Chaos, Solitons and Fractals 160 (2022) 112205

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Exposure-based reputation mechanism promotes the evolution of cooperation

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ARTICLE INFO

Article history:
Received 26 December 2021
Received in revised form 26 April 2022
Accepted 6 May 2022
Available online 24 May 2022

Keywords:
Exposure-based reputation
Evolution of cooperation
Prisoner's dilemma

ABSTRACT

Exposure behavior can affect the reputation of both the exposer and the exposed. In this paper, we consider the effect of exposure-based reputation on the evolution of cooperation. There are three strategies in the population, including exposure cooperation, ordinary cooperation and defection. By undertaking an exposure cost, the exposer will praise the cooperator to increase his reputation and condemn the defector to decrease his reputation, while the reputation of the exposer will also change. Results show that five states emerge under different combinations of parameters, namely, all defection, all exposure cooperation, coexistence of exposure cooperation and ordinary cooperation, coexistence of exposure cooperation and defection, and coexistence of all three strategies. When the population pays moderate attention to payoff, there is a phased dominant phenomenon in three strategies regarding the temptation to defect. In addition, exposure cooperation has the opportunity to exist when the temptation to defect or the exposure cost is high. Thus, the fact that even in a harsh environment there will still be cooperators who make exposure, which provides a new perspective for understanding the emergence of cooperation.

1. Introduction

Human society is a dynamic and interconnected whole, complex individuals have different ways of engaging in social activities, and cooperation is a common way of activity in people's lives. Under Darwin's evolutionary background [1], individuals in nature are selfish and cooperation will eventually disappear. But there are widespread cooperation behaviors in the real world, what induces cooperation has always been a matter of academic discussion [2–5].In the past, scholars have analyzed the reasons from different perspectives, the most widely used is evolutionary game theory [6–8]. In particular, the prisoner's dilemma [9–12] is most often used as a classic example to study the emergence of cooperation, this is because in the prisoner's dilemma, no one can resist the temptation brought by defection, choosing cooperation will only put themselves in a disadvantageous position. In infinite well-mixed and other population structure, cooperators are likely to be weeded out by natural selection [13–15].

As a pioneering work, Nowak and May [16] discovered that cooperators can resist defector invasion by forming clusters on the lattice. Furthermore, Nowak [17] systematically summarized five rules for fostering the emergence of cooperation, including kin selection [18], direct and indirect reciprocity [19–21], group selection [22,23] and network reciprocity [24,25]. In the subsequent works, researchers also considered all sorts of factors existing in the real society in order to facilitate cooperation, such as reputation [26–28], rewards [29–31], punishment [32,33] and so on. Among these results, reputation as a direct expression of indirect reciprocity has attracted the interest of many scholars. Usually, the assessment rules of this reputation system are simple: Give good credit to those who do a good deal and bad credit for misconduct. Such a simple rule is called image-scoring [34] and has been considered in both theoretical analyses and experiments. In recent years, many scholars have studied the effect on the evolution of cooperation when reputation is private based on indirect reciprocity. Christian Hilbe et al. [35] explored indirect reciprocity when information transmission is private and noisy. Cedric Perret et al. [36] identified which rules are evolutionary stable strategies among all possible moral rules while considering private assessment. They developed an analytical model to determine when a strategy can be invaded by another. Marcus Krellner and The Anh Han [37] showed that most strategies can overcome the private assessment problem by applying pleasing. A pleasing agent can achieve better reputation when there is disagreement in the population, thus enhancing cooperation. Isamu Okada [38] pointed out that behaviors with respect to recipients with good reputations should be

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https://doi.org/10.1016/j.chaos.2022.112205
0960-0779/

rigorously evaluated, while behaviors of recipients with bad reputations should be generously assessed for maintaining stable cooperation.

In real life, reputation may originate from a variety of ways, and the exposure behavior is one of the ways in which reputation is sourced. Exposure behavior can make the behavior of the exposed individual known to all players. The exposed cooperation behavior will be praised thus making the reputation increase, and the exposed defection behavior will be condemned thus making the reputation decrease, and the reputation of the exposer will be changed as a result. This reputation that results from exposure and being exposed is called exposure-based reputation and is collectively referred to as reputation. In this paper, we introduce a cooperation strategy with exposure behavior, which exposes the behavior of the individuals with whom interacted by paying a cost, thus causing a variation in their mutual reputation. The aim is to explore the effects of exposure-based reputation mechanism on the evolution of cooperation.

