors being successfully excluded within the group.

In Case II, the exclusion strategy involves a dynamic cost of exclusion. This exclusion strategy has a fixed and higher exclusion probability compared to the EP strategy, but suffers dynamic and higher costs. In a group, the payoffs for the three strategies are as follows:

$$\Pi_C = \frac{r(N_C + N_E + 1)}{G - N_K} - 1, \tag{8}$$

$$\Pi_D = \begin{cases} 0 & \text{expelled with probability } \gamma, \\ \frac{r(N_C+N_E)}{G-N_K} & \text{otherwise,} \end{cases} \tag{9}$$

$$\Pi_E = \frac{r(N_C + N_E + 1)}{G - N_K} - 1 - \phi \frac{N_D}{G-1}. \tag{10}$$

From the above formulas, when $\beta_{\max} = \beta_{\min}$, the exclusion probability α in Case I transitions from dynamic to fixed, simplifying Case I from a model with dynamic exclusion probability to one with fixed exclusion probability. Similarly, when $\theta_{\max} = \theta_{\min}$, the dynamic exclusion cost ϕ in Case II transitions from dynamic to fixed, simplifying Case II from a model with dynamic exclusion cost to one with fixed exclusion cost. Both of these simplified models belong to the classic exclusion models where both exclusion probability and exclusion cost are static, as classic work studied [47].

In the strategy update process, we utilize a Monte Carlo simulation approach, synchronously updating the strategies. This method involves a substantial number of basic Monte Carlo steps (MCS). Within each MCS, every individual engages in a public goods game with 4 neighboring individuals situated around them. Each individual(denoted as i) participates in 5 rounds of the game, accumulating a payoff denoted as Π_i. Subsequently, i randomly selects one of its neighbors, j, and compares the accumulated payoffs Π_j from neighbor j with its own accumulated payoff Π_i. Strategy learning takes place through a probability $P_{i\to j}$ based on the Fermi function:

$$P_{i\to j} = \frac{1}{1 + \exp(-\frac{\Pi_j - \Pi_i}{\kappa})}, \tag{11}$$

where $\kappa > 0$ represents the noise. To remain consistent with previous research, we use $\kappa = 0.5$ [65]. This makes the strategy updates tend to favor learning strategies with higher payoffs, but it may also introduce irrational behavior by potentially learning strategies with poorer performance.

Here, we list all our parameters and variables. (1) input parameters: r, $\beta_{\max}$, $\beta_{\min}$, $\theta_{\max}$, $\theta_{\min}$, α (dynamic), δ (static), γ (static), ϕ (dynamic), N_C, N_D, N_E, N_K, Π_i, Π_j, n_C and n_E. (2) auxiliary parameters: L, c, κ, N, G, β and θ. (3) output variables: Π_C, Π_D, Π_E, $P_{i\to j}$, α (dynamic), δ (static), γ (static) and ϕ (dynamic).

3. Results and discussion

In this study, we conducted experiments on a 100×100 square lattice with periodic boundary conditions, which helps us to avoid finite-size effects. The total number of participants is $N = 10^4$. In both cases, the frequencies of strategies in the population can achieve stability within 10,000 MCSs. Therefore, all our experiments used 10,000 MCSs to get results and generate figures. To ensure the robustness of our findings, we

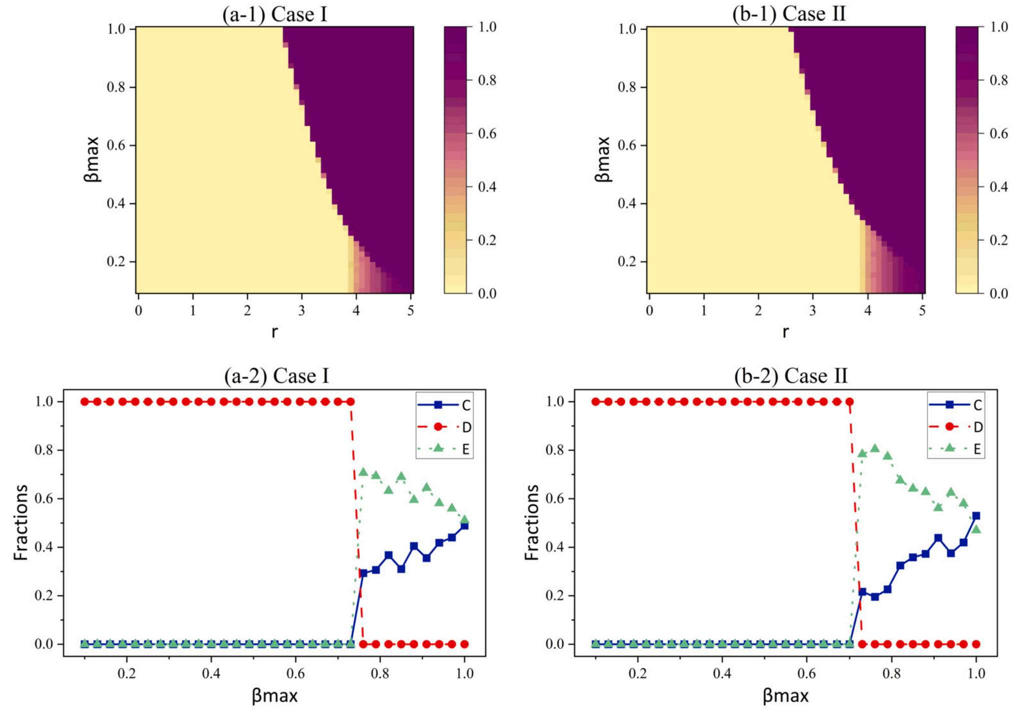

Fig. 2. (top) The two panels (a-1) and (b-1) represent 2D heat maps of the cooperation levels for both cases under different parameters of synergy factor r and maximum probability of exclusion $\beta_{\max}$, with $\theta_{\max}$ fixed at 3. The yellow region indicates low cooperation and the purple region indicates high cooperation. The two panels closely resemble each other, implying a similar impact of r and $\beta_{\max}$ on both cases. (bottom) Stable distributions of different strategies in the two cases are depicted in panels (a-2) and (b-2) while maintaining a fixed value of $r = 3$ and varying exclusion probability $\beta_{\max}$. These panels represent vertical cross-sections of the corresponding top panels, with r held constant at 3.

carried out 10 independent experiments and computed the average results. The strategy updates were executed synchronously, ensuring that each participant had the opportunity to update their strategy at each MCS. The metric employed to evaluate cooperation in our models is the average level of cooperation, denoted as ρ_{C+E}, and defined as:

$$\rho_{C+E} = \frac{n_C + n_E}{N}, \tag{12}$$

where n_C and n_E denote the number of cooperators and excluders among all players.

