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A social monitoring mechanism for third-party judges promotes cooperation in evolutionary games

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ABSTRACT

Corruption of third-party judges seriously undermines the level of cooperation. Without intervention, more corruptors and defectors would emerge, disrupting social harmony. Therefore, introducing an anti-corruption mechanism is crucial for the evolution of cooperation. In this paper, we propose a social monitoring mechanism to monitor third-party judges so that their payoffs are affected by the proportions of cooperators. Monte Carlo simulations on periodic boundary lattices. The results show that the social monitoring mechanism is effective in promoting cooperation and inhibiting corruption, and enhances the effectiveness of zealots in promoting cooperation. This facilitation effect is not only manifested in the Prisoner's Dilemma Game but also in the Snowdrift Game, which confirms the robustness of the results. Our research provides new insights for solving social dilemmas and curbing corruption.

1. Introduction

Cooperation is an important force for human development and promotes the progress of human society [1–3]. Therefore, how to maintain cooperation has aroused discussions among many scholars [4,5]. Evolutionary games have been widely used as an important mechanism to explain and maintain cooperation [5]. Evolutionary game is a mathematical model and theoretical framework for studying the evolutionary process of interaction and decision-making among individuals [6–8].

Evolutionary game models include Prisoner's Dilemma Game [9–13], Public Goods Game [14–16], Snowdrift Game [17–19], Coordination Game [20,21] and so on. Among them, the Prisoner's Dilemma Game is a typical game to solve social dilemmas, which has been widely applied and studied in various fields such as social science, biology, psychology, and other areas [22–24]. The Prisoner's Dilemma Game (PDG) is used to describe the dilemma faced by participants when making decisions. They choose either cooperation or defection at the same time and receive relevant payoffs after the interaction as follows: R for mutual cooperation, P for mutual defection, S for cooperation with the defector, and T for the successful defection of the cooperator [9]. These four payoffs satisfy $T > R > P \geq S$, which means that although cooperation is the best choice for both parties, due to the lack of trust or information asymmetry, each participant will tend to pursue personal interests and ignore the overall interests, which ultimately leads to losses for both parties [25]. In PDG, Nash equilibrium occurs when both parties choose defection because defection is optimal for oneself no matter what the other party chooses [26]. Many researchers and scholars have tried to change the tragedy of the Nash equilibrium in the prisoner's dilemma and find a solution to this dilemma [27–29]. In addition, while PDG is very valuable and significant in a variety of areas, many species exhibit altruism. For example, vampire bats share blood [30], chimpanzees adopt

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orphans [31], and monkeys groom each other [32]. This has led to an increasing discomfort with the PDG as the only model to discuss cooperative behavior [33,34]. The Snowdrift Game (SDG) is a feasible and biologically fascinating alternative [35]. It differs from PDG in that its payoffs S and P are in reverse order, i.e., $T > R > S > P$. This has fundamentally changed the situation and led to the sustainability of cooperation [35]. Therefore, in this paper, we mainly use PDG to explore whether the social monitoring mechanism can change the tragedy of Nash equilibrium in PDG. In addition, in order to test the robustness of the results, we also simulate the SDG to explore whether the level of cooperation can be facilitated in different models.

In 1992, Nowak and May proposed a spatial evolutionary game model [7]. In this model, individuals are assumed to be located in a square grid and interact through neighboring positions in the grid. The interactions between individuals can be modeled by a game, where each individual decides their behavior based on their strategy and that of the surrounding individuals. Each individual's strategy is adjusted according to the payoffs they made in the previous round of the game. Since then, many network models with different topological structures have been studied, such as scale-free networks [36,37], multi-layer networks [38,39], and random networks [40], etc. However, network structures do not always promote cooperation. For example, Konno pointed out [41] that among the three representative networks, regular, scale-free, and random, the regular network is the most conducive to cooperation, while the scale-free network supports cooperation the least and its promotion of cooperation depends on specific conditions. In addition, many other scenarios have had a significant impact on the evolution of cooperation [26,42,43], e.g., personal reputation [44–46], learning rules [10,47,48], aspiration [49,50], exit [51–53], reward and punishment [26,54–58]. Nowak proposes five reciprocity rules [42]: directional reciprocity, indirect reciprocity, kinship selection, group selection, and network reciprocity, which will articulate the conditions under which cooperation can flourish in groups.

Particularly famously, the impact of reward and punishment on the evolution of cooperation has received extensive scholarly attention. It is a method designed to motivate participants to adopt specific behaviors or avoid specific behaviors [59]. It is worth noting, however, that although punishment can promote cooperative behavior in some cases, Nowak points out [42,43] that it is not a mechanism for cooperative evolution, and all evolutionary models of punishment are grounded in underlying mechanisms, and punishment can increase the level of cooperation achieved in such models [42]. In previous mainstream studies, research scholars have only considered the role of reward or punishment in games in isolation [60–64]. In addition, many scholars studied reward or punishment that was direct, i.e., second-party [16,57]. Later, people began to study reward and punishment in conjunction with third-party [26,65]. Third-party rewarders and punishers are those who are independent of the participants, do not interact with the participants, and monitor the participants in the form of reward and punishment. However, in practice, third-party rewarders and punishers may also be corrupt. Corruption is a universal phenomenon in human society, which seriously affects the fairness of society [66]. Many scholars have studied the anti-corruption mechanism. Huang [67] et al. introduced asymmetric punishment as a means of controlling corruption, which effectively promotes cooperation. Liu [68] et al. introduced social exclusion into the public goods game, and proposed three measures for controlling corruption, respectively. Shi [69] et al. introduced a bidirectional monitoring mechanism, in which participants and third-party judges monitor each other in a two-layer network, and ultimately found that the bidirectional monitoring mechanism was effective in curbing corruption and defection. Inspired by these scholars, we introduce a new anti-corruption mechanism, i.e., a social monitoring mechanism, among third-party judges with corruption. The results of the study show that this mechanism works well in curbing corruption and defection. Our study provides new perspectives and design ideas for anti-corruption mechanisms.

In human societies, courts and police forces usually play the role of third parties in punishing defectors [70]. Citizens play the role of participants. However, the strategies of third-party agents are often affected by various external factors. Few studies have examined the impact of social monitoring on third-party agents. Where monitoring is a potentially valuable tool for strengthening the capacity of social workers to perform their duties optimally [71–73]. Many scholars have introduced monitoring mechanisms into the game. Han [74] et al. considered the quality monitoring of major manufacturers and an evolutionary game model is constructed for quality improvement between two suppliers. Shi [69] et al. considered participants and third-party agents monitoring each other in a Prisoner's Dilemma Game. Hu [75] et al. proposed a reputational incentive mechanism with public monitoring to explore the evolution of cooperation. In our study, we first introduce a two-layer network model including a participant layer and a judge layer, where the participant layer has cooperators and defectors, and the judge layer has altruistic judges and corrupt judges. Each judge in the judge layer performs point-to-point monitoring of a participant in the participant layer. It is also worth noting that we have introduced a new mechanism in the judge layer called the social monitoring mechanism (SMM). The introduction of this mechanism aims to reduce corruption in the judge layer. Specifically, in SMM, the society will reward each judge according to a probability value G, which is related to the social reward criterion (γ) and the current moment's proportion of cooperators (f_c) in the participant layer. When f_c exceeds the value of γ by more, then G will be larger, i.e., the more likely that judges will be rewarded. On the contrary, if f_c is very small or even less than the value of γ, judges are very unlikely to be rewarded. The PDG is performed in the participant layer, and the SDG is used to test the robustness of the results. Through Monte Carlo simulation, the results show that SMM can effectively inhibit corruption in the judge layer, which indirectly leads to a higher number of cooperators in the participant layer. In a practical sense, the social monitoring mechanism introduced in this paper is similar to government monitoring of some law enforcers, with law enforcers viewed as third-party judges and ordinary people as participants. When the benefits (similar to f_c in the model) created by ordinary people (similar to participants in the model) under the supervision of law enforcers (similar to judges in the model) exceed a certain level (similar to γ in the model), the government will give more rewards to the fair law enforcers as a way of encouraging them to govern with impartiality, which in turn promotes the ordinary people to create greater benefits for society. Compared with previous studies, we consider that the third-party judges will be influenced by other factors when making decisions, which is closer to real life. A social monitoring mechanism is introduced in the judge layer, which simulates the real-life phenomenon that some law enforcers are also constrained in exercising their power. Exploring the impact on the evolution of cooperation in the participant

Table 1
Meanings of all parameters.

