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Full Length Article

The evolution of cooperation and global synchronization in the evolutionary Kuramoto dilemma combined with the prisoner's dilemma

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ABSTRACT

In this work, we have studied the evolution of the cooperation and the global synchronization on the Kuramoto model upon random networks with two games, in which one is the evolutionary Kuramoto dilemma and the other is the weak prisoner's dilemma. Agents can gain their payoffs from these two games synchronously. In the evolutionary Kuramoto game, cooperators prefer the formation of local synchronization but pay less cooperative cost. Cooperators in weak prisoner's dilemma can form cooperator clusters because of the network reciprocity. Combining these two games shows a mutual promotion effect upon the order of the global synchronization but a competition effect on the cooperation level. The global synchronization begins to rise only when most agents choose cooperation. It is manifested by the fact that the optimal coupling strength for the highest cooperation level is smaller than that for the highest synchronization level. The high cooperation cost makes defection the most advantageous strategy when the coupling strength reaches rather high, which diminishes the effect of cooperator clusters, and ultimately results in the absence of both cooperators and global synchronization. When agents have more neighbors in the random network, it is observed that cooperation is consistently hindered unless the coupling strength is relatively small. Additionally, there is always an optimal average degree that allows for global synchronization.

1. Introduction

Cooperative behavior is a widespread phenomenon in both the natural world and human society, but the phenomenon of the cooperation is contrary to Darwin's evolutionary theory [1], which argues that the cooperative behavior benefiting others can be extinguished by selfish and self-serving defective behaviors. To explain this social dilemma, the evolutionary game theory provides a general research framework [2–7]. As the famous two-player games, the snowdrift game (SDG) [8,9] and the prisoner's dilemma game (PDG) [10–12] have been widely studied. In the PDG, two agents will receive the reward R (the punishment P) if they are cooperators

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(defectors). If one cooperator meets one defector, the defector will receive the temptation to defect T and the cooperator will receive the sucker's payoff S. These payoffs strictly satisfy $T > R > P > S$ and $2R > T + S$. To better understand the social dilemma, many researchers have proposed many mechanisms to study the evolution of the cooperation, including punishment [13–15], reward [16,17], reputation [18], migration [19,20], reinforcement learning [21,22], aspiration [23,24], and so on.

In the previous studies, most works mainly focused on only one game model. However, in the majority of systems that pique our interest, we observe agents engaging in multiple games concurrently, including biological games within ecosystems and social games within human societies. This simultaneous participation in multiple games by players is referred to as multigame [25]. Thus, models with multigames should be taken into consideration. In recent years, an increasing number of researchers have shown great interest in multigames [26,27]. Additionally, multigames often include at least two types of social dilemmas (such as the PDG and the SDG). Based on multigames, Wang et al. investigated evolutionary multigames in structured populations and they found that a larger fraction of the population using a different payoff matrix enhances the cooperation [28]. Hu et al. investigated the effect of conformity on the evolution of the cooperation in multigames and their results demonstrated that increasing the fraction of conformists enhanced the cooperation [29]. Han et al. incorporated the age factor into a multigame to investigate the effect of aging property on the cooperation and they found that the cooperation can be greatly improved [30].

The synchronization phenomena and the cooperative behaviors are closely related, and synchronous behaviors are also universal in human society and nature [31–34]. For the study of the synchronization, the Kuramoto model is one of the most well-known and well-studied models [35], and it is the most fundamental framework for studying synchronization dynamics. Very recently, Antonioni and Cardillo have combined the evolutionary game theory and the global synchronization phenomenon and called it the evolutionary Kuramoto game (EKG) [36], they take into account the cooperative cost on account of mutual interactions and provide some understandings about the co-evolution of the synchronization and the cooperation. Based on EKG, researchers have conducted some further studies about EKG. For instance, Yang et al. found that a moderate average degree promises that the system could achieve its maximum level of synchronization [37]. Li et al. combined PDG and EKG, considering the local synchronization as an effective local temperature determining the agent's strategy update process, and they observed double explosive transition of the synchronization and the cooperation [38]. Zhu et al. combined reinforcement learning with EKG, and they showed that both the cooperation and the dynamic feature can enhance the synchronization, and that the synchronization can enhance the cooperation between agents [39]. More recently, inspired by EKG, Huang et al. [40,41] incorporate the evolutionary games into the Hegselmann-Krause model and opinion changing rate model. Thus, it is known that the emergence of the cooperation is closely linked to the emergence of the synchronization.

