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Reputation-based disconnection-reconnection mechanism in Prisoner's Dilemma Game within dynamic networks

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A R T I C L E I N F O

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A B S T R A C T

Individual reputation significantly influences players' decision-making. Players tend to disconnect from neighbors with lower reputations so as to mitigate potential losses in the game. Therefore, it is crucial and meaningful to explore how to apply reputation in the game to enhance cooperation. In this study, we propose a reputation-based evolution mechanism in the Prisoner's Dilemma Game (PDG) within dynamic complex networks, including square lattices and small-world networks. We introduce the reputation factor α and the selection intensity σ in order to measure the changes in reputation and their impact objectively. Moreover, considering the impact of information connectivity, we define the environmental degree r and investigate the effects of environmental degree r and rewiring probability p on cooperation rates. Simulation results demonstrate that the reputation factor and information connectivity have a positive promoting effect on cooperative behavior; conversely, the increase in selection intensity affects cooperation negatively.

1. Introduction

In concert with Darwin's theory of natural selection, individuals are inherently self-interested. Within the context of strategic interactions, they are inclined to pursue a defection strategy as a means of optimizing personal outcomes. However, cooperative behaviors are pervasive across numerous domains, such as the nature world, behavioral economics, sociological studies [1–3], and so on. To account for this paradox, researchers have developed the framework of evolutionary game theory, which provides insights into the emergence and stability of cooperative phenomena in evolutionary processes.

In recent years, evolutionary games have gradually become a popular research topic, encompassing many classic game models such as the Prisoner's Dilemma Game (PDG) [4], Public Goods Game (PGG) [5], and Snowdrift Game (SDG) [6]. Nowak [7] proposed five relevant rules, including kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection, to investigate issues related to cooperation. Among these rules, the network reciprocity mechanism holds a crucial position. This mechanism regards players as nodes, and the players with gaming relationships are connected by edges, forming a network structure among the gaming groups. The spatial prisoner's dilemma theory proposed by Nowak et al. [8] represents the initial form of grid network games, placing players within a two-dimensional spatial grid and defining rules for game interactions between players and their neighbors each round. Despite the simplicity of this spatial structure, it effectively promotes the emergence and maintenance of collective cooperation in PDG. Szabó et al. [9] investigated the impact of inhomogeneous strategy transfer capabilities on cooperative behavior within the

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spatial PDG on a square lattice. With the development of complex network theory, networks with more intricate interaction structures have been considered. Abramson and Kuperman [10] incorporated the small-world network into the PDG framework to investigate how varying topologies, spanning from square lattices to random graphs, impact player strategies. Hauert and Szabó [11] developed regular small-world networks and random regular graphs based on the square lattices structure. They observed that, because of the increased long-range connections, these two types of networks are more effective in fostering cooperative behavior compared with the square lattices.

In addition, researchers have proposed various incentive mechanisms for evolutionary games, encompassing mechanisms such as exclusion [12,13], rewards [14–16], punishment [17–19], taxation [20], teaching activities [21], memory effects [22,23], emotions [24–26], and reputation [27–37]. These mechanisms serve to facilitate cooperation among participants and mitigate the temptation of betrayal. Notably, reputation emerges as an effective and widely embraced mechanism for fostering the evolution of cooperative behavior. Nowak et al. [27] introduced a straightforward binary reputation mechanism coupled with a first-order assessment rule, delineating the conditions for evolutionary stability in the context of indirect reciprocity. This necessitates that the likelihood of players accurately perceiving the 'image' of their counterparts must surpass the ratio of cost to benefit associated with altruistic actions. Subsequently, Nowak and Sigmund [28] integrated this binary reputation mechanism with a third-order assessment rule, thereby constructing a strategic evolutionary model that operates under indirect reciprocity. Utilizing experimental methods, they explored the influence of indirect reciprocity on cooperative engagement and the attendant conditions. Considering the pivotal role of trust in sustaining social links, Li et al. [34] examined how second-order reputation evaluation impacts the trust game dilemma and how players adjust their reputation to establish an indirect reciprocal relationship. Furthermore, Feng et al. [35] investigated the effect of network topology on the N-player trust game by considering the second-order reputation rule, offering new insights into the evolution of cooperation or trust in structured groups. Moreover, in an effort to inspire a greater number of defectors to adopt cooperative strategies and to solidify the participants' commitment to long-term cooperation, Bi and Yang [32] introduced a novel reputation-based mechanism within the PDG. This mechanism incrementally elevates the reputation of cooperators while concurrently diminishing the reputation of defectors, and also resets the reputation of players who alter their strategic choices. When contrasted with prevailing reputation mechanisms, this approach has been demonstrated to be more effective in fostering cooperation.

The majority of the aforementioned studies assume that connections between players are fixed, implying that players consistently interact with the same neighbors in games. However, in reality, players adjust their strategies according to their surrounding environment and decide whether to maintain connections with their neighbors based on the strategies those neighbors employ. Typically, players prefer to continue interactions with cooperative individuals while breaking ties with defectors. Consequently, dynamic networks, in contrast to static networks, represent the interactive behaviors of human or other biological populations in real-life scenarios more accurately. Fehl et al. [38] compared the cooperative behavior of participants in static and dynamic networks across multiple, yet independent, repeated games. They discovered that the frequent alterations in dynamic networks can enhance cooperation. Pan et al. [39] investigated the impact of spatially adaptive interactions on the evolution of cooperation within the PDG. They found that if a player defects in one round, their neighbors, based on a predetermined probability distribution, might opt not to engage in further games with that player. The study demonstrated that an increased likelihood of such neighborly rejection significantly improves the cooperators' ability to resist escalating temptations, thus enhancing the overall level of cooperation. Du and Wu [40] investigated the coevolution of individual strategies and network structure by introducing a linking dynamics and incorporating a public mechanism during network rewiring. In the PGG, Wang et al. [41] posited that payoff differences serve as the driving force behind the evolution of social networks. They hypothesized that players are likely to disconnect from neighbors with similar earnings and seek to connect with new players with different earnings. This research elucidates how human societies can achieve cooperation even when individuals are pursuing disparate rewards.

