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Effects of exposure-based reward and punishment on the evolution of cooperation in prisoner's dilemma game

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ABSTRACT

Reward and punishment are important factors in maintaining cooperation among selfish individuals. This paper explores the effects of exposure-based reward and punishment on the evolution of cooperation. In this model, players can choose to expose their opponent's cooperation or defection status, leading to the identification of four strategies: exposure cooperation, exposure defection, ordinary cooperation, and ordinary defection. Exposing one's behavior incurs a cost, and exposed cooperators are rewarded while exposed defectors are punished. The exposure is, in turn, rewarded for exposing the defection. The evolutionary results show that three states emerge after the population stabilizes: only ordinary defection remains, exposure cooperation coexists with exposure defection, and all four strategies coexist. Cooperative behavior can occur when the exposure cost is less than the punishment for defection. Simply rewarding exposed cooperators is insufficient to promote cooperation. Simultaneous reward and punishment can significantly promote cooperation. There exists an optimal minimum reward threshold for cooperation, which varies nonlinear with increasing punishment. Larger temptation to defect and smaller cooperation reward favor the survival of exposure-oriented players and can effectively discourage second-order free-riding behavior. Our study provides insights into effective solutions to social dilemmas.

1. Introduction

Human society has a common dilemma that when the immediate individual interest conflicts with the chronic collective interest, selfish individuals will tend to pursue their individual interests and therefore the collective interests will suffer, which is also known as the social dilemma [1,2]. There is no doubt that in infinite well-mixed populations, cooperation is definitely not natural selection if no mechanism interferes [3]. But in the real world, cooperative behaviors are everywhere [4,5], and what exactly drives the emergence of cooperative behaviors among selfish individuals has attracted the attention of scholars [6,7]. Evolutionary game theory has been widely used as an effective framework for studying this problem in fields such as economics, biology, physics, and mathematics [8–12].

In the past few years, more and more mechanisms to promote cooperation have been proposed based on examples of exploring dilemmas such as the prisoner's dilemma game (PDG) [13–15] and the public goods game (PGG) [16]. In 2006, Nowak [17] summarized five rules, namely kin selection, group selection, direct and indirect reciprocity, and network reciprocity. In addition, mechanisms such as reputation [18–21], aspiration [22–24], and voluntary participation [25,26] have also been shown to have an important impact on facilitating cooperation. Besides the above mechanisms, reward and punishment are also major factors in maintaining cooperation among selfish individuals and have been validated experimentally and theoretically [27–31]. Depending on the type of reward and punishment, they are usually divided into pool and peer types [32,33]. In the case of punishment, for example, pool punishment means that punishers contribute a cost to the "punishment pool" before the game, and the outsourcing institution performs the punishment on the wrong-doers after the game, so it can also be called institutional punishment; peer punishment means that punishers impose a fine on the punished at a cost personally is a typical decentralized punishment. These punishments are costly [34,35], entail a cost, and are considered as an altruistic act [36], yet they are also illustrated as a self-interested act [37] due to their ability to punish potential intruders for invasion. But in any case, those who do not impose punishment will indirectly benefit, and this therefore creates a second-order free-riding dilemma [34,38].

Moreover, there has been controversy about whether reward and punishment actually promote cooperation and how well they do so. Sigmud et al. [39] noted that natural selection favors pool punishment only when punishing second-order free-riders. Baumard [40] argued

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that there is no empirical evidence that altruistic punishment has played a role in the evolution of human cooperation. Gao et al. [41] noted that whether and how punishment promotes cooperation depends on the mode of punishment. Boyd et al. [42] noted that punishment can have a stabilizing effect on cooperation, whereas reward may encourage cooperation but not stabilize it. Szolnoki et al. [43] suggested that the cost of punishment often does not offset the benefits of increased cooperation, a fact that has led some to reconsider the use of reward as the primary catalyst behind cooperation. Dong et al. [44] showed that only incentives that lead to higher adaptations can survive social evolution. All of the above studies were able to reveal the influence of reward and punishment mechanisms on the evolution of cooperation.

In addition, scholars also have studied the impact of incentives in different environments. Liu et al. [45] studied the necessary conditions for the evolution of cooperation in a corrupt environment by introducing corrupt enforcers and violators into the public goods game with pool punishment. Sun et al. [46] considered collective-risk in the social dilemma game and revealed that regardless of the value of risk, a local scheme can promote group success more effectively than a global scheme, no matter whether the imposed incentives are fixed or flexible. Wang et al. [47] investigated the system behavior in the case of group interaction where distributed incentives and determined the optimal dynamical punishing and rewarding schemes. Wang et al. [48] considered time-varying positive and negative incentives, and found that the optimal negative and positive incentive protocols are identical and time-invariant for each given strategy update rule. Besides studying incentives in specific environments, certain specific behaviors have rewarding and punishing properties, and the rewards and punishments elicited by these behaviors can also influence the evolution of cooperation. Zhang et al. [49] investigated the effect of conditional cooperators in PGG on the effectiveness of different types of incentives. Wang et al. [50] proposed a tax-based pure punishment and reward strategy and found it to be superior to pure punishment and reward in terms of sustained cooperation. Lee et al. [51] introduced a tax-based mercenary punishment in structured PGG and provided some new solutions to promote cooperation. Pan et al. [52] introduced a special cooperative strategy with punishment for defection and reward for cooperation in structured PDG, and the presence of this special cooperator can significantly contribute to the evolution of cooperation.

Based on the above studies, exposure has attracted our attention as a behavior with both punishing and rewarding properties. Exposure (revealed to others) behaviors are everywhere in real life, people will expose good behaviors to praise them, and also expose bad behaviors to resist them [53]. Exposure-based reputation mechanisms [54] have been shown to promote the evolution of cooperation in past studies, but in that work only the cooperators were assumed to have exposure behavior and the effect produced by exposure was reflected in reputation. In addition, they used a weighted summation approach to combine payoff and exposure reputation in the fitness, where the weighting factor reflects the degree of social concern about payoff or reputation. Unlike that work, this paper assumes that both cooperators and defectors have the opportunity for exposure, which is clearly more realistic. In addition, we directly reflect the effect of exposure in the form of reward and punishment in the fitness, which can better quantitatively assess the effect of exposure. Similar phenomena are very common in social life, for example, in some public exposure platforms, people can choose whether or not to expose the behavior they encounter, and the exposure will have different effects depending on the behavior of the exposed person, with the exposed good behavior being rewarded and the exposed bad behavior being punished. Because those who expose suffer loss or injustice when interacting with bad behavior, those who are brave enough to expose bad behavior are also rewarded. Therefore, how exposure behavior affects the evolution of cooperation deserves further study. In this paper, we develop an exposure-based reward and punishment model, in which all players can choose whether to perform exposure or not, to investigate the effect of this mechanism on the evolution of cooperation in infinite well-mixed populations.