In general, agents in different situations may face different kinds of dilemmas at the same time, which have been appropriately considered when considering the multigame environments. For instance, within an enterprise, employees need to coordinate work, share resources, and reach consensus. However, due to different goals and conflicting benefits among employees, coordination difficulties may arise. Therefore, synchronous collaboration among employees is beneficial for improving overall efficiency, similar to the evolutionary Kuramoto game. At the same time, employees need to balance between pursuing personal benefits and collective benefits, similar to the prisoner's dilemma game. The prisoner's dilemma game emphasizes the conflict of benefits between cooperative and defective behaviors among individuals. In contrast, the evolutionary Kuramoto game focuses on the evolution of phase synchronization behavior within a group. Combining the prisoner's dilemma game and the evolutionary Kuramoto game helps to explore the complex interaction between cooperation and defection, as well as how individual strategic choices influence the phenomenon of phase synchronization within a group.

Inspired by previous works, we are interested in the evolution of the cooperation and the global synchronization in the Kuramoto model with the two combined dilemma games, composed of the evolutionary Kuramoto dilemma and the prisoner's dilemma. In this double dilemma evolutionary Kuramoto game, agents encounter two dilemmas simultaneously and the payoff of each agent is the sum of the EKG dilemma payoff and the prisoner's dilemma payoff. The numerical result shows the relation between the cooperative behavior and the global synchronization is not a monotonous one but the synchronization is highly dependent upon the cooperation level. Only when there exists the cooperation, the global synchronization may emerge. When the cooperation and synchronization of the system reaches the highest level, there exist optimal values for both the coupling strength and the average degree. However, the highest synchronization level always occurs after the peak of cooperation has been reached. This result is independent of the variation of the coupling strength or the average degree of the complex network.

The paper is structured as follows. In Section 2, we will provide a detailed description of our multigame model. Next, simulation results and analysis are provided in Section 3. Finally, we provide the conclusion in Section 4.

2. Model

In this paper, we consider a Erdős-Rényi (ER) random graphs [42] of N interacting agents with an average degree $\langle k\rangle$. The variable θ_l describes the state of the agent sitting on node l, and the dynamics of agent l's state is governed by the following equation:

$$\dot{\theta}_l = w_l + s_l \lambda \sum_{m=1}^{N} A_{lm} sin(\theta_m - \theta_l), \tag{1}$$

in which w_l is the natural frequency of agent l, and s_l represents the strategy of agent l. If agent l is a cooperator (C), $s_l = 1$ and the agent chooses to interact with her neighbors. Otherwise, agent l is a defector (D), she refuses the interaction with others and $s_l = 0$. λ represents the global coupling strength, A_{lm} is the element of the adjacent matrix and describes whether agents l and m are connected to each other. Thus $A_{lm} = A_{ml}$. $A_{lm} = 1$ is for the connected case and $A_{lm} = 0$ otherwise.

The complex order parameter R is introduced to describe the global coherence of the whole population, written as

$$R = \frac{1}{N} \sum_{m=1}^{N} e^{i\theta_m}, \tag{2}$$

where i is the imaginary unit. $R = R_G e^{i\psi}$, where R_G is the module of the order parameter and ψ is the average phase of the whole population. $R_G = 1$ denotes when all agents hold a same phase and the population reaches a fully coherent state. While agents hold different phases at random, the population stays completely incoherent and $R_G = 0$. For each agent, the local order parameter r_l measures the local synchronization degree of agent l's local environment, which is defined as the average of the coherence between agent l and her neighbors, written as

$$r_l = \frac{\sum_{m=1}^{N} A_{lm} r_{lm}}{\sum_{m=1}^{N} A_{lm}}, \tag{3}$$

where r_{lm} describes the coherence between a pair of agents l and m, which is defined as

$$r_{lm} e^{i(\theta_l+\theta_m)/2} = \frac{e^{i\theta_l} + e^{i\theta_m}}{2}. \tag{4}$$

The synchronization cost of agent l is defined as the absolute value of the angular acceleration

$$c_l = |\dot{\theta}_l(t) - \dot{\theta}_l(t-\varepsilon)|, \tag{5}$$

where $\varepsilon = 0.01$ is the discrete time step for the numerical solution Eq. (1) with the fourth-order Runge-Kuntta method. Agent l will obtain the payoff in the evolutionary Kuramoto game (EKG payoff) [36,37], which is defined as

$$\Pi_l^{(EKG)} = r_l - \alpha \frac{c_l}{2\pi}, \tag{6}$$

where the cost is divided by 2π so that it is commensurable with the payoff, and the scalar α is the relative cost scale. Meanwhile, agent l plays the weak prisoner's dilemma game (wPD) with its neighbors in pairwise interactions, in which $R = 1$, $T = b$, and $P = 0$, $S = 0$, and accumulates payoff π_{lm} according to the following matrix

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{7}$$

Two agents will get payoff 1 or 0 respectively if they both choose C or D. If one agent chooses C and the other one chooses D, the C agent will get payoff 0 and the D agent will get temptation to defect $T = b$. In each round, agent l will obtain the average payoff in this wPD, calculated as

$$\Pi_l^{(wPD)} = \frac{\sum_{m=1}^{N} A_{lm} \pi_{lm}}{\sum_{m=1}^{N} A_{lm}}. \tag{8}$$