In this paper, we propose a reputation-based evolutionary mechanism within the context of the PDG in dynamic complex networks, such as square lattices and small-world networks. Specifically, we suggest that players' strategies will influence their reputation, which in turn affects the probability of establishing connections with neighbors. To quantify the impact of players' strategies on their reputation, we introduce the reputation factor α. In addition, we assume that the discrepancy between players' reputation and the environmental reputation influences their likelihood of connecting with neighbors, and we incorporate the selection intensity factor σ to measure this influence. This assumption shifts the level of a player's reputation from a traditional absolute value to a relative value by taking into account the external environment, making our mechanism more in line with reality. Furthermore, considering the impact of information connectivity, we define the environmental degree r and investigate how variations in both the environmental degree r and the rewiring probability p impact cooperation rates in different complex networks. Observing the results, we find that cooperation is superior in small-world networks compared to square lattices. Given that small-world networks are closer to real life, this finding suggests that the mechanism is able to address, to some extent, the social dilemma of individual interests (defection) conflicting with group interests (cooperation). In summary, our innovations are as follows: i) We introduce the selection intensity factor σ when studying the impact of reputation; ii) We design a disconnection-reconnection mechanism that aligns more closely with real-world scenarios; iii) We conduct a comparative analysis between the square lattice network and the small-world network, yielding insightful findings.

The rest of this paper is organized as follows: In Section 2, we provide the background information on the model. We offer a detailed analysis through data simulation in Section 3, and finally draw the conclusions based on the obtained results in Section 4.

2. Model

In this section, we consider the PDG. In each simulation's initial stage, $N = L^2$ ($L = 100$) players are embedded within an $L \times L$

square lattice network (SL) or a small-world network (WS). In the square lattice network, each player is represented by a node, and each node has four neighbors. To allow for evolutionary dynamics, the square lattice is designed with periodic boundary conditions, which means that the nodes at the top and bottom boundaries are considered as neighbors, as are those at the left and right boundaries. This ensures that the network is fully connected. Similarly, the small-world network also employs periodic boundary conditions, ensuring the boundaries of the network are interconnected. However, unlike the square lattice, the small-world network features a variable number and arrangement of neighbors for each node. This results in a significantly reduced average path length between nodes, making the small-world network more akin to real-world social structures.

Each individual in the system is randomly assigned an initial strategy, which can be either cooperation (C) or defection (D). When both players choose a cooperative strategy, they each receive a reward (R). Conversely, if they both defect, they face a punishment (P). In the scenario where a cooperator interacts with a defector, the cooperator receives a sucker's payoff (S), whereas the defector obtains the temptation payoff (T). If these four parameters satisfy $T > R > P > S$, the game is a PDG. To simplify the analysis and without loss of generality, we set $R = 1, T = b, S = 0, P = 0,$ thereby creating a weak Prisoner's Dilemma. The corresponding payoff matrix is expressed as