This paper is organized as follows. Section 2 builds a strategic evolution model through establishment of replication equations. Section 3 presents and analyzes the evolution results. Finally, the main conclusions are given in Section 4.

2. Model

In this paper, we introduce the cooperation strategy with exposure behavior denoted as exposure cooperation (EC), the other two strategies are ordinary cooperation (C) and defection (D) respectively. Here, we consider a Prisoners' Dilemma Games (PDG), where the reward for mutual cooperation is R, the punishment for mutual defection is P, the sucker's payoff is S, and the temptation to defect is T. These payoffs are ordered as follows: $T > P > R > S$. Thus, defection optimizes an individual's payoff, although mutual cooperation yields the highest collective payoff. Like previous works [16], we use weak PDG to study the evolution process of cooperation, where $T = b(1 < b \leq 2)$, $R = 1$, $P = S = 0$. Exposure cooperators pay an exposure cost θ in the course of each round of the game, so the corresponding payoff matrix is presented as in Table 1.

We only consider the impact of EC's exposure behavior on its own reputation as well as those exposed. When cooperator (both C and EC) interacts with EC they are praised and thus receive an increase in reputation, and when defector interacts with EC they are condemned and thus receive a decrease in reputation. In addition, EC gets a sucker's payoff when interacting with D and also fights D through exposure, so EC is given a greater reputation reward and D is given the same degree of reputation penalty. Let δ denotes the variation of reputation, the corresponding reputation matrix is presented as in Table 2.

Assume that $x, y, z = 1 - x - y$ are the fraction of ordinary cooperators, exposure cooperators and defectors respectively. The average payoff of these strategies is given by:

$$
\begin{aligned}
P_C &= x + y,\\
P_{EC} &= x + y - \theta,\\
P_D &= b(x + y),
\end{aligned}
\tag{1}
$$

The average reputation of these strategies is given by:

$$
\begin{aligned}
R_C &= 1 + \delta y,\\
R_{EC} &= 1 + \delta(1 + z),\\
R_D &= 1 - 2\delta y,
\end{aligned}
\tag{2}
$$

Table 1
Payoff matrix.

	C	*EC*	*D*
C	1	1	0
EC	$1-\theta$	$1-\theta$	$-\theta$
D	b	b	0

Table 2
Reputation matrix.

	C	*EC*	*D*
C	1	$1+\delta$	1
EC	$1+\delta$	$1+\delta$	$1+2\delta$
D	1	$1-2\delta$	1

An individual's survival is related to both payoff and reputation, usually the higher the payoff the easier it is to survive, and the higher the reputation the easier it is to survive. The relationship between payoff and reputation has been treated in different ways in previous studies [39–44]. In this paper, we use a weighted summation approach to consider the effect of payoff and reputation on the fitness of the three strategies together. Taking into account the dimensional differences, we normalize the payoff and reputation. The average fitness of these strategies is given by:

$$
\begin{aligned}
\Pi_C &= \alpha\left(\frac{P_C}{P_{\max}}\right) + (1-\alpha)\left(\frac{R_C}{R_{\max}}\right),\\
\Pi_{EC} &= \alpha\left(\frac{P_{EC}}{P_{\max}}\right) + (1-\alpha)\left(\frac{R_{EC}}{R_{\max}}\right),\\
\Pi_D &= \alpha\left(\frac{P_D}{P_{\max}}\right) + (1-\alpha)\left(\frac{R_D}{R_{\max}}\right),
\end{aligned}
\tag{3}
$$

where $\alpha(0 \leq \alpha \leq 1)$ represents the degree of emphasis on payoff. When $\alpha = 1$ indicates that the focus is only on payoff, the change in reputation brought about by the EC has no effect on the selection of the group. When $\alpha = 0$ indicates that the focus is only on reputation, the player has completely given up payoff. P_{max} and R_{max} denote the maximum payoff and the maximum reputation respectively.