The total payoff Π_l for agent l is the sum of the $\Pi_l^{(EKG)}$ and $\Pi_l^{(wPD)}$, which is defined as

$$\Pi_l = \Pi_l^{(EKG)} + \Pi_l^{(wPD)}. \tag{9}$$

After all agents have collected their payoffs, agents will synchronously update their strategies. Agent l randomly chooses one agent m of her neighbors and decides whether to learn her strategy s_m according to the update possibility following the Fermi function [43],

$$W(s_l \leftarrow s_m) = \frac{1}{1 + e^{-\beta(\Pi_m - \Pi_l)}}, \tag{10}$$

where $\beta > 0$ is the overall rationality parameter (larger β corresponding to more rational agents).

In this work, we focus on the fraction of cooperators f_C, defined as the ratio of the number of cooperative agents and the number of the whole population, and the order of global synchronization R_G. We set the population size $N = 1000$, the average degree of ER networks $\langle k \rangle = 6$, and the rationality parameter $\beta = 100$ unless specified. It's worth mentioning that the results of the model are robust to other rationality parameters (e.g., $\beta = 10$), despite the fact that the noise has been found to have a significant impact on the evolution of cooperation [44]. The initial phase θ and the natural frequency w of all agents are uniformly random distributed in the interval $[-\pi, \pi]$. The initial strategy of all agents has the same probability to choose as cooperator (C) or defector (D). The data presented in the figures are averaged over 100 independent realizations.

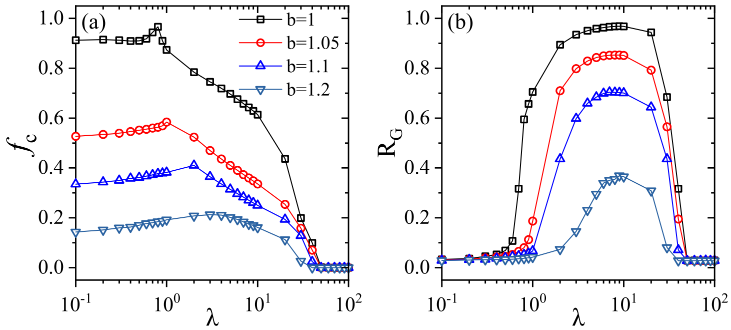

Fig. 1. Dependence of (a) the fraction of cooperation f_c and (b) the global synchronization R_G on the coupling strength λ under different temptations $b = 1, 1.05, 1.1, 1.2$. The average degree <k>= 6 and the relative cost $\alpha = 0.1$.

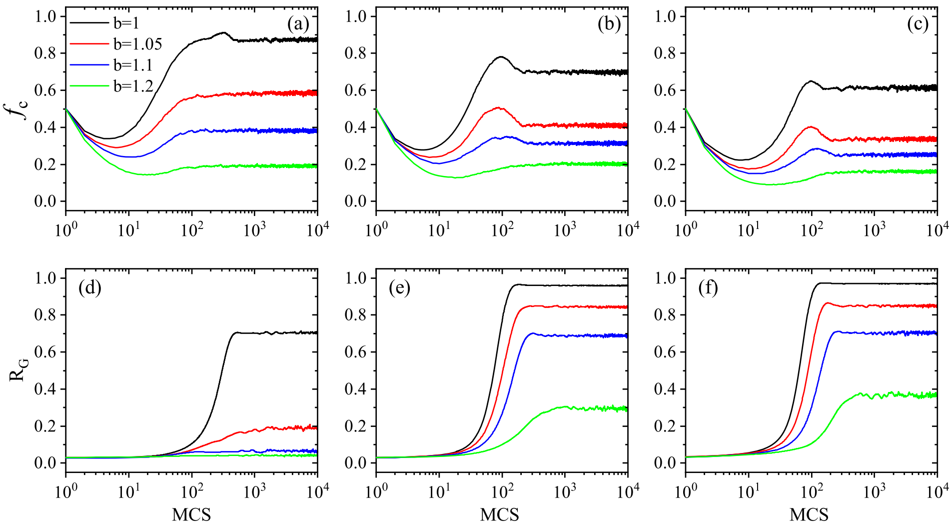

Fig. 2. The time evolution with different temptations b of the fraction of cooperation f_C for the first row and the global synchronization R_G for the second row. From left to right, the coupling strength to be set $\lambda = 1, 5, 10$ respectively. The average degree $\langle k \rangle = 6$ and the relative cost $\alpha = 0.1$.