$$M = \begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{1}$$

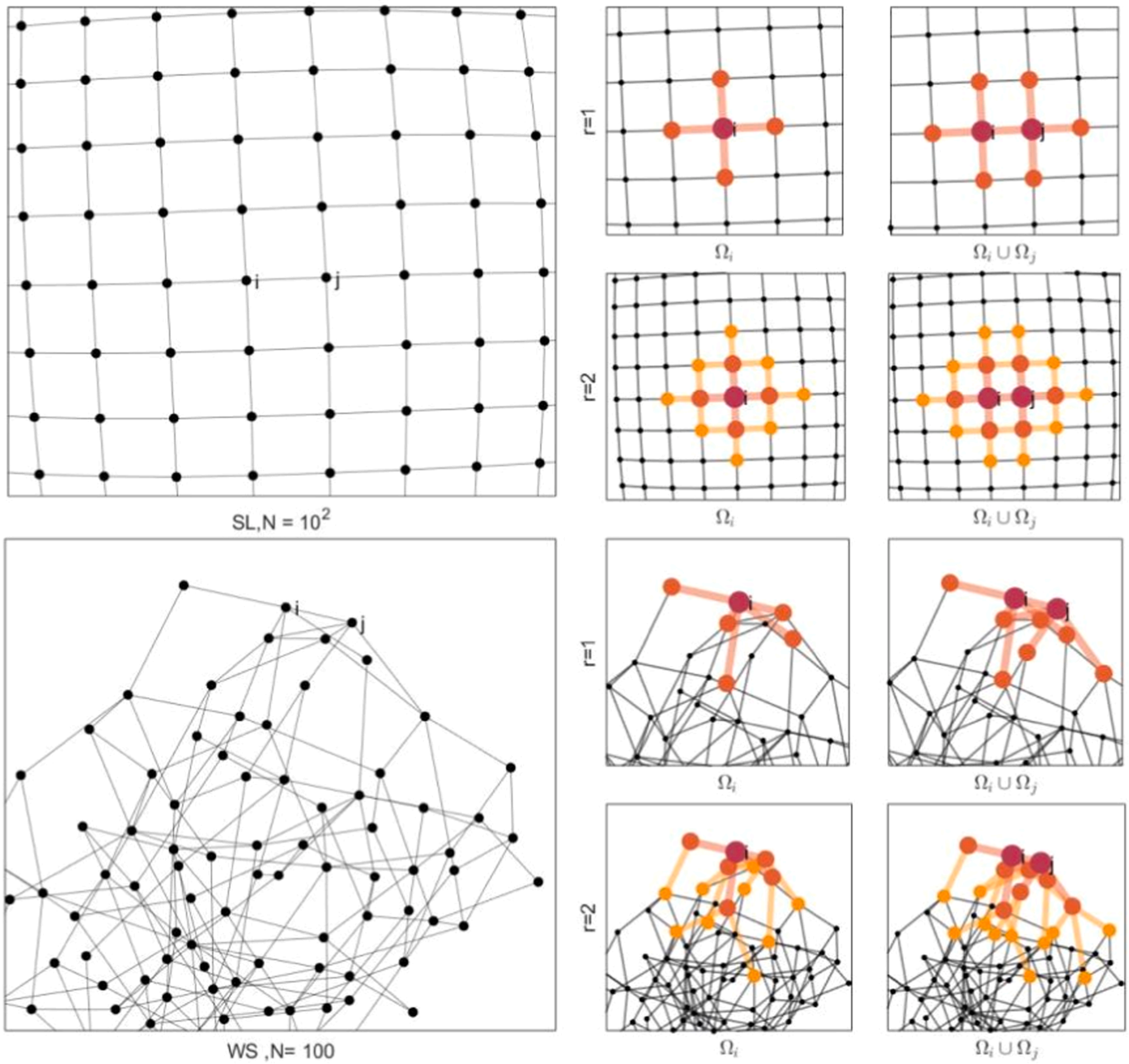


Fig. 1. The r_{th}-degree environment ($r = 1, 2$) of players i and j in the square lattice and the small-world network where $N = 100$.

where the range of b is $1 \leq b \leq 2$.

According to [32], we assume that at the outset, the reputation of player i, denoted as $R_i(0)$, is initialized to 0. At the t-th time step ($t \geq 1$), if player i opts to cooperate, the value of its reputation will increase α, i.e., $R_i(t) = R_i(t-1) + \alpha$; conversely, if it chooses to betray, the value of its reputation will decrease α, i.e., $R_i(t) = R_i(t-1) - \alpha$, where α is the reputation factor.

To ensure fairness and maintain a general framework, we introduce a reputation upper bound, denoted as H. When $R_i(t) \geq H$, we set $R_i(t) = H$; conversely, when $R_i(t) \leq -H$, we set $R_i(t) = -H$. Considering the inherently bounded nature of reputation, we set $H = 20$, unless specified otherwise. In this study, players are influenced by their own reputation as well as the reputations of their neighbors. Consequently, players may choose to either connect or disconnect from their neighbors with a certain probability. When the reputations of both players in the game are lower than the reputation of the surrounding environment, indicating a collectively poor reputation, the probability of choosing to disconnect (and not participate in the game) increases. On the other hand, if both players have reputations that exceed the environmental reputation, indicating a collectively positive reputation, the probability of connecting (and engaging in the game) is higher. To elucidate the importance of environmental reputation, we first provide definitions for the graph structure and the environment.

We consider the network composed of N players as an unweighted undirected graph $G(V,E)$, where V denotes the set of nodes corresponding to all players, and E represents the set of edges that connect pairs of players who have the potential to engage in the game. For example, let (x_i, y_i), where $x_i, y_i \in \{1, 2, \cdots, L\}$, denote the coordinates of player $i \in V$ on an $L \times L$ square lattice. Consequently, the graph $G(V, E)$ satisfies $|V| = N$, representing the total number of players. The set of edges is defined as $E = \{(i,j)|(x_i - x_j)(y_i - y_j) = 0, \max\{|x_i - x_j|, |y_i - y_j|\} \equiv 1 \ (\mathrm{mod} L)\}$, which constructs a square lattice graph comprising $N = L^2$ players, where mod denotes the modulo operator.

We define the r_{th}-degree environment Ω_i^r of the player $i \in V$ as:

$$\Omega_i^r = \{i' \in V | d_G(i, i') \leq r\}, \quad r = 1, \quad 2, \quad \cdots, \tag{2}$$

where $d_G(i, i')$ represents the shortest path between i and i' in the graph $G(V,E)$. Since we treat both the square lattice and the small-world network as fully connected graphs, any node i' in V can be considered as a potential neighbor.

To visually clarify the aforementioned definition, we present the r_{th}-degree environment of players i and j within both the square lattice and the small-world network in Fig. 1. For the subgraph with $r = 1$ in the square lattice part, the left subgraph features player i, indicated by red dots, and its neighbors with distance 1, represented by orange dots. The right subgraph plots players i and j and their neighbors with distance 1 accordingly; while the subgraph with $r = 2$ also plots their neighbors with distance 2 with yellow dots in addition to the above points. The small-world network part is identical to the square lattice.

Therefore, we can define the r_{th}-degree environmental reputation $\overline{R_{ij}^r}(t)$ of players i and j $(i,j \in V)$, which satisfies $d_G(i,j) \leq r$, at time t as follows:

$$\overline{R_{ij}^r}(t) = \frac{\sum_{i' \in \Omega_i^r \cup \Omega_j^r} R_{i'}(t)}{\left|\Omega_i^r \cup \Omega_j^r\right|}, \tag{3}$$

where $|\Omega_i^r \cup \Omega_j^r|$ represents the number of players included in the r_{th}-degree environmental reputation of players i and j.

After establishing the aforementioned definition, we can assess the probability of a disconnection or connection between players i and j. Specifically, when the average reputation of the two players is equal to the environmental reputation, i.e., $\frac{R_i(t)+R_j(t)}{2} = \overline{R_{ij}^r}(t)$, the probability of a successful connection is 0.5. If $\frac{R_i(t)+R_j(t)}{2} < \overline{R_{ij}^r}(t)$, the probability of a successful connection is less than 0.5, whereas if $\frac{R_i(t)+R_j(t)}{2} > \overline{R_{ij}^r}(t)$, the probability exceeds 0.5. In essence, the probability of a successful connection between players relies not only on their individual reputations but also on the reputation of their surrounding neighbors. Consequently, the greater the average reputation of two players surpasses the environmental reputation, the higher the probability of a successful connection.

