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Diverse selection intensities resolve the cooperation dilemma induced by breaking the symmetry between interaction and learning

Wei Chen *, Boyu Tao, Sheng Wang, Lin Geng

School of Electrical Engineering and Automation, Anhui University, Hefei, China

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ABSTRACT

Traditionally, the evolution of cooperation on structured population assumed the uniform interaction partner between gaming and learning. Yet in real-world society, individuals often act different roles in which environments gaming partners differ from learning partners. This investigation studies the evolution of cooperation under the effects of the diverse selection intensity induced by network asymmetry on two-layer networks, where the gaming and learning environments are modeled by different layers, respectively. It is found that heterogeneous selection intensity can alleviate the cooperation dilemma induced by asymmetry between gaming and learning environments. When selection intensity has a correlation with the edge overlap level of two layers, it is found that both positive correlation and negative correlation can optimize the evolution of cooperation for a moderate overlap level. However, positive correlation performs better than negative correlation in promoting the evolution of cooperation. Moreover, the increasing heterogeneity of selection enhances the evolution of cooperation under positive correlation, yet has different effects on cooperation under negative correlation for different temptations. Furthermore, we prove that the results are robust to the deterministic learning process as well as a higher noise.

1. Introduction

As a fundamental cornerstone in the natural world and human society, cooperation always faces the threat of social dilemma [1]. In such a dilemma, maximized overall welfare is reached if all individuals cooperate, yet incentives for individuals run counter to group interests. Cooperators yield benefits at a cost to themselves, but free riders without cost enjoy the benefit from cooperators, thus leading to the collapse of cooperation. The way to explain why cooperation emerges in social dilemma appeals to the evolution game theory [2,3]. In the context of evolution game theory, one of the solutions to solve the cooperation dilemma is structured populations [1,4,5]. In the evolutionary games on structured populations, individuals game only with neighbors to reap payoffs and reproduce according to payoffs. The spatial structures of populations imply that some individuals interact more frequently than others, which creates an advantage for cooperators to aggregate.

Recently, great progress has been made in the evolution of cooperation on network reciprocity, not least regular [6,7], random [8–11], small-world [12–14], scale-free [15–23], and adaptive networks [24–35]. However, population structures are often more complex

* Corresponding author.
E-mail address: wwdchen@163.com (W. Chen).

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than the limited case of single networks. Multi-layer networks or interdependent networks are naturally modeled to characterize more real-life situations of coupled systems [36–38]. The interdependency between different networks plays a vital role in the evolution of cooperation. Interconnectedness [39,40], information sharing [41,42], biased imitation [43,44], as well as coupled evolutionary fitness [45–48], to name a few examples have been proven to favor the alleviation of social dilemmas. Particularly, population structures constrain the choices of interacting partners as well as imitation partners [49–53]. Interacting partners thereby form the interaction network where individuals accumulate their payoffs, while imitation partners constitute the updating network where individuals spread their strategies. When interacting network is identical with imitation network, spatial structures favor the evolution of cooperation. Yet breaking the symmetry between interaction and imitation impedes cooperators to evolve, since the defectors can leap over the boundary of cooperative clusters to invade the inside cooperators [49,50].

In the framework of the evolutionary game theory, heterogeneity has been proved to be an important mechanism in the emergence of cooperation more than heterogeneous networks. Santos et al. [54,55] studied how social diversity provides a context for the emergence of cooperation. Chen et al. [56] established an institutional reward and punishment based on network heterogeneity, and found an optimal distribution of institutions for cooperation. In addition, phenotypic diversity is also proved to enhance the evolution of cooperation [57,58]. Here, this study aims to explore how the diversity of selection intensities induced by network asymmetry affects the evolution of cooperation. Selection means the differences in reproductive success. In the evolutionary game theory, selection intensities only impact the time scales instead of the selection direction on infinite well-mixed populations, since the evolution process is a deterministic replicator dynamics. Yet the evolution on finite populations transforms into stochastic dynamics, that may present different selection directions. Weak selection means small fitness differences. By approximately transforming a nonlinear process into a linear process, numerous important insights can be derived analytically under weak selection approximations [59,60]. However, qualitative results under the condition of weak selection can not extend to moderate and strong selection. On structured populations, either weak selection or strong selection results in different dynamics outcomes on homogeneous networks, yet networks with high heterogeneity are more resilient to selection [61]. Here, by abandoning the traditional assumption of uniform selection intensity, this study proposes diverse selection intensities induced by network asymmetry, and we prove that strong selection and diverse fitness can overcome the cooperation dilemma induced by network asymmetry.