Replicator equations are:

$$
\begin{aligned}
\dot{x} &= x(\Pi_C - \overline{\Pi}),\\
\dot{y} &= y(\Pi_{EC} - \overline{\Pi}),\\
\dot{z} &= z(\Pi_D - \overline{\Pi}),
\end{aligned}
\tag{4}
$$

where $\overline{\Pi} = x\Pi_C + y\Pi_{EC} + z\Pi_D$ is the expected fitness of the whole population.

3. Results

We let $\varphi_1 = \delta b - \left[\left((\mathrm{b} + \frac{1}{2}\theta)\delta + \theta\right]\alpha$, $\varphi_2 = [(3b - 2)\delta + b - 1]\alpha - \delta b$, $\varphi_3 = [(5b + 2\theta - 2)\delta + b + \theta - 1]\alpha - 3\delta b$, $\varphi_4 = b\delta(1 - \alpha)$, $\varphi_5 = \alpha\theta(1 + 2\delta)$, $\varphi_6 = [(b + 2)\delta - b + 1]\alpha - 3\delta b$, Model (4) has six equilibria: $M_1 = (1,0,0)$, $M_2 = (0,1,0)$, $M_3 = (0,0,1)$, $M_4 = \left(0, \frac{2\varphi_1}{\varphi_2}, \frac{\varphi_3}{\varphi_2}\right)$, $M_5 = \left(\frac{\varphi_5}{\varphi_4}, \frac{\varphi_4 - \varphi_5}{\varphi_4}, 0\right)$ and $M_6 = \left(\frac{2\varphi_1(\varphi_5 - \varphi_3)}{\varphi_4\varphi_6}, \frac{2\varphi_1\varphi_5(b-1)}{\varphi_4\varphi_6\theta}, \frac{-(\varphi_3 + 2\varphi_5)}{\varphi_6}\right)$. The stability results are as follows (see appendix for derivation) [45,46]:

1. The equilibrium point M_1 is unstable under any parameter value. It is impossible to have only ordinary cooperation strategy in the system;
2. When $\alpha = 0$ or $\theta = 0$ and $[(5b - 2)\delta + b - 1]\alpha \leq 3\delta b$, M_2 is stable equilibrium point. At this point everyone will choose only exposure cooperation;
3. When $2(1 - \alpha)b\delta \leq \alpha\theta(1 + 2\delta)$, M_3 is stable equilibrium point. The system will reach a stable state of full defection;
4. When $\left[3(b + \theta)\delta + \frac{3}{2}\theta\right]\alpha \leq 3b\delta \leq [(5b - 2)\delta + b - 1]\alpha$, M_4 is stable equilibrium point. Exposure cooperation and defection coexist, and there are no ordinary cooperation in the system;
5. When $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha \leq 3b\delta$, M_5 is stable equilibrium point. Ordinary cooperation and exposure cooperation coexist, and there is no defection in the system;

6. When $[(5b-2)\delta+b-1]\alpha \le 3b\delta \le [(5b+6\theta-2)\delta+b+3\theta-1]\alpha$, M_6 is stable equilibrium point. The three strategies can coexist.

Let the initial value be $(x(0)=y(0)=z(0))=1/3$. When temptation to defect $b=1.4$, exposure cost $\theta=0.1$, the variation of reputation $\delta=0.1$, payoff attention degree $\alpha \in \{0,0.2,0.4,0.6,0.8\}$, the evolution results of ordinary cooperators, exposure cooperators and defectors over time are given in Fig. 1.

From the results of Fig. 1, it is clear that the proportion of the three strategies will eventually stabilize over time. In Fig. 1(a), when $\alpha=0$, there is only exposure cooperators in the system, corresponding to M_1. Let $\alpha=0.2$, in Fig. 1(b), ordinary cooperators and exposure cooperators coexist as time evolves, corresponding to M_5. Let $\alpha=0.4$, in Fig. 1(c), the three strategies coexist as time evolves, corresponding to M_6. Let $\alpha=0.6$, in Fig. 1(d), exposure cooperators and defectors coexist as time evolves, corresponding to M_4. Let $\alpha=0.8$, in Fig. 1(e), ordinary cooperators and exposure cooperators have been reduced to 0, leaving only defectors in the system, corresponding to M_3. Thus, for a given combination of parameters, the different degree of emphasis on payoff and reputation can directly affect the state of the system. Greater emphasis on reputation favors cooperators, greater emphasis on payoff favors defectors, and cooperators and defectors can coexist when there is little difference in the emphasis on payoff and reputation.