We can calculate the probability of a successful connection between players i and j $(i \neq j)$ using the following formula:

$$P_{ij}(t) = \frac{1}{2}\left[1 + erf\left(\frac{\left(R_i(t) + R_j(t)\right)/2 - \overline{R_{ij}^r}(t)}{\sigma\sqrt{2}}\right)\right]. \tag{4}$$

Here, $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$. In particular, when player i has no neighbors, there is $P_{ii}(t) = 0$. The selection intensity σ represents the standard deviation of a normal distribution and reflects the impact of the reputations of players i and j, as well as their environmental reputations, on the probability of a successful connection between them. When $\sigma \to 0$, if the average reputation of players i and j is less than the environmental reputation, then $P_{ij}(t) \to 0$; conversely, if the average reputation is greater, $P_{ij}(t) \to 1$. This demonstrates the significant influence of both individual and environmental reputations on the connection. However, when $\sigma \to \infty$, $P_{ij}(t) \to 0.5$, indicating that the connection between players becomes entirely random, and both individual and environmental reputations lose their impact.

The sum of the payoffs obtained by player i from all successfully connected neighbors during the t-th time step constitutes its payoff for that time step. Let $\pi_i(t)$ represent this payoff, such that

$$\pi_i(t) = \sum_{j \in N_i(t)} C_{ij}(t), \tag{5}$$

where $N_i(t)$ represents the set of all neighbors successfully connected to player i at the t-th time step, and $C_{ij}(t)$ represents the payoff obtained by player i from playing the game with its neighbor j. To achieve higher payoffs, players continuously refine their strategies, which may involve learning from their neighbors. In this model, we employ the Fermi function as the strategy updating rule. Specifically, the probability that player i learns the strategy of neighbor j is

$$W_{j \to i}(t) = \frac{1}{1 + e^{\frac{\pi_i(t) - \pi_j(t)}{k}}}, \tag{6}$$

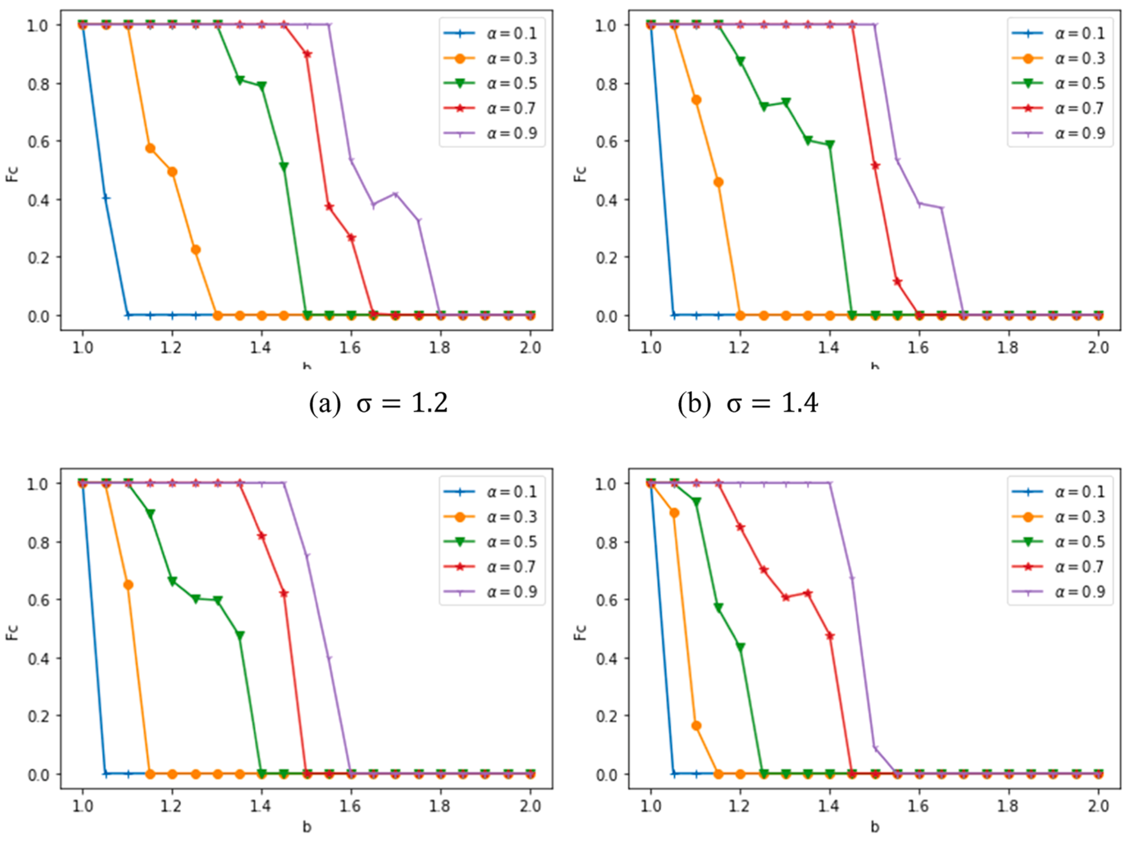
where k denotes noise intensity. When $k \to 0$, player i becomes entirely rational and prefers to learn from neighbors with the highest payoffs. Conversely, as $k \to \infty$, with $W_{j \to i}(t) \to 0.5$, it indicates that player i's strategy selection is entirely random.

3. Simulation results and discussion

In this section, we carry out simulation experiments to investigate the dynamics of cooperation in our proposed mechanism. To ensure the precision of our simulation results, the evolutionary process was allowed to iterate over 10^5 time steps.

3.1. The analysis of reputation factor α and selection intensity σ

Here, we primarily concentrate on three key parameters: α, σ, and b, which respectively represent the influence of player strategies on reputation, the effect of both individual and environmental reputation on the probability of forming connections, and the magnitude of the temptation to betray. Although our model is applicable to both types of complex networks, computational complexity considerations lead us to focus primarily on the square lattice. The insights provided in this section are equally applicable to small-



(a) σ = 1.2 (b) σ = 1.4

(c) σ = 1.6 (d) σ = 2.0

Fig. 2. The relationship between the frequency of cooperator F_c and the temptation factor b for various reputation factors α under different selection intensities σ.

world networks.

