

The effect of nonlinear environmental feedback on the outcomes of evolutionary dynamics

Jiaquan Huang^a, Yuying Zhu^b, Chengyi Xia^{b,*}, Jun Tanimoto^c

^a School of Electronics and Information Engineering, Tiangong University, Tianjin, 300387, China

^b School of Artificial Intelligence, Tiangong University, Tianjin, 300387, China

^c Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka, 816-8580, Japan

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ABSTRACT

In this paper, we construct a nonlinear evolutionary game model to analyze the cooperation mechanisms of the population based on a nonlinear relationship among environment and strategies. In the model, replicator dynamics and aspiration dynamics are used to explore the evolutionary outcomes of collective decision, respectively. The results suggest that the environment tends to become progressively more affluent as the number of cooperators increases, if there is a smaller intensity of environmental destruction of defectors. Interestingly, the enriched environments may attract more defectors. Hence, the population requires a higher level of vigilance against plentiful environments in response to the emergence of defectors. As opposed to replicator dynamics, aspiration dynamics can avoid the persistent oscillatory loops due to the level of aspiration. Further, we investigate the effect of complexity between the population strategy and the environment on the evolutionary outcomes. It is found that higher level of complexity can drive the environment closer to a state of affluence, but the population's strategy structure will not be modified. These insights into the relationship between environment and strategies further our understanding of the evolutionary mechanism of population and society.

1. Introduction

With the continuous development of the evolutionary theory of cooperation, it has been found that cooperation is a major factor in maintaining social stability [1–5] and analysing nonlinear system [6–8]. Therefore, analysing and understanding cooperative behaviours among different groups has become the key to dealing with various social problems [9–12], such as resource allocation [13,14], information sharing [15–18] and global climate [19,20]. According to the cooperative evolution rules of populations, some better solutions may be able to be formulated to solve these problems, which in turn can promote the sustainable development of the society and maintain the ecological balance. In biology, cooperation is an important survival strategy [21–23]. Mutual cooperation enables individuals to improve their adaptation to environmental changes and to cope with challenges of survival [24–26]. As an example, there is a mutualistic relationship between bees and flowers [27,28]. The process of nectar collection by bees helps flowers to spread pollen, while flowers provide food for bees, and this type of mutualistic relationship is known as ‘symbiosis’ in biology.

Interestingly, a bidirectional feedback mechanism prevails between the population's environment and individual's survival strategies [29–32]. The changes of the environment can affect the individual strategy choices, which, in turn, also can reshape the state of

* Corresponding author.

E-mail address: cyxia@tiangong.edu.cn (C. Xia).

the environment [33,34]. For instance, birds typically nest in areas with plentiful food, minimal predators, and favourable conditions. Meanwhile, behaviour such as pruning of branches during the nesting process may have a negative impact on the ecosystem, which may lead birds to change their nesting sites. In addition, the quality of the environment is closely related to the lives of individuals in the population [35–37]. Especially, it is important to consider regeneration within a given time frame when it relates to the management of the population's renewable resources [38–40]. Insufficient and untimely positive feedback on personal strategies towards the environment may have adverse effects on individuals' lives. Therefore, research on the bidirectional feedback mechanism between strategies and environment of the population can provide important theoretical guidance for improving environmental quality and promoting harmonious coexistence between humans and nature [41].

In recent years, scholars have shown extensive interest in the dynamics of environmental feedback on evolutionary games due to the crucial role that such feedback plays in the stability and development of society, as well as the reproduction and growth of organisms in nature. Weitz et al. [42] proposed a dynamical model that unifies environmental feedback and evolutionary games. The model shows how the strategic choices of individuals in a population explicitly affect the environment, which in turn impacts the outcome of the evolutionary game. Meanwhile it was found that players oscillated cyclically in both rich and depleted environments. Andrew et al. [43] developed a linear eco-evolutionary game framework with two strategies to analyse the dynamics of strategy and environment in population, and they found that the key to the evolutionary outcome is the relevant time scale of strategic choice and environmental change. In order to avoid the tragedy of the commons, Bairagya et al. [44] presented a special framework that combined individual payoffs and logistic growth of the population into eco-evolutionary based on the environment feedback. Under a finite carrying capacity, the tragedy of the commons could be avoided or generated. Su et al. [45] analysed the effects on cooperative evolutionary behaviour of influencing social environmental factors. They discovered that an individual's ability to adapt to social context significantly influences the development of cooperative behaviour, and the specific social context is the primary catalyst for the emergence of cooperative behaviour. Additional studies on evolutionary games have been conducted, such as the three-strategy evolutionary game with environmental feedback [46], network adaptation based on environmental feedback [47], and institutional rewards in public goods games [48].

Previous studies have mainly concentrated on the linear connections between strategies and environments, the nonlinear relationships between them are less considered. In fact, the strategy–environment relationships tend to be more complex in realistic scenarios. For example, the impacts of greenhouse gas emissions on human societies tend to be nonlinear and more complex in the climate system. An increase in greenhouse gases can cause global warming, which may lead to more frequent extreme weather events. These events, in turn, may prompt changes in production behaviours in human societies. Moreover, there also exist nonlinear relationships between individual gains and the environment for population [49,50]. Based on the nonlinear relationship between environment and strategy (or the nonlinear relationship between environment and payoff matrix, which means that the nonlinear grouping between two game payoff matrices), this paper presents a nonlinear evolutionary game model to analyse the mechanism of cooperation in populations. To clarify the interaction mechanism between the population environment and strategies, we utilize replication dynamics [51,52] and aspiration dynamics [53,54] to probe the outcomes of the population's evolutionary game, respectively. Furthermore, we examine repeated games for individuals, where each player participates in two rounds of the game. But the game model may differ for each participation, so we consider a nonlinear combination of the two game modes, i.e., a nonlinear grouping of the two game payoff matrices. Our studies indicate that populations' environments can reach a state of affluence, if there is a higher intensity of collaborators improving their surroundings and a lower intensity of betrayers destroying them. Nevertheless, rich environment may attract more defectors, as it may provide more resources, allowing some individuals to obtain better returns without relying on the cooperative behaviour of the population. Therefore, when the environment is in a state of prosperity, it may be more necessary to watch out for defectors. In replicator dynamics, there is a persistent oscillation. Fortunately, the oscillation can be prevented by adjusting the initial individual gains. With regard to aspiration dynamics, the oscillations may be avoided due to the presence of aspiration factors. In the meantime, our results still show that an increase of α (the nonlinear coefficient about the environment and payoff matrix) can have a positive effect on the environment.

This paper is organized as follows. Section 2 delineates nonlinear evolutionary game models with replicator and aspiration dynamics. Section 3 provides a detailed discussion of the models' evolutionary outcomes. Finally, Section 4 concludes the findings of this paper.

2. Model

Here, replicator dynamics and aspiration dynamics are used to analyse the impact of environmental feedback on evolutionary outcomes, respectively, where a nonlinear relationship between environment and strategy is assumed. Note that all our analyses are presented in the context of an infinite, well-mixed population.

2.1. Replicator dynamics for evolutionary games with environmental feedback

To analyse the impact of environmental feedback on the evolutionary outcomes of populations, Weitz et al. [42] considered the following form of payoff matrix:

$$A = (1 - n) \begin{bmatrix} R_0 & S_0 \\ T_0 & P_0 \end{bmatrix} + n \begin{bmatrix} R_1 & S_1 \\ T_1 & P_1 \end{bmatrix}, \quad (1)$$

where $n \in [0, 1]$ denotes the state of the environment, $n = 0$ denotes the depleted state, and $n = 1$ denotes the rich state. R and P demonstrate the payoffs for mutual cooperation and mutual defection, respectively. $S(T)$ is the payoff for cooperation (defection) against defection (cooperation).

Eq. (1) represents the relationship between the payoff matrix and the environment, where a change in the environment will inevitably lead to a change in A , which in turn will lead to a shift in the individual's strategy. It can be seen that A is a linear combination of two payoff matrices with respect to the environment in Eq. (1), and Weitz et al. [42] also pointed out that if other nonlinear combinations are considered, other types of games may emerge, leading to different evolutionary outcomes. Therefore, we propose one type of nonlinear combinations between the environment and the payoff matrix to explore the different evolutionary outcomes, where the final payoff matrix A can be written as follows:

$$A = (1 - n^\alpha) \begin{bmatrix} R_0 & S_0 \\ T_0 & P_0 \end{bmatrix} + n^\alpha \begin{bmatrix} R_1 & S_1 \\ T_1 & P_1 \end{bmatrix}, \quad (2)$$

where $\alpha \in (0, +\infty)$ denotes the nonlinear coefficient about the environment and payoff matrix. When $\alpha = 1$, Eq. (2) is identical to Eq. (1), so Eq. (2) is an expansion of Eq. (1). n^α represents the partial environment for the second payoff matrix of A in the population, and $1 - n^\alpha$ is the remaining environment for the first payoff matrix of A . If $n \neq 0$ and $n \neq 1$, A will be changed as α changes, and the player will choose other strategy which offer higher yields. In an ecosystem, organisms must adapt their strategies to the changing natural environment in order to survive and reproduce. For example, animals will adjust their foraging and reproduction strategies according to seasonal changes, food availability and so on. Where α denotes the complexity of environmental change, with a larger α indicating more complex environmental change.

In order to reduce the parameter in Eq. (2), we note that $D_g = T - R$ and $D_r = P - S$, D_g and D_r denote the 'gamble intended dilemma' and the 'risk averted dilemma', respectively. Assuming $R = 1$ and $P = 0$, which allow us to have $D_g = T - 1$, $D_r = P - 0$. Then, we can rewrite the Eq. (2) as follows:

$$A = (1 - n^\alpha) \begin{bmatrix} 1 & -D_{r0} \\ 1 + D_{g0} & 0 \end{bmatrix} + n^\alpha \begin{bmatrix} 1 & -D_{r1} \\ 1 + D_{g1} & 0 \end{bmatrix}, \quad (3)$$

where $D_{g0} = T_0 - R_0$, $D_{r0} = P_0 - S_0$, $D_{g1} = T_1 - R_1$ and $D_{r1} = P_1 - S_1$. If the proportion of cooperators in the population is denoted by x , the average gain of cooperators and defectors in the population is shown below:

$$\begin{cases} \pi_c = x + (1 - x) \left[n^\alpha (D_{r0} - D_{r1}) - D_{r0} \right] \\ \pi_d = x \left[1 + D_{g0} + n^\alpha (D_{g1} - D_{g0}) \right] \end{cases}, \quad (4)$$

where π_c and π_d indicate the cooperators' and defectors' average payoff, respectively.

The replicator dynamics describes how strategies evolve over time, and the proportion of cooperators in the population is represented by $\dot{x} = x(\pi_c - \phi)$, where $\phi = x\pi_c + (1 - x)\pi_d$. So the replicator dynamics for evolutionary games with environmental feedback based on the nonlinear correlation between payoff matrix and environment can be written as follows:

$$\begin{cases} \varepsilon \dot{x} = x(1 - x)(\pi_c - \pi_d) \\ \dot{n} = n(1 - n)f(x) \end{cases}, \quad (5)$$

where $\varepsilon \in (0, 1)$ represents the relative speed of the strategy dynamics in relation to the environmental dynamics. $f(x)$ serves as a feedback mechanism, determining whether the environment will evolve towards affluence ($n \rightarrow 1$) or barrenness ($n \rightarrow 0$) based on the positivity or negativity of its value. The $f(x)$ is hypothesised as follows:

$$f(x) = \theta_c x - \theta_d (1 - x), \quad (6)$$

where θ_c and θ_d represent the strength with which collaborators improve the environment and defectors destroy it, respectively. Obviously, changes in the feedback function are affected by both θ_c and θ_d . As x increases, the population environment may become rich when θ_c is larger and θ_d is smaller.

From Eq. (5), there are 4 border fixed points, which are $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$, and an internal fixed point (x^*, n^*) , where

$$x^* = \frac{\theta_d}{\theta_c + \theta_d} \text{ and } n^* = \left[\frac{x^*(D_{g0} - D_{r0}) + D_{r0}}{(1 - x^*)(D_{r0} - D_{r1}) - x^*(D_{g1} - D_{g0})} \right]^{\frac{1}{\alpha}}.$$

In Eq. (2), it is assumed that $R_1 < T_1$ and $S_1 < P_1$ (the second matrix of Eq. (2) on the right is the payoff matrix of prisoner's dilemma), which enable us to have $D_{r1} > 0$ and $D_{g1} > 0$. If $n = 1$, the only Nash equilibrium for the individual's strategy choice in Eq. (2) is mutual defection, as it is dominated by defectors. While $n = 0$, the strategic choices of players are dominated by the first matrix on the right-hand side. Then, we will explore the evolutionary results for Eq. (2) under $D_{r1} > 0$ and $D_{g1} > 0$, and there four cases about D_{r0} and D_{g0} .

- Case 1: $R_0 < T_0$ and $S_0 > P_0$, i.e. $D_{g0} > 0$ and $D_{r0} < 0$;
- Case 2: $R_0 > T_0$ and $S_0 < P_0$, i.e. $D_{g0} < 0$ and $D_{r0} > 0$;
- Case 3: $R_0 > T_0$ and $S_0 > P_0$, i.e. $D_{g0} < 0$ and $D_{r0} < 0$;
- Case 4: $R_0 < T_0$ and $S_0 < P_0$, i.e. $D_{g0} > 0$ and $D_{r0} > 0$.