Next, we pay attention to the fraction of the three strategies regarding the temptation to defect b when $\alpha=0.5$, $\theta=0.1$, $\delta=0.1$, corresponding results are provided in Fig. 2.

As shown in Fig. 2, as b increases, the state of the population is divided into two phases. When $1<b\le 1.33$, the three strategies coexist, with C decreasing and EC and D increasing as b increases, at which point EC increases at a faster rate than D. When $1.33<b\le 2$, EC coexists with D. As b increases EC starts to decrease and D continues to increase. Thus, there exists an optimal value of b such that EC reaches the highest ratio when C just vanishes. It is worth noting that with the increase of b, the three strategies have obvious phased advantages. In particular, when $1<b\le 1.13$, ordinary cooperators prevail, indicating that less temptation to defect will make it easier for ordinary cooperators to hitchhike. For other cases, when $1.13<b\le 1.2$, defectors prevail; when $1.2<b\le 1.37$, exposure cooperators prevail; when $1.37<b\le 2$, defectors prevail again.

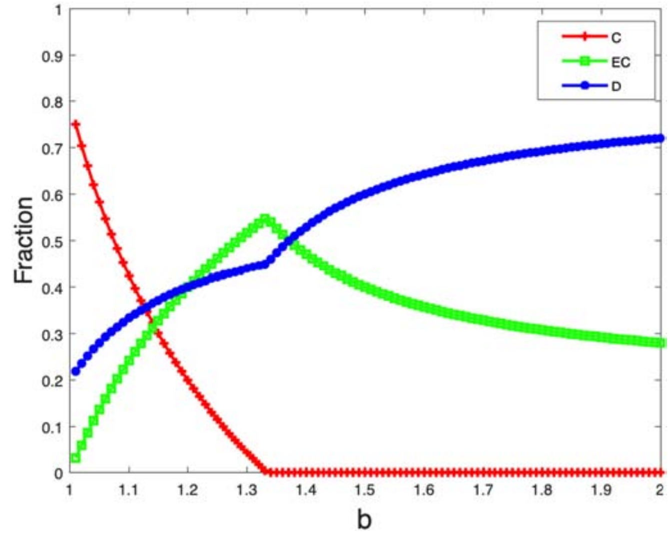

Fig. 2. The evolution of the fraction of ordinary cooperators, exposure cooperators and defectors in dependence on the variation of temptation to defect b. The fixed parameters are $\alpha=0.5$, $\theta=0.1$, $\delta=0.1$.