3.1.1. The effect of α for given σ

Fig. 2 illustrates the relationship between the cooperation frequency (F_c) and the temptation to betray (b) for varying reputation factors (α), under four different selection intensities (σ). As depicted in Fig. 2(a), a higher reputation factor (α) corresponds to a slower overall rate of betrayal in the population. This indicates that the cooperation frequency is influenced by individual assessments; specifically, an increased sensitivity to individual betrayal within the population can effectively enhance the overall cooperation rate. By examining the four plots in Fig. 2 collectively, it becomes visually evident that with an increase in σ, the decline in cooperation rates for all curves becomes more pronounced as the temptation to betray rises. In other words, lower selection intensities are conducive to the maintenance of group cooperation.

During the evolutionary process, snapshot images offer a visual representation of individual strategy shifts within the system. To elucidate this, we captured the microevolution of player strategies and their corresponding reputation values at parameters $b = 1.2$ and $\sigma = 1.2$, as depicted in Fig. 3. In the images depicting strategies, which are Figs. 3(a) and 3(c), yellow indicates cooperators, whereas blue signifies defectors. Correspondingly, in the images depicting reputation, which are Figs. 3(b) and 3(d), yellow indicates players with high reputation, and blue is used for those with low reputation. Our observations reveal that at the 10th time step, under two distinct α values, the distribution of cooperating players is scattered and largely surrounded by defectors, signifying the prevalence of betrayal. However, as time progresses, cooperators gradually influence their neighbors, eventually coalescing into a large cooperative cluster. Moreover, we observe that over time, groups with high reputation gradually converge into several dense clusters, which correlates with cooperative behavior. This observation implies a synergistic relationship between cooperative strategies and reputation, with each facilitating the other's evolution. Additionally, our findings suggest that as α increases, the time necessary for co-operators to prevail over defectors diminishes, and the steady-state cooperation rate rises. This highlights the pivotal role of α in fostering cooperative behavior.

3.1.2. The effect of σ for given α

To further investigate the impact of selection intensity, we present Fig. 4, which depicts the relationship between selection intensity and cooperation rates at each Monte Carlo step. As previously discussed, with the increase of selection intensity σ, the impact of individual and environmental reputation on connections decreases progressively. Specifically, when $\sigma = 2$, the cooperation rates

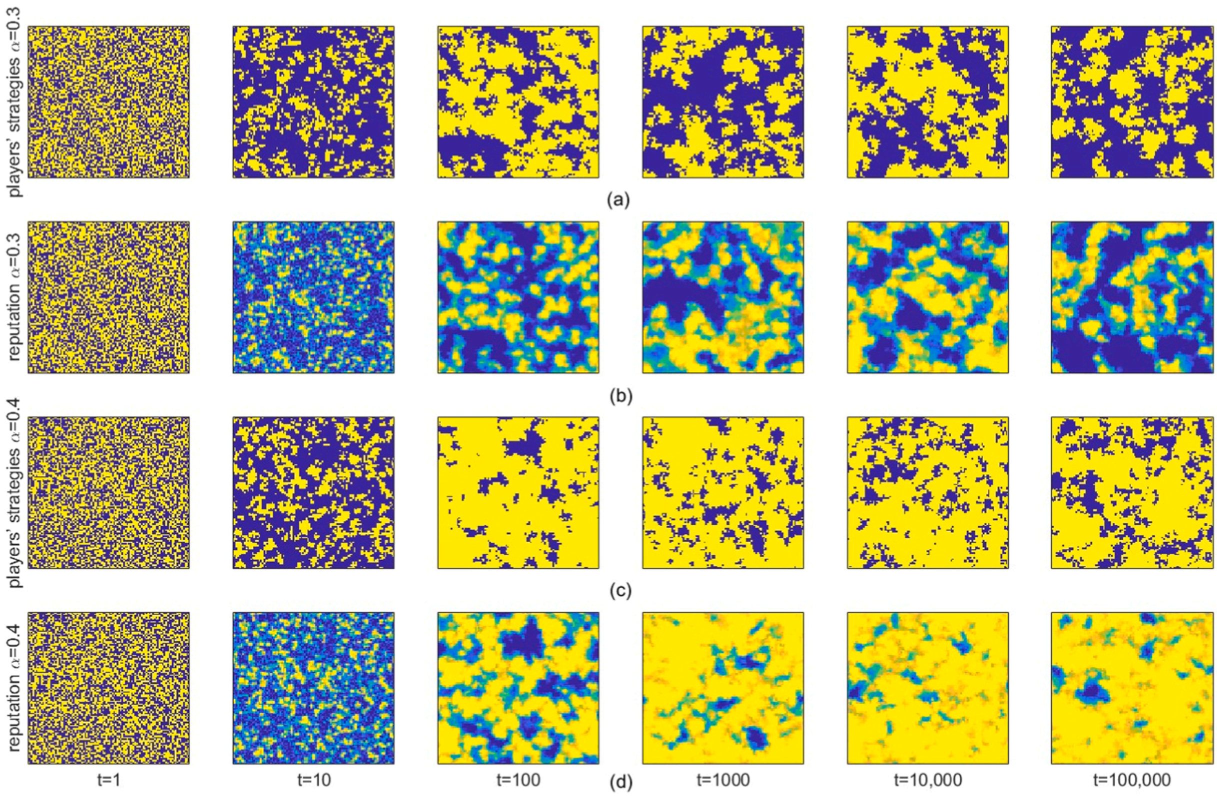


Fig. 3. Spatial snapshots of cooperators and the corresponding reputation value snapshot maps at various weights α and time steps t with $b = 1.2$ and $\sigma = 1.2$. The values of α for the first two rows are 0.3, whereas those for the last two rows are 0.4. The time steps progress from the first to the last column as 1, 10, 100, 1000, 10,000 and 100,000.