The stability of the boundary point and the internal point of the Eq. (5) in the four cases mentioned above is shown below:

Theorem 1.1. (Case 1). $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$ are unstable. If $\frac{D_{r0}}{D_{r0}-D_{g0}} < \frac{\theta_d}{\theta_c+\theta_d}$, $\left(\frac{D_{r0}}{D_{r0}-D_{g0}}, 0\right)$ is a stable fixed point, and if $\frac{D_{r0}}{D_{r0}-D_{g0}} > \frac{\theta_d}{\theta_c+\theta_d}$, then (x^*, n^*) is a stable fixed point.

Theorem 1.2. (Case 2). $(0, 0)$ is a stable fixed point, while other border points are unstable. If $\frac{D_{r0}}{D_{r0}-D_{g0}} > \frac{\theta_d}{\theta_c+\theta_d}$, $\left(\frac{D_{r0}}{D_{r0}-D_{g0}}, 0\right)$ is unstable, and $n^* \notin (0, 1)$. If $\frac{D_{r0}}{D_{r0}-D_{g0}} < \frac{\theta_d}{\theta_c+\theta_d}$, then (x^*, n^*) is unstable.

Theorem 1.3. (Case 3). $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$ are unstable. And (x^*, n^*) is a stable point, if $\frac{D_{r0}}{D_{g0}} > \frac{D_{r1}}{D_{g1}}$.

The proofs of these theories are similar, and we will do the proof of **Theorem 1.3**. The proofs of the other two theories have been discussed in Appendix A.

Proof of Theorem 1.3. Without loss of generality, we set $\varepsilon = 1$, $g(x, n) = (\pi_c - \pi_d)$ and

$$\begin{cases} p(x, n) = x(1-x)g(x, n) \\ q(x, n) = n(1-n)f(x) \end{cases}, \quad (7)$$

so the Jacobian matrix is:

$$J = \begin{bmatrix} \frac{\partial p}{\partial x} & \frac{\partial p}{\partial n} \\ \frac{\partial q}{\partial x} & \frac{\partial q}{\partial n} \end{bmatrix}, \quad (8)$$

where

$$\begin{cases} \frac{\partial p}{\partial x} = (1-2x)g + x(1-x)\frac{\partial g}{\partial x} \\ \frac{\partial p}{\partial n} = x(1-x)\frac{\partial g}{\partial n} \\ \frac{\partial q}{\partial x} = n(1-n)(\theta_c + \theta_d) \\ \frac{\partial q}{\partial n} = n(1-2n)[\theta_c x - \theta_d(1-x)] \end{cases}. \quad (9)$$

Then, the Jacobian matrices of $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$ are:

$$\begin{cases} J(0, 0) = \begin{bmatrix} -D_{r0} & 0 \\ 0 & -\theta_d \end{bmatrix} \\ J(0, 1) = \begin{bmatrix} -D_{r1} & 0 \\ 0 & \theta_d \end{bmatrix} \\ J(1, 0) = \begin{bmatrix} D_{g0} & 0 \\ 0 & \theta_c \end{bmatrix} \\ J(1, 1) = \begin{bmatrix} D_{g1} & 0 \\ 0 & -\theta_c \end{bmatrix} \end{cases}. \quad (10)$$

In Case 3, there are $D_{r0} < 0$ and $D_{g0} < 0$. From Eq. (10), it is easily found that positive eigenvalues exist for $J(0, 0)$, $J(0, 1)$, $J(1, 0)$ and $J(1, 1)$, and thus $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$ are unstable.

The Jacobian matrices of (x^*, n^*) are:

$$J(x^*, n^*) = \begin{bmatrix} \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial x}(x^*, n^*) & \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial n}(x^*, n^*) \\ n^*(1-n^*)(\theta_c + \theta_d) & 0 \end{bmatrix}, \quad (11)$$

where $g(x^*, n^*) = 0$. The eigenvalues of $J(x^*, n^*)$ are:

$$\lambda = \frac{J(x^*, n^*)_{11}}{2} \pm \sqrt{\left(\frac{J(x^*, n^*)_{11}}{2}\right)^2 + J(x^*, n^*)_{12} \times J(x^*, n^*)_{21}}, \quad (12)$$

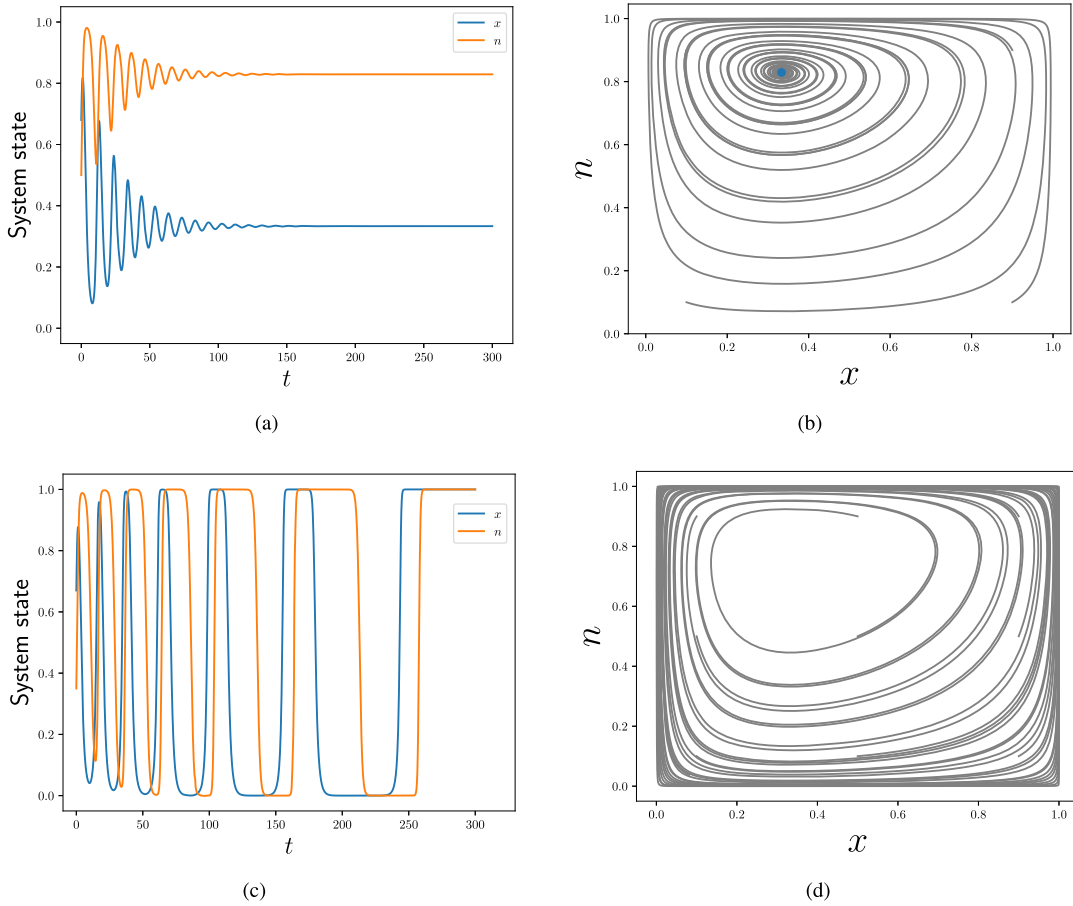


Fig. 1. The evolutionary outcomes for Eq. (5) in Case 3. (a) and (b) $D_{r0}/D_{g0} > D_{r1}/D_{g1}$, $D_{g0} = -0.25$ and $D_{r0} = -0.15$. (c) and (d) $D_{r0}/D_{g0} < D_{r1}/D_{g1}$, $D_{g0} = -0.25$ and $D_{r0} = -0.05$. Where the blue dot is the stable point, and all panels share the common parameters, $\alpha = 2$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, $\theta_c = 2$ and $\theta_d = 1$. Panels (a) and (b) show that (x^*, n^*) is a stable point when $\frac{D_{r0}}{D_{g0}} > \frac{D_{r1}}{D_{g1}}$. While $\frac{D_{r0}}{D_{g0}} < \frac{D_{r1}}{D_{g1}}$, there's a constant oscillation in panels (c) and (d).

where

$$\begin{cases} J(x^*, n^*)_{11} = \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial x} \\ J(x^*, n^*)_{12} = \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial n} \\ J(x^*, n^*)_{21} = n^* (1 - n^*) (\theta_c + \theta_d) \end{cases} \quad (13)$$

From Eq. (9) and $g(x^*, n^*) = 0$, we can have $\frac{\partial g}{\partial x} = \frac{\theta_c + \theta_d}{\theta_d} [D_{r0} - (n^*)^\alpha (D_{r0} - D_{r1})] < 0$, then

$$\frac{\theta_d (D_{r0} D_{g1} - D_{g0} D_{r1})}{\theta_c D_{r0} + \theta_d D_{g0} - (\theta_c D_{r1} + \theta_d D_{g1}) (D_{r0} - D_{r1})} < 0, \quad (14)$$

where $[\theta_c D_{r0} + \theta_d D_{g0} - (\theta_c D_{r1} + \theta_d D_{g1}) (D_{r0} - D_{r1})] > 0$, and thus $\theta_d (D_{r0} D_{g1} - D_{g0} D_{r1}) < 0$, namely $\frac{D_{r0}}{D_{g0}} > \frac{D_{r1}}{D_{g1}}$. So we have $\frac{\partial g}{\partial x}(x^*, n^*) < 0$ when $\frac{D_{r0}}{D_{g0}} > \frac{D_{r1}}{D_{g1}}$, which means that $J(x^*, n^*)_{11} < 0$, and $\lambda < 0$ in Eq. (12). Consequently, (x^*, n^*) is a stable point, if $\frac{D_{r0}}{D_{g0}} > \frac{D_{r1}}{D_{g1}}$.

Therefore, we derive **Theorem 1.3**. \square

In Case 4, strategic choices within the population are likely to be dominated by defectors. As evolution progresses, the number of cooperators in the population will gradually converge to zero ($x \rightarrow 0$), leading to depletion of the environment ($n \rightarrow 0$). Therefore, $(0, 0)$ is a stable point, and the remaining equilibrium points are unstable.

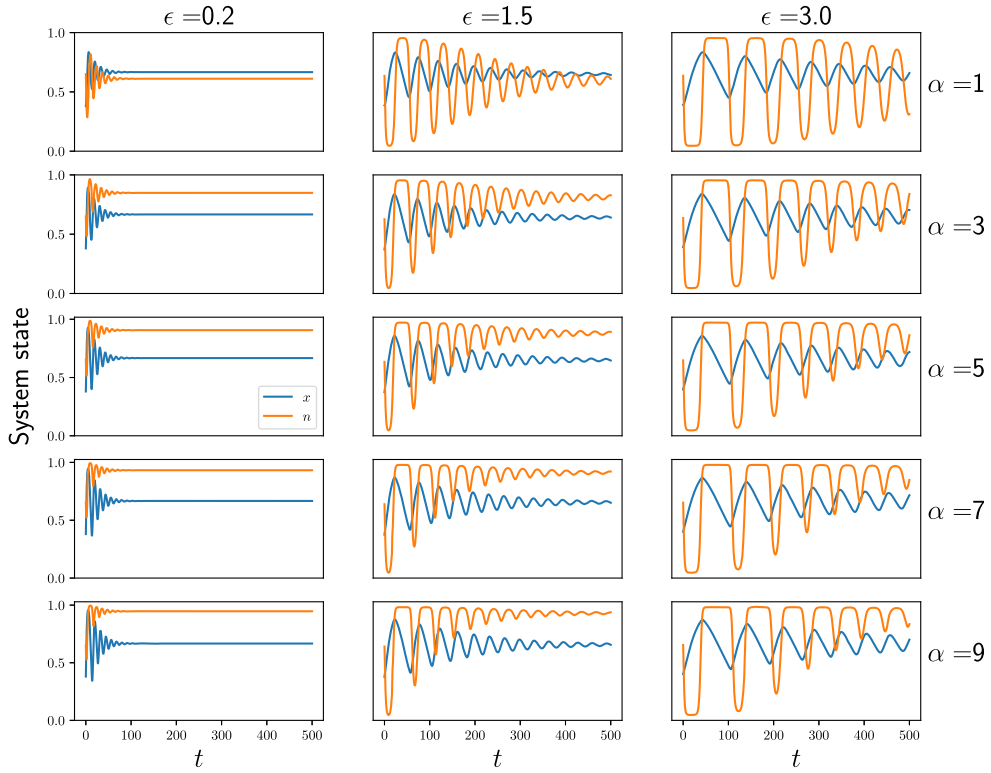


Fig. 2. Impact of the interaction of α and ϵ for Eq. (5). There are 15 panels with 3 columns (denote $\epsilon = 0.2$, $\epsilon = 1.5$ and $\epsilon = 3.0$, respectively) and 5 rows (denote $\alpha = 1$, $\alpha = 3$, $\alpha = 5$, $\alpha = 7$, and $\alpha = 9$, respectively). ϵ can't change the dynamics's steady state, and it will reduce the convergence speed of dynamics. α can enhance the environmental homeostasis, and the strategy composition on population will not be reorganised. The parameters are set to be $\alpha = [1, 3, 5, 7, 9]$, $\epsilon = [0.2, 1.5, 3.0]$, $D_{r0} = -0.25$, $D_{g0} = -0.15$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\theta_c = 1$ and $\theta_d = 2$.