3. Simulation results and analysis

Firstly, we show cooperation level f_C and the global synchronization R_G as a function of the coupling strength λ. It can be observed that with the increase of λ, both f_C and R_G increase to their peaks and then decrease to zero. However, the critical coupling strengths for peaks of f_C and peaks of R_G are different. This observation is robust for different temptations to defect b, shown in Fig. 1. When the coupling strength λ is relatively small, the interaction effect among agents is rather small, phases of agents obey the dynamics mainly determined by their natural frequencies, and the population is completely incoherent with $R_G = 0$. This is independent of the temptation b for the wPD. As the payoff of EKG is rather small for small λ, the choice of strategy is mainly determined by the wPD. Thus, a larger temptation b makes agents prefer the choice of the defection as shown in Fig. 1 (a). The dependence of the coupling term in Eq. (1) on λ becomes more significant as λ increases, some pairs of agents with similar natural frequencies evolve to similar phases but no global coherence arises. Thus, the existence of agent interactions leads to an increase in the EKG payoffs of some cooperative agents. It makes agents prefer the cooperation strategy when the temptation to defect is fixed. As the further increase of the coupling strength, the local synchronization expands, synchronized clusters merge with each other, and the global synchronization emerges. Cooperative agents gain more local benefits due to a higher level of local synchronization, but they also incur higher costs due to the adaptation to their locally synchronized frequencies. Meanwhile, defectors get their higher local coherence passively due to the adaptation of cooperators, similar with that zealots hold their opinions unchanged and conformists follow them [45]. Once cooperator clusters survive, they make the global synchronization R_G increase but with f_C slowly decreasing. When the coupling strength λ increases further, cooperators cannot afford the cost of the local synchronization, the global synchronization shrinks and fades away, and the defection expands to the whole population.

Fig. 2 shows the detail of the time evolution of the cooperation level f_C in the first row and the global synchronization R_G in the second row. From Fig. 2, one can find that the time dynamics of f_C are first down and later up, which is a clear evidence of network reciprocity [46,47]. More specifically, Fig. 2 (a) and (d) show that phases of agents are incoherent, meanwhile, the cooperation level f_C decreases in the first stage. Although there exist cooperative agents, no global synchronization emerges because agents

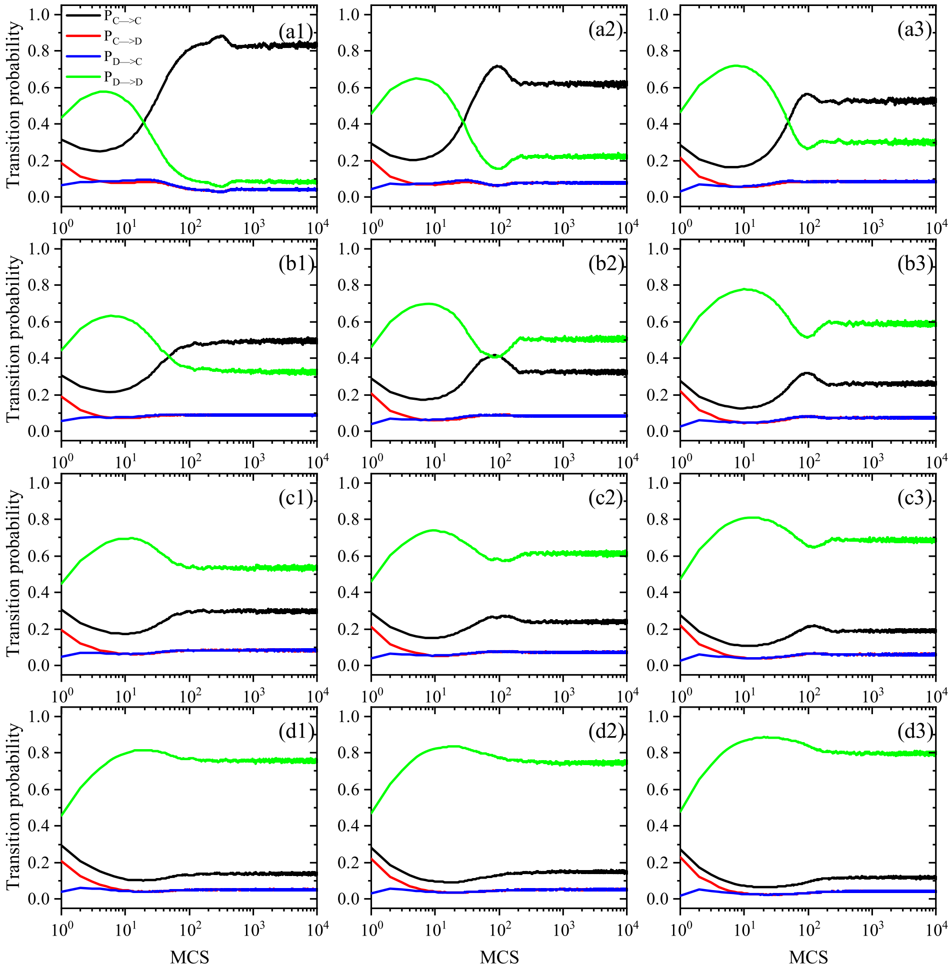

Fig. 3. The time evolution of the transition ratio of strategies with different temptations $b = 1, 1.05, 1.1, 1.2$ for different rows. The first column is for the coupling strength $\lambda = 1$, the second column for $\lambda = 5$, and the third column for $\lambda = 10$. In each panel, the black line is for cooperators willing to be cooperators $P_{C\to C}$, the red is for cooperators turning to be defectors, the blue is for defectors transiting to be cooperators, and the green is for defectors remaining the defection strategy. The average degree $\langle k \rangle = 6$ and the relative cost $\alpha = 0.1$.