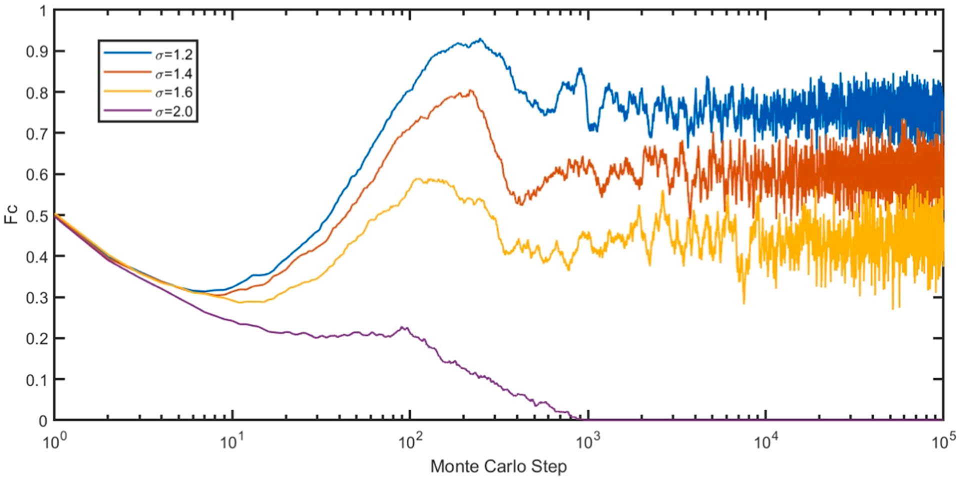


Fig. 4. Frequency of cooperator F_c at each step of the Monte Carlo simulation, with varying selection intensity σ, where the parameters are set at $b = 1.2$ and $\alpha = 0.4$.

diminish to 0 as the step length increases, resembling the classic PDG. This suggests that when the influence of individual and environmental reputation on the probability of connection is minimal, the proposed evolutionary mechanism fails to encourage group cooperation. However, as σ gradually decreases ($\sigma \leq 1.4$), even though defectors initially invade the population, over time, cooperators become increasingly prevalent. They succeed in converting more defectors into cooperators, ultimately causing the cooperation rate to fluctuate around a higher value. Furthermore, we note that at lower selection intensities, the rate of cooperation at the steady state is significantly higher. Hence, reputation serves a positive role in promoting cooperative behavior, and the establishment of a robust reputation evaluation system is a crucial factor in nurturing cooperation within populations.

3.1.3. The combined effect of α and σ

To gain a deeper understanding of the combined effects of the betrayal temptation b, the reputation factor α, and the selection intensity σ on cooperation, Fig. 5 presents separate panels for (α, b) and (σ, b) that depict the proportion of cooperators. Notably, the impacts of α and σ on maintaining cooperation are exactly opposite. Figs. 5(a) and 5(b) illustrate the collective influence of the betrayal temptation and the reputation factor, with σ set to 1.5. When α tends to 0, nearly all the individuals in the population become defectors. However, as the reputation factor α increases, cooperators begin to emerge. Consequently, when α is low, the collective system exhibits a weak resistance to betrayal. Even with a low betrayal temptation (below 1.1), a considerable number of defectors persist in the population.

However, the impact of the selection intensity is precisely the opposite. As depicted in Figs. 5(c) and 5(d), we set $\alpha = 0.5$. When σ is small, a considerable number of cooperators persists in the population, despite the relatively high temptation to defect. This suggests that, even with a significant betrayal temptation, cooperation within the population can be maintained to some degree, as long as the influence of individual and environmental reputation on connection probabilities is significant. As σ increases, however, the overall system's resistance to betrayal progressively diminishes. When σ becomes large, defectors come to dominate the population, even in the face of minor temptation to defect. Concurrently, as the temptation to betray increases, the rate of decline in cooperation accelerates, causing the cooperation rate to plummet to 0, even when the temptation to betray is relatively small. Therefore, Figs. 5(c) and 5 (d) illustrate that the population's resistance to temptation is significantly enhanced by the influence of reputation.

To gain a clearer understanding of how the proposed influencing factors affect the cooperation rate, we investigate the evolutionary process across various values of α and σ, as depicted in Fig. 6. When α is very small, the entire population tends toward betrayal, regardless of the value of σ. As α increases, the proportion of cooperators incrementally increases. This is attributable to the fact that when α is small, individual acts of betrayal have minimal impact on their reputation, and similarly, cooperative acts have little effect on increasing reputation. Consequently, players are motivated to prioritize betrayal over cooperation for personal gain. When σ is relatively large, the influence of reputation on connection probability may diminish, yet as long as alpha retains a high value—signifying a substantial effect of betrayal on reputation evaluation—the cooperation rate among players remains comparatively high. This demonstrates that, in contrast to the selection intensity σ, the reputation factor α exerts a more significant influence on player behavior.

3.2. The analysis of environmental degree r and rewiring probability p

In complex networks, information connectivity serves as an indicator for evaluating the efficiency and connectivity of information transmission between nodes. It encompasses the flow of information between nodes as well as the impact of the network structure on the speed and breadth of information propagation. The degree of information connectivity within a network can potentially affect the