Fig. 1 depicts the evolutionary results for Eq. (5) in Case 3. From panels (a) and (b), we can find that (x^*, n^*) is a stable point, which is consistent with **Theorem 1.3**. While $\frac{D_{r0}}{D_{g0}} < \frac{D_{r1}}{D_{g1}}$, there's a constant oscillation here, as shown in panels (c) and (d). According to Eq. (3), when n is close to 0 (1), individuals tend to prefer high-reward cooperative behaviour (defective behaviour). So, it will exhibit oscillating phenomena (which is a heteroclinic cycle on the boundary) when n appears cyclically between 0 and 1. And we prove that the oscillations will converge to an asymptotically stable heteroclinic cycle (see **Theorem 1.4** on Appendix A). The results of other cases are shown in the Appendix A.

Subsequently, the interaction effect between α and ϵ in the system dynamics has been analysed in detail, as shown on Fig. 2. In Fig. 2, it is intuitively clear that an increase of ϵ leads to a slowdown in the rate of convergence of the dynamics, and it can not interfere with the system dynamics' steady state. On the other hand, α demonstrates the ability to optimise the steady state of the system dynamics environment, although this improvement is reflected at different ϵ .

2.2. Aspiration dynamics for evolutionary games with environmental feedback

In nature, species adapt and seek greater chances of survival and reproduction by adapting to changing environments, which can be thought of as an aspiration dynamic [53]. In contrast to replicator dynamics, the individual comparisons of gains are not considered during the strategy update, and it heavily relies on self-assessment in aspiration dynamics [54]. Suppose there is a desired payoff (p_i) for player (i). If $p_i' < p_i$, player (i) will continue with the original strategy, where p_i' denotes the payoff of players (i) in the next round game. Otherwise, they replace it with a new strategy. As the master equation of aspiration dynamics on [41,53,54], the aspiration dynamics for evolutionary games with environmental feedback based on the nonlinear correlation between strategies and environment can be written as follows:

$$\begin{cases} \epsilon \dot{x} = (1-x)P_d - xP_c \\ \dot{n} = n(1-n)f(x) \end{cases}, \quad (15)$$

where, x and ϵ have the same meaning as in Eq. (5), P_c and P_d indicate the strategy transition probabilities of cooperators and defectors, respectively, and $f(x)$ as shown in Eq. (6). The probabilities can be calculated by using the Fermi function.

$$\begin{cases} P_c = \frac{1}{1+\exp[k(\pi_c - A_c)]} \\ P_d = \frac{1}{1+\exp[k(\pi_d - A_d)]} \end{cases}, \quad (16)$$

where, π_c and π_d have the same meaning as in Eq. (4), A_c and A_d indicate the aspiration level of cooperators and defectors, respectively. And k represents the sensitivity of individual earnings in different aspiration levels. If $k \ll 1$, k is a weak selection level, and vice versa for a strong selection level. From Eq. (15), there are three fixed points, $(x^*, 0)$, $(x^*, 1)$ and $(\frac{\theta_d}{\theta_c + \theta_d}, n^*)$, where (Eq. (17) is based on the weak selection level)

$$x^* \approx \frac{1}{2} + \frac{k}{8} [(A_d - A_c) - (\pi_d - \pi_c)]. \quad (17)$$

With different A_c and A_d , this paper analyses the stability of Eq. (15) when $k \rightarrow 0$ and $k \rightarrow \infty$, as it is more challenging to be analysed when $k \in (0, +\infty)$.

If $A_c = A_d$, there are **Theorem 2.1** and **Theorem 2.2** about the stability of Eq. (15) under four cases.

Theorem 2.1. Suppose that $\frac{\theta_d}{\theta_c + \theta_d} < 0.5$. $(x^*, 1)$ is a stable point when $x^* > \frac{\theta_d}{\theta_c + \theta_d}$. And $(\frac{\theta_d}{\theta_c + \theta_d}, n^*)$ is unstable, if $\pi_d < \pi_c$.

Theorem 2.2. Suppose that $\frac{\theta_d}{\theta_c + \theta_d} > 0.5$. $(x^*, 0)$ is a stable point when $x^* < \frac{\theta_d}{\theta_c + \theta_d}$. And $(\frac{\theta_d}{\theta_c + \theta_d}, n^*)$ is unstable, if $\pi_d > \pi_c$.

If $A_c \neq A_d$, there is **Theorem 2.3** about the stability of Eq. (15) under four cases.

Theorem 2.3. If $x^* > \frac{\theta_d}{\theta_c + \theta_d}$, $(x^*, 1)$ is a stable point. While $x^* < \frac{\theta_d}{\theta_c + \theta_d}$, $(x^*, 0)$ is a stable point. And $(\frac{\theta_d}{\theta_c + \theta_d}, n^*)$ is unstable, if $\pi_d - A_d < \pi_c - A_c$ and $\frac{\theta_d}{\theta_c + \theta_d} < 0.5$, or $\pi_d - A_d > \pi_c - A_c$ and $\frac{\theta_d}{\theta_c + \theta_d} > 0.5$.

Proofs of these theorems are discussed in supplementary Appendix 2.

To validate the persuasiveness of the aforementioned theory, the results of Eq. (15) under Case 3 are illustrated in Fig. 3. Where $\alpha = 2$, $D_{r0} = -0.05$, $D_{g0} = -0.15$, $D_{r1} = 0.25$, $D_{g1} = 0.15$, $\epsilon = 0.1$, $k = 0.1$ and $A_c = A_d = 0.1$.

Fig. 3 shows that aspiration dynamics does not exhibit the oscillation, unlike the replicator dynamics (as shown in Fig. 1). The aspiration dynamics will converge to a steady state and a stationary point will appear during the evolutionary game progresses as seen from panel (a) to (d), which implies that an appropriate level of aspiration may prevent the oscillation. And results for other scenarios are provided in supplementary Appendix 2. And the four cases are shown on below:

Case 1: $R_0 < T_0$ and $S_0 > P_0$, i.e. $D_{g0} > 0$ and $D_{r0} < 0$;

Case 2: $R_0 > T_0$ and $S_0 < P_0$, i.e. $D_{g0} < 0$ and $D_{r0} > 0$;

Case 3: $R_0 > T_0$ and $S_0 > P_0$, i.e. $D_{g0} < 0$ and $D_{r0} < 0$;

Case 4: $R_0 < T_0$ and $S_0 < P_0$, i.e. $D_{g0} > 0$ and $D_{r0} > 0$.

3. Results

In this section, the results of the environmental feedback evolution game based on replicator dynamics and aspiration dynamics are provided. We first demonstrate the stability of the equilibrium points of dynamics under different circumstances. Then, the analysis focuses on the effects of θ_c , θ_d , and α on the evolutionary outcomes.

3.1. The evolutionary outcomes on replicator dynamics

In Case 3, there is a constant oscillation when $\frac{D_{r0}}{D_{g0}} < \frac{D_{r1}}{D_{g1}}$. Therefore, we investigate the evolutionary outcomes of replicator dynamics under $\frac{D_{r0}}{D_{g0}} > \frac{D_{r1}}{D_{g1}}$, as shown in Fig. 4. The parameters of Fig. 4 are set to be $\alpha = 2$, $D_{r0} = -0.25$, $D_{g0} = -0.15$, $D_{r1} = 0.05$, $D_{g1} = 0.15$ and $\epsilon = 0.1$. And results of Case 1, Case 2 and Case 4 are discussed in Appendix A.

In Fig. 4, panels (a) and (b) express the evolutionary outcomes for x (the proportion of cooperators) and n (the environmental state) with different θ_c , respectively. While panels (c) and (d) show the evolutionary results for x and n with different θ_d , respectively. From Fig. 4 (a) and (b), it is observed that as θ_c increases, the number of cooperators is on the decline, and the tendency of the environment evolves prosperity. This points out the fact that the intensity with which cooperators improve the environment can contribute to its restoration, but this may increase the number of traitors or defectors. However, the number of cooperators will increase and the environment is becoming infertile when θ_d increases, as shown in 4 (c) and (d). The intensity of the defectors' destruction of the environment increases, the state of the environment will move towards a barren state, and the proportion of cooperators will gradually increase.

In Case 3, the stationary strategy is dominated by defectors under an enriched environmental state, which will lead to a decrease in the proportion of cooperators. As the number of defectors increases, the environment is damaged and begins to become unproductive.

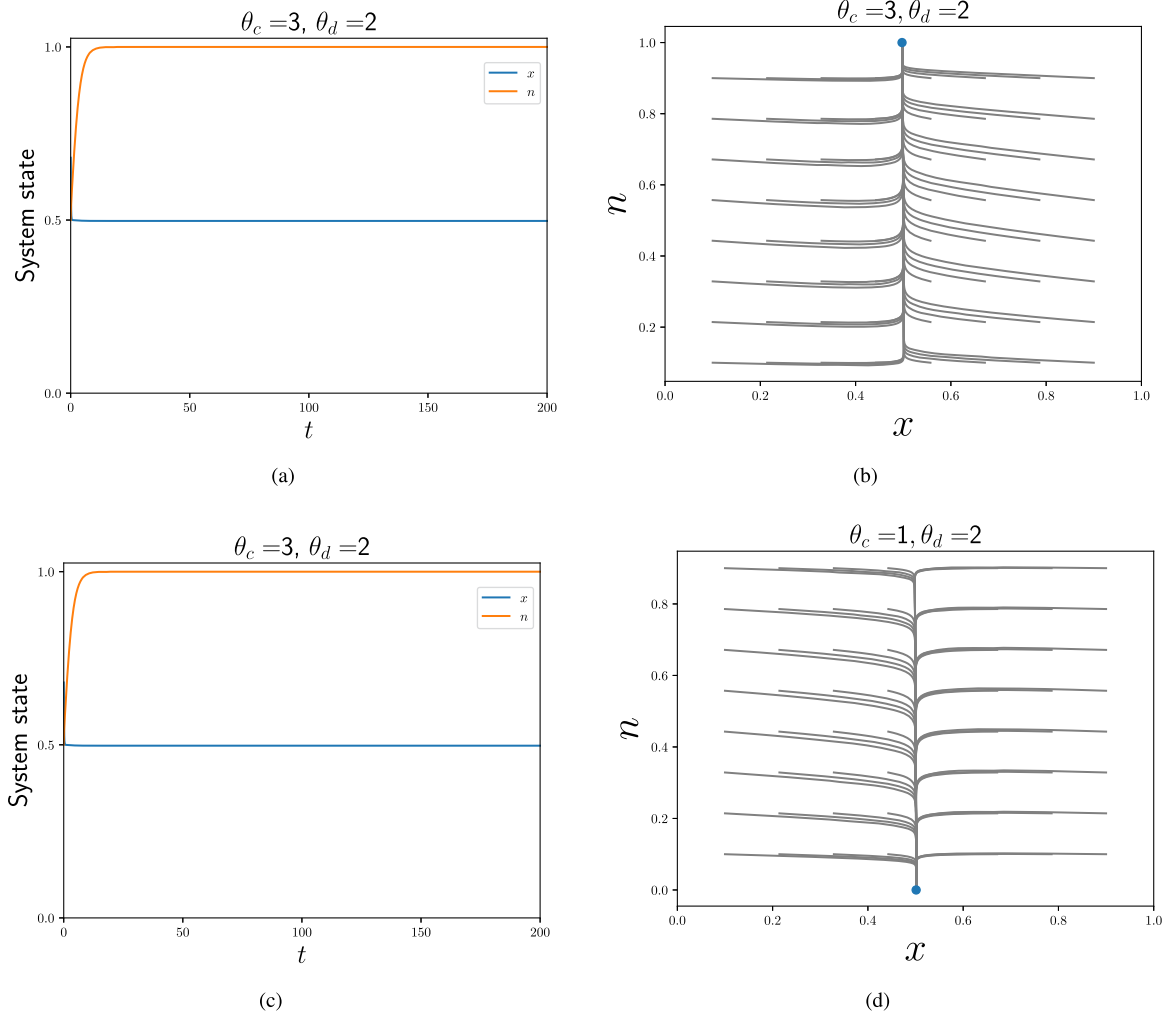


Fig. 3. The evolutionary outcomes for Eq. (15) in Case 3, where the blue dot is the stable point, x denotes the proportion of cooperators and n denotes the environmental state. In panels (a) and (b), $\frac{\theta_c}{\theta_c + \theta_d} = 0.4 < 0.5$ and $x^* \approx 0.50 > 0.4$, then $(x^*, 1)$ is a stable point, which correspond to **Theorem 2.1**. In panels (c) and (d), $\frac{\theta_c}{\theta_c + \theta_d} = 2/3 > 0.5$ and $x^* \approx 0.50 < 2/3$, then $(x^*, 0)$ is a stable point, which correspond to **Theorem 2.2**. Moreover, we can find that aspiration dynamics does not exhibit the oscillation from panel (a) to (d).

At this point, the players' strategy will gradually shift from defection to cooperation, which will increase the proportion of cooperators. Therefore, the change in the state of the environment will affect the structure of the distribution of benefits among the population, which will cause the change in the strategies of the players, and finally the environment will be altered again.