SMM	Social Monitoring Mechanism
C	Cooperator
D	Defector
AJ	Altruistic Judge
CJ	Corrupt Judge
L	Edge length of the lattice
F	Participant's payoff
P	Judge's payoff
γ	Social reward criterion
g	Social reward factor
b	Temptation to defect
q	Altruistic probability
f_c	Proportion of cooperators
e_1	Reward amounts for judge to participant
e_2	Fine amounts for judge to participant
h	Bribe amounts for defector to judge
m	Base payoff of judge
v	Update speed
α	zealots ratio

layer by doing corrupt interference into the judge layer enables a deeper understanding of the coupling between the two layers. In addition, we use a two-layer network structure in the model, which adds complexity to the model and better reflects the complex interactions in society.

2. Model

We introduce a two-layer square network model that includes a participant layer and a judge layer. Each participant (judge) occupies a node in the participant (judge) layer, and the participant layer corresponds one-to-one with the nodes in the judge layer. Specifically, each judge monitors only one participant at its corresponding node location. Each participant interacts with their four von Neumann neighbors. We use the periodic boundary lattices of $L * L$ and fix $L = 100$.

The parameters involved in this paper are first described in Table 1.

Participants' strategies are categorized into cooperation and defection, and we refer to the participant who chooses cooperation as the cooperator (C) and the participant who chooses defection as the defector (D). Participants play the Prisoner's Dilemma Game (PDG) in the participant layer. The payoffs of the PDG are set as follows: $T = b > 1$, $R = 1$, $P = S = 0$, which satisfies $T > R > P \geq S$. Judges in the judge layer play the role of enforcers, and each judge has two optional strategies: altruism and corruption. The judge who chooses altruism is called the altruistic judge (AJ), and the judge who chooses corruption is called the corrupt judge (CJ). Specifically, a cooperator who encounters AJ receives reward amounts(e_1), a defector who encounters AJ pays fine amounts(e_2); a cooperator who encounters CJ pays fine amounts(e_2), and a defector who encounters CJ is not fined, but only pays a few bribe amounts(h). Here e_1 is paid for by taxes and e_2 acts as a tax. AJ is the equivalent of the altruistic judge who maintains social justice [26]; CJ is the equivalent of the corrupt judge who allows defectors to be more unrestrained [67]. In addition, each judge is born in the group with an altruistic probability ($0 \leq q \leq 1$), which is equivalent to a judge's preference to choose altruism. Specifically, each judge is an AJ with probability q and a CJ with probability $1 - q$. $q = 1$ means that the judge must be an AJ and $q = 0$ means that the judge must be a CJ.

We introduce a social monitoring mechanism (SMM) in the judge layer. Specifically, SMM is equivalent to the role of society, which is independent of judges and participants, to monitor judges by checking how the cooperation rate is under the guidance of judges. A criterion will be set in the SMM to compare with the proportion of cooperators in the participant layer to decide whether judges can be rewarded or not. Specifically, in any given round, each judge is rewarded with a probability:

$$G(t) = \frac{1}{1 + exp(\frac{\gamma - f_c(t)}{\eta})} \tag{1}$$

where $\eta \geq 0$ describes a kind of uncertainty, as the judges may not receive the reward due to some unexpected factors. In this paper, fix $\eta = 0.1$. Whether judges are rewarded is also related to the social reward criterion ($0 \leq \gamma \leq 1$) and the proportion of cooperators ($0 \leq f_c \leq 1$) at the current moment. The social reward criterion is equivalent to a criterion set by society for judges, and it can be interpreted as a minimum of the proportions of cooperators that the judges need to monitor the participants and promote them. For example, when $f_c > \gamma$, this means that because of judges' monitoring, f_c in the participant layer exceeds the criterion set by society. And the more the value of f_c exceeds the value of γ, the more the value of G approaches 1, i.e., the greater the probability that judges are rewarded. Conversely, when $f_c < \gamma$, it implies that judges do not reach the criterion, at which point the probability of judges being rewarded is small. In addition, when $\eta = 0$, judges are rewarded if and only if $f_c > \gamma$. When η approaches infinity or $f_c = \gamma$, whether judges are rewarded or not is determined by a coin toss. The expression for G is similar to the Fermi function [9,76], where we replace the payoff in the Fermi function by γ and f_c, respectively. G adopts this expression mainly to better imitate the regularity of reality by taking into account the occasional Error [76]. Nowak [76] et al. show that in finite populations, the Fermi function

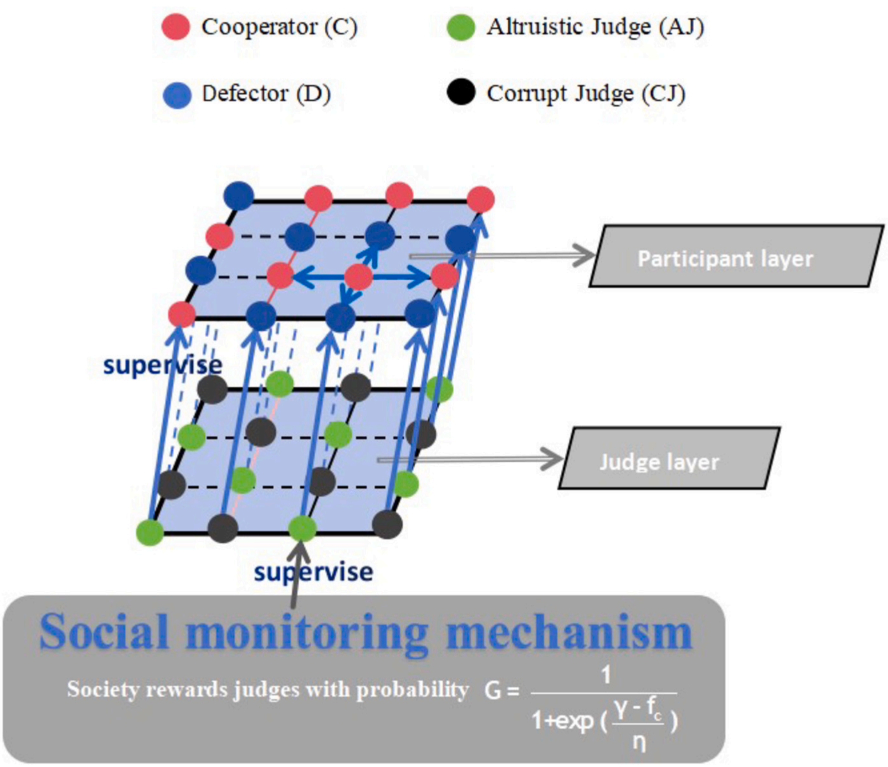

Fig. 1. The top layer is the participant layer, where each node represents a participant, red denotes the cooperator, and blue denotes the defector; the lower layer is the judge layer, where green denotes the altruistic judge, and black denotes the corrupt judge. The judges monitor participants, and society monitors judges. The parameter γ denotes the social reward criterion, f_c denotes the proportion of cooperators, and η denotes a kind of uncertainty.

captures the impact of occasional errors under dynamic selection of replicators. Dean et al. state in [9] that the Fermi function is used to simulate the probability of information diffusion and influence propagation within a network. In addition, if society rewards judges in SMM, the reward amount is set as the product of the social reward factor ($g \geq 0$) and the altruistic probability (q), i.e., $g * q$. The social reward factor represents the strength of the judges being rewarded, e.g., the larger the value of g, the higher the reward amounts that judges will get. In the game, we fix the value of g such that it acts as a constant and every judge has the same value of g. For q represents each judge's preference to choose altruism. In order to encourage more judges to increase their q-values and become AJs, the reward amounts are set to be $g * q$, which is a positively proportional relationship with q. Judges with higher q-values tend to get higher reward amounts than judges with lower q-values.