In order to study the impact of exclusion under cost constraints or probability constraints, we fixed the minimum exclusion probability $\beta_{\min}$ and the minimum exclusion cost $\theta_{\min}$ at 0.1 and 0.3, respectively. Initially, we examined the combined effects of the maximum exclusion probability $\beta_{\max}$ and the synergy factor r. As illustrated in panels (a-1) and (b-1) of Fig. 2, it becomes apparent that in Case I, cooperation begins to emerge when r is above 2.7, while in Case II, cooperation starts to appear when r is above 2.6. The synergy factors at which cooperation occurs are very close in both cases, and they are highly sensitive to the value of $\beta_{\max}$. Additionally, in Case II, the cooperation region is slightly larger than that in Case I when $\beta_{\max}$ is high and r is low, but it is smaller when $\beta_{\max}$ is low and r is high. When $\theta_{\max}$ is fixed at 3, the combined effects of $\beta_{\max}$ and r on the two cases are remarkably similar.

Moreover, the panels (a-2) and (b-2) in Fig. 2 show when $r = 3$, cooperation emerges in Case I at $\beta_{\max} = 0.76$, while in Case II, cooperation appears at $\beta_{\max} = 0.73$. It is noteworthy that once cooperation survives, it rapidly gains dominance within the system. The proportions of the three strategies undergo profound changes at this juncture, underscoring the pronounced sensitivity of both Case I and Case II to fluctuations in the maximum exclusion probability $\beta_{\max}$.

Next, with $\beta_{\max}$ fixed at 1, the effects of the maximum unit cost of exclusion and dilemma strength on the proposed two cases were investigated. In Fig. 3, the cooperation occurrences in different parameters r and $\theta_{\max}$ are portrayed through heat maps (a-1) and (b-1). In contrast to Fig. 2, there are noticeable differences in Fig. 3. Specifically, Case II exhibits cooperation at smaller values of r compared to Case I, and the high cooperation region in Case II is more extensive. Consequently, it is plausible to preliminarily infer that Case II exhibits a higher propensity for cooperation when compared to Case I. This interpretation is further supported by the vertical cross-sectional panels (a-2) and (b-2) in Fig. 3, while r is fixed at 2.4.

In the cross-sectional panels of Fig. 2 and Fig. 3, an intriguing phenomenon comes to light. When the maximum exclusion probability $\beta_{\max}$ is high and the maximum unit cost of exclusion $\theta_{\max}$ is low, the stable fractions of cooperators and excluders are similar. However, as $\beta_{\max}$ decreases and $\theta_{\max}$ increases, the fraction of excluders undergoes a gradual rise while the fraction of cooperators experiences a corresponding decline. Eventually, both reach a critical threshold where they extinct, paving the way for the dominance of defection. This signifies that when the system environment is "good" (with a high $\beta_{\max}$ and low $\theta_{\max}$), cooperation prevails, and excluders don't have a significant advantage over pure cooperators. Conversely, as the system environment deteriorates, exclusion strategies gradually manifest their survival advantage over pure cooperation strategies, leading to an increase in their fraction.

Based on our prior investigations, even slight modifications in parameters have been observed to result in notable fluctuations in cooperation levels. To delve deeper into the abrupt changes within the system at critical junctures where cooperation emerges, we have meticulously analyzed the distinctive evolutionary processes leading from defection to cooperation under minor variations in the synergy factor r, while keeping other factors constant. In order to identify the typical scenarios, we set $\beta_{\max}$ at 0.7 and $\theta_{\max}$ at 2.7. For Case I, when the synergy factor $r = 3$, the evolution stable state remains in the defection phase; however, with a mere 0.1 increment in r to 3.1, the evolution stable state transitions to cooperation. As for Case II, the demarcation between the (all D) and the (C+E) states is observed at 2.9 and 3.0. We present snapshots of the evolution process in Fig. 4 and depict the fractions of strategies over time in Fig. 5.