Fig. 5 (a) and (b) present the heat map of the proportion of cooperators and the environment with respect to θ_c and θ_d , respectively, and all parameters are the same as Fig. 4. From Fig. 5 (a), smaller θ_c and larger θ_d would encourage population's cooperation, and vice versa would increase the level of defection. Nevertheless, a lower θ_c and a higher θ_d would have a negative effect on the population's environment in Fig. 5 (b), and otherwise it would have a positive effect, which means that cooperators will be in the minority for rich environments. Furthermore, if $\theta_c = \theta_d$, both the environmental state of the population and the proportion of cooperators are at an intermediate level, which implies that the population may reach a relatively stable equilibrium at the moment.

Fig. 6 (a) and (b) represent the evolutionary outcomes for Eq. (5) as a function of α in Case 1 and Case 3, respectively. From Fig. 6, we can find that the environment tends to be rich in Case 3, while the outcomes under Case 1 and the proportion of cooperators in Case 3 remain intact when α changes. In other words, increasing the complexity of a population's strategic composition and environmental state may improve the environment, but it may not increase the level of cooperation. The analytical results in Section 2.1 show that $(0, 0)$ is the stability point of Eq. (5) in Case 2 and Case 4, and thus the variation of α does not alter the evolutionary outcome.

3.2. The evolutionary outcomes on aspiration dynamics

Under weak and strong selection intensities, we investigate the results of Case 3, respectively, as shown in Fig. 7 and Fig. 8. Here, the parameters are set to be $\alpha = 2$, $D_{r0} = -0.25$, $D_{g0} = -0.15$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, and $\theta_c = 2$.

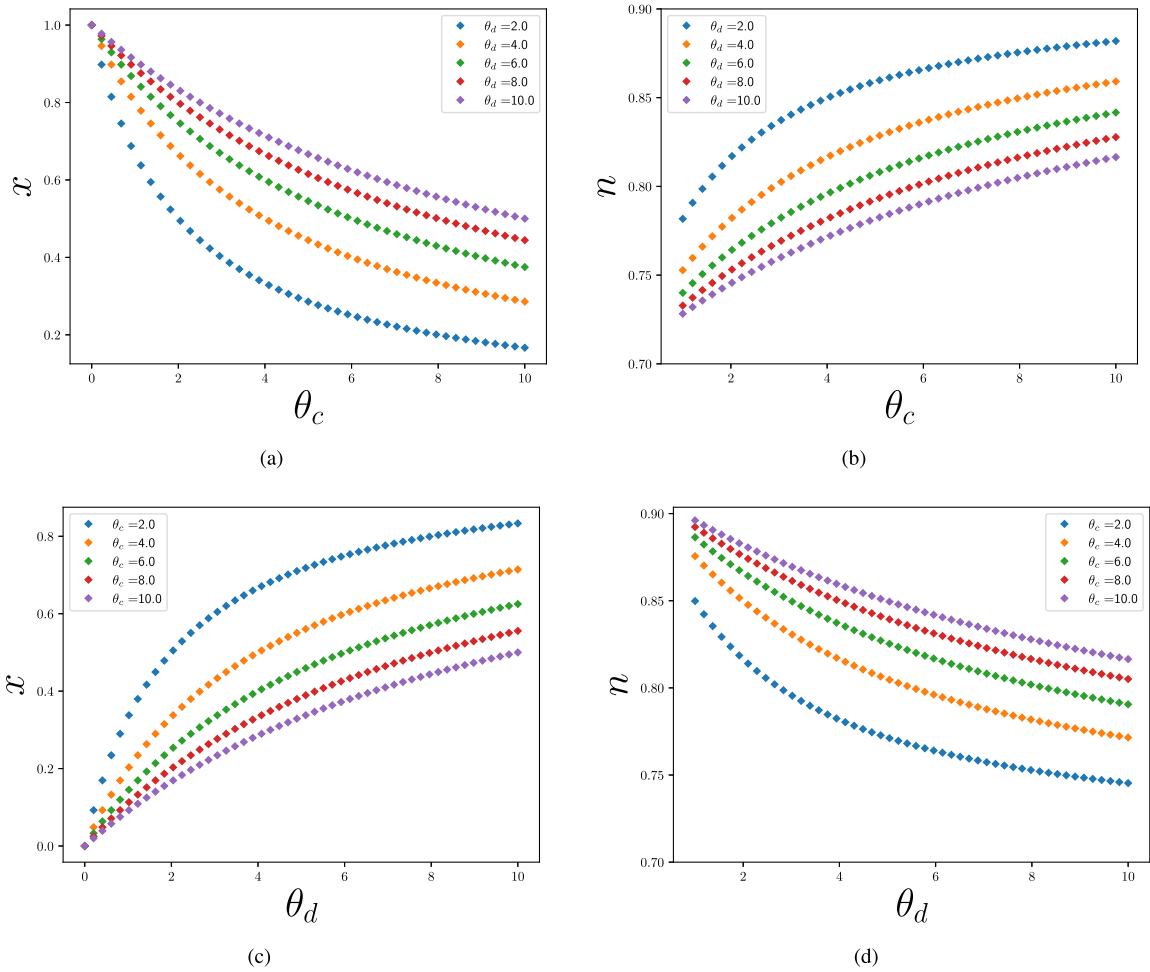


Fig. 4. The evolutionary outcomes for Eq. (5) in Case 3 with respect to θ_c and θ_d , where x denotes the proportion of cooperators and n denotes the environmental state. From **Theorem 1.3**, we can find that (x, n^*) is the stable point, where $x^* = \theta_d / (\theta_c + \theta_d)$, which means that x^* is a convex function with respect to θ_d and a concave function with respect to θ_c . And $n^* = \sqrt{(0.25 - 0.1x^*) / 0.3}$, which means that n^* is a concave function about x^* . Thus, it is easily found that the state of the environment will move towards a barren state and the proportion of collaborators will gradually increase from panel (a) to (d), if there are a smaller θ_c and a larger θ_d .

In Fig. 7, the composition of the strategy remains the same when θ_d , A_c and A_d are varied under a weak selection strength ($k = 0.1$), and it is found that the fraction of cooperators has always tended to be around 0.5 (x^* will tend to 0.5 when $k \ll 1$, as shown in Eq. (17)). Then, the environment slowly changes from replete state to deplete state as a result of the increase of θ_d , which is implied that the environment could be improved for Case 3 with a weak selection strength and a smaller θ_d .

In contrast to the weak selection strength, the proportion of cooperators in the population and the state of the environment are affected by θ_d , A_c and A_d under a strong selection strength ($k = 10$), as shown in Fig. 8. First, if $A_c = A_d$, the proportion of cooperators tends to increase from a smaller value ($x < 0.5$) to a larger value ($x > 0.5$) as θ_d continues to increase (as shown in panels (a) and (b) of Fig. 8), while the environmental state will shift from rich to poor. Then, the environment is approaching a rich state and the cooperators occupy a major position, if $A_c < A_d$. While $A_c > A_d$, the population is leaving the poor environment and the defection is the players' main strategy. In a brief, increasing A_c (A_c) means that the cooperators' (defectors') expected return is higher, the cooperators (defectors) will be replaced by defectors (cooperators) when the actual payoff of the cooperators (defectors) may be harder to achieve. Thus, A_d can be raised appropriately to facilitate the development of cooperation.

Fig. 9 illustrates the heat map of the proportion of cooperators and the environment with respect to θ_c and θ_d . Under a weak selection strength ($k = 0.1$), the proportion of cooperators is always close 0.5 even for different θ_c and θ_d . While environmental state will change as θ_c and θ_d are changed, and it keeps switching between rich and poor state. According to **Theorem 2.1** and **Theorem 2.2**, if $x^* > \frac{\theta_d}{\theta_c + \theta_d}$, the state of the environment will be in a state of abundance and vice versa, as shown in Fig. 9 (b). With a strong selection strength ($k = 10$), if the environmental state of the population is similar to a state of affluence, the number of cooperators is lower than that of defectors, while conversely, the number of cooperators is higher than that, which can be observed in Fig. 9 (c) and (d). Meanwhile, the results of Case 1, Case 2 and Case 4 have been considered in supplementary Appendix 2.

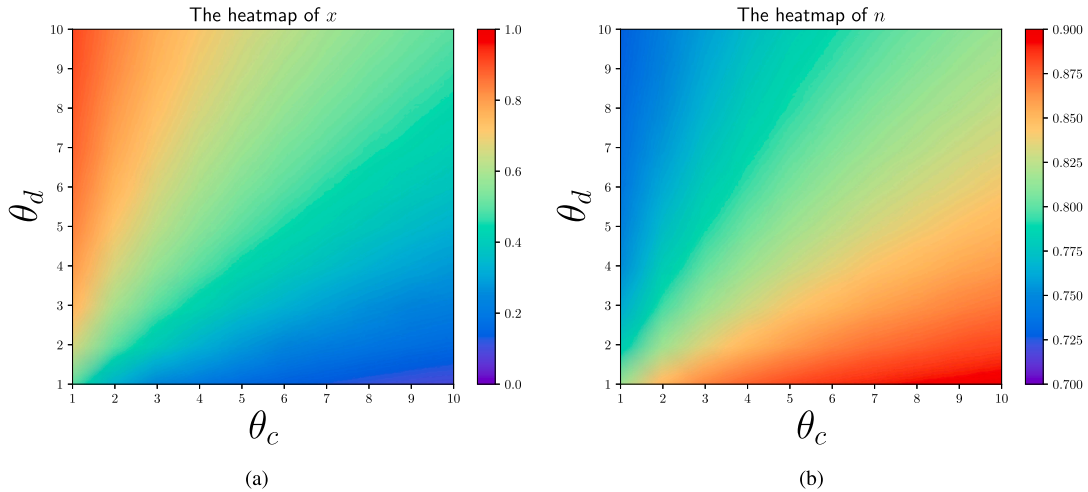


Fig. 5. The heat map of the proportion of cooperators and the environment with respect to θ_c and θ_d , where x denotes the proportion of cooperators and n denotes the environmental state. Panels (a) and (b) show that cooperators will be in the minority under a smaller θ_d and larger θ_c , while the environment will tend to be a rich state. Interestingly, the environment is always in a better state in panel (b) ($n > 0.7$).

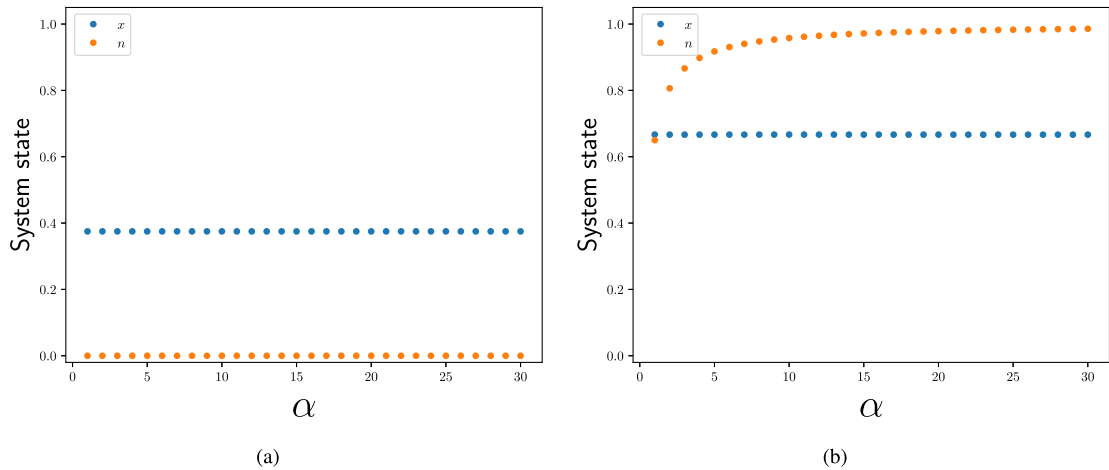


Fig. 6. The evolutionary outcomes of Eq. (5) as a function of α . Among them, panels (a) and (b) denote the evolutionary results of Case 1 and Case 3, respectively. Where x denotes the proportion of cooperators and n denotes the environmental state. In panels (a) and (b), it is easy to observe that x is not affected as α increases. On the other hand, the environment is restored to a state of abundance with a larger α in panel (b), while it is in a poor state and there is no effect on panel (a). The parameters are $\alpha = 2$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, $\theta_c = 1$ and $\theta_d = 2$, (a) $D_{g0} = 0.25$ and $D_{r0} = -0.15$, (b) $D_{g0} = -0.25$ and $D_{r0} = -0.05$.

From panel (a) to (d), it is found that there are no fluctuations in the curve of x . Meanwhile, the environment will tend to a replete state in panels (b) and (c), and it is in a deplete state in panels (a) and (d).

Fig. 10 depicts the evolutionary outcomes of Eq. (15) about α under four cases. In Case 1 (as shown in Fig. 10 (a)), changes of α do not significantly alter either the proportion of cooperators or the state of the environment. And the environment tends to be rich and the population's strategic structure has not changed as α increases in Case 2 (as shown in Fig. 10 (b)). The results for Case 3 (as shown in Fig. 10 (c)) is the same as those under Case 2. Interestingly, the outcomes in Case 4 are similar to Case 1, but the proportion of cooperators of Case 4 is significantly lower. Because populations may have the ability to adapt to environmental changes and maintain the stability of their strategic structure, this ability is a key that allows them to remain competitive in complex and changing environments.

4. Conclusions

In this paper, we examine the mutual feedback effects among the environmental state and individual strategy. In order to bring the relationships between the environment and individual strategy closer to reality, we expand the linear combination of two different payoff matrices into a nonlinear combination to analyse the complex relationship based on the game model proposed by Weitz et al. [42]. Then, replicator dynamics and aspiration dynamics are employed to obtain the evolutionary outcomes of the population, respectively. The main results are outlined as follows.