The structure of participants and judges is shown in Fig. 1.

The payoffs of participants and judges are discussed below. In a two-layer network, each participant corresponds to a judge. Each participant interacts with their four von Neumann neighbors, and after each interaction, the corresponding judge rewards or punishes the participant once. In each round of the game, participants interact four times, which means they are rewarded or penalized four times. Additionally, the defector only needs to bribe the corresponding CJ once to be exempt from the penalty. Each participant's payoff is affected by whether the corresponding judge is an AJ or a CJ. Each judge's payoff is mainly affected by the current moment's f_c and γ. And CJ only has the opportunity to take bribe amounts when monitoring a defector.

The participant (x) corresponds to the judge AJ, and the payoff of x at moment t is given in equations (2) (3).

$$F_{x=C}(t) = \sum_{x_i \in \Omega_x} S_x^T A S_{x_i} + 4e_1 \tag{2}$$

$$F_{x=D}(t) = \sum_{x_i \in \Omega_x} S_x^T A S_{x_i} - 4e_2 \tag{3}$$

The participant (x) corresponds to the judge CJ, and the payoff of x at moment t is given in equations (4) (5).

$$F_{x=C}(t) = \sum_{x_i \in \Omega_x} S_x^T A S_{x_i} - 4e_2 \tag{4}$$

$$F_{x=D}(t) = \sum_{x_i \in \Omega_x} S_x^T A S_{x_i} - h \tag{5}$$

where Ω_x is the set of four Von Neumann neighbors of participant x. Bribe amounts for defectors to judges need to satisfy: $h < 4e_2$. S_x denotes the strategy matrix of participant x, and S_x when x is C or D, respectively:

$$S_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad or \quad S_x = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

S_{x_i} denotes the strategy matrix of x's neighbors x_i at moment t. A denotes the payoff matrix, which can be expressed as:

$$A = \begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{7}$$

If the judge(y) is rewarded, the payoff for y at moment t is shown in Equations (8) and (9).

$$P_{y=AJ}(t) = m + gq_y(t) \tag{8}$$

$$P_{y=CJ}(t) = \begin{cases} m + gq_y(t) & y \overset{sup}{\to} C \\ m + gq_y(t) + h & y \overset{sup}{\to} D \end{cases} \tag{9}$$

If the judge(y) is not rewarded, the payoff for y at moment t is shown in Equations (10) and (11).

$$P_{y=AJ}(t) = m \tag{10}$$

$$P_{y=CJ}(t) = \begin{cases} m & y \overset{sup}{\to} C \\ m + h & y \overset{sup}{\to} D \end{cases} \tag{11}$$

where m denotes the base payoff of judges, which is fixed to 1; $q_y(t)$ denotes the altruistic probability of y at moment t. $y \overset{sup}{\to} C$ denotes the judge y monitoring cooperator, and $y \overset{sup}{\to} D$ denotes the judge y monitoring defector.

Regarding the strategic update, each participant x randomly chooses a neighbor x_i $(i = 1, 2, 3, 4)$ and imitates x_i's strategy through the Fermi rule:

$$W_{x\to x_i}(t) = \frac{1}{1 + exp(\frac{F_x(t) - F_{x_i}(t)}{\kappa})} \tag{12}$$

Where κ^{-1} describes the imitation strength such that $\kappa^{-1} > 0$

The strategy update on the judge y corresponds to the altruistic probability (q) is updated for each round, and y decides to become an AJ or a CJ based on the q-value. In the initial phase of the simulation, each judge is randomly assigned a q-value, and then the payoffs of judge y to its four direct neighbors are compared. Assuming y_0 denotes the judge with the largest payoff among them, i.e., the y_0 satisfies $P_{y_0} = max\{P_y, P_{y_i}, i = 1, 2, 3, 4\}$. If y_0 is an AJ, y updates the q_y value at the next moment with a probability of $V_{y\to y_0}$ and the updated q_y value is shown in Eq. (14). If y_0 is a CJ, when $q_y(t) < \frac{v - vf_c(t)}{1 + v - vf_c(t)}$, y chooses corruption at the next moment; when $q_y(t) \geq \frac{v - vf_c(t)}{1 + v - vf_c(t)}$, y updates the q_y value at the next moment with a probability of $V_{y\to y_0}$, and the updated q_y value is shown in equation (15).

$$V_{y\to y_0(t)} = \frac{1}{1 + exp(\frac{P_y(t) - P_{y_0}(t)}{\kappa})} \tag{13}$$

$$q_y(t+1) = q_y(t) + v(1 - q_y(t))f_c(t) \tag{14}$$

$$q_y(t+1) = q_y(t) - v(1 - q_y(t))(1 - f_c(t)) \tag{15}$$

where v denotes the update speed, which simulates the finite learning rate of judges, and v is fixed to 0.1. When each judge has determined the value of $q_y(t+1)$, the judge chooses AJ at the next moment with probability $q_y(t+1)$ and CJ at the next moment with probability $1 - q_y(t+1)$.

3. Results

Monte Carlo simulations in a two-layer network model with a social monitoring mechanism (SMM) to explore the level of cooperation in the participant layer and the inhibition of corruption in the judge layer. First, we studied the effects of reward amounts (e_1) and fine amounts (e_2) given by judges to participants on the evolution of participants and judges. The results of the 50th iteration are shown in Fig. 2, where Fig. 2(a) shows the evolution of participants in the participant layer and Fig. 2(b) shows the evolution of judges in the judge layer. Fig. 2 shows that the evolution results of the participant layer and judge layer are almost the same, which indicates that the SMM directly affects the judge layer and indirectly affects the participant layer. An increase in the number of altruistic judges in the judge layer leads to an increase in the number of cooperators in the participant layer. When $e_1 + e_2 < 0.5$, essentially defectors and corrupt judges dominate, indicating that smaller rewards and fines are not powerful enough to maintain cooperation and curb corruption. As the values of $e_1 + e_2$ increase, cooperators in the participant layer and altruistic judges in the