The paper is organized as follows: in Section 2 we introduce a four-strategy game model and establish the replication equations; in Section 3 we analyze the equilibrium points and stability of the model and give the evolution results. The main conclusions are summarized in Section 4.

Table 1
Payoff matrix.

	C	EC	D	ED
C	1	$1+\alpha$	0	α
EC	$1-\theta$	$1+\alpha-\theta$	$\beta-\theta$	$\beta+\alpha-\theta$
D	b	$b-\beta$	0	$-\beta$
ED	$b-\theta$	$b-\beta-\theta$	$\beta-\theta$	$-\theta$

2. Model

In this paper, we present a four-strategy game model that incorporates players' exposure behavior in addition to their cooperation and defection strategies. Players who perform exposure are called exposure players and players who do not perform exposure are called ordinary players. Thus, four strategies exist, namely exposure cooperation (EC), exposure defection (ED), ordinary cooperation (C) and ordinary defection (D).

For the exposure players, their exposure behavior entails a cost θ. Their strategy is to expose cooperators so that the exposed cooperators are rewarded α (called cooperation reward) on the one hand, and to expose defectors so that the exposed defectors are punished β (called defection punishment) on the other hand. When an exposure player exposes a defector, the punishment β to the exposed defector is used as a reward to the exposure player due to his own payoff deficiency and his behavior against defector.

In prisoner's dilemma game, if both players hold cooperation (defection) strategy, they each gain payoff R (P). If one player holds cooperation strategy and the other holds defection strategy, the former gains payoff S and the latter gains payoff T. The payoff satisfies $T > P > R > S$ and $2R > T + S$[3]. Like previous work, we use weak PDG [15], i.e., $T = b\ (1 < b \leq 2), R = 1, P = 0, S = 0$. The corresponding payoff matrix is presented as in Table 1.

Playing the above game in an infinite well-mixed population, let x, y, z, h represent the fraction of ordinary cooperation, exposure cooperation, ordinary defection, exposure defection, respectively, and $x + y + z + h = 1$. The average payoff of the four strategies can be expressed as:

$$\begin{aligned}
&P_C = x + (1+\alpha)y + \alpha h,\\
&P_{EC} = (1-\theta)x + (1+\alpha-\theta)y + (\beta-\theta)z + (\beta+\alpha-\theta)h,\\
&P_D = bx + (b-\beta)y - \beta h,\\
&P_{ED} = (b-\theta)x + (b-\beta-\theta)y + (\beta-\theta)z - \theta h.
\end{aligned} \tag{1}$$

According to the payoff calculation formula (1), the replication equations, describing the effect of frequency on choice when the number of strategies is fixed, are given as:

$$\begin{aligned}
&\dot{x} = x\left(P_C - \bar{P}\right),\\
&\dot{y} = y\left(P_{EC} - \bar{P}\right),\\
&\dot{z} = z\left(P_D - \bar{P}\right),\\
&\dot{h} = h\left(P_{ED} - \bar{P}\right),
\end{aligned} \tag{2}$$

where $\bar{P} = xP_C + yP_{EC} + zP_D + hP_{ED}$ is the expected payoff of the whole population.

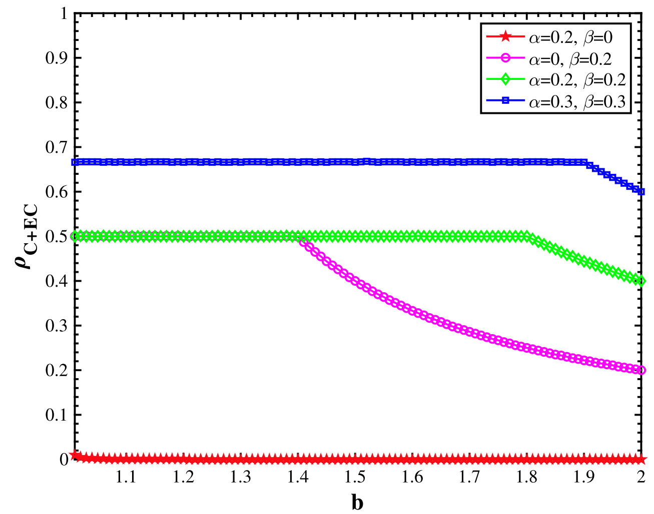


Fig. 1. The evolution results of the overall cooperation proportion ρ_{C+EC} with respect to b under different α and β parameter combinations. The fixed parameter is $\theta = 0.1$.

3. Results

We first analyze the equilibrium points of the model (2) and their stability conditions [55–58]. There are eight equilibrium points: $M_1 = (0,0,0,1)$, $M_2 = (0,0,1,0)$, $M_3 = (0,1,0,0)$, $M_4 = (1,0,0,0)$, $M_5 = \left(\frac{a+\theta}{b+a-1}, 0, 0, \frac{b-\theta-1}{b+a-1}\right)$, $M_6 = (0, \frac{\theta-\beta}{a-b+1}, \frac{\beta-\theta+a-b+1}{a-b+1}, 0)$, $M_7 = (0, \frac{a+\beta}{b-1}, 0, \frac{b-1-a-\beta}{b-1})$, $M_8 = (\frac{\beta-\beta y-\theta}{\beta}, y, \frac{\beta^2 y-\beta(b-1-ay-\theta)+\theta(b+a-1)}{\beta(a+\beta)}, \frac{\beta(b-1-ay)-\beta^2 y-\theta(b-1)}{\beta(a+\beta)})$.

The stability results are as follows (see Appendices A–C for derivation):

1. The equilibrium points M_1, M_3, M_4, M_5 and M_6 are all unstable equilibrium points, i.e., the system will not appear in the state corresponding to that equilibrium points.
2. When $\beta < \theta$, the equilibrium point M_2 is stable. That is, when the parameters satisfy this condition, all in the system will be ordinary defection.
3. When $\beta^2 + \theta(b-1) - \beta(b-\alpha-1) < 0$, the equilibrium point M_7 is stable. That is, when the parameters satisfy this condition, the exposure cooperation and exposure defection coexist in the system.
4. The equilibrium point M_8 is too complex in form to give its stability condition, but it is stable and has multi-stability as found by numerical simulation. For different initial values, all four types of strategies converge to different values, but the overall cooperation proportion (the sum of the fraction of exposure cooperation and ordinary cooperation) or the overall defection proportion (the sum of the fraction of exposure defection and ordinary defection) converge to the same value. For a given parameter condition and initial proportion, these four strategies can coexist in the system.