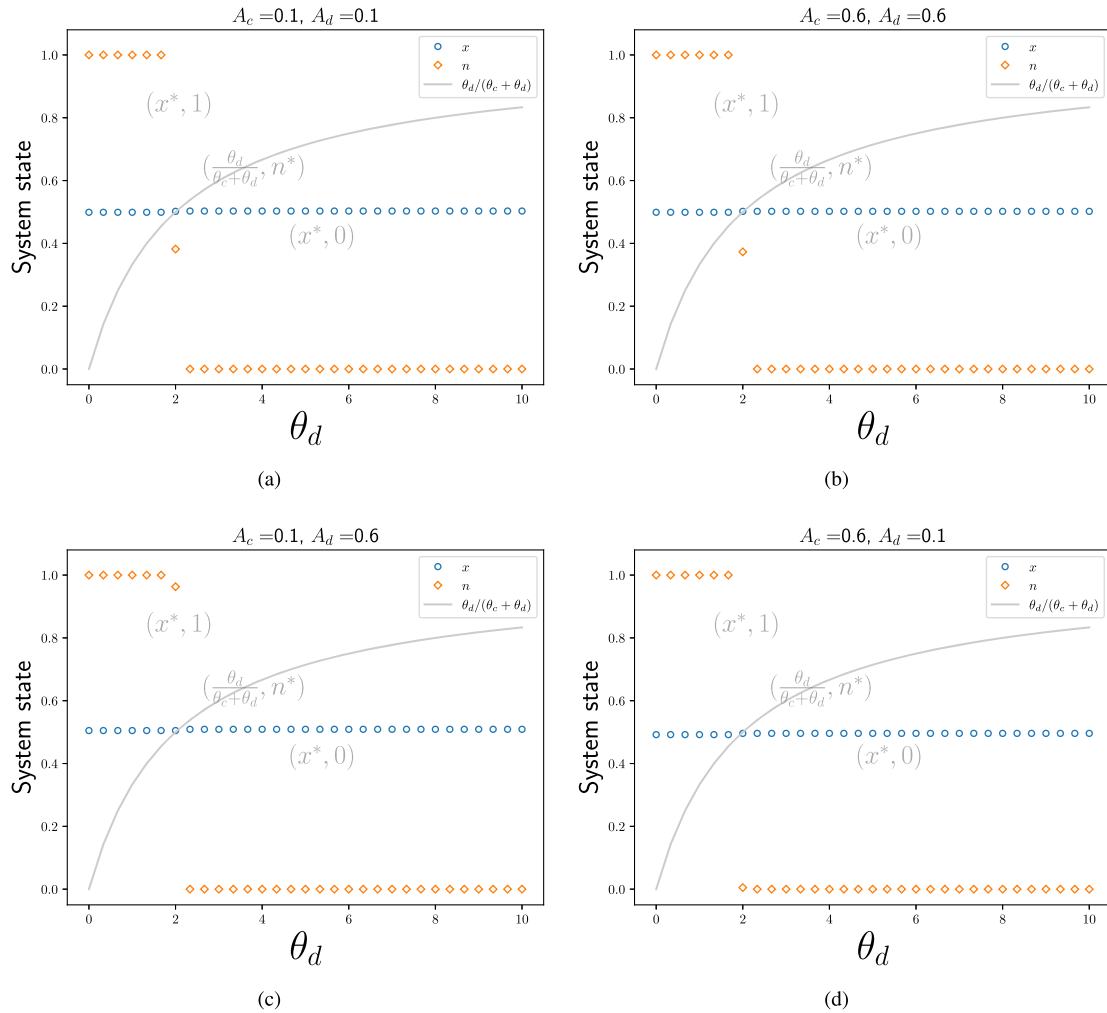


Fig. 7. The evolutionary results of Eq. (15) when a weak selection strength is assumed ($k = 0.1$). Panels (a), (b), (c) and (d) denote the evolutionary results with different A_c and A_d , respectively. (a) $A_c = A_d = 0.1$, (b) $A_c = A_d = 0.6$, (c) $A_c = 0.1$ and $A_d = 0.6$, (d) $A_c = 0.6$ and $A_d = 0.1$. If x and n are situated above the curve of $\theta_d / (\theta_c + \theta_d)$, Eq. (15) converges towards $(x^*, 1)$. Conversely, if they lie below the curve, Eq. (15) converges towards $(x^*, 0)$. In the event that x and n reside precisely on the curve, Eq. (15) converges to (x^*, n^*) .

In replicator dynamics, the evolutionary results of Eq. (5) under four cases (outlined in Section 2.1) are investigated. In Case 1, the cooperators are going to be in a weak position, no matter how θ_c and θ_d change (as shown in Fig. 11 of Appendix A). Which means that the efforts of cooperators may not promote the cooperation in the population even with a smaller θ_d , and the individual strategy is dominated by defection. For the environment, it is in a deplete state when the defector destroys the environment with greater intensity (θ_d). Therefore, reducing the intensity of environmental destruction seems to be somewhat more important than increasing the intensity of environmental improvement for Case 1. In the case of plastic waste pollution, it is particularly important to reduce the intensity of environmental damage. While all players chose defection in Case 2 (as shown in Fig. 12 of Appendix A), and the environment remains a poor state even with $\theta_d = 0$. These results show that the benefits of defection may be far greater than cooperation and the environment may be a burden for the cooperators, prompting a shift from cooperators to defectors. Consequently, it may be that the only way to break this situation is to change the players' initial gains in the population. Then in Case 3, the population will experience the persistent oscillations when $D_{r0}/D_{g0} < D_{r1}/D_{g1}$, resulting in a depletion of resources, but it can be avoided by adjusting the initial payoff of the players. Therefore, the development of reasonable resource management and conservation strategies can prevent the waste of public resources. If $D_{r0}/D_{g0} > D_{r1}/D_{g1}$, the proportion of cooperators will begin to decrease and the environment will tend to become richer as the intensity (θ_d) of environmental improvements increases (as shown in Fig. 5). Interestingly, the only game model of the population is the Prisoner's Dilemma in Case 4 ($D_{r0} > 0$, $D_{g0} > 0$, $D_{r1} > 0$ and $D_{g1} > 0$). As evolution progresses, cooperators are gradually transformed into defectors until they eventually become extinct, and the state of the environment is rapidly damaged until it becomes depleted (as shown in Fig. 13 of Appendix A). Finally, we explored the effect of α (the nonlinear coefficient about the environment and payoff matrix) on the evolutionary outcomes. It is found that an increase in α can drive the environment towards a state of affluence in Case 3, and it has little effect on other cases.

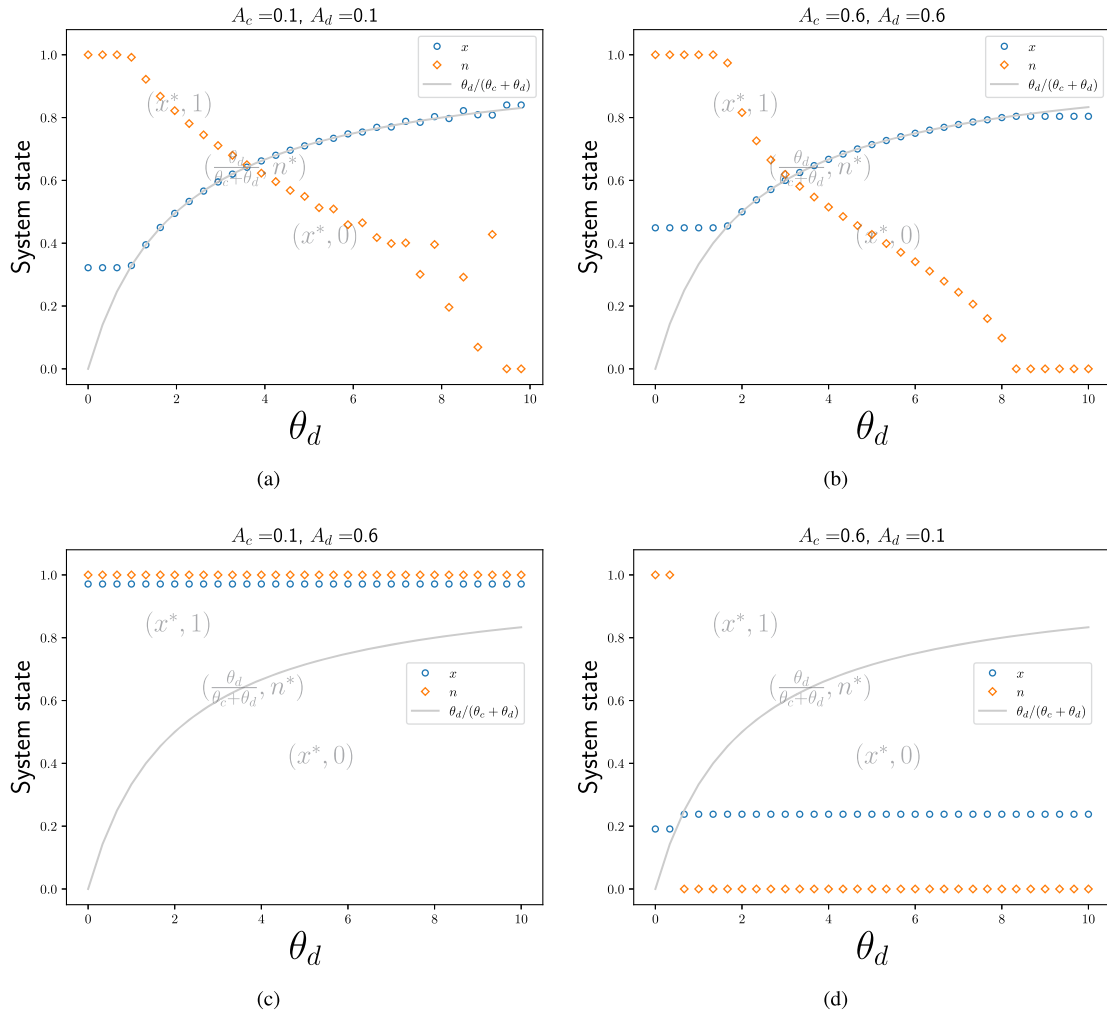


Fig. 8. The evolutionary results of Eq. (15) when a strong selection strength is assumed ($k = 10$). Where (a), (b), (c) and (d) denote the evolutionary results with different A_c and A_d , respectively. (a) $A_c = A_d = 0.1$, (b) $A_c = A_d = 0.6$, (c) $A_c = 0.1$ and $A_d = 0.6$, (d) $A_c = 0.6$ and $A_d = 0.1$. Similar to Fig. 6, Eq. (15) converges towards $(x^*, 1)$ when the point x and n are located above the curve of $\theta_d / (\theta_c + \theta_d)$. If it is below the curve, Eq. (15) converges towards $(x^*, 0)$. And Eq. (15) converges to (x^*, n^*) , if x and n are exactly on the curve.

In addition, we explore evolution outcomes of Eq. (15) with the different levels of aspiration in the aspiration dynamics. In Case 1, it is shown that environment cannot reach a rich state under a larger θ_d (as shown in Fig. 2.2 of supplementary Appendix 2). Meanwhile, θ_c and θ_d have a minimal impact on the strategic composition of the population. In Case 2, we find that population require a smaller θ_d and a bigger θ_c to achieve the replete environmental state, but there is a lower number for cooperators than defectors (as shown in Fig. 2.3 of supplementary Appendix 2). Accordingly, if we attempt to increase the level of cooperation among populations may have a negative influence on the environment. Different from replicator dynamics, permanent oscillations are not found in Case 3, and the environment and strategies of the population converge towards a stable state as time evolves (as shown in Fig. 3). In other words, oscillations can also be avoided by aspiration dynamics. As with replicator dynamics, it is easy to perceive that Prisoner's Dilemma is the unique game model in Case 4, but there is still a small number of cooperators in the population due to the aspiration levels, rather than all of them becoming defectors. Moreover, increasing the level of aspiration for defectors may disrupt the population environment under a weak selection strength. Ultimately, compared to replicator dynamics, we find that α has a positive impact on the environment under Case 2 and Case 3 for aspiration dynamics.

In summary, it tends to contribute to the population's environment to reach the state of affluence with a larger θ_c and a lower θ_d , but this may cause more defectors. As a consequence, it may be necessary to exercise caution regarding the transformation of cooperators into defectors under a rich environmental state. As well, it is important to note that the environment is going to a replete state as increasing α . Finally, our work highlights the interplay between environmental feedback and individual strategic choices, and we wish that the current work can inspire further research into the mechanisms of cooperation with evolutionary dynamics.