2. Model

In the model, a simplified version of the prisoner's dilemma game is considered to characterize the interactions, the key point featuring the conflict of interest between individual and group is preserved yet its strength is only controlled by a single parameter. In particular, mutual cooperation yields a reward $R = 1$, whereas mutual defection causes a punishment $P = 0$. The defection enjoys the temptation $T > 1$ by exploiting cooperation with a sucker's payoff $S = 0$. Thus, the payoff matrix can be rescaled as:

$$\begin{array}{ccc} & C & D \\ C & 1 & 0 \\ D & T & 0 \end{array}$$

The evolution is carried out on two-layer random networks. One acts as an interaction layer where individuals game and reap payoffs, and the other serves as an imitation layer specifying spreading strategies. Each layer has $N = 5000$ nodes with the average degree $\bar{k} = 8$. Since interaction partners can differ from imitation partners, the definition of the overlap level ol between two layers is in accordance with the ratio between the number of overlap edges and total edges. Initially, each individual has an equal probability to be a cooperator (C) or defector (D).

The evolution is processed in accordance with the standard Monte Carlo simulation by synchronous updating. Each individual plays the prisoner's dilemma game with his all interaction partners. After all individuals accumulate their payoffs π, all individuals update their strategies by learning more successful imitation partners synchronously. Specifically, an individual x randomly selects an imitation partner y to learn his strategy with a probability given by the Fermi function:

$$W(s_x \leftarrow s_y) = \frac{1}{1 + exp[(f_x - f_y)/\kappa]} \tag{1}$$

where the noise $\kappa = 0.1$ measures the uncertainty. f_x (f_y) measure the fitness of x (y), given by $f_x = 1 - w_x + w_x \pi_x$. Significantly, the selection intensity w_x can either have a positive correlation with overlap level of x, in which case $w_x = (1 + k_x^{ol}/k_x^I)^\alpha$, or negative correlation, in which case $w_x = (2 - k_x^{ol}/k_x^I)^\alpha$, where k_x^{ol} denotes the number of overlap neighbors adjacent to x and k_x^I is the number of interaction partners of x. When two layers do not coincide, different local overlap levels lead to diverse selection, and the parameter α features the strength of selection. The case $\alpha = 0$ recovers the situation that payoff equals fitness, and the increment of α implies the enhancement of selection as well as its heterogeneity.

We focus on how the diverse selection induced by network asymmetry affects the evolution of cooperation, and evaluate the mean cooperative level ρ_c in a stable state. For the sake of accuracy, the evolution is processed up to 10^5 standard Monte Carlo steps (MCS), and the cooperative levels are obtained over the last 5×10^3 MCS. Moreover, the final results are averaged over 50 independent runs.

3. Results and discussion

Fig. 1 shows the fraction of cooperators as a function of temptation for different strengths of selection. The up panel presents the results of a positive correlation. For $ol = 1$, in which case the interaction network and learning network are identical, all individuals

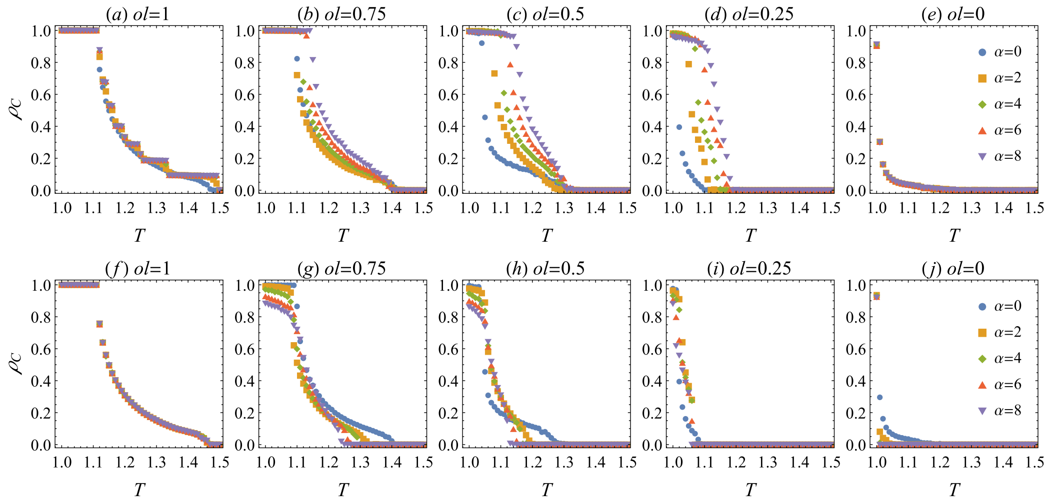

Fig. 1. The evolution of cooperation as a function of temptation T for different α and different overlap levels ol under positive correlation (up panel) $w_x = (1 + k_x^{ol}/k_x^I)^\alpha$ and negative correlation (down panel) $w_x = (2 - k_x^{ol}/k_x^I)^\alpha$.