Let the initial value be $(x(0), y(0), z(0), h(0)) = (0.25, 0.25, 0.25, 0.25)$, we give the main evolutionary results of the model (2) using numerical simulations.

When the exposure cost θ is equal to 0.1, considering different cooperation reward α and defection punishment β parameter combinations, the evolution results of the overall cooperation proportion ρ_{C+EC} with respect to temptation to defect b are shown in Fig. 1.

As can be seen from Fig. 1, when $\alpha = 0.2, \beta = 0$, only the exposed cooperators will be rewarded, and the exposed defectors will not be punished. At this time, the ρ_{C+EC} is always equal to 0. Therefore, only taking incentive measures is not enough to support cooperation. When $\alpha = 0, \beta = 0.2$, only the exposed defectors will be punished, and the ρ_{C+EC} will be significantly improved. Therefore, taking punitive measures can promote cooperation. At this time, when $b \le 1.4$, the ρ_{C+EC} remains unchanged with the increase of b, and when $b > 1.4$, the ρ_{C+EC} decreases with the increase of b. When $\alpha = 0.2, \beta = 0.2$, both the exposed cooperators will be rewarded and the exposed defectors will be punished. At this time, compared with only punishing the exposed defectors, when $b \le 1.4$, the ρ_{C+EC} will remain unchanged, but when $b > 1.4$, the ρ_{C+EC} will be increased. Therefore, when the temptation to defect is small, the same effect can be achieved without rewarding the exposed cooperators. When the temptation to defect is large, the promotion effect of taking reward and punishment measures is more obvious. When $\alpha = 0.3$ and $\beta = 0.3$, compared with $\alpha = 0.2$ and $\beta = 0.2$, the ρ_{C+EC} is further improved, and the corresponding temptation to defect becomes larger when the ρ_{C+EC} starts to decline. Therefore, to a certain extent, increasing the strength of cooperation reward and defection punishment simultaneously can promote cooperation.

The payment of exposure behavior is reflected in the exposure cost and Fig. 2 explores the effect of changes in exposure costs on the evolution of cooperation under different temptations to defect, cooperation rewards and defection punishments. From Fig. 2(a), we can see that there are three states of the population with the increase of θ. When $\theta \le 0.07$, there are only exposure cooperation and exposure defection in the population, when $0.07 < \theta < 0.3$, four types of strategies coexist in the population, and when $\theta \ge 0.3$, only ordinary defection remains in the population. For ordinary cooperation, the fraction shows nonlinear changes with the increase of exposure cost. In Fig. 2(b), the fraction of ordinary cooperation increases first and then decreases with the increase of θ, so there is an optimal exposure cost θ^* to maximize the fraction of ordinary cooperation. When $b = 1.2, \theta^* = 0.01$; when $b = 1.5, \theta^* = 0.12$; when $b = 1.8, \theta^* = 0.19$. Therefore, with the increase of the temptation to defect, the exposure cost corresponding to the optimal fraction of ordinary cooperation is also increasing, and the maximum fraction that can be reached is decreasing. Assuming that the cooperation reward is the same as the defection punishment, it can be found in Fig. 3(c) that, under different temptation to defect, with the increase of reward and punishment, the exposure cost corresponding to the optimal fraction of ordinary cooperation can increase first and then decrease. Therefore, there is a set of reward and punishment that require a higher exposure cost to make the fraction of ordinary cooperation optimal. In addition, we can find that when the reward and punishment are large enough, the fraction of ordinary cooperation can reach the optimum when the exposure cost is 0.01.

The amount of defection punishment affects the payoff of exposure cooperation and two types of defection. It can be seen from Fig. 3(a) that with the increase of β, there are two states of the population. When $\beta \le 0.1$, only ordinary defection exists in the population, and when $\beta > 0.1$, the four types of strategies coexist in the population. In Fig. 3(b), corresponding to different exposure costs, when the defection punishment is greater than the exposure cost, cooperative players begin to appear, and the overall cooperation proportion increases with the increase of β. The smaller the θ, the higher the overall cooperation proportion. Therefore, the increase in defection punishment and the decrease in exposure cost facilitate the overall cooperation. In Fig. 3(c), we can find that, for different exposure costs, the fraction of exposure defection increases first and then decreases, but with the increase of defection punishment, the fraction of exposure defection corresponding to different exposure costs shows a trend of phased dominance. When $0.05 < \beta \le 0.17, \theta = 0.05$ prevails; when $0.17 < \beta \le 0.3, \theta = 0.1$ prevails; when $0.3 < \beta \le 0.51, \theta = 0.2$ prevails; when $\beta > 0.51, \theta = 0.3$ prevails. Thus, for the exposure defection, under different defection punishments, there is always an optimal exposure cost making it optimal, and the greater the defection punishment the greater the exposure cost is for its survival.

The amount of cooperation reward affects the payoff of both types of cooperation. It can be seen from Fig. 4(a) that with the increase of α,

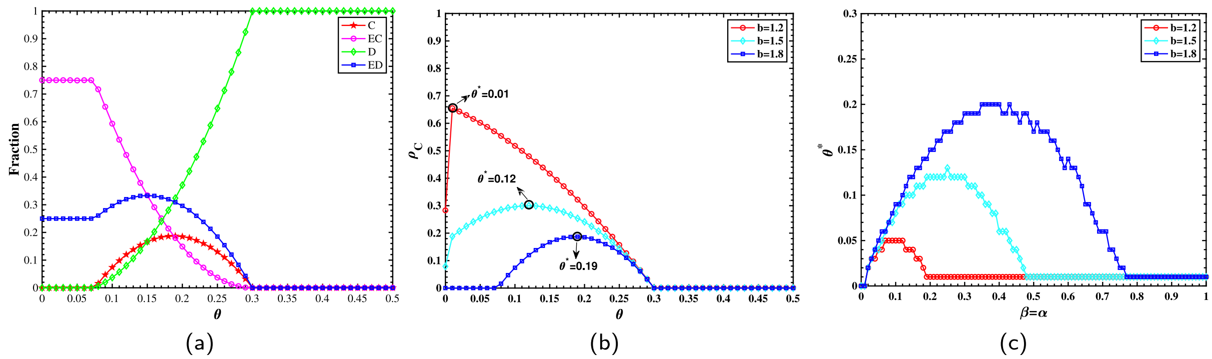


Fig. 2. (a) Evolutionary results of the fraction of four strategies with respect to θ when $\alpha = \beta = 0.3$, $b = 1.8$. (b) Evolutionary results of the fraction of ordinary cooperation with respect to θ when $\alpha = \beta = 0.3, b \in \{1.2, 1.5, 1.8\}$. (c) The evolution of the exposure cost θ^* corresponding to the optimal fraction of ordinary cooperation in the population with respect to $\alpha = \beta$ when $b \in \{1.2, 1.5, 1.8\}$.