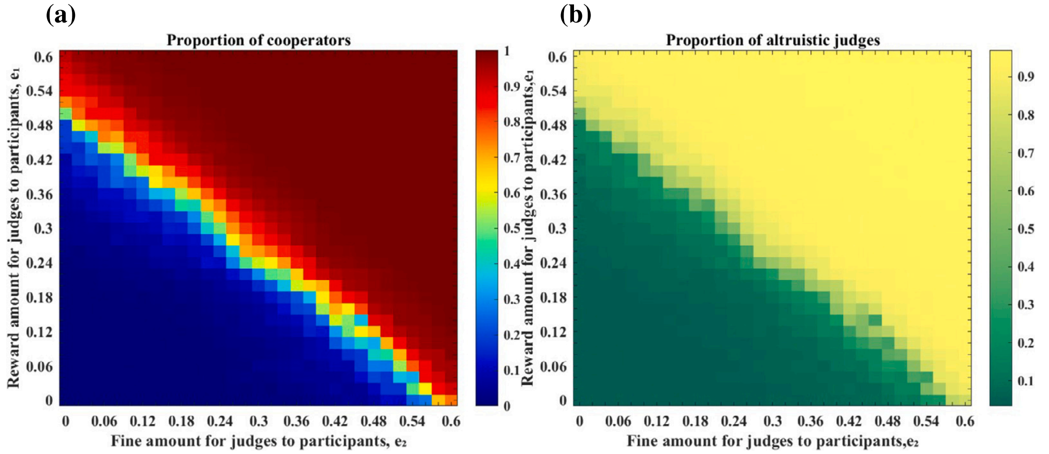

Fig. 2. (a) is the proportion of cooperators as a function of e_1 and e_2. A deeper red indicates a higher proportion of cooperators; a deeper blue indicates a lower proportion of cooperators. (b) is the proportion of altruistic judges as a function of e_1 and e_2. A deeper yellow indicates a higher proportion of altruistic judges; a deeper green indicates a lower proportion of altruistic judges. The other parameter values are: $\kappa = 0.1$, $\alpha = 0$, $h = 0.1$, $g = 1$, $b = 1.5$, $\gamma = 0.5$.

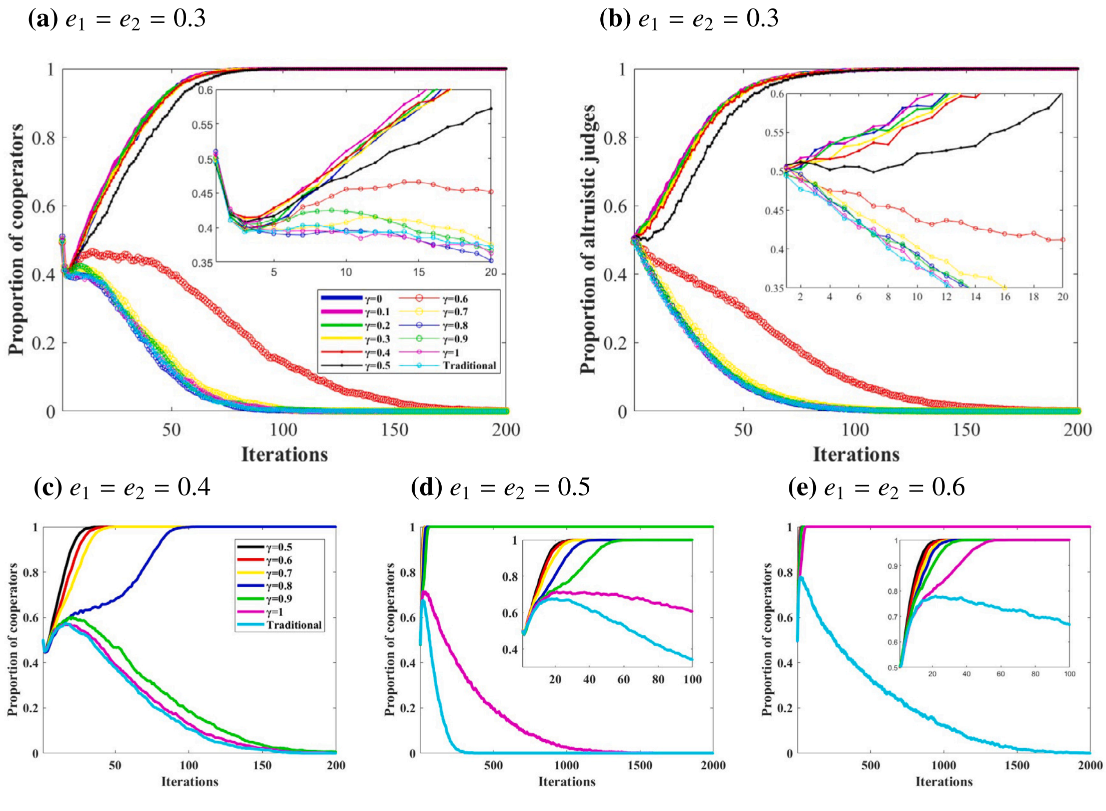

Fig. 3. (a), (c), (d), and (e) are the proportions of cooperators as a function of iterations for different values of γ and for the traditional case. (b) is the proportion of altruistic judges as a function of iterations for different values of γ and the traditional case. The other parameter values are: $\kappa = 0.1$, $\alpha = 0$, $h = 0.1$, $g = 1$, $b = 1.5$.

judge layer begin to emerge. It is worth noting that rewards are more effective than fines in promoting cooperation and eliminating corruption. This is because when $e_2 = 0$, the value of e_1 only needs to be around 0.52 for cooperators and altruistic judges to dominate. Conversely, when $e_1 = 0$, the value of e_2 reaches about 0.58 before more cooperators and altruistic judges become visible.

In the next study, set the reward amounts and fine amounts the same, i.e., $e_1 = e_2 = e$. Discuss how participants and judges evolve as iterations increase for different values of e and different values of γ. In addition, due to the existence of corrupt judges in the judge layer, this paper wants to explore whether SMM can inhibit corruption well. Therefore, we set "traditional case" as a "two-layer network structure without SMM". The evolutionary results are shown in Fig. 3, where (a), (c), (d), and (e) all show the results for the proportion of cooperators; (b) shows the results for the proportion of altruistic judges. From Fig. 3(a) and Fig. 3(b), it can be seen that the results of the participant layer and judge layer in a steady state are almost the same. When $\gamma \leq 0.5$, in the steady state, cooperators occupy all the nodes in the participant layer and altruistic judges occupy all the nodes in the judge layer. The evolution of the two-layer network differs in that at the beginning of the iteration, the proportion of cooperators in the participant layer decreases regardless of the value of γ, whereas the proportion of altruistic judges in the judge layer continues to rise or continues to fall for different values of γ. In fact, the cluster of cooperators in the initial phase is too fragmented to resist the aggression of defectors despite the introduction of SMM, and as the iterations increase, the proportion of cooperators starts to be related to the γ value. Fig. 3(a) illustrates that for $e = 0.3$, cooperation can be revealed only for $\gamma \leq 0.5$. From Fig. 3(c), Fig. 3(d), and Fig. 3(e), it can be seen that increasing the value of e allows cooperation to be revealed even if γ is high. This suggests that enhancing the rewards and fines in the participant layer would temporarily increase the proportion of cooperators. But without the introduction of the SMM, i.e., the traditional case, this temporary increase would just slow down the rate at which the defector occupies all the nodes, and would not eliminate the corruption in the judge layer. However, with the introduction of the SMM, this temporary increase makes the value of G larger, i.e., the probability of a judge being rewarded is greater, which encourages more judges to choose altruism. Once the number of altruistic judges in the judge layer is increased, this also leads, in turn, to an increase in the number of cooperators in the participant layer. This creates a virtuous circle where the two-layer network positively influences each other, as a result of which the cooperators can occupy the whole network even if $\gamma = 1$, as long as there is a large e value. Moreover, in our model, the steady state is either all cooperators or all defectors in the participant layer. Therefore, in the next study, in order to better explore the effect of different parameters on the speed of evolution, we will discuss the results for the moments before the steady state, but close to reaching the steady state.