have uniform selection intensity. In Fig. 1(a), the situation of $\alpha = 0$ recovers to the evolution of the conventional prisoner's dilemma game. The increasing α leads to an almost deterministic reproduction process, indicating that a strong selection effect favors the evolution of cooperation. When two networks partly coincide (Figs. 1(b)-(d)), each individual has diverse overlap levels. The increment of α enhances the heterogeneity of fitness. The individuals with more overlap neighbors have higher fitness under the condition of positive correlation, and the evolution of cooperation is favored with the increment of α. Particularly, for a high overlap level, the increasing α enhances the threshold of temptation for cooperation to dominate the whole population, as well as the temptation for cooperators to survive. If two networks are totally different (Fig. 1(e)), the selection intensity keeps constant as 1 regardless of α. With increasing temptation, the cooperation levels drop rapidly first, then reduce to zero slowly.

In the down panel of Fig. 1, the evolution of cooperation is presented under negative correlation. $ol = 1$ (Fig. 1(f)) leads to the constant selection intensity, and the results regardless of α are just accordance with the case of $\alpha = 0$ in Fig. 1(a). When two networks partly coincide (Figs. 1(g)-(i)), the effects of the selection intensity on the evolution of cooperation rely on the levels of temptations. Both weak and strong temptations lead to the inhibition of cooperation with the increment of α, while a moderate temptation is more likely to favor the evolution of cooperation with increasing heterogeneity of fitness. If two networks do not coincide (Fig. 1(j)), the evolution of cooperation is suppressed with increasing α, and reduces to zero rapidly.

Fig. 2 shows the evolution of cooperation as a function of overlap levels for different temptations. It has been proved that breaking the symmetry between interaction and imitation suppresses the evolution of cooperation, since the defectors can immediately invade cooperators inside the cooperative clusters. This conclusion is just accordance with the observations in Fig. 1. When the heterogeneity of fitness is not very strong, the increment of overlap level promotes the evolution of cooperation regardless of positive correlation (Figs. 2(a)-(c)) and negative correlation (Figs. 2(e)-(g)). However, under the condition of extremely heterogeneous fitness, the evolution of cooperation is best favored for a moderate overlap level particularly for a small temptation under positive correlation (Fig. 2(d)). Yet for negative correlation (Fig. 2(h)), both too small and too high temptation lead to the monotonous increment of cooperation as the function of overlap level. For a moderate temptation, the emergence of optimum cooperation levels needs a moderate overlap level. As a consequence, breaking the symmetry between interaction and imitation can facilitate the evolution of cooperation under high α, and the positive correlation has more relaxed conditions to optimize the evolution of cooperation than the negative correlation.

To further understand why positive correlation promotes the evolution of cooperation, the time evolution of cooperation is shown in Fig. 3. When two networks are completely identical (Fig. 3(a)), cooperators can rapidly stabilize around 0.68, while the fraction of the cooperators (C_I) along the interface of the cooperative clusters on the learning network is around 0.59, thus cooperators inside the clusters are less abundant. It indicates that cooperative clusters are large in number but small in scale. In addition, C_I can overcome defectors (D_I) along the interface of the cooperative clusters for the vast income gap between two types of individuals (Fig. 3(e)). For a moderate overlap level (Fig. 3(b)), the heterogeneity of selection induced by asymmetric networks leads to the vast income gap between individuals with different overlap levels. Particularly, the cooperators with high overlap levels have an advantage over defectors with low overlap levels on fitness, thus leading to the domination of cooperation. In Fig. 3(f), the mean income of C_I overcomes that of D_I at the outset of the evolution. However, with the decrement of C_I and D_I, D_I have more favorable conditions to exploit cooperators, thus the average payoff of D_I can be higher than C_I temporarily. Yet the expansion of defectors makes defectors lose the payoff advantage, and the optimal site for defectors to exploit cooperators may also be occupied by cooperators, thus cooperators can dominate the whole population. When the overlap level is low (Fig. 3(c)), defectors have more ways to invade cooperative clusters, and thus the evolution of cooperation needs more time to be stabilized. The decrement of overlap level can improve the heterogeneity of selection, but also reduces the income difference (Fig. 3(g)). Low overlap levels and fitness