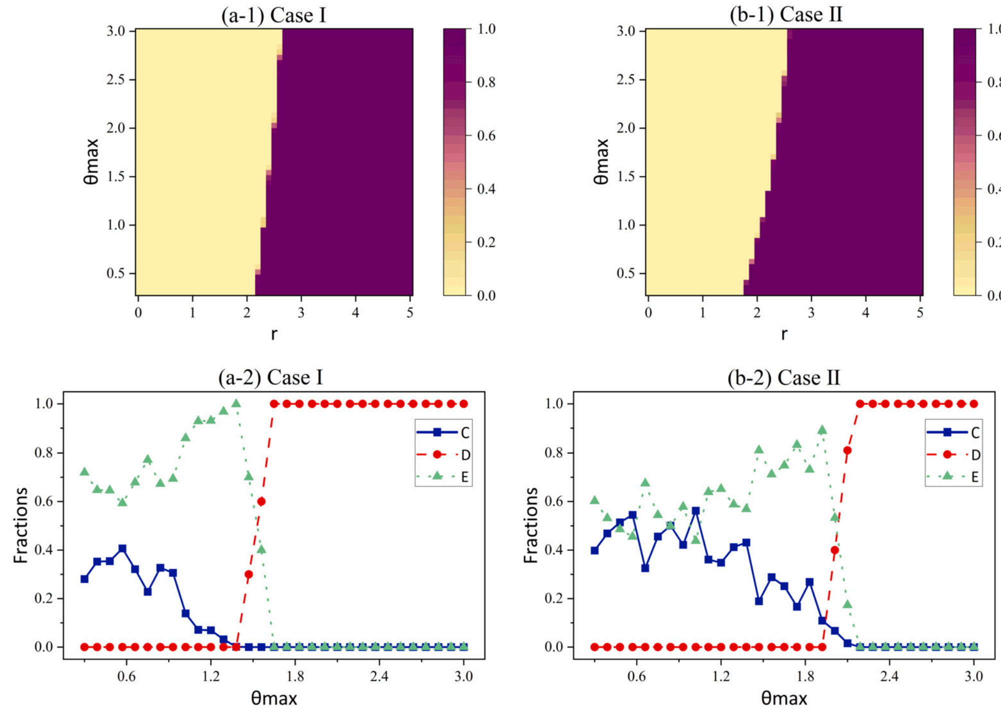

Fig. 3. (top) The two panels (a-1) and (b-1) represent 2D heat maps of the cooperation levels for both cases under different parameters of synergy factor r and maximum unit cost of exclusion θ_{max}, with β_{max} fixed at 1. (bottom) Stable distribution of different strategies in the two cases with a fixed value of $r = 2.4$ while the unit cost of exclusion θ_{max} varies. These panels present vertical cross-sections of the corresponding top panels, with r maintained at 2.4.

In both scenarios, we can observe a distinct pattern during the evolutionary process, where cooperators and excluders tend to form cohesive clusters as a defense mechanism against the invasion of defectors, as illustrated in Fig. 4. When the system ultimately reaches a state of full cooperation, excluders have a significant advantage over cooperators in numbers. This trend is evident in Fig. 5, which reveals an observable pattern: as the system transitions to an (all D) state, the number of cooperators initially increases, only to subsequently decrease, while the number of excluders consistently declines. This suggests that in unfavorable system environments, excluders struggle to carry out their exclusion strategies effectively and are gradually overrun and eliminated.

While pure cooperators experience a temporary surge in membership initially, owing to their role as second-order free-riders, they cannot withstand the onslaught of defectors and ultimately perish alongside excluders. However, in situations where the evolutionarily stable state is one of cooperation, the number of cooperators follows a similar pattern of initial increase, subsequent decrease, and stabilization at a low level with fluctuations. The number of defectors initially declines, then undergoes a significant increase before ultimately decreasing to zero. In contrast, the number of excluders experiences an initial decrease, followed by an increase, and then remains at a high level with fluctuations. This phenomenon underscores that when the system conditions favor excluders, their exclusion strategies exhibit a robust capacity to repel defectors, and excluders demonstrate superior survival capabilities compared to cooperators in such circumstances.

After previous studies, our investigation delved into the combined effects of the maximum exclusion probability β_{max} with the maximum unit cost of exclusion θ_{max} in shaping cooperative behavior. To facilitate a comprehensive evaluation of the influence of β_{max} and θ_{max} on cooperation, we selected three progressively increasing values of r, specifically, 2.1, 2.3, and 2.5. Subsequently, We conducted experiments for both cases and generated heat maps, as shown in Fig. 6. These heat maps unveiled a discernible partition of the entire 2D parameter space into two distinct regions: a region of complete cooperation and a region of complete defection. This suggests that the evolutionary stable state tends to either stabilize towards cooperative strategies(C and E) or towards a defection strategy(D). This indicates that constrained exclusion strategies can still exert a strong punitive influence, making their coexistence with defection an arduous proposition.

Furthermore, it is evident that the high cooperation parameter region in Case II surpasses that in Case I by a substantial margin. When $r = 2.1$, in Case I, cooperation cannot occur under any conditions of θ_{max} and β_{max} parameters. In contrast, Case II already exhibits a distinct high-cooperation parameter region. When $r = 2.5$, both cases can achieve cooperation under moderate θ_{max} conditions, with Case II exhibiting cooperation in the case of moderate β_{max} conditions. However, Case I can only achieve cooperation in the high β_{max} region. These observations confirm our earlier findings that Case II outperforms Case I in promoting cooperation.

To further validate our earlier findings, we established three different system exclusion environments: a "Good" system environment (high β_{max}, low θ_{max}), a "Medium" system environment (moderate β_{max}, moderate θ_{max}), and a "Poor" system environment (low β_{max}, high θ_{max}). Within these diverse system environments, we investigated the threshold of synergy factor r, necessary for the emergence of cooperative behavior in both Case I and Case II, as illustrated in Fig. 7. It is evident that under all three system exclusion environments, Case II can achieve cooperation at a higher strength of the game dilemma. Furthermore, as the system exclusion environment improves, Case II is more capable of achieving cooperation at even lower values of r compared to Case I. These studies confirm our conclusion that Case II excels in promoting cooperative outcomes.