The above studies have shown that increasing the values of e1 and e2 can make cooperation manifest, even at very high values of γ. Next, we will explore whether the introduction of committed individuals in the participant layer facilitates the evolution of cooperation. The terminology for committed individuals varies across different fields, in social dilemma games, they are referred to as “zealots” or “stubborn players” [77]. In this paper, we will refer to these committed individuals as “zealots”. These zealots all share a common characteristic: they always stick to their beliefs or strategies and are not affected by outside factors [77]. These committed individuals have been extensively studied by many scholars [78–80]. These studies suggest that committed individuals tend to be more likely to promote cooperation at weak imitation intensities than at strong imitation intensities. In our study, two different imitation intensities are set, i.e., $\kappa^{-1} = 10$ and $\kappa^{-1} = 0.1$. Explore whether the introduction of zealots can promote cooperation in our model for both imitation intensities. In Fig. 4, fixing $e_1 = e_2 = 0.3$, we study the evolution at the 50th iteration for different zealots ratio (α) and different values of γ. Fig. 4(a) and Fig. 4(b) show that the introduction of SMM enables zealots to facilitate the manifestation of cooperators even under strong imitation conditions. Even when e is relatively small, an increase in zealots ratio can make cooperators appear at very high values of γ. In Fig. 4(a), it can be seen that without the introduction of zealots, cooperation is not enough to be well maintained when $\gamma > 0.56$, at which point increasing the number of zealots can bring the cooperators back to dominance. Higher values of γ require larger values of α to maintain cooperation. From Fig. 4(b), it can be found that in the traditional case, the proportions of cooperators remain low even with the addition of zealots. This suggests that introducing SMM into the judge layer increases the effectiveness of zealots in promoting cooperation. Fig. 4(c) and Fig. 4(d) demonstrate the evolution of cooperation under the weak imitation condition. From Fig. 4(c), it can be seen that the proportions of cooperators in the weak imitation condition are almost unaffected by the value of γ. And it only takes about 6% of zealots in the participant layer to make the number of cooperators exceed that of defectors. Fig. 4(d) shows that even in the traditional case, the weak imitation condition made zealots effective in promoting cooperation, which is consistent with previous findings. Overall, zealots promoted cooperation more in the weak imitation condition than in the strong imitation condition. To further explore the effect of zealots on the evolution of cooperation, we set a very high social reward criterion, i.e., fixed $\gamma = 1$. The evolution of participants and judges at the 50th iteration was investigated for different values of α and different values of e, see Fig. 5. From Fig. 5, it can be seen that even at very high criteria, cooperators and altruistic judges can still dominate, as long as a high value of e and the corresponding value of α are set. When $e < 0.18$, the introduction of zealots hardly eliminates defectors. This is because a very high criterion like $\gamma = 1$ is already very unfavorable to the existence of altruistic judges. If a small e value is set, this will allow defectors to be even looser so that even the introduction of zealots does not curb corruption and defection. When e reaches or exceeds 0.32, the introduction of zealots at this point allows cooperators and altruistic judges to have the opportunity to occupy almost the entire network. The higher the value of e, the smaller the value of α is required to reach a high level of cooperation. In addition, comparing with Fig. 3, we find that the introduction of zealots can allow a high level of cooperation with $\gamma = 1$ even if the value of e does not reach 0.6.

In addition, the strength of judges being rewarded in SMM also affects the evolution of cooperation, so we studied the social reward factor (g). Fig. 6 illustrates the effect of g and γ values on the evolution of participants and judges at the 50th iteration. Fig. 6(a) shows the three-dimensional plot of the proportion of cooperators, and it can be visualized that g plays a crucial role in the evolution of cooperation. It can be seen that when the value of γ is not too large, an increase in the value of g leads to an increase in the proportions of cooperators. This is because when the value of g increases, those judges who are rewarded receive more payoffs, which leads more judges to want to receive rewards from society rather than greedy bribe amounts from defectors. This allows more judges to work to maintain the manifestation of cooperators in the participant layer as a way to increase the probability that they will be rewarded. In addition, the value of g cannot be too small, otherwise, judges are more willing to receive bribe amounts from defectors to increase their payoffs. It is shown in Fig. 6(b) that when $g \leq 0.1$, the defectors dominate the whole network regardless of the γ value. As the value of g increases, the effect of the γ on the proportions of cooperators starts to become obvious. When $\gamma \leq 0.5$, as the g-value increases from 0 by 0.5, the proportions of cooperators also increase eventually approaching 90%. However, when $\gamma \geq 0.7$, g values in the range of 0 to 0.5 are not sufficient to maintain cooperation, and higher g values are needed. For $\gamma = 0.6$, the proportions of cooperators are still relatively low, although there is an overall increasing trend. Fig. 6(c) shows the evolution

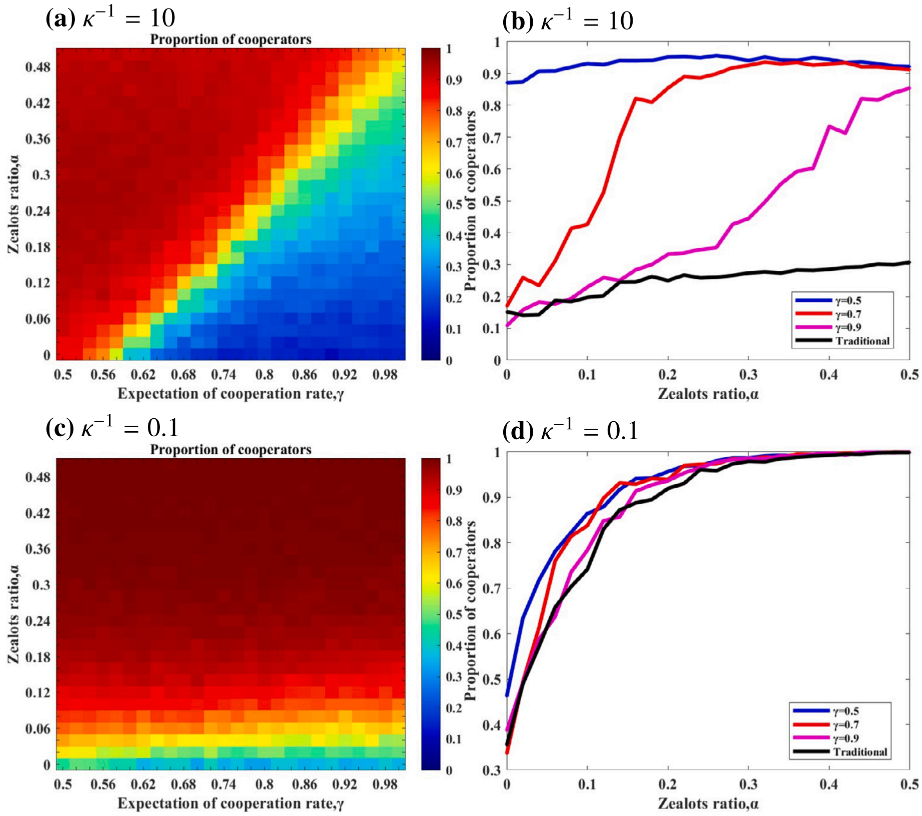

Fig. 4. (a) and (c) are the proportions of cooperators as a function of α and γ. A deeper red indicates a higher proportion of cooperators; a deeper blue indicates a lower proportion of cooperators. (b) and (d) are the proportions of cooperators as a function of α for different values of γ and the traditional case. (a) and (b) are the strong imitation condition, i.e., $\kappa^{-1} = 10$. (c) and (d) are the weak imitation condition, i.e., $\kappa^{-1} = 0.1$. The other parameter values are: $h = 0.1$, $e_1 = 0.3$, $e_2 = 0.3$, $g = 1$, $b = 1.5$.