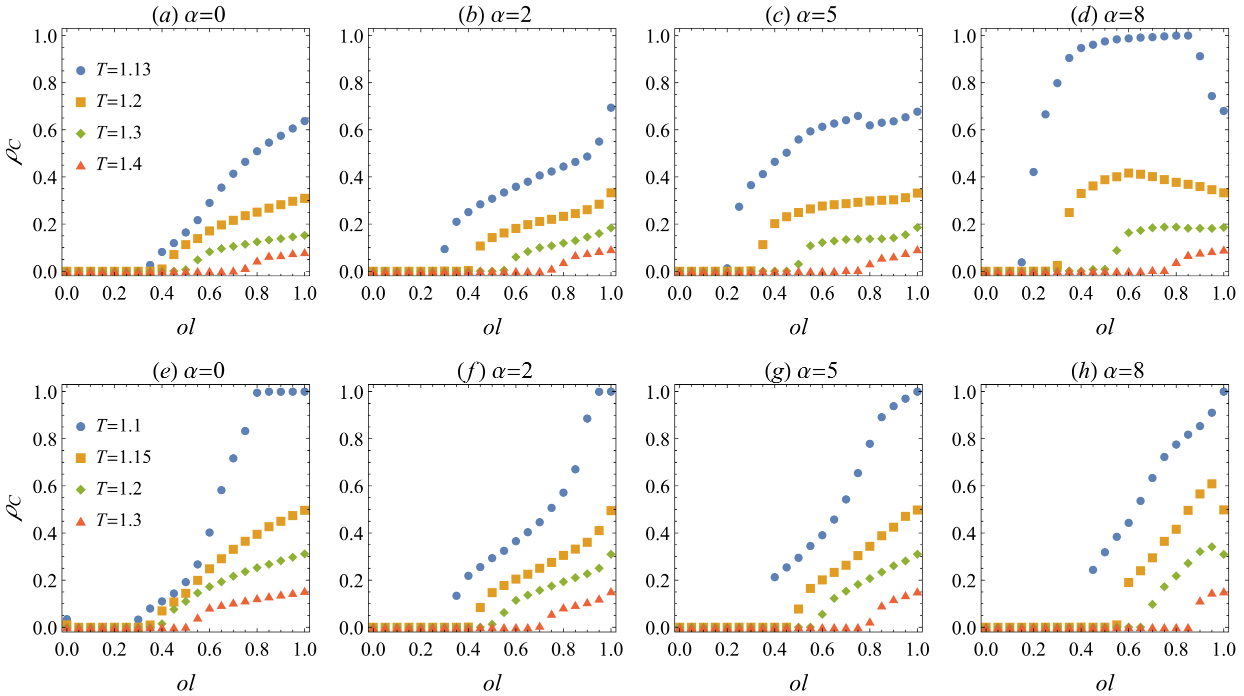

Fig. 2. The evolution of cooperation as a function of overlap levels ol for different α and different temptation T under positive correlation (up panel) and negative correlation (down panel).