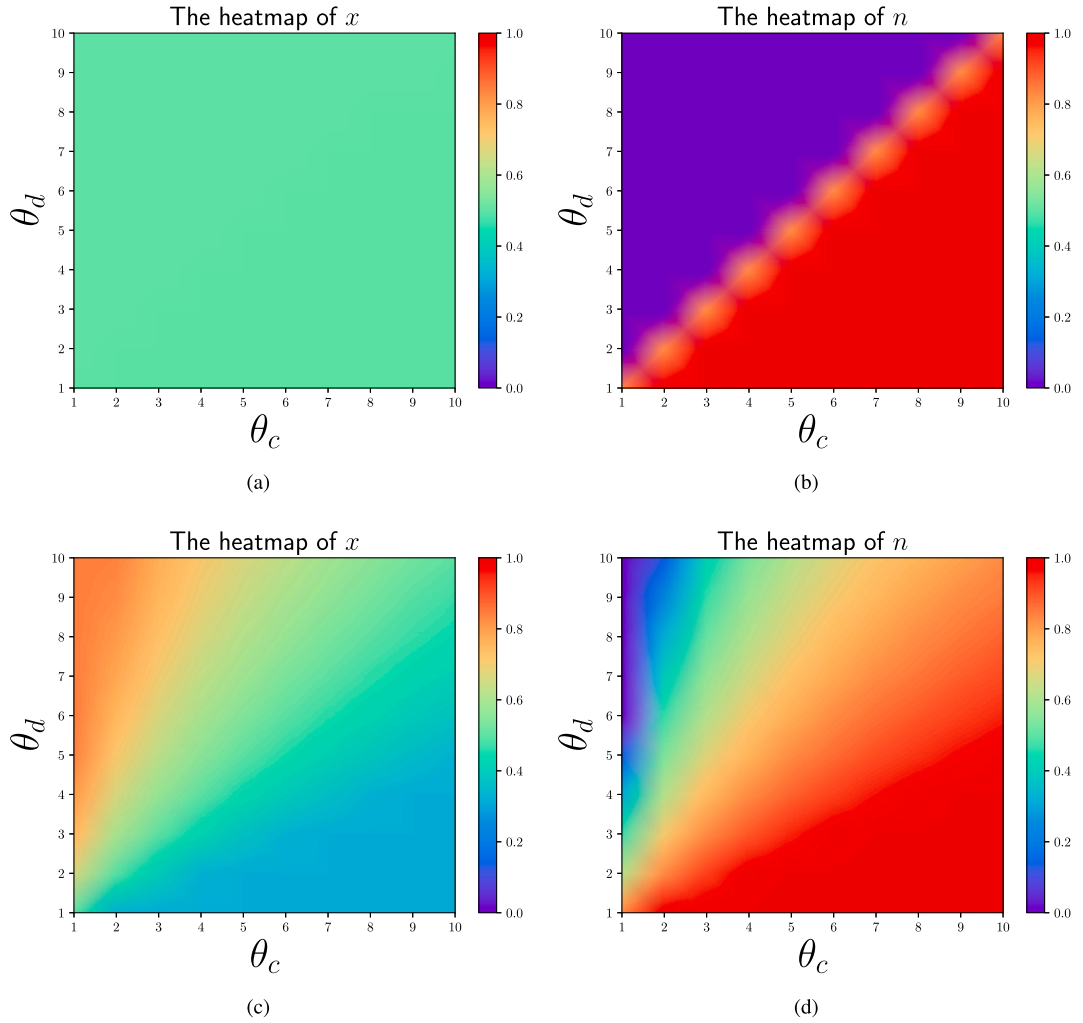


Fig. 9. The heat map of the proportion of cooperators and the environment with respect to θ_c and θ_d . (a) the heat map of x in $k = 0.1$, (b) the heat map of n in $k = 0.1$, (c) the heat map of x in $k = 10$, (d) the heat map of n in $k = 10$. In panel (a), x tends to 0.5 even with different θ_c and θ_d . According to **Theorem 2.1** and **Theorem 2.2**, n tends to 0 when $\theta_d / (\theta_c + \theta_d) > 0.5$ in panel (b), and n tends to 0 when $\theta_d / (\theta_c + \theta_d) < 0.5$. In panels (c) and (d), the environment will be in a rich state with a smaller θ_d and a larger θ_c , while defectors will be dominant. The parameters are set to be $\alpha = 2$, $D_{r0} = -0.15$, $D_{g0} = -0.25$, $D_{r1} = 0.15$, $D_{g1} = 0.05$, $\varepsilon = 0.1$, $A_c = A_d = 0.1$, $k = 0.1$ in (a) and (b), $k = 10$ in (c) and (d).

Data availability

No data was used for the research described in the article.

Acknowledgements

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Appendix A. Proofs for environmental feedback evolutionary games based on replicator dynamics

In this section we show that replicator dynamics with nonlinear correlation between environment and strategies (payoff matrix) will convergence to an equilibrium point. The payoff matrix is shown on Eq. (3). Then we consider these cases as follow:

- Case 1: $R_0 < T_0$ and $S_0 > P_0$, i.e. $D_{g0} > 0$ and $D_{r0} < 0$;
- Case 2: $R_0 > T_0$ and $S_0 < P_0$, i.e. $D_{g0} < 0$ and $D_{r0} > 0$;
- Case 3: $R_0 > T_0$ and $S_0 > P_0$, i.e. $D_{g0} < 0$ and $D_{r0} < 0$;
- Case 4: $R_0 < T_0$ and $S_0 < P_0$, i.e. $D_{g0} > 0$ and $D_{r0} > 0$.

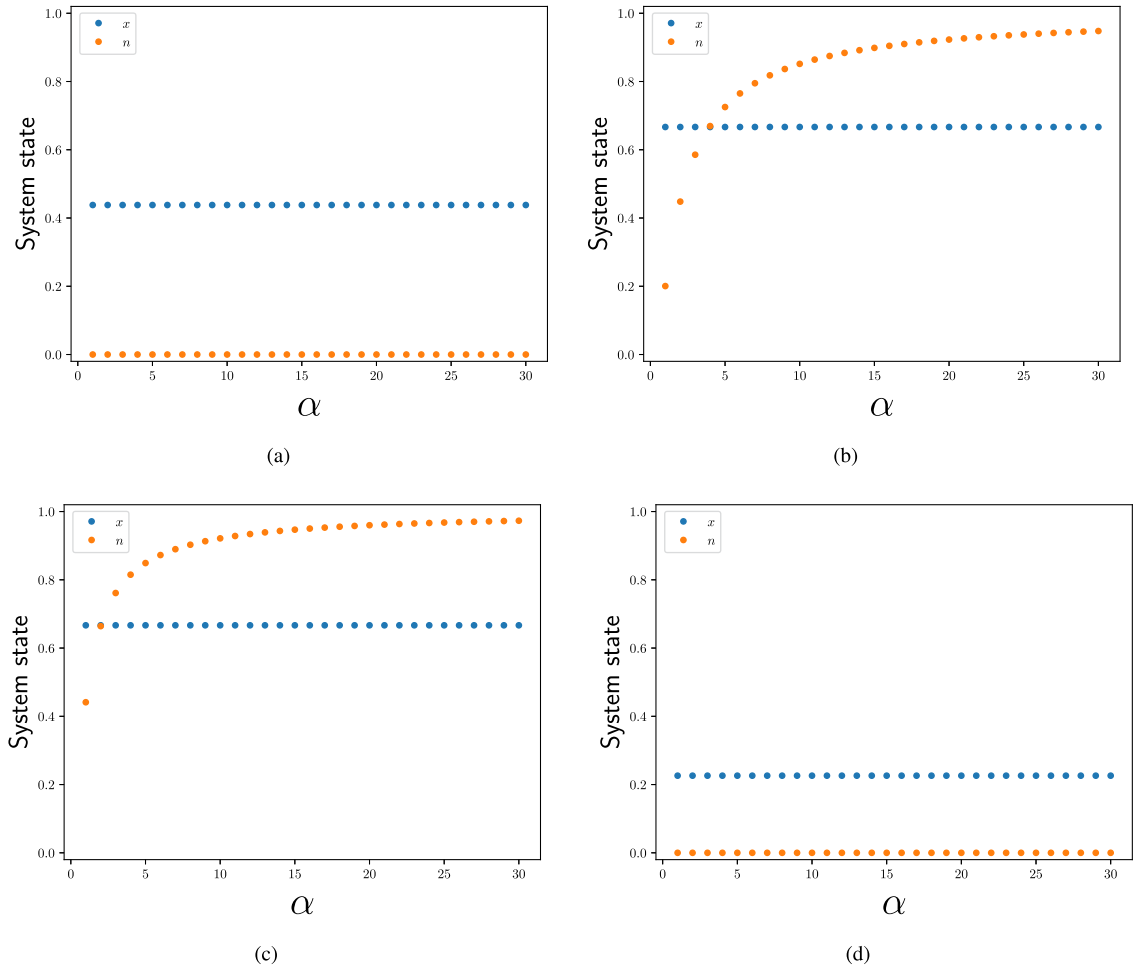


Fig. 10. The evolutionary outcomes of Eq. (15) as a function of α . Where (a), (b), (c) and (d) denote the evolutionary results of Case 1, Case 2, Case 3 and Case 4, respectively. According to Theorem 2.2, we can find that $(\frac{\theta_d}{\theta_c + \theta_d}, 0)$ is a stable point in panels (b) and (c), so the proportion of cooperators will remain unchanged, which means that the strategy structure irrelevant to the complexity of the environment at this moment. While in panels (a) and (d), $(x^*, 0)$ is a stable point. And we can find that x^* will be unchanged as α changes from Eq. (17), so the proportion of cooperators will also stay the same. Therefore, the strategy structure irrelevant to the complexity of the environment. (a) $D_{g0} = 0.25$ and $D_{r0} = -0.15$, (b) $D_{g0} = -0.25$ and $D_{r0} = 0.15$, (c) $D_{g0} = -0.25$ and $D_{r0} = -0.15$, (d) $D_{g0} = 0.25$ and $D_{r0} = 0.15$. Other parameters are set to be $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, $k = 10$, $A_c = A_d = 0.1$, $\theta_c = 1$ and $\theta_d = 2$.

Without loss of generality, we set $\varepsilon = 1$ to explore the dynamics in Eq. (5). So, Eq. (5) can be rewritten as follows:

$$\begin{cases} \varepsilon \dot{x} = x(1-x)(\pi_c - \pi_d) \\ \dot{n} = n(1-n)[(\theta_c + \theta_d)x - \theta_d] \end{cases} \quad (18)$$

Note that $p(x, n) = \dot{x}$, $q(x, n) = \dot{n}$, and $g(x, n) = (\pi_c - \pi_d)$, so we have:

$$\begin{cases} p(x, n) = x(1-x)(\pi_c - \pi_d) \\ q(x, n) = n(1-n)[(\theta_c + \theta_d)x - \theta_d] \end{cases} \quad (19)$$

From Eq. (19), there are 4 border fixed points, which are (0, 0), (0, 1), (1, 1) and (1, 0), and an internal fixed point, (x^*, n^*) , where

$$x^* = \frac{\theta_d}{\theta_c + \theta_d} \text{ and } n^* = \left[\frac{x^*(D_{g0} - D_{r0}) + D_{r0}}{(1-x^*)(D_{r0} - D_{r1}) - x^*(D_{g1} - D_{g0})} \right]^{\frac{1}{\alpha}}.$$

The Jacobian matrix of Eq. (19) as follows:

$$J = \begin{bmatrix} \frac{\partial p}{\partial x} & \frac{\partial p}{\partial n} \\ \frac{\partial q}{\partial x} & \frac{\partial q}{\partial n} \end{bmatrix}, \quad (20)$$

where

$$\begin{cases} \frac{\partial p}{\partial x} = (1-2x)g + x(1-x)\frac{\partial g}{\partial x} \\ \frac{\partial p}{\partial n} = x(1-x)\frac{\partial g}{\partial n} \\ \frac{\partial q}{\partial x} = n(1-n)(\theta_c + \theta_d) \\ \frac{\partial q}{\partial n} = n(1-2n)[\theta_c x - \theta_d(1-x)] \end{cases} \quad (21)$$

The Jacobian matrices of (0,0), (0,1), (1,0) and (1,1) are:

$$\begin{cases} J(0,0) = \begin{bmatrix} -D_{r0} & 0 \\ 0 & -\theta_d \end{bmatrix} \\ J(0,1) = \begin{bmatrix} -D_{r1} & 0 \\ 0 & \theta_d \end{bmatrix} \\ J(1,0) = \begin{bmatrix} D_{g0} & 0 \\ 0 & \theta_c \end{bmatrix} \\ J(1,1) = \begin{bmatrix} D_{g1} & 0 \\ 0 & -\theta_c \end{bmatrix} \end{cases} \quad (22)$$

Case 1: $R_0 < T_0$ and $S_0 > P_0$, i.e. $D_{g0} > 0$ and $D_{r0} < 0$.

Theorem 1.1. (0,0), (0,1), (1,0) and (1,1) are unstable. And $(x_m, 0)$ is a stable point, if $x_m < \frac{\theta_d}{\theta_c + \theta_d}$. If $x_m > \frac{\theta_d}{\theta_c + \theta_d}$, $(x_m, 0)$ is unstable and (x^*, n^*) is a stable point, $n^* \in (0, 1)$.