Finally, we depict heatmaps to illustrate the disparity in the ratio of excluders to cooperators, denoted as Diff, and defined as:

$$\mathrm{Diff} = \frac{n_E - n_C}{N}, \tag{13}$$

where n_C and n_E denote the number of cooperators and excluders among all participants (N). As depicted in Fig. 8, regions shaded orange denote scenarios where the count of excluders (E) surpasses that of cooperators (C), while blue regions indicate the converse. Darker

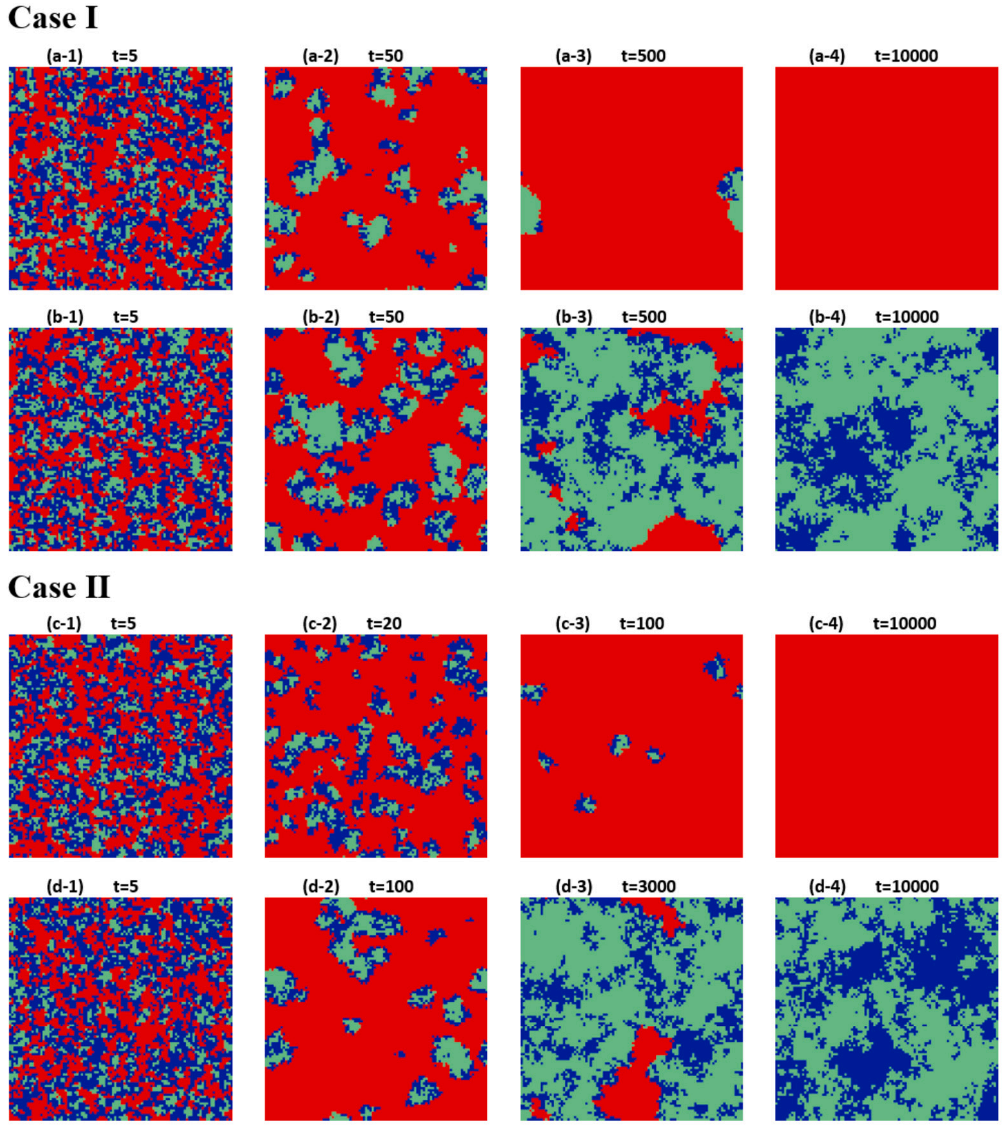

Fig. 4. In each case, upper snapshots show the evolution process to the (all D) state and lower snapshots represent the evolution process to the (C + E) state. Blue, red, and green squares denote C, D, and E, respectively. All snapshots have fixed parameters $\beta_{\max} = 0.7$ and $\theta_{\max} = 2.7$. The different parameters are set as follows: (a) $r = 3$; (b) $r = 3.1$; (c) $r = 2.9$; (d) $r = 3$. In snapshots (a) and (c), it is shown that in both Case I and Case II, even if excluders and cooperators can form small clusters, they cannot resist the invasion of defection when r is low. However, in snapshots (b) and (d), after a slight increase in r, excluders and cooperators can initially form small clusters to survive, and these clusters continue to grow, eventually enabling the cooperative strategy clusters to defeat defectors in the system.

hues signify greater absolute disparities. Notably, under conditions of complete cooperation, predominant orange hues prevail across most regions. Deep orange hues cluster proximate to the demarcation between complete cooperation and defection, implying that in bad environments, cooperators can easily be defeated by defectors, whereas excluders exhibit resilience, leading the overall evolution eventually. Conversely, blue regions predominantly manifest in the lower right quadrant of the heatmaps, expanding with increasing r. This region signifies environments conducive to cooperative strategies (C and E), effectively defeating defectors. Our findings underscore that only under high values of r, high $\beta_{\max}$, and low $\theta_{\max}$ does the prevalence of cooperators over excluders emerge, as evidenced by the blue sectors on the heatmaps. This observation suggests that a surplus of cooperators relative to excluders arises solely under exceptionally favorable environmental conditions. Conversely, in other parameter domains, excluders maintain numerical superiority over cooperators, with this advantage accentuated in harsher environments, as exemplified by the deeper orange hues on the heatmaps. This phenomenon further supports our early finding that excluders exhibited a pronounced advantage for survival when compared to pure cooperators in both Case I and Case II.