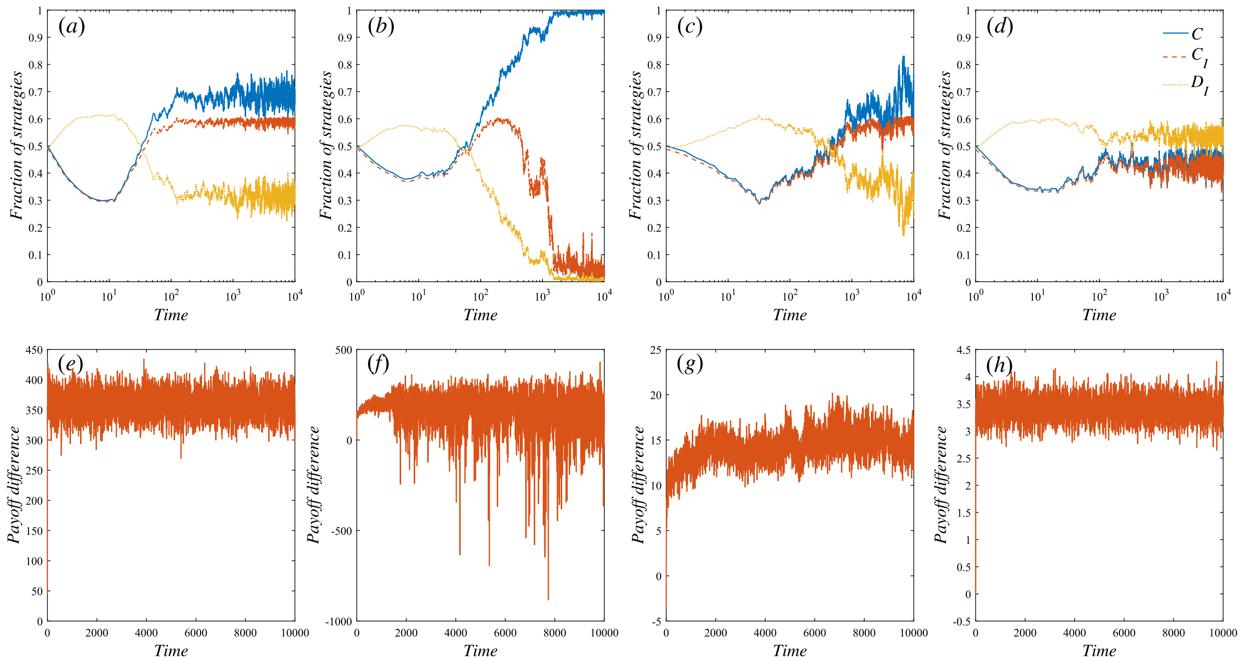

Fig. 3. Time evolution of strategies and payoff difference under positive correlation with $T = 1.13$ for parameters: (a) and (e) $ol = 1, \alpha = 8$, (b) and (f) $ol = 0.75, \alpha = 8$, (c) and (g) $ol = 0.25, \alpha = 8$, (d) and (h) $ol = 0.75, \alpha = 2$. The up panel shows the evolution of cooperators (C), cooperators (C_I) adjacent to defectors on learning layer and defector (C_I) adjacent to cooperators on learning layer. The down panel shows the evolution of mean payoff difference between C_I and D_I.

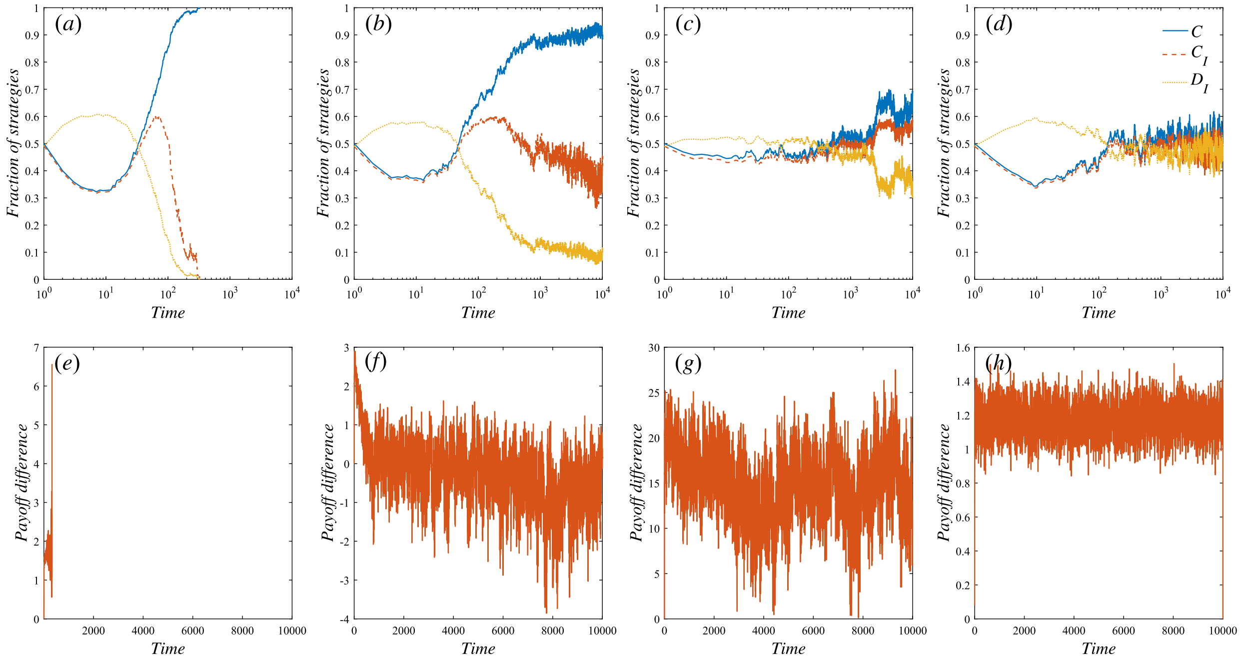

Fig. 4. Time evolution of strategies and payoff difference under negative correlation with $T = 1.08$ for parameters: (a) and (e) $ol = 1, \alpha = 8$, (b) and (f) $ol = 0.95, \alpha = 8$, (c) and (g) $ol = 0.6, \alpha = 8$, (d) and (h) $ol = 0.6, \alpha = 1$. The up panel shows the evolution of cooperators (C), cooperators (C_I) adjacent to defectors on learning layer and defectors (C_I) adjacent to cooperators on learning layer. The down panel shows the evolution of mean payoff difference between C_I and D_I.

differences are unfavorable for the evolution of cooperation. Figs. 3(d) and (h) show the results under low strength of selection $\alpha = 2$. Low strength of selection leads to the low fitness difference, which goes against the expansion of cooperators. Fig. 3(d) also shows that the amount of C_I approaches the sum of cooperators, indicating that cooperators hardly form large-scale clusters.