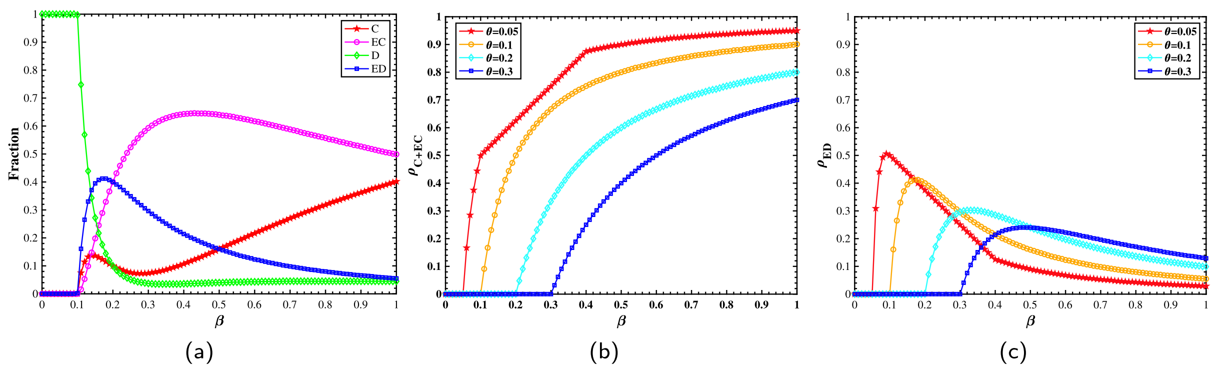


Fig. 3. (a) Evolutionary results of the fraction of four strategies with respect to β when $\theta = 0.1$. (b) Evolutionary results of the overall cooperation proportion ρ_{C+EC} with respect to θ when $\theta \in \{0.05, 0.1, 0.2, 0.3\}$. (c) Evolutionary results of the fraction of exposure defection ρ_{ED} with respect to β when $\theta = 0.1$. The fixed parameters are $b = 1.8, \alpha = 0.3$.

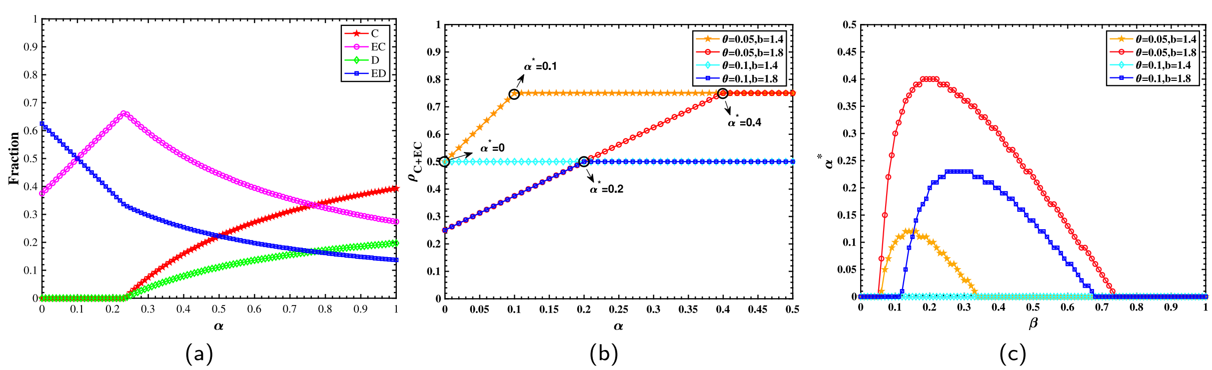


Fig. 4. (a) Evolutionary results of the fraction of four strategies with respect to α when $b = 1.8, \theta = 0.1, \beta = 0.3$. (b) Evolutionary results of the overall cooperation proportion ρ_{C+EC} with respect to α when $\beta = 0.3, \theta \in \{0.05, 0.1\}$, $b \in \{1.4, 1.8\}$. (c) The evolution of the cooperation reward threshold α^* corresponding to the optimal overall cooperation proportion ρ_{C+EC} in the population with respect to β when $\theta \in \{0.05, 0.1\}, b \in \{1.4, 1.8\}$.

there are two states of the population. When $\alpha \leq 0.23$, exposure cooperation and exposure defection coexist. At this time, with the increase of α, exposure cooperation increases, and exposure defection decreases. When $\alpha > 0.23$, four types of strategies coexist. With the increase of α, the fraction of exposure players decreases, while the fraction of ordinary players increases. Therefore, larger cooperation reward will promote the generation of "second-order free-riders". For cooperators or defectors, the decrease in exposure players is equal to the increase in ordinary players, i.e., continuing to increase α at this point does not affect the overall fraction of cooperation or defection. In Fig. 4(b), when $\theta = 0.1, b = 1.4$, the overall cooperation proportion is not affected by the α change. For other case, the overall cooperation proportion increases first and then remains unchanged with the increase of α, and for the same θ, the overall cooperation proportion will eventually be the same with the increase of α. Moreover, there exists a minimum cooperation reward threshold α^* that makes the overall cooperation proportion optimal. Further, in Fig. 4(c), we can find that when the cooperation reward can affect the overall cooperation proportion, with the increase of β, the minimum cooperation reward threshold α^* corresponding to the optimal overall cooperation proportion will increase first and then decrease. In the stage when the defection punishment is small, increasing the defection punishment requires a simultaneous increase in the cooperation reward to make the overall cooperation proportion optimal, but when the defection punishment increases to a certain value, continuing to increase the defection punishment only requires a smaller cooperation reward to make the overall cooperation proportion optimal. When the defection punishment is large enough, the change of cooperation reward has no effect on the overall cooperation proportion.