with the cooperation strategy are scattered in the network. After the reorganization of states and strategies of agents, cooperative agents merge into clusters, cooperators gain more PDG payoffs due to the network reciprocity [27,28,48], clustered cooperators also get localized synchronization and expand to a global one, which makes the improvement of the global synchronization. As for the temptation to defect, cooperators with a larger b gain lower PDG payoffs, fewer cooperators could resist the invasion of the defector, and the global synchronization is also weakened. When the coupling strength λ becomes larger, the case with $\lambda = 5$ in Fig. 2 (b) and (e) is much similar with the former one but with a higher global synchronization and fewer cooperators. This is because of that fewer cooperation clusters are formed. When $\lambda = 1$ cooperators merge into many cooperation clusters and hold different synchronized phases, while for $\lambda = 5$ cooperators merge into fewer clusters, which makes a higher R_G. Furthermore, a larger λ makes cooperators pay more synchronization cost and more cooperators turn to be defectors. However, it is quite interesting how a decrease in cooperation supports a higher level of global synchronization (e.g., Fig. 2 (c) and (f)). In the case with $\lambda = 10$, unlike with that phases of cooperators are not just locked with phases of defectors drifting as in the synchronization with $\lambda = 1$, cooperators are chasing defectors. That's why fewer cooperators support a much higher R_G. As shown in Fig. 1, the global synchronization finally fades away. This is because even though λ increases further, only fewer agents still hold the cooperation strategy, so they cannot fail to chase the phase variation of defectors and maintain the formation of cooperation clusters.

To better understand the transition of agent's strategies, we show the evolution process of their strategies in Fig. 3 with different temptations to defect $b = 1, 1.05, 1.1, 1.2$ for different rows and coupling strengths $\lambda = 1, 5, 10$ for different columns. In each panel, there are four transition ratios between the cooperation strategy and the defection strategy. The black is for $P_{C\to C}$ where cooperators will hold the cooperative strategy, the red is for $P_{C\to D}$ in which cooperators will turn to be defectors, the blue is for $P_{D\to C}$ where defectors will become cooperators, and the green is for $P_{D\to D}$ in which defectors will hold the defection strategy unchanged. Of course, the fraction of cooperators $f_C = P_{C\to C} + P_{D\to C}$. Fig. 3 shows clearly that the defection expands, where $P_{D\to D}$ always increases along with $P_{C\to C}$ decreases, and $P_{D\to C}$ and $P_{C\to D}$ reach a balance in the first stage, as shown in Fig. 2. This is due to the lack of network

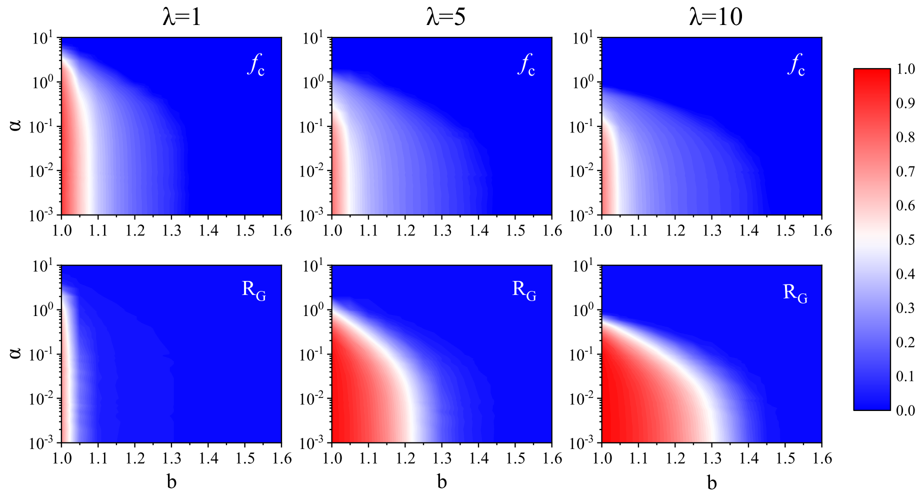

Fig. 4. Contour graphs of the fraction of cooperation f_C and the global synchronization R_G on $\alpha - b$ panel. From left to right, the coupling strength to be set $\lambda = 1, 5, 10$ respectively. The average degree <k>= 6.