Under the impact of negative correlation, individuals with lower overlap levels have higher selection intensity. Fig. 4 shows the evolution of cooperation with time for small temptation $T = 1.08$. On the completely symmetrical networks (Fig. 4(a)), cooperators can rapidly expand to occupy the whole population. The evolution of payoff difference ends with the domination of cooperation. When two networks are slightly asymmetry, the mean payoff difference oscillates around the interval of $[-2, 1]$, and the cooperation levels stabilize around 0.91. Compared with the results shown in Figs. 4(a) and (b), it is found that negative correlation fails to enhance the payoff advantage with the increment of overlap levels. For a small temptation, the heterogeneity of fitness fails to weigh the effects of symmetrical networks, and thus the evolution of cooperation is suppressed. For a higher temptation $T = 1.15$, it is found that the evolution of cooperation is best favored for a moderate overlap level and a strong selection intensity. Particularly, Fig. 5(b) and (f) show the persistence of larger cooperative clusters and the higher mean payoff difference when two networks are slightly asymmetrical. However, the decreasing overlap levels make cooperators inside the clusters more easily to exchange strategies. Slight asymmetry of two networks results in the payoff advantage of C_I, the heterogeneity of fitness overcomes the effects of symmetrical networks to optimize the evolution of cooperation, yet too asymmetrical networks suppress the evolution of cooperation regardless of heterogeneous selection. These are also held by Figs. 6(a)-(c), where temptation is even higher.

However, the impact of selection intensity is more complex for negative correlation. In most cases, the increment of α suppresses the evolution of cooperation, yet a moderate temptation is more likely to favor the evolution of cooperation with increasing α. Fig. 4(d), Fig. 5(d), and Fig. 6(d) show the time evolution of cooperators, and present a similar microcosmic process that cooperators persist in small-scale clusters. These phenomena are induced by the combination of multiple factors. On the one hand, breaking the symmetry between interaction and imitation inhibits the evolution of cooperation, indicating that individuals with higher overlap levels have advantages in maintaining cooperation. For positive correlation, the fitness of individuals with higher overlap is enhanced, thus the increasing α promotes the evolution of cooperation. Yet for negative correlation, enhancing the fitness of individuals with lower overlap fails to provide the environment for cooperation to spread. On the other hand, the asymmetry between interaction and imitation leads to diverse fitness of individuals. Increasing α enhances the heterogeneity of fitness, and the enhancement of heterogeneity promotes the evolution of cooperation. Thus, the increment of α leads to different evolutionary outcomes for negative correlation.

To test the robustness of our results, both the cases of zero noise and higher noise ($\kappa = 1$) are considered here. The results in Fig. 7 are just accordance with the previous observations. The increasing α favors the evolution of cooperation under positive correlation. Yet for negative correlation, a small α can enhance the resilience of cooperation for both high and low temptation. In addition, a moderate overlap level also enhances the evolution of cooperation regardless of noise.