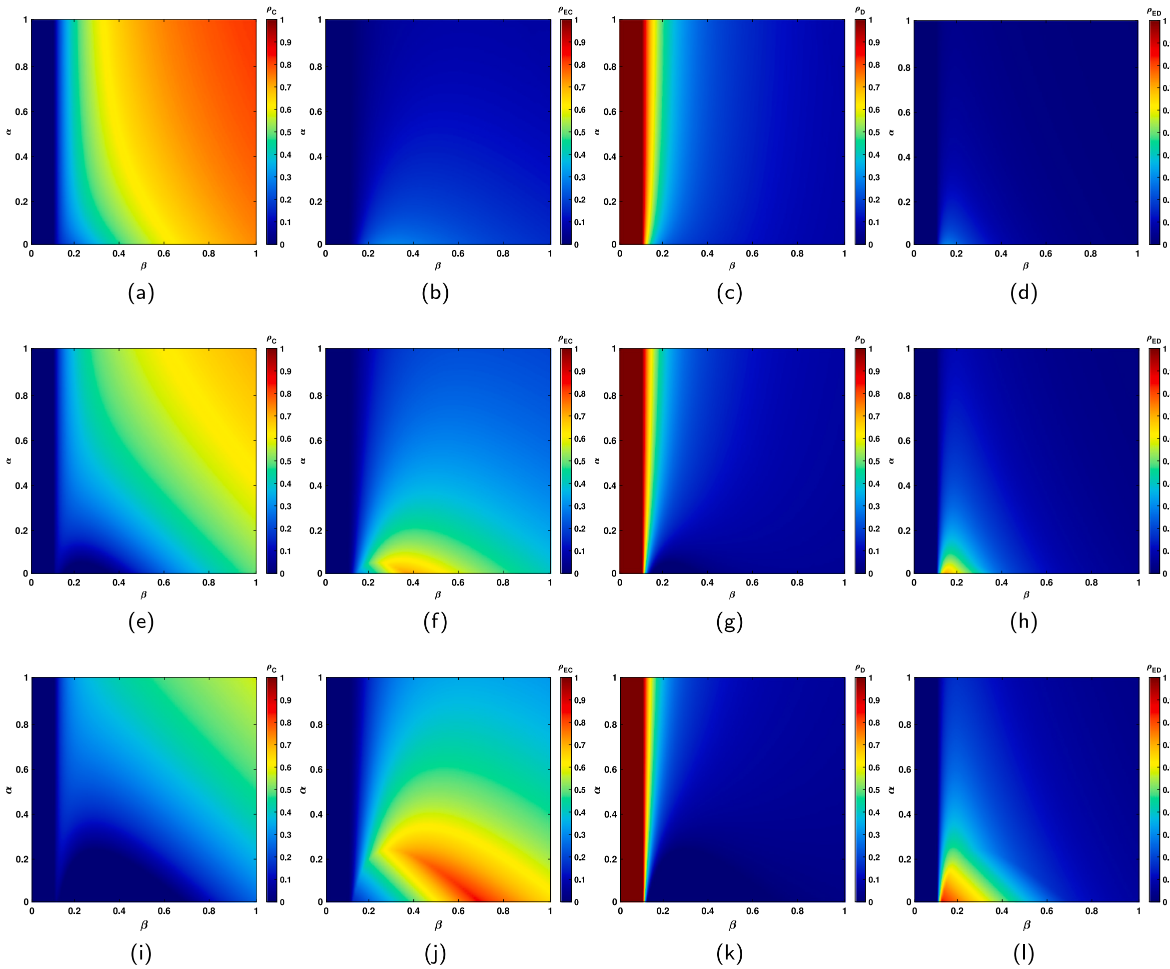


Fig. 5. Heat map of the fraction of the four types of strategies with respect to the $\alpha - \beta$ parameter panel. (a–d) $b = 1.2$, (e–h). $b = 1.5$, (i–l) $b = 1.8$. The fixed parameter is $\theta = 0.1$.

To further investigate the effect of the combined effect of cooperation reward and defection punishment on the four types of strategies under different temptations to defect, the heat maps of the fraction of the four types of strategies with respect to the $\alpha - \beta$ parameter panel when $\theta = 0.1$, $b \in \{1.2, 1.5, 1.8\}$ are shown in Fig. 5.

In Fig. 5(a–d) with $b = 1.2$, we can find that ordinary cooperation is well promoted when β is larger than 0.1, and the larger the defection punishment and cooperation reward, the higher the fraction of ordinary cooperation. Ordinary defection is more sensitive to the change of defection punishment and occupies a higher fraction only when the defection punishment is small, but even if the defection punishment is large enough, ordinary defection will not disappear. Exposure cooperation and exposure defection are both insensitive to changes in cooperation reward and defection punishment, and maintain a low proportion. In Fig. 5(e–h) with $b = 1.5$, there exists a region in $\beta \in [0.2, 0.4]$, $\alpha \in [0, 0.1]$ making ordinary cooperation and ordinary defection disappear, leaving only exposure cooperation and exposure defection in the population. Exposure players are facilitated when the cooperation reward is small, with exposure cooperation occupying a higher proportion at $\beta = 0.4$ and exposure defection occupying a higher proportion at $\beta = 0.2$. When $\beta > 0.4$, the change in cooperation reward does not affect the fraction of ordinary defection and exposure defection, but only the fraction of the two types of cooperation, the larger the cooperation reward, the higher the fraction of ordinary cooperation. In Fig. 5(i–l), $b = 1.8$, the coexistence area of exposure cooperation and exposure defection becomes larger, that is, the increase of temptation to defect will promote the survival of exposure players. At this time, the proportion of ordinary cooperation and ordinary defection are reduced, so the increase of temptation to defect can inhibit the “free-riding” behavior, and has a greater impact on the “second-order free-riding” behavior of ordinary cooperation.

4. Conclusion

In this paper, we analyze the effects of exposure-based reward and punishment on the evolution of cooperation in the prisoner’s dilemma. Exposure behavior entails a cost to expose the players with whom it is played. The exposed cooperator is rewarded, the exposed defector is punished, and the punishment serves as a reward for the exposed player for exposing the defection.