4. Conclusion

We investigate two limiting cases of exclusion strategies based on the Broken Windows Theory. These cases were applied to Spatial Public Goods Game models featuring cooperation, defection, and exclusion strategies, and we conducted simulation experiments on a square lattice. Considering the Broken Windows Theory and reality, we introduced the concept of constrained exclusion strategies subject to the influence of disordered environmental constraints. We applied the idea that disordered environments can reduce a community's management capacity to the SPGG. The exclusion strategies might be dynamically limited by the community's disordered environment, represented by the number of defectors in the group. To explore the impact of different types of limitations on cooperation, we proposed two cases. In Case I, the exclusion strategy is limited by dynamic exclusion probabilities. As the number of defectors in the community increases, the ability of excluders to carry

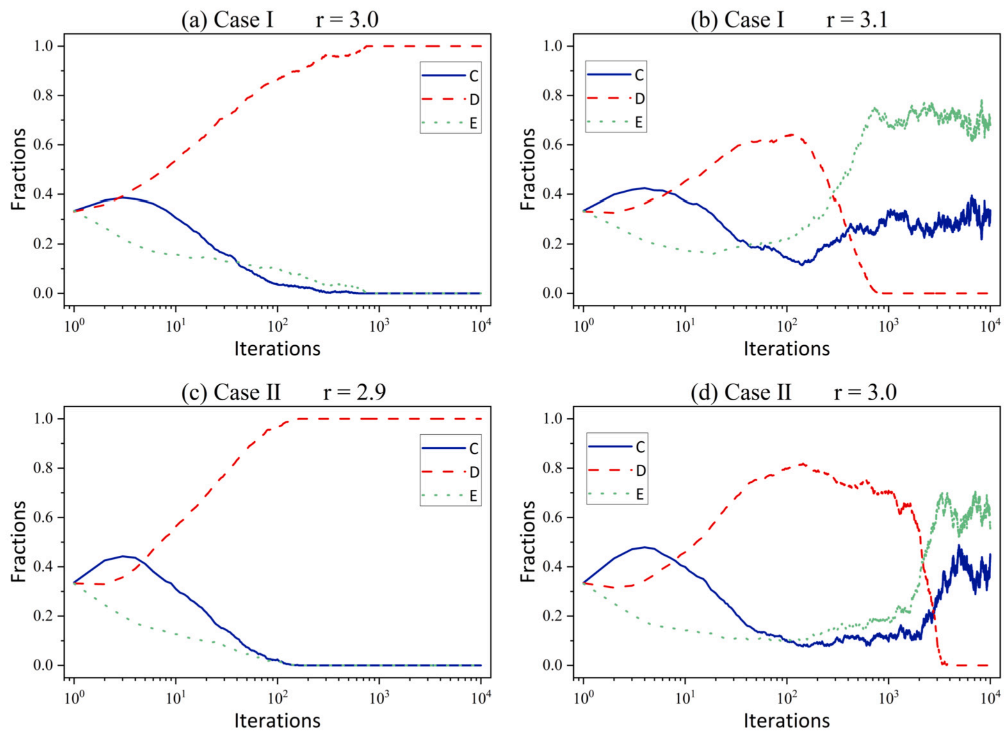

Fig. 5. (top) Panels (a) and (b) represent the strategy fractions as a function of time in Case I at $r = 3.0$ and 3.1, respectively. (bottom) Panels (c) and (d) portray the temporal evolution of strategy fraction in Case II at $r = 2.9$ and 3.0. The fixed parameters are as follow: $\beta_{max} = 0.7$, $\theta_{max} = 2.7$. The left panels (a) and (c) illustrate the evolution process terminated into the state of (all D), while just a slight increase in the synergy factor r results in the disappearance of defectors, as shown in right panels (b) and (d). These images correspond to the evolutionary process shown in Fig. 4.

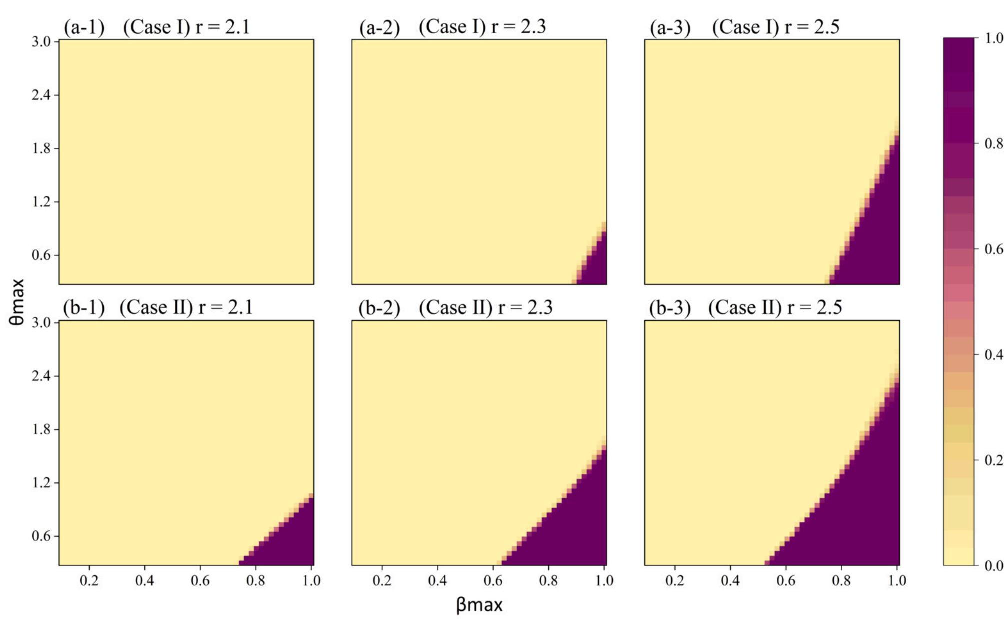

Fig. 6. (top) Panel (a) depicts the heat maps of the synergistic effects of θ_{max} and β_{max} on Case I. (bottom) Panel (b) illustrates the heat map of the combined effects of θ_{max} and β_{max} on Case II. From left to right in both cases, the synergy factor $r = 2.1$, 2.3, and 2.5, respectively. As the synergy factor r increases, the area where cooperation emerges gradually expands.

out exclusion diminishes, resulting in a dynamic reduction in the success rate of exclusion within the group. In Case II, the exclusion strategy is constrained by dynamic unit exclusion costs. With an increasing number of defectors in the game group, excluders' success rate in carrying out exclusion remains constant, but the unit cost of exclusion dynamically escalates.