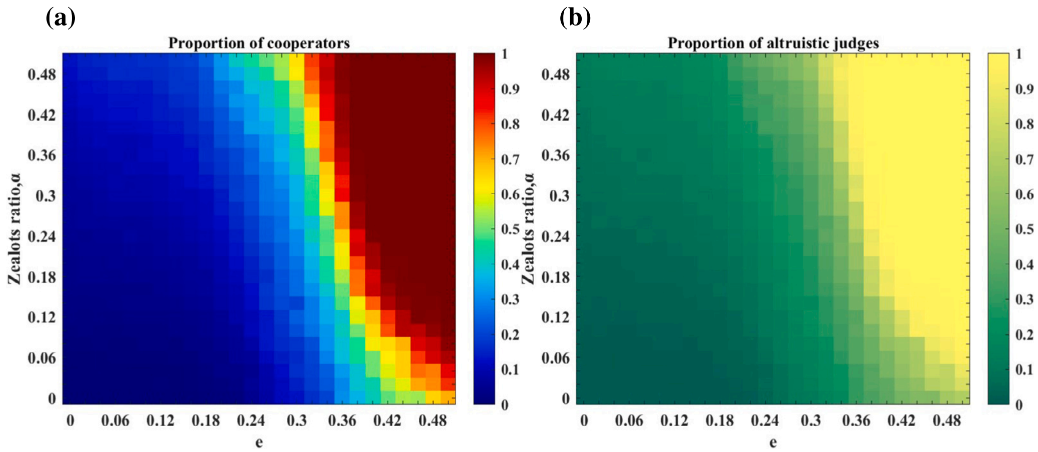

Fig. 5. (a) is the proportion of cooperators as a function of α and e. A deeper red indicates a higher proportion of cooperators; a deeper blue indicates a lower proportion of cooperators. (b) is the proportion of altruistic judges as a function of α and e. A deeper yellow indicates a higher proportion of altruistic judges; a deeper green indicates a lower proportion of altruistic judges. The other parameter values are: $\kappa = 0.1$, $h = 0.1$, $g = 1$, $b = 1.5$, $\gamma = 1$.

of judges. It can be seen that when $\gamma = 1$, a g-value of not more than 0.5 causes corrupt judges to occupy most of the nodes in the network; whereas when $\gamma = 0.5$ or 0, some corrupt judges start to disappear as the g-value increases from 0 to 0.5. When $g = 0$, this means that judges will not be rewarded by society and will be more willing to accept bribe amounts to increase their payoffs regardless of the value of γ, which leads to a large number of corrupt judges; whereas when $g = 0.2$ or 0.5, some corrupt judges will disappear as the value of γ decreases. Overall, just by increasing g to 0.5, the level of cooperation in those smaller γ cases approaches 90%, and a large number of corrupt judges in the participant layer disappear.

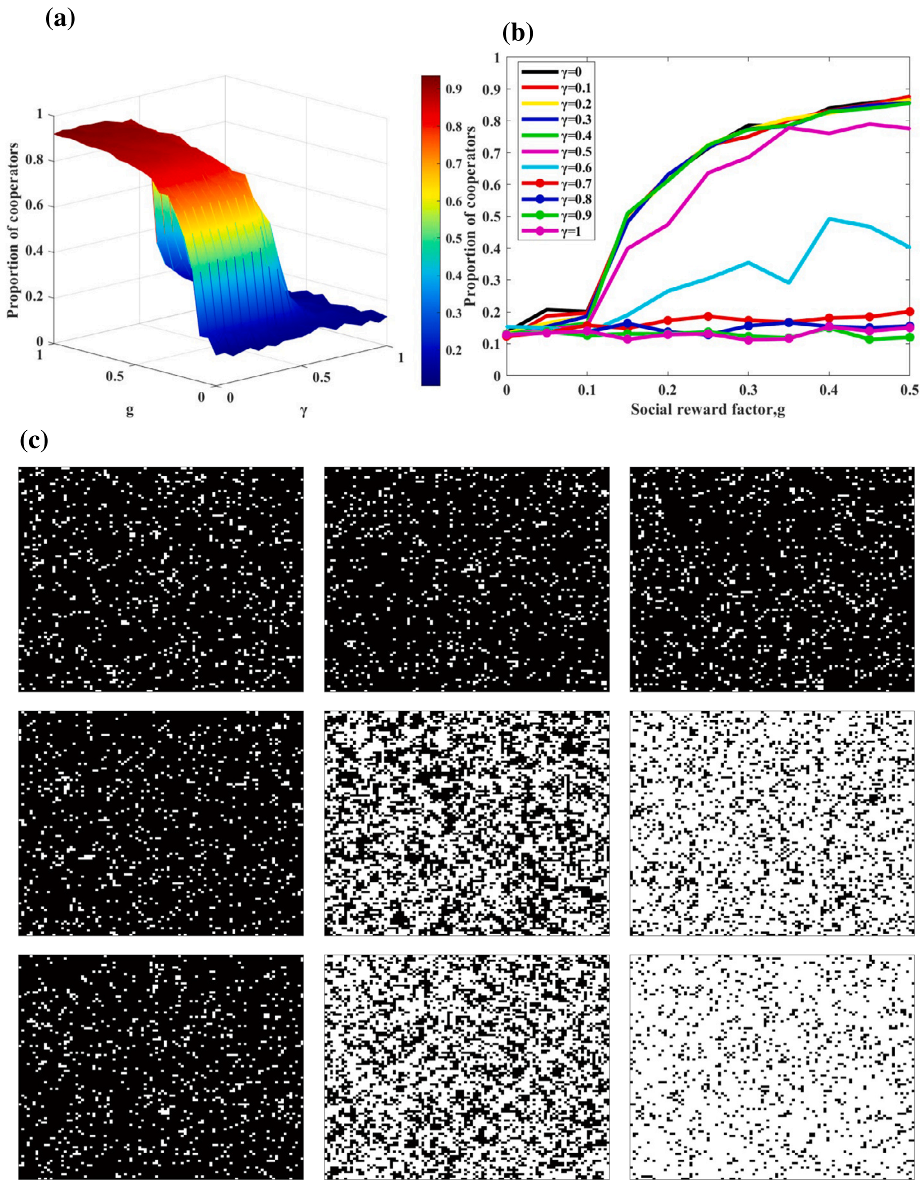

Fig. 6. (a) is a three-dimensional plot depicting the proportions of cooperators at different values of g and different values of γ. A deeper red indicates a higher proportion of cooperators; a deeper blue indicates a lower proportion of cooperators. (b) is the proportion of cooperators as a function of g for different values of γ and the traditional case. (c) are snapshots of the spatial distribution of altruistic judges and corrupt judges with three typical values of γ and g, which from left to right are $g = 0$, $g = 0.2$, $g = 0.5$, and from top to bottom are $\gamma = 1$, $\gamma = 0.5$, and $\gamma = 0$. The snapshots show white for an altruistic judge and black for a corrupt judge. The other parameter values are: $\kappa = 0.1$, $\alpha = 0$, $h = 0.1$, $e_1 = 0.3$, $e_2 = 0.3$ $b = 1.5$.