Utilizing stability analysis and numerical simulations we find that the system emerges in a total of three states, which are all ordinary defection, exposure cooperation coexists with exposure defection, and all four strategies coexist. The evolutionary results show that cooperative behavior can emerge as long as the exposure cost is less than the defection punishment. In addition, we find that simply rewarding cooperation is not sufficient to promote cooperation, and simply punishing defection can promote cooperation but the promotion effect is not as obvious as the simultaneous reward and punishment. There is an exposure cost that makes the fraction of ordinary cooperation optimal, and a very small exposure cost can make the fraction of ordinary cooperation optimal when both the cooperation reward and the defection punishment are large enough. The larger the defection punishment, the more favorable the cooperation, but not the larger the cooperation reward. When the cooperation reward increases to a certain level, continuing to increase will not affect the overall cooperation fraction, but only the fraction between the two types of cooperation, as reflected by the larger the cooperation reward, the more ordinary cooperation and the less exposure cooperation. In addition, there exists a minimum cooperation reward that makes the overall cooperation proportion optimal, and as the defection punishment increases, the cooperation reward that can drive the overall cooperation proportion to be optimal shows a non-linear change. Moreover, smaller cooperation reward and larger temptation to defect is conducive to the survival of exposure players, which can effectively prevent "free-riding" behavior, when the defection punishment is small is conducive to exposure defection, when the defection punishment is larger is conducive to exposure cooperation.

Exposure behavior is everywhere, and our research can provide a new perspective on solving social dilemmas. In this work, we consider the interactions between individuals in the well-mixed populations. However, interactions between individuals are usually not random but limited by network structure. Therefore, explore the impact of this mechanism in a more complex network [59] is important and valuable. Also, combining this reward and punishment mechanism with reputation mechanism deserves further research. In addition, whether the time lag effect [60] has an impact on punishment or reward measures is also a topic worth investigating.

CRediT authorship contribution statement

Wenqiang Zhu: Conceptualization, Methodology, Software, Formal analysis, Visualization, Writing – original draft. **Qiuhui Pan:** Conceptualization, Methodology, Supervision, Validation, Writing – review & editing. **Sha Song:** Methodology, Investigation, Formal analysis, Writing – review & editing. **Mingfeng He:** Methodology, Validation, Writing – review & editing.

Declaration of competing interest

No conflict of interest exits in the submission of this manuscript, and manuscript is approved by all authors for publication. I would like to declare on behalf of my co-author that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the manuscript that is enclosed.

Data availability

No data was used for the research described in the article.

Appendix A. Equilibrium stability analysis

Due to $h = 1 - x - y - z$, the four formulas in model (2) can be simplified into three formulas, as follows,

$$
\begin{aligned}
\dot{x} = f(x,y,z) &= (b+\alpha-1)x^3 + (2b+\alpha-2)x^2 y \\
&+ \alpha x^2 z + (b-1)xy^2 + \alpha xyz - (b+\alpha-1)xy \\
&- (\alpha+\theta)xz - (b+2\alpha+\theta-1)x^2 + (\alpha+\theta)x, \\
\dot{y} = g(x,y,z) &= (b-1)y^3 + (2b+\alpha-2)xy^2 \\
&+ (b+\alpha-1)x^2 y + ay^2 z - (b+\alpha+\beta-1)y^2 \\
&- (b+2\alpha+\beta+\theta-1)xy + axyz - (\alpha+\theta)yz + (\alpha+\beta)y, \\
\dot{z} = w(x,y,z) &= (b+\alpha-1)x^2 z + \alpha(x+y)z^2 + (b-1)y^2 z \\
&+ (\beta-\theta)z^2 + (2b+\alpha-2)xyz + (\beta-\alpha-\theta)xz - ayz - (\beta-\theta)z.
\end{aligned}
\tag{A.1}
$$

The system has eight equilibrium points: $M_1 = (0,0,0,1), M_2 = (0,0,1,0), M_3 = (0,1,0,0), M_4 = (1,0,0,0), M_5 = \left(\frac{a+\theta}{b+a-1}, 0, 0, \frac{b-\theta-1}{b+a-1}\right)$, $M_6 = (0, \frac{\theta-\beta}{a-b+1}, \frac{\beta-\theta+a-b+1}{a-b+1}, 0), M_7 = \left(0, \frac{a+\beta}{b-1}, 0, \frac{b-1-a-\beta}{b-1}\right), M_8 = (\frac{\beta-\beta y-\theta}{\beta}$, $y, \frac{\beta^2 y-\beta(b-1-ay-\theta)+\theta(b+a-1)}{\beta(a+\beta)}, \frac{\beta(b-1-ay)-\beta^2 y-\theta(b-1)}{\beta(a+\beta)})$.

According to the Jacobian matrix to examine the stability of these equilibrium points:

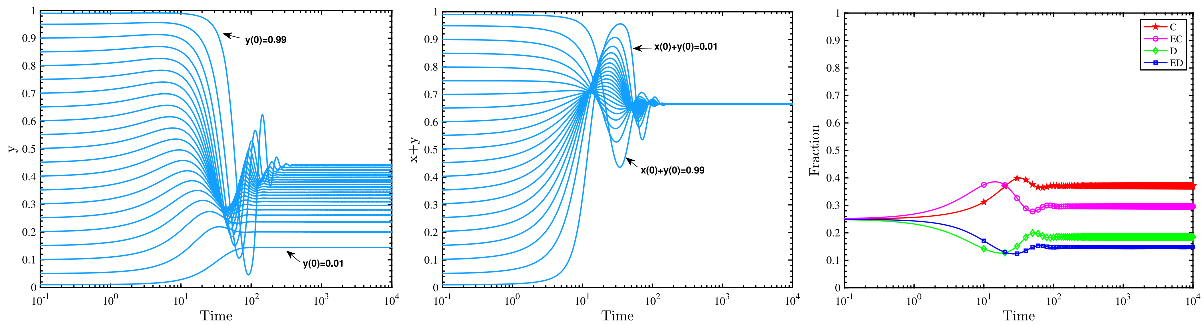
$$
J = \begin{bmatrix}
\frac{\partial f(x,y,z)}{\partial x} & \frac{\partial f(x,y,z)}{\partial y} & \frac{\partial f(x,y,z)}{\partial z} \\
\frac{\partial g(x,y,z)}{\partial x} & \frac{\partial g(x,y,z)}{\partial y} & \frac{\partial g(x,y,z)}{\partial z} \\
\frac{\partial w(x,y,z)}{\partial x} & \frac{\partial w(x,y,z)}{\partial y} & \frac{\partial w(x,y,z)}{\partial z}
\end{bmatrix}
\tag{A.2}
$$