reciprocity, which hinders the formation of cooperation clusters. Moreover, the fact that $P_{C\to D}$ is larger than $P_{D\to C}$ indicates the expansion of the defection and the decrease of f_C. This shows that scattered cooperators are easy to learn the defection strategy unless the cooperators get clustered. As time goes by, cooperation clusters are formed and it is shown by the increase of $P_{C\to C}$ and the decrease of $P_{D\to D}$, which also indicates that $P_{D\to C}$ is larger than $P_{C\to D}$. This means clustered cooperators show the cooperation is a more advantageous strategy for agents beside them. At the final stage, not only the effect of wPD but also the cost of the synchronization should be considered. While cooperators benefit from reciprocity, they still incur costs as long as at least one of their neighbors chooses to be a defector. Once the payoff gained by wPD is lower than the cost of EKG, cooperators will tend to be defectors. Fig. 3 (a1) with $\lambda = 1$, (a2) with $\lambda = 5$, and (a3) with $\lambda = 10$ show it clearly. The cost described by the variation of phase velocity caused by the interaction increases as the coupling strength is larger, leading to more cooperators turn to be defectors. It shows much clearly that $P_{C\to C}$ decreases after getting its peak, which also manifested by the similar behavior of f_C in Fig. 2. While for the increase of b, as with $b = 1$ in Fig. 3 (a1), $b = 1.05$ in Fig. 3 (b1), $b = 1.1$ in Fig. 3 (c1), $b = 1.2$ in Fig. 3 (d1), the effect of defection is very obvious for the suppression of network reciprocity and it suppresses the expansion of cooperator clusters. It is much similar with the case in $\lambda = 5$ and $\lambda = 10$.

The effect of the relative cost scale α is investigated and the result is shown in Fig. 4. The diagram of Fig. 4 shows the competition between the wPG and the EKG. Fig. 4 (a) shows that when the coupling strength is small (e.g., $\lambda = 1$), the effect of synchronization is small until the relative cost is rather high. When the temptation to defect b is small, even if the cooperation level f_C is very high, cooperators still cannot merge into one synchronized cluster and the global synchronization R_G is small. When the temptation to defect b is larger, the fraction of cooperators f_C is still high due to cooperators gaining more payoffs from the EKG to defeat the invasion of defection caused by the wPD. To be noticed, the payoff of the EKG is mainly determined by the local synchronization as opposed to the global synchronization. Thus, clustered synchronization can provide a high EKG payoff but with a low R_G. When the coupling strength is larger $\lambda = 5$ in Fig. 4 (b), the cooperation level f_C is more sensitive to the relative cost scale α. It is because the effect of the interaction term for cooperators in Eq. (1) is more significant, the phase variation in Eq. (5) is larger, and the cost paid by the EKG Eq. (6) is higher. However, synchronized clusters tend to merge into one as the coupling strength is higher, shown by a high R_G, which makes cooperators gain more payoffs due to the EKG than defectors when the cost scale α is small. Thus, it makes cooperator clusters more robust to the invasion of defection due to the wPD and it is manifested by the numerical result shown in Fig. 4 (b), where f_C is higher for $\lambda = 5$ than that for $\lambda = 1$ around $b \approx 1.3$. It is much similar with the case of $\lambda = 10$ in Fig. 4 (c). From this perspective, Fig. 4 shows the EKG and the wPD have a mutual promotion effect. A higher global synchronization supports the robustness of cooperator clusters and the cooperation helps agents to foster synchronization. However, a larger coupling strength λ always causes a larger cooperation cost and it still causes the defection is a more advantageous strategy than the cooperation.

The effect of the average degree $\langle k\rangle$ is investigated and the result is shown in Fig. 5. In line with the above analysis, Fig. 5 (a) shows a smaller coupling strength with a higher level of cooperation f_C, which is caused by the large phase variation unless the complete global synchronization is reached. In the case of low coupling strength, cooperators will choose to form different synchronized clusters, which makes the global synchronization is low when the average degree $\langle k\rangle$ is small. A large $\langle k\rangle$ makes cooperator clusters merge into one and it will make a high R_G. However, when the average degree $\langle k\rangle$ is rather large, cooperators are surrounded by more defectors and vice versa. As defectors pay nothing but gain more payoffs from the wPD and even from the EKG, cooperator clusters are finally been invaded $f_C = 0$ and the global synchronization disappears.

Furthermore, the cooperation level f_C and the global synchronization R_G against the coupling strength λ with different average degrees $\langle k\rangle$ is studied and the result is shown in Fig. 6. Similar with the result in Fig. 5, a smaller average degree $\langle k\rangle$ leads to a higher cooperation level f_C. And increasing the coupling strength λ still first supports the establishment of cooperator clusters, then decreases f_C due to a higher EKG cooperation cost, and finally makes f_C drop to 0 thanks to the collapse of the cooperation cluster.