To further study the effect of SMM on the evolution of cooperation, we discuss the effect of temptation to defect on the level of cooperation when γ is different values. To test the robustness of the model, we perform Monte Carlo simulations in the PDG and SDG respectively, see Fig. 7. Fig. 7(a) and Fig. 7(b) depict the results of the PDG, where (a) denotes the proportion of cooperators; and (b) denotes the proportion of altruistic judges. As the value of b increases, there are fewer and fewer cooperators and altruistic judges. This is because the greater the value of b, the greater the social dilemma, which favors the survival of more defectors. Smaller values of γ can withstand higher values of b, allowing cooperators and altruistic judges to survive. For example, when $\gamma = 0$, the cooperators did not disappear completely until $b \geq 2.1$, and the altruistic judges never disappeared completely. However, in the traditional case, the cooperators become weaker against the defectors, and the cooperators and altruistic judges have completely disappeared when $b = 1.5$. This suggests that under traditional network reciprocity, for very strong social dilemmas ($b > 1.5$), it is difficult for cooperators to defend themselves against defectors. However, in the two-layer network model with SMM, the level of cooperation is also better than in the traditional case, and setting the right value of γ also allows the cooperators to survive even under very strong social dilemmas. The results of the SDG are depicted in Fig. 7(c) and Fig. 7(d). The payoffs of the SDG are set as

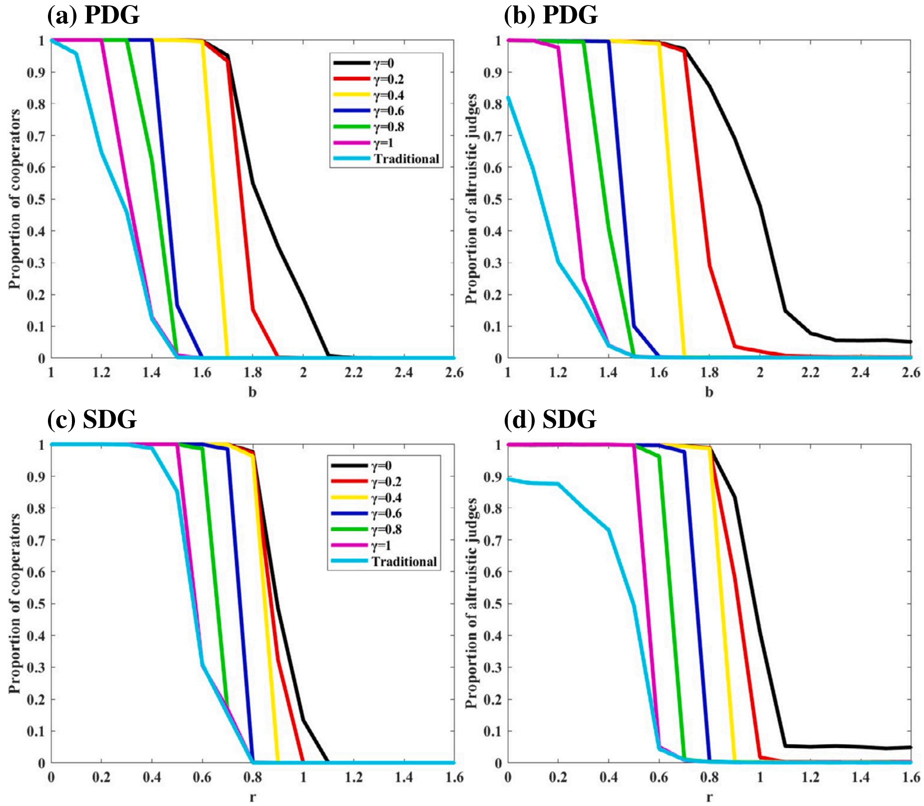

Fig. 7. (a) is the proportion of cooperators as a function of b for different values of γ and the traditional case. (b) is the proportion of altruistic judges as a function of b for different values of γ and the traditional case. (c) is the proportion of cooperators as a function of r for different values of γ and the traditional case. (d) is the proportion of altruistic judges as a function of r for different values of γ and the traditional case. The other parameter values are: $\kappa = 0.1$, $\alpha = 0$, $h = 0.1$, $e_1 = 0.3$, $e_2 = 0.3$, $g = 1$.

follows: $T = r + 1$ ($r > 0$), $R = 1$, $S = 1 - r$, and $P = 0$, which satisfies $T > R > S > P$. It can be found that the two-layer network model with SMM promotes cooperation not only in PDG but also in SDG. When $r \leq 0.5$, the cooperators in SDG face relatively small risks, and the level of cooperation remains high even under the two-layer network model without SMM. However, it can be seen from Fig. 7(d) that regardless of the r value, the number of altruistic judges in the judge layer is higher when SMM is introduced than in the traditional case. When the value of r increases, the greater the risk of cooperation faced by the participants, at which point the advantages of SMM begin to emerge in the participant layer. And the smaller the value of γ, the higher the value of r that can be afforded to make the cooperators visible. Overall, the introduction of SMM leads to stronger cooperation than in the traditional case, both in PDG and SDG. And the low γ-value sustains cooperation under strong dilemmas. In fact, strong dilemmas are unfriendly to cooperators in the participant layer. Yet when the number of altruistic judges in the judge layer increases, more cooperators can be encouraged to appear in the participant layer even under strong dilemmas. Setting a lower value of γ in the SMM would imply that the criteria set for the judges to be rewarded are low, i.e., the judges could easily receive rewards from society and the amount of the rewards would be positively proportional to the q-values of the judges. So lower γ-values may encourage judges to increase their q-values, which leads to an increase in the number of altruistic judges and indirectly promotes the level of cooperation in the participant layer. Fig. 8 shows snapshots of the spatial distribution of the judges' q-values at a certain time, where (a) and (b) show the results of the two-layer network model with SMM, and (c) shows the results of the two-layer network model without SMM (i.e., the traditional case). It can be found that after the introduction of SMM, the q-values of most of the judges are higher than those of the traditional case, and some of them even have q-values as high as 1. Moreover, the q-values of the judges at $\gamma = 0.5$ are higher than those at $\gamma = 0.6$. This suggests that SMM can be a good incentive for judges to increase their q-values, and the lower the value of γ the more effective in promoting judges to increase their q-values.

Next, we describe the evolution of participants and judges. Fig. 9 shows snapshots of the spatial distribution of participants and judges, comparing the two scenarios with and without SMM. Fig. 9(a) depicts the evolution of cooperators, with red indicating cooperators and blue indicating defectors, where in the steady state with $\gamma = 0.3$ the cooperators occupy the entire participant layer, while in the traditional case the defectors occupy the entire participant layer. For $\gamma = 0.6$ the cooperators ultimately cannot resist the defectors, this is because the values of e_1 and e_2 are set relatively small, from Fig. 3 it can be seen that increasing the values of e_1 and e_2 also allows the cooperators at $\gamma = 0.6$ to occupy the entire network. Before stabilization, it can be noticed that the number of red in the second row is more than that in the third row, and in conjunction with Fig. 3(a), this suggests that $\gamma = 0.6$ slows down the rate at which defectors take over the whole network relative to the traditional case. It is also well illustrated that the introduction of SMM

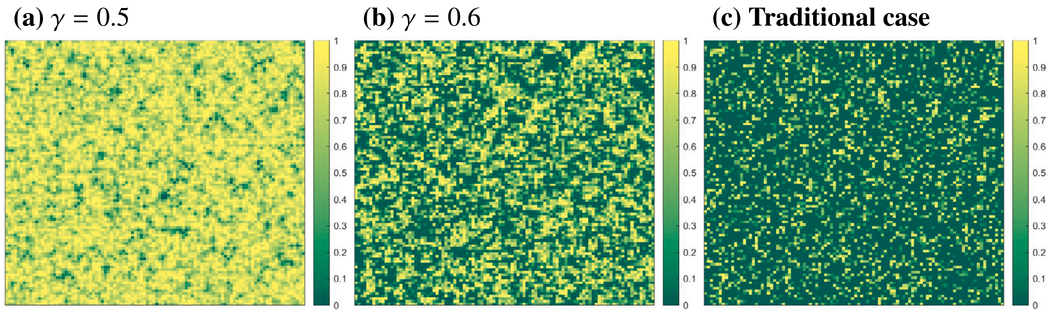

Fig. 8. Snapshots of the spatial distribution of the judges' q-values. A deeper yellow indicates higher q-values; a deeper green indicates lower q-values. (a) SMM is introduced and $\gamma = 0.5$ is set, (b) SMM is introduced and $\gamma = 0.6$ is set, and (c) is the traditional case. The other parameter values are: $\kappa = 0.1$, $\alpha = 0$, $h = 0.1$, $e_1 = 0.3$, $e_2 = 0.3$, $g = 1$, $b = 1.5$.

can influence the level of cooperation in the participant layer and promote an increase in the number of cooperators. Fig. 9(b) depicts the evolution of judges, with altruistic judges in white and corrupt judges in black. It can be found that the evolution of altruistic judges in the judge layer is synchronized with the evolution of cooperators in the participant layer. With the increase of iteration, the altruistic judges occupy almost the whole network at $\gamma = 0.3$. Although corrupt judges dominate both the $\gamma = 0.6$ and the traditional case, $\gamma = 0.6$ is less corrupt than the traditional case. It shows that the introduction of SMM can suppress the corruption in the judge layer and increase the number of altruistic judges.