Proof of Theorem 1.1. From Eq. (22), it is easy to perceive that $J(0,0)$, $J(0,1)$, $J(1,0)$ and $J(1,1)$ have eigenvalues with positive real parts, so these points are unstable. When $n = 0$, there is a mixed Nash equilibrium at $x_m \in (0, 1)$ in Eq. (3), where

$$x_m = \frac{D_{r0}}{D_{r0} - D_{g0}}. \quad (23)$$

The Jacobian matrix of $(x_m, 0)$ is shown as below.

$$J(x_m, 0) = \begin{bmatrix} \frac{D_{g0}D_{r0}}{D_{r0} - D_{g0}} & 0 \\ 0 & (\theta_c + \theta_d)x_m - \theta_d \end{bmatrix}. \quad (24)$$

From Eq. (24), there are two eigenvalues, namely $\lambda_1 = \frac{D_{g0}D_{r0}}{D_{r0} - D_{g0}}$ and $\lambda_2 = (\theta_c + \theta_d)x_m - \theta_d$. Because $D_{r0} < 0$ and $D_{g0} > 0$, then we can obtain that $\lambda_1 < 0$. And $\lambda_2 < 0$ when $x_m < \frac{\theta_d}{\theta_c + \theta_d}$. So, it is easily found that $(x_m, 0)$ is a stable point, if $x_m < \frac{\theta_d}{\theta_c + \theta_d}$, and $(x_m, 0)$ is unstable if $x_m > \frac{\theta_d}{\theta_c + \theta_d}$.

If $x_m > \frac{\theta_d}{\theta_c + \theta_d}$, the Jacobian matrix of (x^*, n^*) is shown below:

$$J(x^*, n^*) = \begin{bmatrix} \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial x}(x^*, n^*) & \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial n}(x^*, n^*) \\ n^*(1 - n^*)(\theta_c + \theta_d) & 0 \end{bmatrix}. \quad (25)$$

The eigenvalues of $J(x^*, n^*)$ are:

$$\lambda = \frac{J(x^*, n^*)_{11}}{2} \pm \sqrt{\left(\frac{J(x^*, n^*)_{11}}{2}\right)^2 + J(x^*, n^*)_{12} \times J(x^*, n^*)_{21}}, \quad (26)$$

where, $J(x^*, n^*)_{11} = \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial x}(x^*, n^*)$, $J(x^*, n^*)_{12} = \frac{\theta_c \theta_d}{(\theta_c + \theta_d)^2} \frac{\partial g}{\partial n}(x^*, n^*)$ and $J(x^*, n^*)_{21} = n^*(1 - n^*)(\theta_c + \theta_d)$.

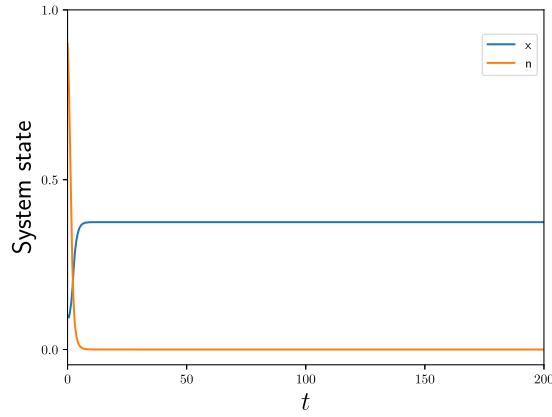
In $g(x, n)$, we have

$$\begin{aligned} g(x, n) &= \pi_c - \pi_d \\ &= [D_{r0} - n^\alpha(D_{r0} - D_{r1})](x - 1) - x[D_{g0} - n^\alpha(D_{g0} - D_{g1})]. \end{aligned} \quad (27)$$

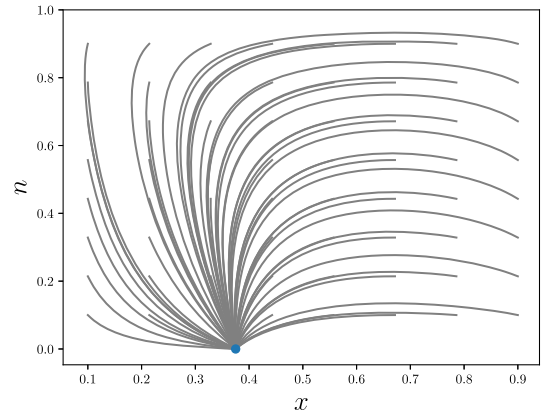
So,

$$\frac{\partial g}{\partial x} = [D_{r0} - n^\alpha(D_{r0} - D_{r1})] - [D_{g0} - n^\alpha(D_{g0} - D_{g1})], \quad (28)$$

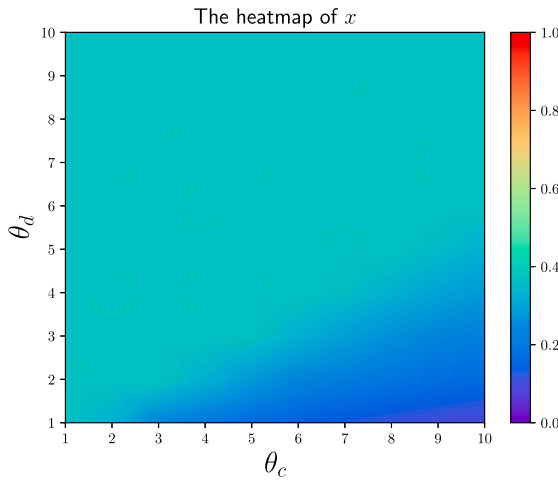
because $g(x^*, n^*) = 0$ and $x^* = \frac{\theta_d}{\theta_c + \theta_d}$, then



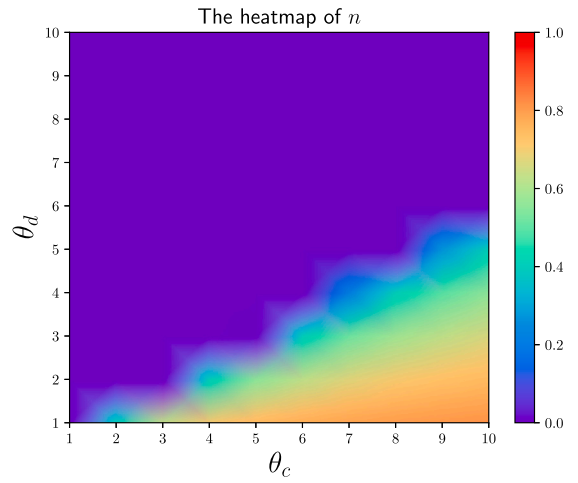
(a)



(b)



(c)



(d)

Fig. 11. The evolutionary results of replicator dynamics in Case 1. (a) time evolution. (b) phase diagram. (c) the heat map of x . (d) the heat map of n . Where the blue dot is the stable point. And $\alpha = 2$, $D_{r0} = -0.15$, $D_{g0} = 0.25$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, $\theta_c = 1$ and $\theta_d = 2$.

$$-\theta_c [D_{r0} - (n^*)^\alpha (D_{r0} - D_{r1})] = \theta_d [(D_{g1} - D_{g0})(n^*)^\alpha + D_{g0}]. \quad (29)$$

From Eq. (28) and Eq. (29),

$$\begin{aligned} \frac{\partial g}{\partial x}(x^*, n^*) &= \frac{\theta_c + \theta_d}{\theta_d} [D_{r0} - (n^*)^\alpha (D_{r0} - D_{r1})] \\ &= -\frac{\theta_c + \theta_d}{\theta_d} [D_{g0} + (n^*)^\alpha (D_{g1} - D_{g0})], \end{aligned} \quad (30)$$

because $D_{r0} < 0$ and $D_{r1} > 0$, then $\frac{\partial g}{\partial x}(x^*, n^*) < 0$, namely $J(x^*, n^*)_{11} < 0$.

While $g(x^*, n^*) = 0$ and $n^* = \left(\frac{x^*(D_{g0} - D_{r0}) + D_{r0}}{(1-x^*)(D_{r0} - D_{r1}) - x^*(D_{g1} - D_{g0})} \right)^{\frac{1}{\alpha}}$, then

$$\begin{aligned} \frac{\partial g}{\partial n}(x^*, n^*) &= \alpha(n^*)^{\alpha-1} [(1-x^*)(D_{r0} - D_{r1}) - x^*(D_{g1} - D_{g0})] \\ &= \frac{\alpha}{n^*} [x^*(D_{g0} - D_{r0}) + D_{r0}], \end{aligned} \quad (31)$$

where $\alpha > 0$, $n^* \in (0, 1)$, $D_{g0} > 0$ and $D_{r0} < 0$. When $x^* < \frac{-D_{r0}}{D_{g0} - D_{r0}} = \frac{D_{r0}}{D_{r0} - D_{g0}} = x_m$, i.e. $x_m > \frac{\theta_d}{\theta_c + \theta_d}$, $\frac{\partial g}{\partial n}(x^*, n^*) < 0$, so $J(x^*, n^*)_{12} < 0$.

In addition, $J(x^*, n^*)_{21} > 0$. Then (x^*, n^*) is a stable point, if $x_m > \frac{\theta_d}{\theta_c + \theta_d}$.

Therefore, we derive **Theorem 1.1**. \square

The evolutionary results for replicator dynamics in Case 1 are shown in Fig. 11. In Fig. 11 (a) and (b), there is a mixed Nash equilibrium when the environment is poor ($n = 0$), so the dynamics in Eq. (18) will coverage to $(x_m, 0)$. Fig. 11 (c) and (d) indicate the heat map of the proportion of cooperators and the environment with respect to θ_c and θ_d . We find that proportion of cooperators is always lower than defectors as θ_c and θ_d changing in Fig. 11 (c). While a larger θ_d will destroy the environment and a higher θ_c can improve it in Fig. 11 (d), which means that cooperators are the major contributors in boosting the environmental state.

Case 2: $R_0 > T_0$ and $S_0 < P_0$, i.e. $D_{g0} < 0$ and $D_{r0} > 0$.

Theorem 1.2. $(0, 0)$ is a stable fixed point, while other border points are unstable. If $\frac{D_{r0}}{D_{r0}-D_{g0}} > \frac{\theta_d}{\theta_c+\theta_d}$, $\left(\frac{D_{r0}}{D_{r0}-D_{g0}}, 0\right)$ is unstable, and $n^* \notin (0, 1)$. If $\frac{D_{r0}}{D_{r0}-D_{g0}} < \frac{\theta_d}{\theta_c+\theta_d}$, then (x^*, n^*) is unstable.

Proof of Theorem 1.2. Because $D_{r0} > 0$ and $D_{g0} < 0$, then the eigenvalues of $J(0, 0)$ are non-negative. Thus, $(0, 0)$ is a stable point. While $(0, 1)$, $(1, 1)$ and $(1, 0)$ are unstable, because there are not positive eigenvalues in $J(1, 0)$, $J(0, 1)$ and $J(1, 1)$. Similar to Case 1, there also exists a mixed Nash equilibrium $(x_m, 0)$ when $n = 0$, where

$$x_m = \frac{D_{r0}}{D_{r0} - D_{g0}}. \quad (32)$$

Because $D_{r0} > 0$ and $D_{g0} < 0$, then $\lambda_1 = \frac{D_{g0}D_{r0}}{D_{g0}-D_{r0}} > 0$ in Eq. (24). And $\lambda_2 = (\theta_c + \theta_d)x_m - \theta_d > 0$, when $x_m > \frac{\theta_d}{\theta_c+\theta_d}$, which means that $(x_m, 0)$ is unstable. While $\frac{\theta_d}{\theta_c+\theta_d} < x_m = \frac{D_{r0}}{D_{r0}-D_{g0}}$, we have $\theta_d(D_{r0} - D_{g0}) - (\theta_c + \theta_d)D_{r0} < 0$ and $\theta_c D_{r0} + \theta_d D_{g0} > 0$, then it is easy to obtain that $n^* = \left[\frac{x^*(D_{g0}-D_{r0})+D_{r0}}{(1-x^*)(D_{r0}-D_{r1})-x^*(D_{g1}-D_{g0})} \right]^{\frac{1}{\alpha}} > 1$. So, $(x_m, 0)$ is unstable and $n^* \notin (0, 1)$, if $x_m > \frac{\theta_d}{\theta_c+\theta_d}$.

From Eq. (30), we can find that $\frac{\partial g}{\partial x}(x^*, n^*) = \frac{\theta_c+\theta_d}{\theta_d} [D_{r0} - (n^*)^\alpha (D_{r0} - D_{r1})]$, where $\theta_c > 0$, $\theta_d > 0$ and $D_{r1} > 0$. So, $[D_{r0} - (n^*)^\alpha (D_{r0} - D_{r1})]$ is the main term that determines the sign of $\frac{\partial g}{\partial x}(x^*, n^*)$. There are three scenarios (S1, S2 and S3) about D_{r0} and D_{r1} in $\frac{\partial g}{\partial x}(x^*, n^*)$.

(S1) When $D_{r0} < D_{r1}$, $D_{r0} - (n^*)^\alpha (D_{r0} - D_{r1}) > 0$, then $\frac{\partial g}{\partial x}(x^*, n^*) > 0$.

(S2) When $D_{r0} > D_{r1}$, $(n^*)^\alpha < \frac{D_{r0}}{D_{r0}-D_{r1}}$, namely $\frac{\partial g}{\partial x}(x^*, n^*) > 0$, then $\frac{\partial g}{\partial x}(x^*, n^*) > 0$.

(S3) When $D_{r0} = D_{r1}$, $\frac{\partial g}{\partial x}(x^*, n^*) = \frac{\theta_c+\theta_d}{\theta_d} > 0$.

From (S1), (S2) and (S3), we have $\frac{\partial g}{\partial x}(x^*, n^*) > 0$, then $J(x^*, n^*)_{11} > 0$. And $\theta_c D_{r0} + \theta_d D_{g0} < 0$, if $x_m < \frac{\theta_d}{\theta_c+\theta_d}$, namely $x^*(D_{g0} - D_{r0}) + D_{r0} < 0$, so $\frac{\partial g}{\partial n}(x^*, n^*) < 0$ in Eq. (31) and $J(x^*, n^*)_{12} < 0$ in Eq. (26), which means that the eigenvalues of $J(x^*, n^*)$ are greater than 0. Hence, (x^*, n^*) is unstable when $x_m < \frac{\theta_d}{\theta_c+\theta_d}$.