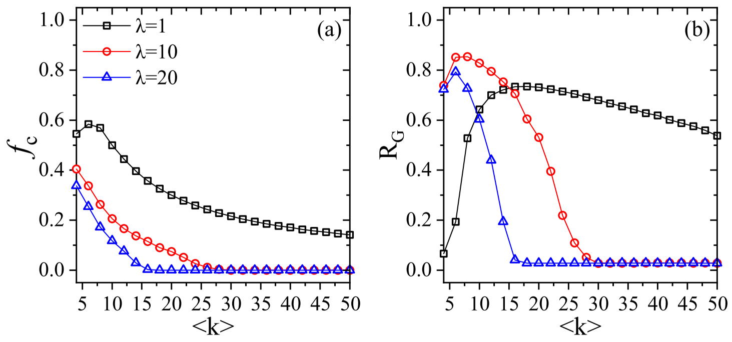

Fig. 5. Dependence of (a) the fraction of cooperation f_c and (b) the global synchronization R_G on the average degree $\langle k\rangle$ under different values of the coupling strength $\lambda = 1, 10, 20$. The temptation to defect $b = 1.05$ and the relative cost $\alpha = 0.1$.

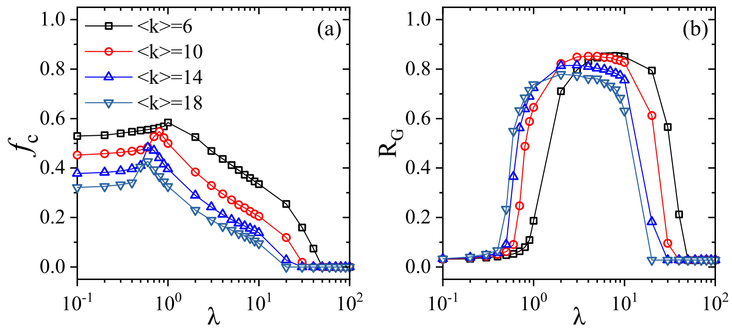

Fig. 6. Dependence of (a) the fraction of cooperation f_c and (b) the global synchronization R_G on the coupling strength λ under different values of the average degree $\langle k\rangle = 6, 10, 14, 18$. The temptation to defect $b = 1.05$ and the relative cost $\alpha = 0.1$.

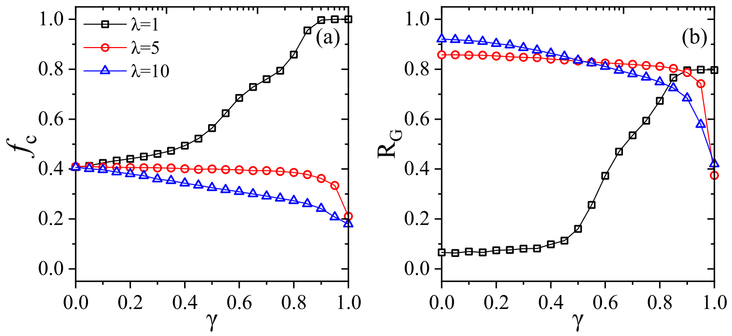

Fig. 7. Dependence of (a) the fraction of cooperation f_c and (b) the global synchronization R_G on the weighting coefficient γ under different values of the coupling strength $\lambda = 1, 5, 10$. The average degree $\langle k\rangle = 6$, the temptation to defect $b = 1.05$ and the relative cost $\alpha = 0.1$.

This is much similar with what happens in Fig. 1 (a). However, in Fig. 6 (b), a larger $\langle k\rangle$ will meet the peak of R_G for a lower coupling strength. This also indicates that defectors with more neighbors are more likely to meet cooperators and gain higher payoffs both due to the EKG and the wPD and that cooperators with more neighbors are easy to be invaded by defectors when agents are sitting on a random network.

Finally, to explore the dynamics of the system with the unequal weight for the total payoff from the two types of games, we utilize the new formulation for the individual's total payoff, $\Pi_l = \gamma\Pi_l^{(EKG)} + (1-\gamma)\Pi_l^{(wPD)}$. The adjustable parameter γ represents the weighting coefficient of the agent's payoff in the EKG. $\gamma = 0$ means that individuals' payoffs depend entirely on the payoffs obtained in wPD game, while $\gamma = 1$ means that individuals' payoffs depend entirely on the payoffs obtained in EKG. The fraction of cooperation f_c and the global synchronization R_G as a function of γ are shown in Fig. 7. For small coupling strength (e.g., $\lambda = 1$), f_c and R_G increase with the increase of γ. While for relatively large coupling strength (e.g., $\lambda = 5$ and 10), f_c and R_G decrease with the increase of γ. Moreover, for different values of γ, the optimal λ corresponding to the highest level of cooperation may vary. To explore the effect of the weighting coefficient γ on the optimization of f_c and R_G against λ, we show the dependence of f_c and R_G on the coupling strength λ under two different γ in Fig. 8. Similar to the case with equal weight for the total payoff from the two types of games (Eq. (9)), the optimization of f_c and R_G against λ can still exist, and the critical coupling strengths for peaks of f_c and peaks