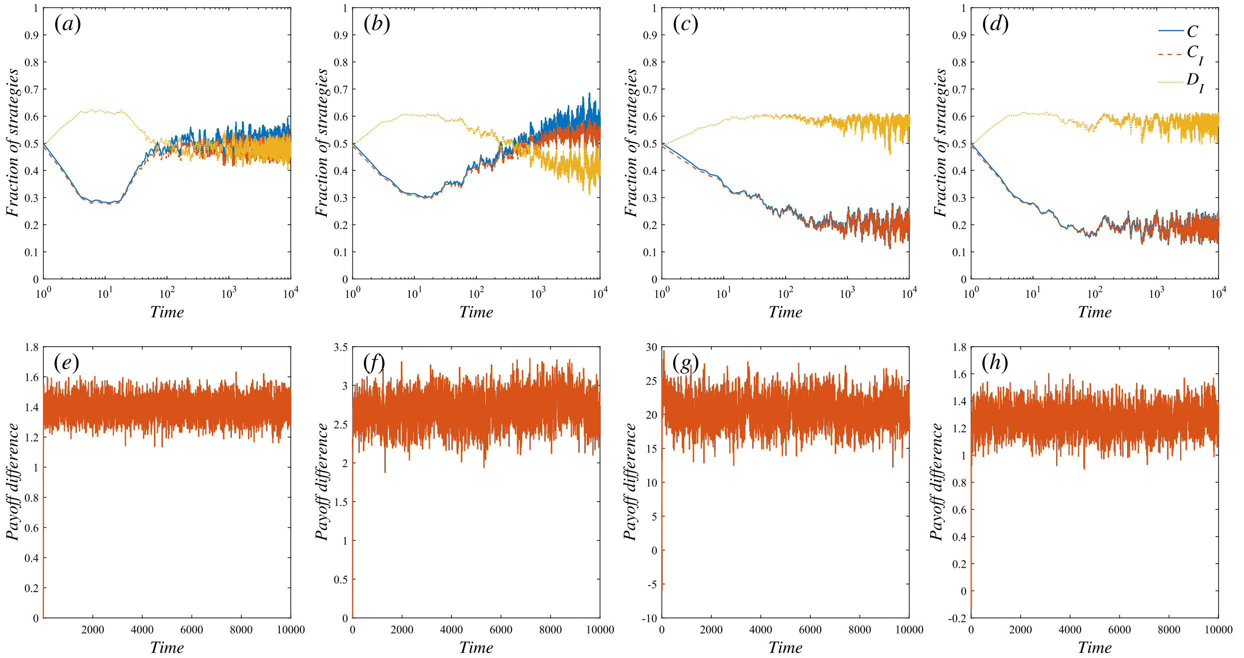

Fig. 5. Time evolution of strategies and payoff difference under negative correlation with $T = 1.15$ for parameters: (a) and (e) $ol = 1, \alpha = 8$, (b) and (f) $ol = 0.95, \alpha = 8$, (c) and (g) $ol = 0.6, \alpha = 8$, (d) and (h) $ol = 0.6, \alpha = 1$. The up panel shows the evolution of cooperators (C), cooperators (C_I) adjacent to defectors on learning layer and defectors (C_I) adjacent to cooperators on learning layer. The down panel shows the evolution of mean payoff difference between C_I and D_I.

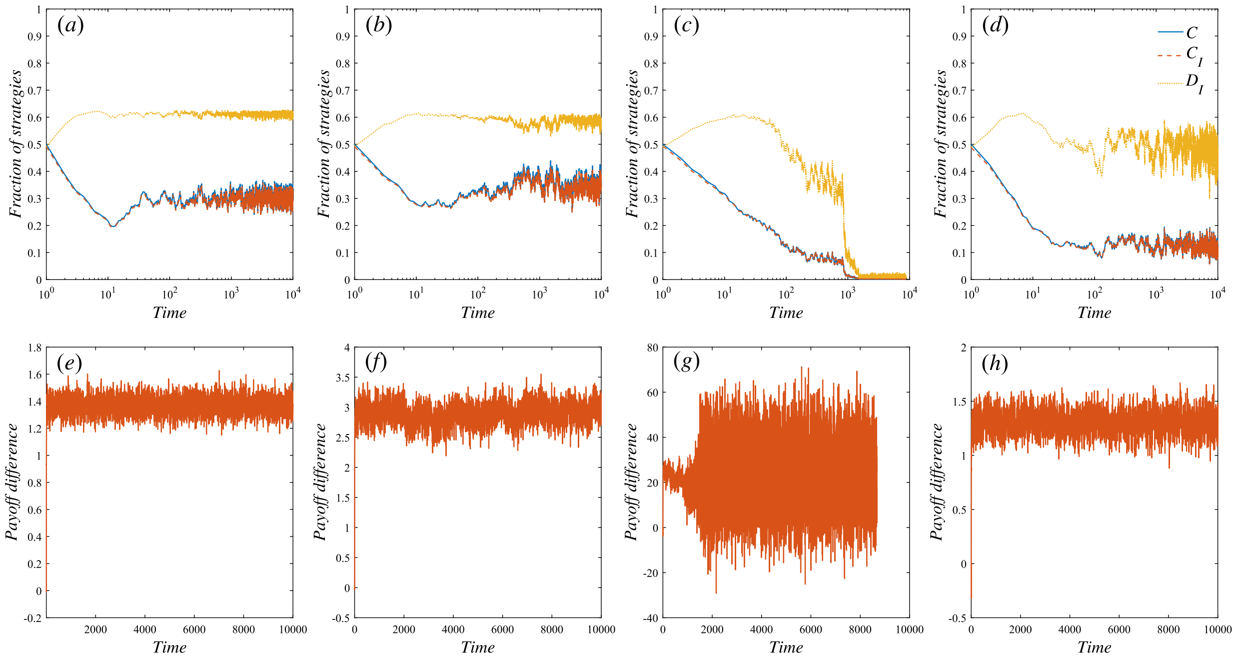

Fig. 6. Time evolution of strategies and payoff difference under negative correlation with $T = 1.2$ for parameters: (a) and (e) $ol = 1, \alpha = 8$, (b) and (f) $ol = 0.95, \alpha = 8$, (c) and (g) $ol = 0.6, \alpha = 8$, (d) and (h) $ol = 0.6, \alpha = 1$. The up panel shows the evolution of cooperators (C), cooperators (C_I) adjacent to defectors on learning layer and defectors (C_I) adjacent to cooperators on learning layer. The down panel shows the evolution of mean payoff difference between C_I and D_I.