We studied different parameter combinations to compare Case I and Case II under various parameter conditions. Our experiments revealed several key findings. Firstly, even under constrained circumstances, exclusion strategies proved adept at fulfilling their role in fostering cooperation and mitigating defection across a wide spectrum of parameter values. Notably, in both Case I and Case II, excluders exhibited a pronounced advantage for survival when compared to pure cooperators, with this advantage becoming more pronounced as the environmental conditions of the system deteriorated. Moreover, our experimental results indicate that, in the majority of parameter conditions, Case I is

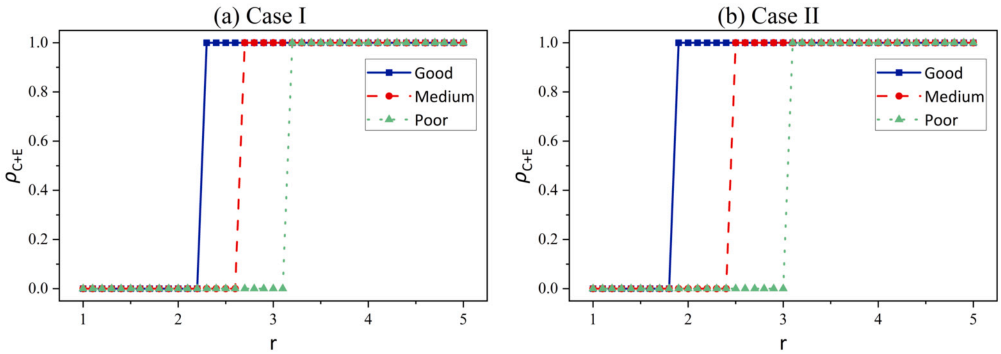

Fig. 7. Comparison curves of the average level of cooperation in the population with r under different exclusion environments: "Good" ($\beta_{max} = 1$, $\theta_{max} = 0.6$), "Medium" ($\beta_{max} = 0.8$, $\theta_{max} = 1.2$), and "Poor" ($\beta_{max} = 0.6$, $\theta_{max} = 2.4$). In panel (a), cooperation emerges at $r = 2.3$, 2.7, and 3.2, respectively. In panel (b), cooperation emerges at $r = 1.9$, 2.5, and 3.1, respectively.

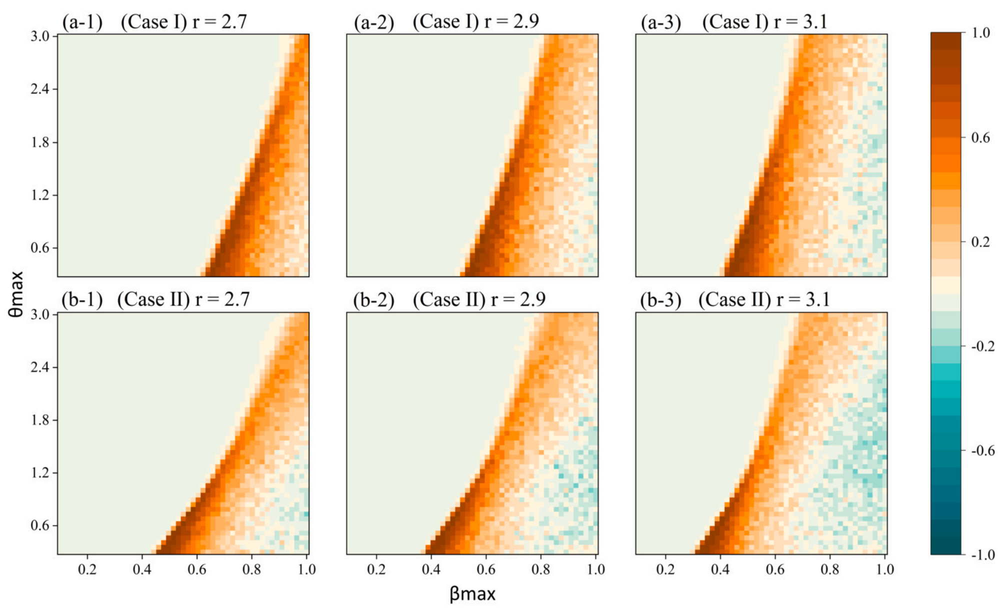

Fig. 8. Heatmaps to illustrate the disparity between E and C. From left to right in both cases, the synergy factor $r = 2.7$, 2.9, and 3.1, respectively. The situation where the number of cooperators exceeds that of excluders only occurs when there are high values of r, high β_{max}, and low θ_{max}.

more severely constrained than Case II. This also means Case II performs better in promoting cooperation. Thus, we can conclusively assert that within the realm of constrained exclusion strategies, the dynamic reduction in exclusion probabilities (Case I) renders the deterrence of defectors more challenging. Conversely, the approach of expending higher costs to prevent defection (Case II) emerges as a more effective strategy in mitigating the detrimental consequences of the "Broken Windows" phenomenon, compared to a strategy that allows defectors to escape more readily (Case I). In this context, the age-old proverb rings true: "It is never too late to mend."

It is worth noting that our current study did not simultaneously integrate both of these constrained strategies into a unified model for comprehensive analysis. Additionally, our research did not account for the potential presence of additional strategies, such as the emergence of punishment strategies, and how these constraints might introduce new dynamics to the game. Therefore, further analysis is necessary. In our future work, we will start with a theoretical investigation of constrained exclusion strategies utilizing mathematical methodologies. We will then explore other research directions, such as expanding the network structure and comparing them in different types of networks, or introducing other mechanisms for network reciprocity or indirect reciprocity. Through these advancements and new mechanisms, we will continue to analyze the effects of constrained exclusion strategies on promoting cooperation under these novel conditions.