Therefore, we derive **Theorem 1.2**. \square

The evolutionary outcomes for replicator dynamics in Case 2 are shown in Fig. 12. The outcomes show that the proportion of cooperators converge to extinction and the state of the environment converge to poor in Fig. 12 (a) and (b). This suggests that the defectors will dominate and the environment is in a deplete state in Case 2. Moreover, the defectors have no reduction and the environment can't be restored even with a smaller θ_d and a larger θ_c , as shown in Fig. 12 (d). It may be that the only way to break this situation is to change the gains of the population.

Case 3: $R_0 > T_0$ and $S_0 > P_0$, i.e. $D_{g0} < 0$ and $D_{r0} < 0$.

The results of the evolution of the dynamics in Case 3 have been discussed in the main text. Then we will prove that the oscillations converge to an asymptotically stable heteroclinic cycle Λ . Here, we consider the planar dynamical system.

$$\begin{cases} \dot{x} = \frac{1}{\epsilon} x(1-x)g(x, n) \\ \dot{n} = n(1-n)[(\theta_c + \theta_d)x - \theta_d] \end{cases}. \quad (33)$$

The state space of the system is the unit square $X = [0, 1]^2$ with boundary ∂X (the invariant set of points on the edges). Observe that X is the intersection of four half-spaces, then $X = x \geq 0 \cap 1-n \geq 0 \cap n \geq 0 \cap 1-x \geq 0$. Suppose $x_1 = x$, $x_2 = 1-n$, $x_3 = n$ and $x_4 = 1-x$. The augmented dynamics can be written as

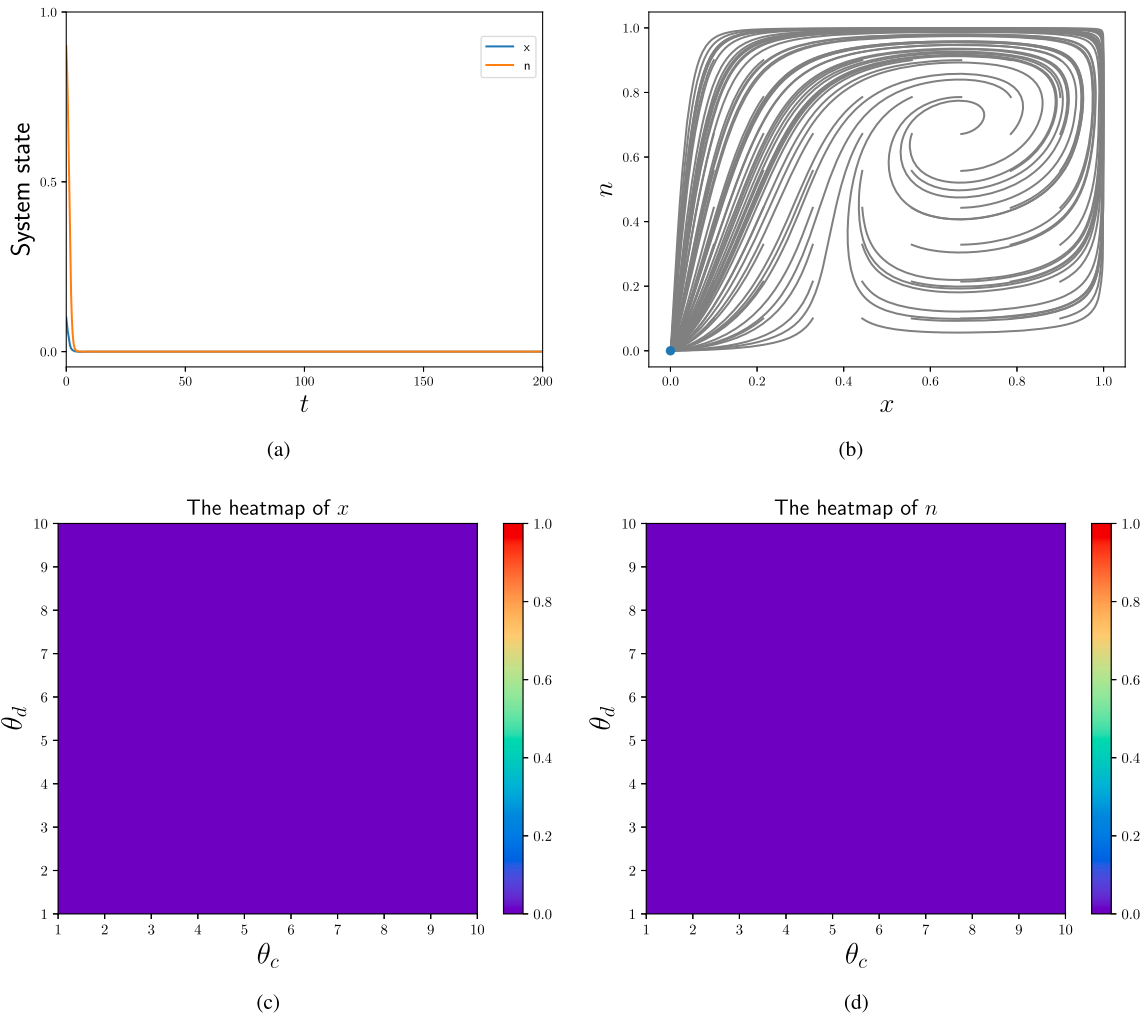


Fig. 12. The evolutionary results of replicator dynamics in Case 2. (a) time evolution. (b) phase diagram. (c) the heat map of x . (d) the heat map of n . Where the blue dot is the stable point. And $\alpha = 2$, $D_{r0} = 0.15$, $D_{g0} = -0.25$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, $\theta_c = 1$ and $\theta_d = 2$.

$$\begin{cases} x_1' = \frac{1}{\varepsilon} x_1 x_4 g(x_1, x_3) \\ x_2' = -x_2 x_3 [(\theta_c + \theta_d)x - \theta_d] \\ x_3' = x_2 x_3 [(\theta_c + \theta_d)x - \theta_d] \\ x_4' = -\frac{1}{\varepsilon} x_1 x_4 g(x_1, x_3) \end{cases} \quad (34)$$

The elements of characteristic matrix C are defined by

$$C_{ij} = \frac{\dot{x}_j}{x_j} \Big|_{z_i}, i, j = 1, 2, 3, 4, \quad (35)$$

where z_i is the i th corner fixed point $((0,0), (0,1), (1,0)$ and $(1,1))$. Then, the characteristic matrix C is below:

$$C = \begin{bmatrix} -\frac{1}{\varepsilon} D_{r0} & 0 & -\theta_d & 0 \\ -\frac{1}{\varepsilon} D_{r1} & \theta_d & 0 & 0 \\ 0 & 0 & \theta_c & \frac{1}{\varepsilon} D_{g0} \\ 0 & -\theta_c & 0 & \frac{1}{\varepsilon} D_{g1} \end{bmatrix}. \quad (36)$$

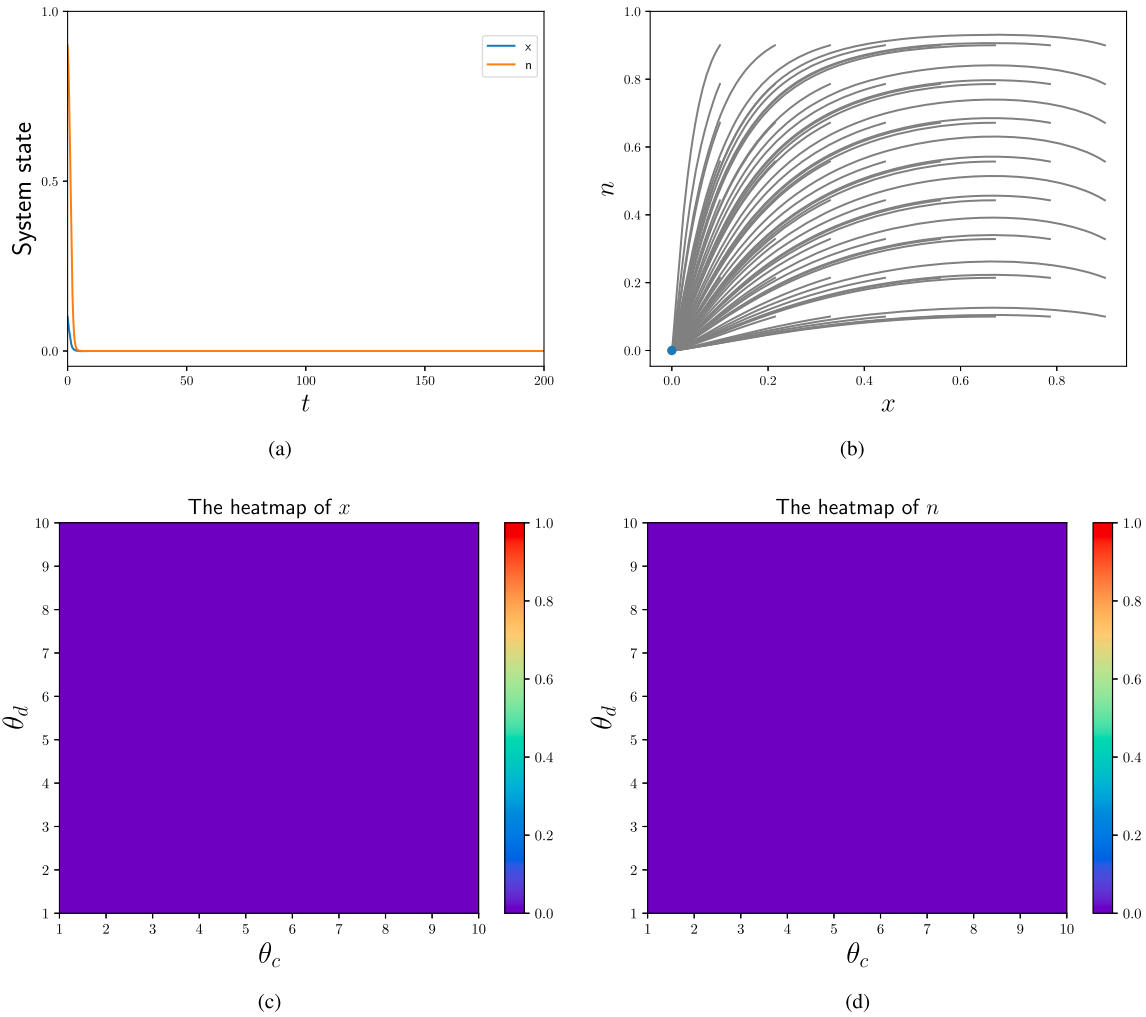


Fig. 13. The evolutionary results of replicator dynamics in Case 4. (a) time evolution. (b) phase diagram. (c) the heat map of x . (d) the heat map of n . Where the blue dot is the stable point. And $\alpha = 2$, $D_{r0} = 0.15$, $D_{g0} = 0.25$, $D_{r1} = 0.05$, $D_{g1} = 0.15$, $\varepsilon = 0.1$, $\theta_c = 1$ and $\theta_d = 2$.

In Case 3, we have $D_{r0} < 0$, $D_{g0} < 0$, $D_{r1} > 0$ and $D_{g1} > 0$. It is easily found that positive entries for C appear only on the diagonal and each row and column contains only one positive entry. Therefore, \wedge is a simple heteroclinic cycle [55]. From [42] and [55], the stability for \wedge is following from:

Theorem 1.4. Let \wedge be a simple heteroclinic cycle, which is asymptotically stable within ∂X . Then C is a square matrix (after elimination of superfluous columns) with positive entries occurring only in the main diagonal (after a suitable rearrangement of the rows or columns). Let $\det C \neq 0$. If C is not an M -matrix (at least one leading principal minor is negative), then \wedge is asymptotically stable.

Proof of Theorem 1.4. From Eq. (36), we can know that C is a square matrix with positive entries occurring only in the main diagonal, and

$$\det C = \frac{\theta_c \theta_d}{\varepsilon^2} (D_{r0} D_{g1} - D_{r1} D_{g0}). \quad (37)$$

Because $\frac{D_{r0}}{D_{g0}} < \frac{D_{r1}}{D_{g1}}$, i.e. $D_{r0} D_{g1} - D_{r1} D_{g0} < 0$, then $\det C < 0$. So, C is not an M -matrix, and \wedge is asymptotically stable.

Therefore, we derive Theorem 1.4.

Case 4: $R_0 < T_0$ and $S_0 < P_0$, i.e. $D_{g0} > 0$ and $D_{r0} > 0$.

Because $D_{r0} > 0$, $D_{g0} > 0$, $D_{r1} > 0$ and $D_{g1} > 0$, then the only game model of the population is the Prisoner's Dilemma in Case 4. Therefore, defection will be a major strategy for individual selection. From Eq. (22), we can know that $(0,0)$ is a stable point.

Additionally, (0, 1), (1, 1) and (1, 0) are unstable. The evolutionary results of replicator dynamics in Case 4 are shown in Fig. 13. As evolution progresses, the cooperators will decrease until it disappears ($x = 0$), and eventually the population's environmental state will become infertile ($n = 0$).

Appendix B. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.amc.2024.128990>.

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