4. Discussion

The setting of asymmetric interaction and learning is a universal phenomenon in human society, since humans can receive a great deal of information without direct interaction from such as internet, television, paper media, and even gossip. Yet the mechanisms supporting cooperation to evolve on the asymmetric networks are still under exploration. We thus explore whether there are any

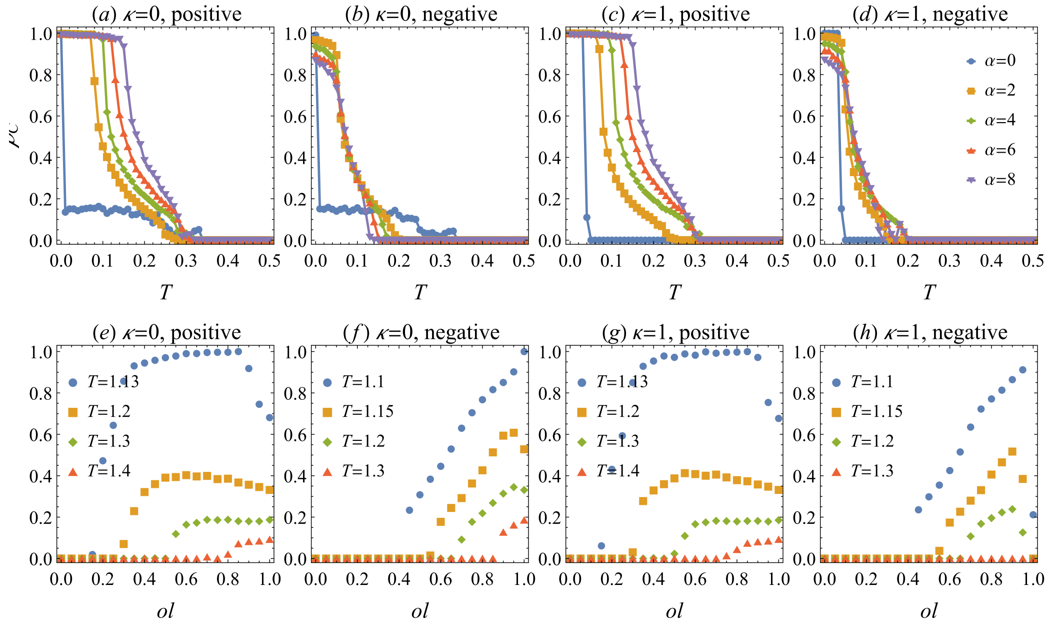

Fig. 7. The evolution of cooperation under the condition of zero noise and $\kappa = 1$. The up panel shows the evolution of cooperation as a function of temptation T for different α with overlap level $ol = 0.5$. The down panel shows the evolution of cooperation as a function of overlap levels ol for $\alpha = 8$ with different temptation T.

rules to overcome the spatial disadvantage of asymmetric networks in favoring the evolution of cooperation. Previous investigations have proved that breaking the symmetry between interaction and imitation impedes cooperators to evolve [49–51]. For instance, Wang et al. [53] proves that degree mixing fails to resolve the cooperation dilemma induced by asymmetry networks. To explore the mechanism favoring the evolution of cooperation, Xia et al. [62] found that properly enlarging the range of interaction favors cooperators to spread. Chen et al. [63] found that adaptive networks can alleviate the social dilemma induced by network asymmetry. In the seminal work of Su et al. [64], an analytical model is established to predict the evolution of cooperation when gaming objects are partly overlapped with learning objects, and found that when a player interacts more frequently with a next nearest than with a nearest neighbor, asymmetric interaction could provide more advantages for cooperators to evolve.

Here, this investigation studied the effects of heterogeneous selection intensity induced by network asymmetry on the evolution of cooperation based on asymmetric gaming and learning environments. Two types of correlation are proposed including positive correlation and negative correlation, where the former case assumes that selection intensity increases monotonously with overlap levels, and the latter case assumes the opposite situation. It is found that the evolution of cooperation is best favored whenever gaming and learning environments are properly asymmetric for both positive correlation and negative correlation. However, positive correlation provides more relaxed conditions for cooperation to evolve. In addition, the increasing heterogeneity of selection intensity favors the evolution of cooperation for positive correlation. Yet under the effects of negative correlation, the increasing heterogeneity of selection intensity suppresses the evolution of cooperation, except for the conditions of moderate overlap levels and moderate temptations. Our results are also robust against different noises. Overall, we hope our investigation does help to understand how cooperation evolves in the asymmetric gaming environment and learning environment, and may contribute to understanding the evolution of cooperation under network reciprocity.

Data availability

The data that has been used is confidential.

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