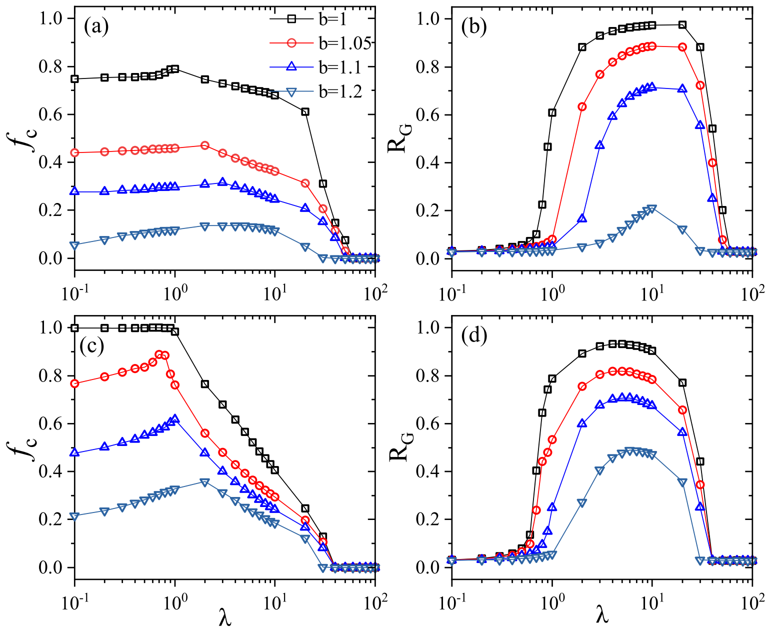

Fig. 8. Dependence of (a, c) the fraction of cooperation f_c and (b, d) the global synchronization R_G on the coupling strength λ under different temptations $b = 1, 1.05, 1.1, 1.2$. The weighting coefficient: (a) and (b) $\gamma = 0.3$, (c) and (d) $\gamma = 0.7$. The average degree <k>= 6 and the relative cost $\alpha = 0.1$.

of R_G are also different. However, a smaller γ can lead to the optimal λs corresponding to the peaks of f_c and peaks of R_G being closer together. Additionally, a smaller γ corresponds to larger values of optimal λ. The unequal weight between the payoff obtained in EKG and wPD game has a significant impact on the dynamics of the system.

4. Conclusion

In this work, the cooperation and the global phase synchronization are studied on the Kuramoto model with two dilemma games, where the weak prisoner's dilemma game (wPD) is combined with the evolutionary Kuramoto dilemma game (EKG). Increasing the coupling strength helps the formation of the local synchronization if the coupling strength is small, where cooperative agents get much payoffs of the EKG, and promote the global synchronization. While the coupling strength is large, too much cooperation cost should be paid by cooperators and no cooperators could survive, where both the cooperation level and the global synchronization are zero. Thus, from this point of view, the coupling strength of EKG has both the positive effect and the negative effect. However, as it is known to all, the increase of the temptation to defect always hinders the cooperation. When both EKG and wPD take effect, they will show the competition effect on the cooperation level but the mutual promotion effect on the global synchronization. Consistent with former EKG works, there exist optimal coupling strengths for the cooperation and the synchronization [36]. However, this work clearly illustrates that there are two distinct optimal coupling strengths, with the highest cooperation level marking the onset of achieving the highest synchronization level. As for the average degree, it shows the result robust, where in Yang et al.'s former work [37] a medium average degree is the optimal one for the synchronization. Additionally, the similar results can also be found in earlier studies [49,50], where there exist an optimal cooperation for intermediate interaction intensity. Moreover, we find that results with unequal weighting for the total payoff from the two types of games exhibit qualitative similarities to those obtained with equal weighting.

To emphasize, the model proposed in this paper aims to examine the coupling dynamics of synchronization and cooperation within the framework of multigames. In this model, we assume that individuals are confronted with both the Kuramoto dilemma and the weak prisoner's dilemma simultaneously, aiming to investigate how the interaction between these two dilemmas influences the emergence of collective behaviors. However, individuals in real-life scenarios often encounter various other social dilemmas, such as the multiplayer prisoner's dilemma, when making decisions. Subsequent research is necessary to examine the coupling dynamics of synchronization and cooperation using different game types (e.g., the snowdrift games [8] and the public goods games [51]) to better understand individual interactions. Moreover, Tripp et al. have presented and analyzed a novel evolutionary game theoretic framework that models the evolutionary dynamics of coupled oscillators [52], they shown that the type of games oscillators play can go beyond the Prisoner's Dilemma. The above work has offered a novel perspective on the application of evolutionary game theory in synchronization and might inspire further investigations in this respect.

Data availability

No data was used for the research described in the article.

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