4. Conclusion

In a previous study, Wu [26] et al. showed that when participants meet altruistic judges with a certain probability, it makes cooperation visible in the PDG and SDG models. In fact, judges are also likely to be corrupt. Therefore, in our model, both altruistic judges and corrupt judges are considered. It can be found that the existence of corrupt judges will make a large number of defectors exist, so it is necessary to set up an anti-corruption mechanism to inhibit corruption and defection. Many scholars have studied anti-corruption mechanisms [67–69]. In our study, a social monitoring mechanism (SMM) is introduced to the judge layer in the two-layer network model, so that judges will be influenced by the SMM when making decisions. The results of the study show that SMM can well inhibit corruption in the judge layer and promote the level of cooperation in the participant layer.

We find that smaller rewards and fines are not strong enough to maintain cooperation and discourage corruption, i.e., both reward amount (e_1) and fine amount (e_2) being small at the same time leads to non-survival of cooperators and grows corruption among judges. Increasing the values of e_1 and e_2 in the model with SMM allows cooperation to be revealed even if the value of the social reward criterion (γ) is high, but in the traditional case increasing the values of e_1 and e_2 only slows down the rate at which defectors take over the whole network. In addition, we also found that injecting a certain percentage of zealots into the participant layer after the introduction of SMM could enhance the level of cooperation not only in the weak imitation conditions but also in the strong imitation conditions. The introduction of zealots can make the cooperation level of $\gamma = 1$ rise even if the values of e_1 and e_2 are not very high. The social reward factor (g) set in the SMM also plays a crucial role in the evolution of the participant and judge layers, and it has been found that for not too large values of γ (e.g., $\gamma \leq 0.5$), the value of g only needs to be taken as 0.5 in helping the cooperation in the participant layer to reach a high level and suppressing the corruption in the judge layer. The SMM enhances cooperation not only in the PDG but also in the SDG, and a low value of γ sustains cooperation under strong dilemmas. SMM was also able to encourage judges to increase their altruistic probability (q), with lower values of γ being more effective in encouraging judges to increase their q values. Finally, we describe snapshots of the spatial distribution of participants and judges, visualizing the advantages of SMM.

Our study enhances network reciprocity and provides new ideas and methods for the evolution of cooperation. In addition, the SMM mainly monitors and encourages the emergence of more altruistic judges employing rewards. In future research, the evolution of cooperation can be continued to be explored by adding punishment factors to the mechanism.

Data availability

Data will be made available on request.

Acknowledgements

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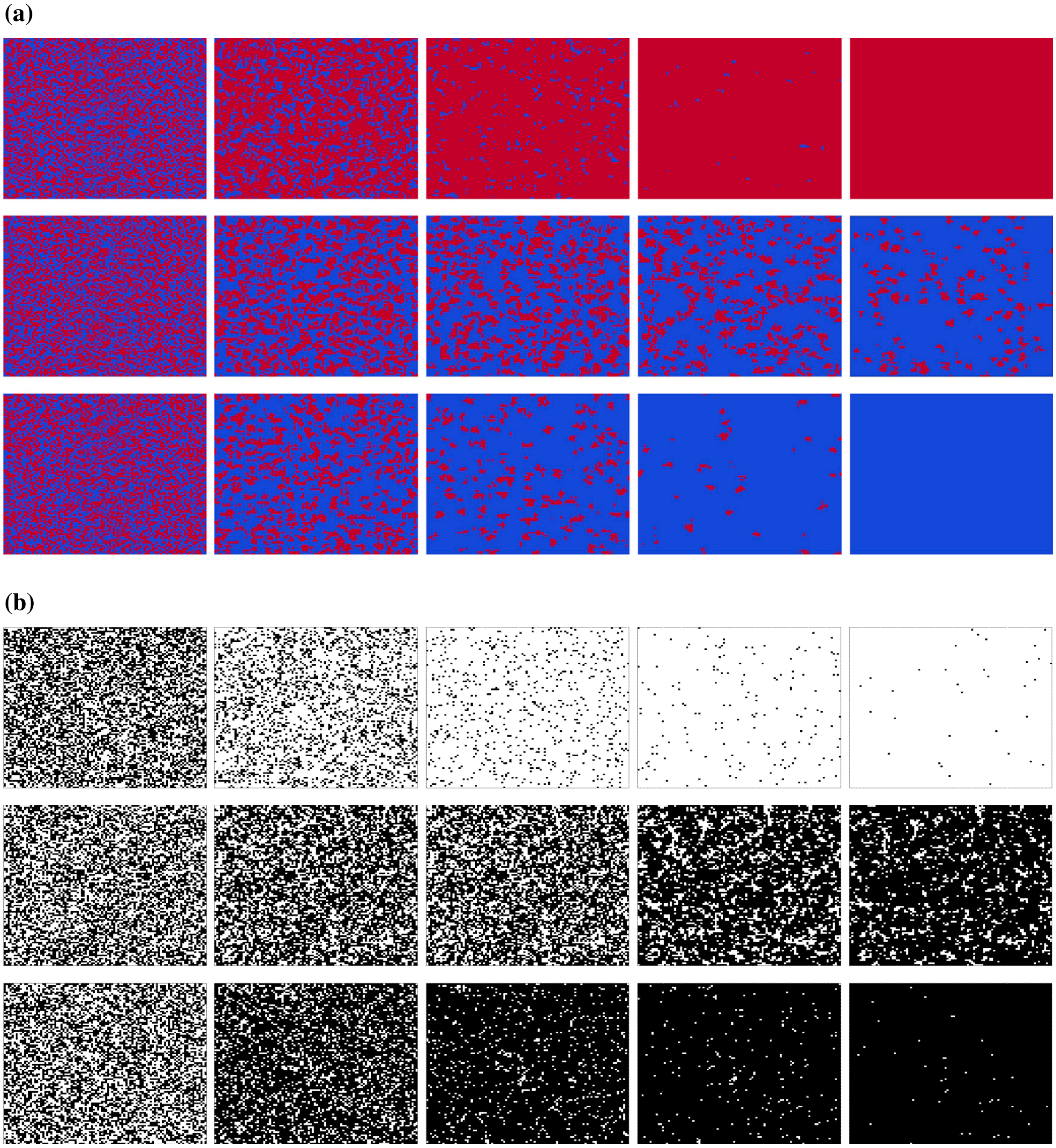

Fig. 9. (a) are snapshots of the spatial distribution of participants with three typical values of γ and traditional case, where red denotes a cooperator and blue denotes a defector. (b) are snapshots of the spatial distribution of judges with three typical values of γ and traditional case, where white denotes an altruistic judge and black denotes a corrupt judge. From left to right are iterations = 1, iterations = 25, iterations = 50, iterations = 75, and iterations = 100. The first two rows simulate the two-layer network with SMM, setting $\gamma = 0.3$ in the first row and $\gamma = 0.6$ in the second row. The third row represents the traditional case. The other parameter values are: $\kappa = 0.1$, $\alpha = 0$, $h = 0.1$, $e_1 = 0.3$, $e_2 = 0.3$, $g = 1$, $b = 1.5$.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.amc.2024.128991.

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