CRediT authorship contribution statement

Xingping Sun: Methodology, Investigation, Conceptualization. **Zhiyuan Huang:** Writing – original draft, Formal analysis. **Hongwei Kang:** Writing – review & editing, Funding acquisition. **Zhekang Li:** Methodology. **Yong Shen:** Validation, Funding acquisition. **Qingyi Chen:** Writing – review & editing, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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References

[1] R. Axelrod, W.D. Hamilton, The evolution of cooperation, Science 211 (4489) (1981) 1390–1396.
[2] R. Axelrod, D. Dion, The further evolution of cooperation, Science 242 (4884) (1988) 1385–1390.
[3] M.A. Nowak, A. Sasaki, C. Taylor, D. Fudenberg, Emergence of cooperation and evolutionary stability in finite populations, Nature 428 (6983) (2004) 646–650.
[4] D.G. Rand, M.A. Nowak, Human cooperation, Trends Cogn. Sci. 17 (8) (2013) 413–425.
[5] D.D. Johnson, P. Stopka, S. Knights, The puzzle of human cooperation, Nature 421 (6926) (2003) 911–912.
[6] E. Pennisi, How did cooperative behavior evolve?, Science 309 (5731) (2005) 93.
[7] K. Sigmund, The Calculus of Selfishness, Princeton University Press, 2010.
[8] M.W. Macy, A. Flache, Learning dynamics in social dilemmas, Proc. Natl. Acad. Sci. USA 99 (suppl_3) (2002) 7229–7236.
[9] G. Hardin, The tragedy of the commons: the population problem has, no technical solution; it requires a fundamental extension in morality, Science 162 (3859) (1968) 1243–1248.
[10] H.-J. Li, J. Zhang, Z.-P. Liu, L. Chen, X.-S. Zhang, Identifying overlapping communities in social networks using multi-scale local information expansion, Eur. Phys. J. B 85 (2012) 1–9.
[11] H.-J. Li, L. Wang, Y. Zhang, M. Perc, Optimization of identifiability for efficient community detection, New J. Phys. 22 (6) (2020) 063035.
[12] K. Sigmund, Games of Life: Explorations in Ecology, Evolution and Behavior, Courier Dover Publications, 2017.
[13] M.A. Nowak, Evolutionary Dynamics: Exploring the Equations of Life, Harvard University Press, 2006.
[14] G. Szabó, G. Fath, Evolutionary games on graphs, Phys. Rep. 446 (4–6) (2007) 97–216.
[15] M. Wang, H. Kang, Y. Shen, X. Sun, Q. Chen, The role of alliance cooperation in spatial public goods game, Chaos Solitons Fractals 152 (2021) 111395.
[16] H. Kang, X. Zhou, Y. Shen, X. Sun, Q. Chen, Influencer propagation model promotes cooperation in spatial public goods game, Phys. Lett. A 417 (2021) 127678.
[17] R. Jiménez, H. Lugo, J.A. Cuesta, A. Sánchez, Emergence and resilience of cooperation in the spatial prisoner’s dilemma via a reward mechanism, J. Theor. Biol. 250 (3) (2008) 475–483.
[18] A. Szolnoki, M. Perc, Reward and cooperation in the spatial public goods game, Europhys. Lett. 92 (3) (2010) 38003.
[19] A. Szolnoki, M. Perc, Evolutionary advantages of adaptive rewarding, New J. Phys. 14 (9) (2012) 093016.
[20] A. Szolnoki, M. Perc, Antisocial pool rewarding does not deter public cooperation, Proc. R. Soc. B 282 (1816) (2015) 20151975.
[21] A. Szolnoki, M. Perc, Effectiveness of conditional punishment for the evolution of public cooperation, J. Theor. Biol. 325 (2013) 34–41.
[22] R. Boyd, H. Gintis, S. Bowles, P.J. Richerson, The evolution of altruistic punishment, Proc. Natl. Acad. Sci. USA 100 (6) (2003) 3531–3535.
[23] C. Hauert, A. Traulsen, H. Brandt, M.A. Nowak, K. Sigmund, Via freedom to coercion: the emergence of costly punishment, Science 316 (5833) (2007) 1905–1907.
[24] C. Zhu, S. Sun, L. Wang, S. Ding, J. Wang, C. Xia, Promotion of cooperation due to diversity of players in the spatial public goods game with increasing neighborhood size, Physica A 406 (2014) 145–154.
[25] M. Perc, A. Szolnoki, Self-organization of punishment in structured populations, New J. Phys. 14 (4) (2012) 043013.
[26] A. Szolnoki, G. Szabó, M. Perc, Phase diagrams for the spatial public goods game with pool punishment, Phys. Rev. E 83 (3) (2011) 036101.
[27] D. Helbing, A. Szolnoki, M. Perc, G. Szabó, Punish, but not too hard: how costly punishment spreads in the spatial public goods game, New J. Phys. 12 (8) (2010) 083005.
[28] X. Chen, A. Szolnoki, M. Perc, Competition and cooperation among different punishing strategies in the spatial public goods game, Phys. Rev. E 92 (1) (2015) 012819.
[29] X. Sun, M. Li, H. Kang, Y. Shen, Q. Chen, Combined effect of pure punishment and reward in the public goods game, Appl. Math. Comput. 445 (2023) 127853.
[30] M. Perc, A. Szolnoki, Social diversity and promotion of cooperation in the spatial prisoner’s dilemma game, Phys. Rev. E 77 (1) (2008) 011904.
[31] J. Gao, Z. Li, T. Wu, L. Wang, Diversity of contribution promotes cooperation in public goods games, Physica A 389 (16) (2010) 3166–3171.
[32] X.-B. Cao, W.-B. Du, Z.-H. Rong, The evolutionary public goods game on scale-free networks with heterogeneous investment, Physica A 389 (6) (2010) 1273–1280.
[33] L.-L. Tian, M.-C. Li, K. Lu, X.-W. Zhao, Z. Wang, The influence of age-driven investment on cooperation in spatial public goods games, Chaos Solitons Fractals 54 (2013) 65–70.