where,

$$
\begin{aligned}
\frac{\partial f(x,y,z)}{\partial x} &= 3(b+\alpha-1)x^2 + 2(2b+\alpha-2)xy + 2\alpha xz + (b-1)y^2 \\
&+ \alpha yz - (b+\alpha-1)y - (\alpha+\theta)z \\
&- 2(b+2\alpha+\theta-1)x + \alpha + \theta, \\
\frac{\partial f(x,y,z)}{\partial y} &= (2b+\alpha-2)x^2 + 2(b-1)xy + \alpha xz - (b+\alpha-1)x, \\
\frac{\partial f(x,y,z)}{\partial z} &= \alpha x^2 + \alpha xy - (\alpha+\theta)x, \\
\frac{\partial g(x,y,z)}{\partial x} &= (2b+\alpha-2)y^2 + 2(b+\alpha-1)xy \\
&- (b+2\alpha+\beta+\theta-1)y + \alpha yz, \\
\frac{\partial g(x,y,z)}{\partial y} &= 3(b-1)y^2 + 2(2b+\alpha-2)xy + (b+\alpha-1)x^2 \\
&+ 2\alpha yz - 2(b+\alpha+\beta+\theta-1)y \\
&- (b+2\alpha+\beta+\theta-1)x + \alpha xz - (\alpha+\theta)z + \alpha + \beta, \\
\frac{\partial g(x,y,z)}{\partial z} &= \alpha y^2 + \alpha xy - (\alpha+\theta)z, \\
\frac{\partial w(x,y,z)}{\partial x} &= 2(b+\alpha-1)xz + \alpha z^2 + (2b+\alpha-2)yz \\
&+ (\beta-\alpha-\theta)z, \\
\frac{\partial w(x,y,z)}{\partial y} &= \alpha z^2 + 2(b-1)yz + (2b+\alpha-2)xz - \alpha z, \\
\frac{\partial w(x,y,z)}{\partial z} &= (b+\alpha-1)x^2 + 2\alpha(x+y)z + (b-1)y^2 \\
&+ 2(\beta-\theta)z + (2b+\alpha-2)xy \\
&+ (\beta-\alpha-\theta)x - \alpha y - \beta + \theta.
\end{aligned}
\tag{A.3}
$$

Stability analysis is carried out according to the eigenvalues of the linearized Jacobian matrix of system ((A.2)) at the equilibrium points.

1. For point $M_1 = (0,0,0,1)$, $\lambda_1 = \theta - \beta$, $\lambda_2 = \alpha + \theta$, $\lambda_3 = \alpha + \beta$, therefore, the equilibrium M_1 is unstable.
2. For point $M_2 = (0,0,1,0)$, $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = \beta - \theta$, since there is a zero value in the eigenvalues, stability cannot be determined simply by judging the sign of the eigenvalues. We use the center manifold theory [61] to analyze this equilibrium point (see Appendix B).
3. For point $M_3 = (0,1,0,0)$, $\lambda_1 = \theta$, $\lambda_2 = b - 1 - \alpha - \beta$, $\lambda_3 = b - 1 - \alpha - \beta + \theta$, therefore, the equilibrium point M_3 is unstable.
4. For point $M_4 = (1,0,0,0)$, $\lambda_1 = b - 1$, $\lambda_2 = -\theta$, $\lambda_3 = b - \theta - 1$, therefore, the equilibrium point M_4 is unstable.
5. For point $M_5 = \left(\frac{\alpha+\theta}{b+\alpha-1}, 0, 0, \frac{b-\theta-1}{b+\alpha-1}\right)$, $\lambda_1 = -\frac{(b-\theta-1)(\alpha+\theta)}{b+\alpha-1}$, $\lambda_2 = -\frac{\theta(b+\alpha-1)-\beta(b-\theta-1)}{b+\alpha-1}$, $\lambda_3 = \frac{\theta(b+\alpha-1)-\beta(b-\theta-1)}{b+\alpha-1}$, therefore, the equilibrium point M_5 is unstable.
6. For point $M_6 = \left(0, \frac{\theta-\beta}{\alpha-b+1}, \frac{\beta-\theta+\alpha-b+1}{\alpha-b+1}, 0\right)$, $\lambda_1 = \frac{(\theta-\beta)(b-\alpha-\beta-1+\theta)}{b+\alpha-1}$, $\lambda_2 = -\frac{(\beta-\theta)(b-\alpha-\beta-1)}{b+\alpha-1}$, $\lambda_3 = \frac{(\beta-\theta)(b-\alpha-\beta-1)}{b+\alpha-1}$, therefore, the equilibrium point M_6 is unstable.
7. For point $M_7 = \left(0, \frac{\alpha+\beta}{b-1}, 0, \frac{b-1-\alpha-\beta}{b-1}\right)$, $\lambda_1 = \frac{(\alpha+\beta)(\alpha+\beta-b+1)}{b-1}$, $\lambda_2 = \lambda_3 = \frac{\beta^2-\beta(b-\alpha-1)+\theta(b-1)}{b-1}$. Since $b-1-\alpha-\beta > 0, \lambda_1 < 0$. Therefore, when $\beta^2 + \theta(b-1) - \beta(b-\alpha-1) < 0, \lambda_2 = \lambda_3 < 0$, the equilibrium point M_7 is stable.
8. For point $M_8 = (\frac{\beta-\beta y-\theta}{\beta}, y, \frac{\beta^2 y-\beta(b-1-\alpha y-\theta)+\theta(b+\alpha-1)}{\beta(\alpha+\beta)}, \frac{\beta(b-1-\alpha y)-\beta^2 y-\theta(b-1)}{\beta(\alpha+\beta)})$, related to the proportion y of exposure cooperation in the population. $\lambda_1 = 0$, $\lambda_2 = \frac{\Phi+\sqrt{\Psi}}{2(\alpha+\beta)\beta}$, $\lambda_3 = \frac{\Phi-\sqrt{\Psi}}{2(\alpha+\beta)\beta}$,



(a) (b) (c)

Fig. A.1. (a) Evolutionary results of the exposure cooperation proportion y over time for different initial proportions $y(0)$. Set the initial proportions of the other three types of strategies as $\frac{1-y(0)}{3}$. (b) Evolutionary results of the sum of the proportions of ordinary cooperation and exposure cooperation x+y over time for different initial proportions $x(0)+y(0)$. Set the initial proportions $x(0)=y(0), z(0)=h(0)=\frac{1-x(0)-y(0)}{2}$. (c) Evolutionary results of the four types of strategies over time when the initial proportion of the four types of strategies is taken to be the same, i.e., $x(0)=y(0)=z(0)=h(0)=0.25$. The fixed parameters are $b=1,4,\theta=0.1,\alpha=0.3,\beta=0.3$.