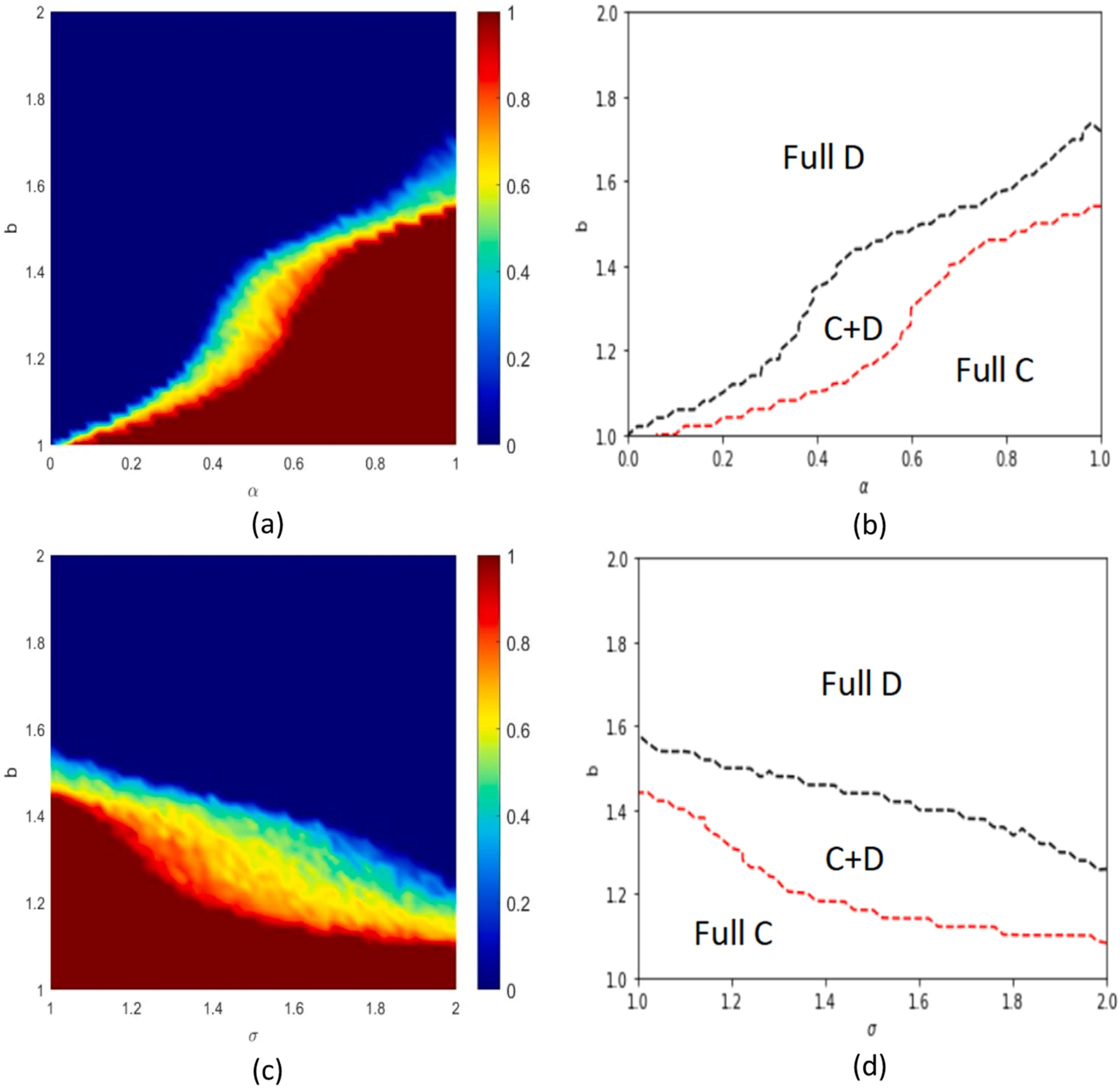


Fig. 5. Fraction of cooperators F_c (left) and phase separation lines on the parameter plane (right). Panel (a) and (b) vary with betrayal temptation b and the reputation factor α, and panels (c) and (d) are influenced by betrayal temptation b and the selection intensity σ.

efficacy of reputation mechanisms, which in turn can impact cooperative behavior across a population. To delve into this effect, we conduct a comparative analysis of these two complex networks (the square lattice network and the small-world network), with the detailed results presented in Fig. 7.

We set $\alpha = 0.5$ and $\sigma = 2$. Upon examining Fig. 7(a) and (b), we note that, at the same level of betrayal temptation, the cooperation rate of the population tends to increase with the environmental degree. This can be attributed to the fact that as the environmental degree rises, a greater number of neighbors are taken into account when calculating environmental reputation, which in turn enhances the information connectivity across the network. A horizontal comparison of the two graphs indicates that, for a given environmental degree, the propensity of players in the small-world network to succumb to betrayal is significantly lower than that in the square lattice. This is because the small-world network reduces the characteristic path length by incorporating random connections that disrupt the regular structure, thereby substantially improving information connectivity. Consequently, Fig. 7 suggests that enhancing information connectivity within the network is an effective way to promote cooperation.

The rewiring probability p [42] governs the likelihood of rewiring a link. Specifically, $p = 0$ yields a regular graph (including a square lattice), while $p = 1$ generates a random network. When p lies within the interval $(0,\ 1)$, the resulting network may possess small-world characteristics, with a small normalized characteristic path length between distant units, akin to that of a random network, and a large normalized clustering coefficient, similar to that of a regular nearest-neighbor graph. Therefore, the parameter p also influences the network's characteristic path length, which consequently impacts information connectivity. As depicted in Fig. 8, we visualize the evolution of cooperation rates in both square lattice and small-world networks with different rewiring probabilities at each Monte Carlo step. When $p = 0$, the node connections are orderly arranged, creating a structure akin to a square lattice without

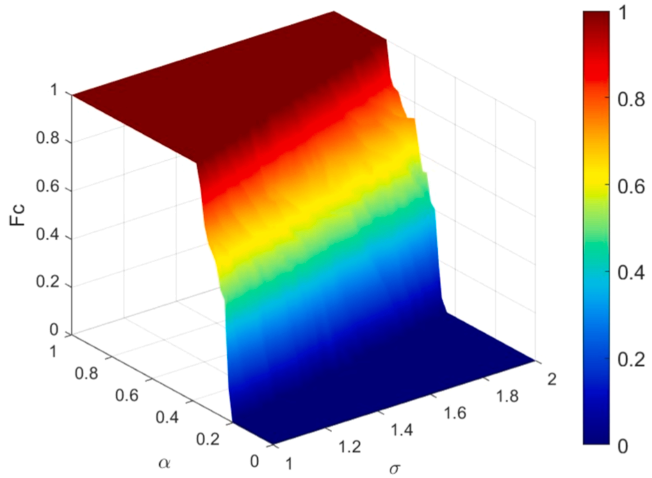


Fig. 6. A three-dimensional heat map illustrating the proportion of cooperators as determined by α and σ, with the betrayal temptation b set at a value of 1.2.