[34] J. Quan, Y. Zhou, M. Zhang, C. Tang, X. Wang, The impact of heterogeneous scale return coefficient between groups on the emergence of cooperation in spatial public goods game, J. Stat. Mech. Theory Exp. 2019 (4) (2019) 043402.
[35] R. Fan, Y. Zhang, M. Luo, H. Zhang, Promotion of cooperation induced by heterogeneity of both investment and payoff allocation in spatial public goods game, Physica A 465 (2017) 454–463.
[36] H. Ding, Y. Zhang, Y. Ren, B. Shi, K.-K.R. Choo, Heterogeneous investment in spatial public goods game with mixed strategy, Soft Comput. 22 (2018) 1287–1294.
[37] H.-X. Yang, J. Yang, Reputation-based investment strategy promotes cooperation in public goods games, Physica A 523 (2019) 886–893.
[38] M. Li, H. Kang, X. Sun, Y. Shen, Q. Chen, Replicator dynamics of public goods game with tax-based punishment, Chaos Solitons Fractals 164 (2022) 112747.
[39] Y. Shen, W. Lei, H. Kang, M. Li, X. Sun, Q. Chen, Evolutionary dynamics of public goods game with tax-based rewarding cooperators, Chaos Solitons Fractals 175 (2023) 114030.
[40] K. Sigmund, C. Hauert, M.A. Nowak, Reward and punishment, Proc. Natl. Acad. Sci. USA 98 (19) (2001) 10757–10762.
[41] B. Rockenbach, M. Milinski, The efficient interaction of indirect reciprocity and costly punishment, Nature 444 (7120) (2006) 718–723.
[42] C. Hauert, S. De Monte, J. Hofbauer, K. Sigmund, Volunteering as red queen mechanism for cooperation in public goods games, Science 296 (5570) (2002) 1129–1132.
[43] S. Mathew, R. Boyd, When does optional participation allow the evolution of cooperation?, Proc. R. Soc. B 276 (1659) (2009) 1167–1174.
[44] C. Wang, M. Perc, A. Szolnoki, Evolutionary dynamics of any multiplayer game on regular graphs, Nat. Commun. 15 (1) (2024) 5349.
[45] L. Liu, X. Chen, A. Szolnoki, Evolutionary dynamics of cooperation in a population with probabilistic corrupt enforcers and violators, Math. Models Methods Appl. Sci. 29 (11) (2019) 2127–2149.
[46] S. Wang, X. Chen, Z. Xiao, A. Szolnoki, V.V. Vasconcelos, Optimization of institutional incentives for cooperation in structured populations, J. R. Soc. Interface 20 (199) (2023) 20220653.
[47] T. Sasaki, S. Uchida, The evolution of cooperation by social exclusion, Proc. R. Soc. B 280 (1752) (2013) 20122498.
[48] J. Quan, Z. Pu, X. Wang, Comparison of social exclusion and punishment in promoting cooperation: who should play the leading role?, Chaos Solitons Fractals 151 (2021) 111229.
[49] L. Liu, X. Chen, A. Szolnoki, Competitions between prosocial exclusions and punishments in finite populations, Sci. Rep. 7 (1) (2017) 46634.
[50] K. Li, R. Cong, T. Wu, L. Wang, Social exclusion in finite populations, Phys. Rev. E 91 (4) (2015) 042810.
[51] K. Li, R. Cong, L. Wang, Cooperation induced by random sequential exclusion, Europhys. Lett. 114 (5) (2016) 58001.
[52] J. Quan, J. Zheng, X. Wang, X. Yang, Benefits of asynchronous exclusion for the evolution of cooperation in stochastic evolutionary optional public goods games, Sci. Rep. 9 (1) (2019) 8208.
[53] L. Liu, X. Chen, M. Perc, Evolutionary dynamics of cooperation in the public goods game with pool exclusion strategies, Nonlinear Dyn. 97 (2019) 749–766.
[54] J. Quan, X. Li, X. Wang, The evolution of cooperation in spatial public goods game with conditional peer exclusion, Chaos 29 (10) (2019).
[55] J. Quan, J. Zheng, X. Wang, X. Yang, The effect of increasing returns to scale in public goods investment on threshold values of cooperation under social exclusion mechanism, Physica A 532 (2019) 121866.
[56] J. Quan, W. Yang, X. Li, X. Wang, J.-B. Yang, Social exclusion with dynamic cost on the evolution of cooperation in spatial public goods games, Appl. Math. Comput. 372 (2020) 124994.
[57] S. Lv, X. Wang, The impact of heterogeneous investments on the evolution of cooperation in public goods game with exclusion, Appl. Math. Comput. 372 (2020) 124960.
[58] X. Sun, L. Han, M. Wang, S. Liu, Y. Shen, Social exclusion with antisocial punishment in spatial public goods game, Phys. Lett. A 474 (2023) 128837.
[59] M.R. D’Orsogna, M. Perc, Statistical physics of crime: a review, Phys. Life Rev. 12 (2015) 1–21.
[60] M. Perc, A. Szolnoki, A double-edged sword: benefits and pitfalls of heterogeneous punishment in evolutionary inspection games, Sci. Rep. 5 (1) (2015) 11027.
[61] W.G. Skogan, Disorder and Decline: Crime and the Spiral of Decay in American Neighborhoods, University of California Press, 1992.
[62] G.L. Kelling, J.Q. Wilson, et al., Broken windows, Atl. Mon. 249 (3) (1982) 29–38.
[63] R.J. Bursik Jr, H.G. Grasmick, et al., Neighborhoods & Crime, Lexington Books, 1999.
[64] R.J. Sampson, S.W. Raudenbush, F. Earls, Neighborhoods and violent crime: a multilevel study of collective efficacy, Science 277 (5328) (1997) 918–924.
[65] A. Szolnoki, X. Chen, Alliance formation with exclusion in the spatial public goods game, Phys. Rev. E 95 (5) (2017) 052316.