where $\Phi=\alpha^2\beta y+\alpha\beta^2 y-(\theta+\alpha)(\beta-\theta)(b-1)$ and $\Psi=4y^2\beta^6+[12\alpha y^2-8y(b-1)]\beta^5+\left[16\left(y-\frac{1}{4}\right)(b-1)\theta+13y^2\alpha^2-16y(b-1)\alpha+4(b-1)^2\right]\beta^4+[-8(y-1)(b-1)\theta^2-12(b-1)\left(\frac{2}{3}-\frac{5}{2}y\right)\alpha\theta+b\theta-\theta+4(b-\alpha y-1)\left(b-\frac{3}{2}\alpha y-1\right)\alpha]\beta^3+[(4-4b)\theta^3+13(b-1)\left(\frac{16}{13}-\frac{14}{13}y\right)\alpha\theta^2+b\theta^2-\theta^2-10\left(\frac{2}{5}-\frac{8}{5}y\right)\alpha^2(b-1)\theta+(b-1)^2\alpha\theta+\alpha^2(b-\alpha y-1)^2]\beta^2-2[(3b+4\alpha-3)\theta^2-4(1-\frac{3}{4}y)\alpha^2\theta+b\alpha\theta-\alpha\theta+\alpha^2(b-\alpha y-1)]\theta(b-1)\beta+[\theta^2(b-1)-2\alpha(b+2\alpha-1)\theta+(b-1)\alpha^2]\theta^2(b-1)$. The form is too complex to judge its stability by simple judgment conditions, and the stable existence of this equilibrium point is discussed through the method of numerical simulation (see Appendix C).

Appendix B. Stability analysis of equilibrium point M_2

According to Appendix A, $M_2=(0,0,1,0)$ and $\lambda_1=0$, $\lambda_2=\lambda_3=\beta-\theta$. Using the center manifold theory, let $z^*=z-1$, M_2 is equivalent to the equilibrium point $(0,0,0)$ of the following equation:

$$\begin{aligned}
\dot{x}&=(b+\alpha-1)x^3+(2b+\alpha-2)x^2y+\alpha x^2z^*+(b-1)xy^2\\
&\quad+\alpha xyz^*+(1-b)xy-(\alpha+\theta)xz^*-(b+\alpha+\beta-1)x^2,\\
\dot{y}&=(\beta-\theta)y+(b+\alpha-1)x^2y+(2b+\alpha-2)xy^2+\alpha xyz^*\\
&\quad-(b+\beta+\theta-1)xy+(b-1)y^3+\alpha y^2z^*-(b+\beta-1)y^2\\
&\quad-(\alpha+\theta)yz^*-\alpha xy,\\
\dot{z}^*&=(b+\alpha-1)x^3+(2b+\alpha-2)xy+\alpha xz^*+(b-1)y^2+\alpha yz^*\\
&\quad+(\beta-\theta)\left(x+z^*\right).
\end{aligned} \tag{B.1}$$

The corresponding Jacobi matrix at this equilibrium point is:

$$J=\begin{bmatrix} 0 & 0 & 0\\ 0 & \beta-\theta & 0\\ \beta-\theta & 0 & \beta-\theta \end{bmatrix} \tag{B.2}$$

According to the central manifold theory, the central manifold of the system is one-dimensional, which can be expressed as:

$$\begin{gathered}
y=\phi(x), z^*=\psi(x), \phi(0)=\psi(0)=0, \phi'(0)=\psi'(0)=0\\
\phi(x)=c_1x^2+c_2x^3+c_3x^4+\cdots\\
\psi(x)=d_1x^2+d_2x^3+d_3x^4+\cdots
\end{gathered} \tag{B.3}$$

Substituting $y=\phi(x), z^*=\psi(x)$ into $\dot{x}$, and the least term on the right-hind side of the equation is the quadratic $-(b+\alpha+\beta-1)x^2$. Because $b+\alpha+\beta-1$ is always greater than 0, the quadratic coefficient is negative. Since the initial value of the system is non-negative, $\lambda_1=0$ will not affect the stability, so only $\lambda_2=\lambda_3<0$, that is, $\beta<\theta$, the equilibrium point M_2 is stable.

Appendix C. Numerical simulation of equilibrium point M_8

According to Appendix A, $M_8=(\frac{\beta-\beta y-\theta}{\beta}, y, \frac{\beta^2y-\beta(b-1-\alpha y-\theta)+\theta(b+\alpha-1)}{\beta(\alpha+\beta)}, \frac{\beta(b-1-\alpha y)-\beta^2y-\theta(b-1)}{\beta(\alpha+\beta)})$. If we sum the proportion of the two types of cooperation (defection) in M_8, the final state of the system will be independent of the proportion of exposure cooperation y. Translating the equilibrium point into the overall cooperation proportion and the overall defection proportion, the equilibrium point becomes $M_8^*=(x+y,\ z+h)=(1-\frac{\theta}{\beta},\ \frac{\theta}{\beta})$.

To explore the properties of equilibrium M_8, when $b=1.4,\theta=0.1,\alpha=0.3,\beta=0.3$ (which is not satisfy $\beta<\theta$ and $\beta^2+\theta(b-1)-\beta(b-\alpha-1)<0$), Fig.A1 shows the evolutionary results of each for different initial proportions of exposure cooperation and for different initial proportions of overall cooperation over time. As well as the evolution results of the four types of strategies over time when taking the same initial proportion of the four types of strategies, i.e., $x(0)=y(0)=z(0)=h(0)=0.25$.

From Fig. A.1(a), it can be seen that, corresponding to different initial proportions $y(0)$, the exposure cooperation eventually exists in the population, but converges at different values. This situation can be considered as multi-stability of the system, but not asymptotic stability, that is, when that equilibrium point is disturbed, as the evolution time increases, the trajectory does not return to that point, but stays in some small domain of that point. From Fig. A.1(b), it can be seen that the overall cooperation proportions eventually converge to the same value corresponding to different $x(0)+y(0)$, so the change in the initial proportions does not affect the convergence values of the cooperation or defection groups in the system. As can be seen from Fig. A.1(c), for a given parameter condition and initial proportion, the fraction of the four types of strategies tend to stabilize with the evolution of time, and eventually the four types of strategies are able to coexist.

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