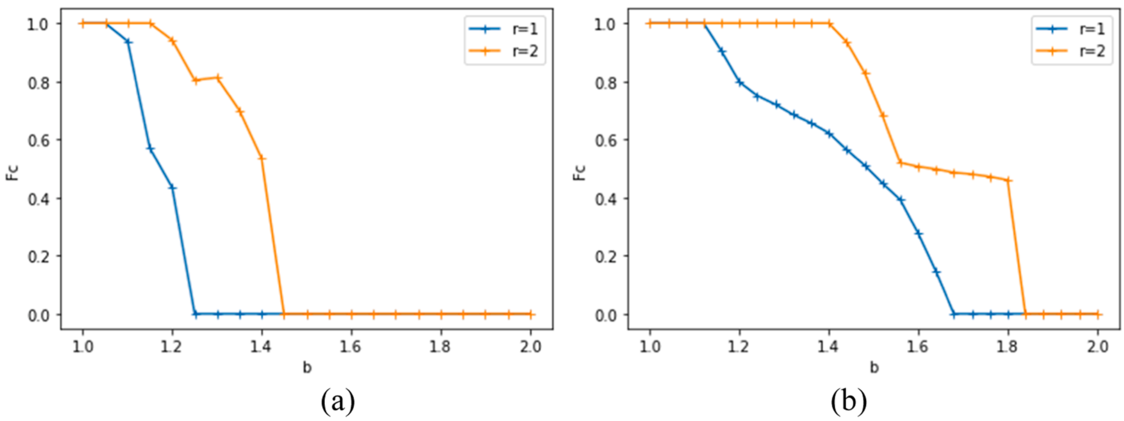


Fig. 7. The relationship between the frequency of cooperation F_c and the temptation factor b for different environmental degree r. Panel (a) represents a square lattice, whereas panel (b) corresponds to a small-world network.

any random rewiring. Consequently, similar to the *SL* curve, the cooperation rate curve in the *WS* ultimately declines to 0 in the stable state at $p = 0$. At $p = 0.1$, approximately $t = 10^4$, the final cooperation rate fluctuates around a higher value, indicating that the introduction of random connections substantially enhances the effectiveness of the reputation mechanism within the population. As p rises to 0.9, there is a slight improvement in the cooperation rate at the final state. Therefore, enhancing the information connectivity of the network structure itself (by shortening the characteristic path length of small-world networks) can also effectively promote cooperation.

4. Conclusion

We have proposed a reputation-based evolution mechanism within dynamic complex networks. In this mechanism, a player's strategy affects their reputation, which in turn affects the probability of connecting with neighbors. The simulation results indicate that both the reputation factor α and information connectivity (influenced by environmental degree r and rewiring probability p) exhibit a positive promoting effect on cooperative behavior. As these factors increase, so does the prevalence of cooperation within the population. Conversely, the impact of selection intensity σ is inversely related; an increase in selection intensity leads to a decrease in cooperative behavior. Therefore, in order to foster a positive cooperative atmosphere in society, it is necessary to establish a comprehensive reputation evaluation system in the population and simultaneously enhance information transmission efficiency.

Our current work focuses on the effects of reputation; however, future studies could investigate additional factors, such as the external environment, that may influence cooperation. Furthermore, it would be beneficial to adapt our model to other types of complex networks, such as scale-free networks, temporal networks and adaptive networks, to assess whether our mechanism can effectively promote cooperation in these contexts as well.

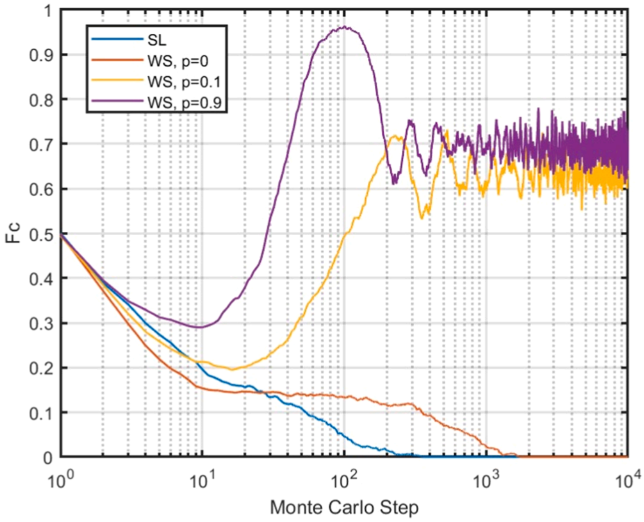


Fig. 8. Frequency of cooperator F_c in a square lattice and small-world networks with different rewiring probabilities at each Monte Carlo step, where the parameters are set as follows: $b = 1.3, \alpha = 0.8, \sigma = 4$.

CRediT authorship contribution statement

Qianwei Zhang: Writing – review & editing, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization. **Jiaqi Liu:** Writing – original draft, Visualization, Validation, Software, Formal analysis, Data curation. **Xinran Zhang:** Writing – review & editing, Data curation.

Declaration of Competing Interest

This is to declare that the work described in this manuscript is our original research that has not been published previously, and it is not under consideration for publication elsewhere, in whole or in part. No conflict of interest exists in the submission of this manuscript, and all the authors listed have approved the manuscript that is enclosed.

Data availability

No data was used for the research described in the article.

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Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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