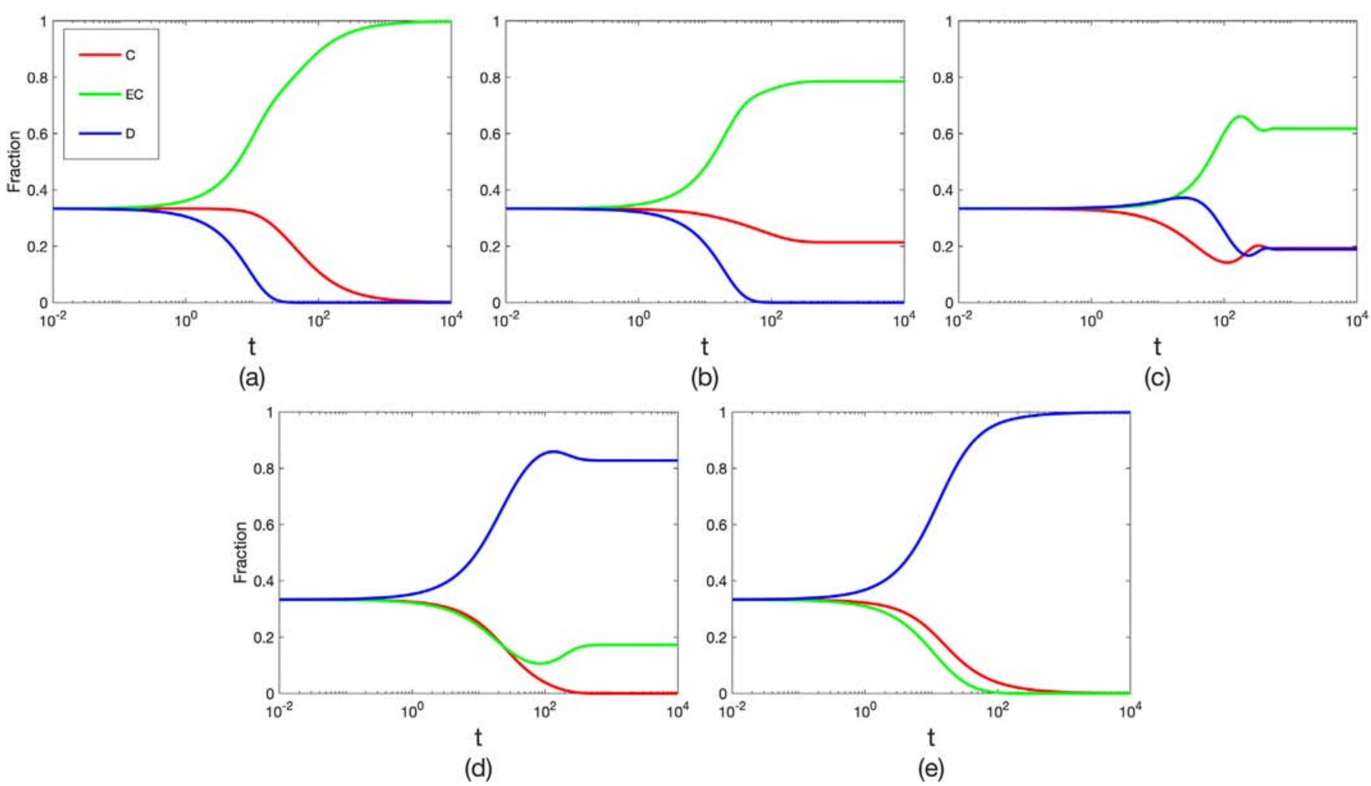

Fig. 1. With $b=1.4$, δ=0.1, θ=0.1, (a) $\alpha=0$, (b) $\alpha=0.2$, (c) $\alpha=0.4$, (d) $\alpha=0.6$, (e) $\alpha=0.8$, the evolution of fraction of ordinary cooperators, exposure cooperators and defectors over time.

The different degree of the group's attention to payoff and reputation reflects a value bias. Heat map of the fraction of ordinary cooperators, exposure cooperators and defectors on the $\alpha - b$ parameter plane, and the phase diagram on the $\alpha - b$ parameter plane is shown in Fig. 3.

In Fig. 3(a), as α increases, C increases and then decreases, and C occupies a higher proportion when the b is small and the α is moderate. In Fig. 3(d), red and green are the regions where C exists, and it can be seen that the α region where C can survive decreases when the b is larger, indicating that for a given δ and θ, the increased temptation to defect limits C's survival in a more payoff-focused environment. For exposure cooperators, in Fig. 3(b), EC tends to decrease with increasing α, but when the α is moderate, the fraction of EC increases to different degrees for different b, and is most obvious around $b = 1.3$. In Fig. 3(d), EC is existed in all regions except mazarine. When $\alpha = 0$, the yellow region shows that there are only exposure cooperators in the population no matter how large b is. The α of the junction of the green and wathet region with the mazarine region becomes progressively larger as b increases, suggesting that the increased temptation to defect allows EC to survive in an environment that is more attention to payoff. For defectors, in Fig. 3(c), D becomes progressively larger with increasing α, is absent when the α is small, and occupies the entire population when the α is large. In Fig. 3(d), the junction between the red region and the green region is the value of the parameter corresponding to the emergence of D. It can be seen that as b increases, the α that can induce the emergence of D becomes progressively smaller, indicating that the increased temptation to defect allows D to survive in an environment that is more attention to reputation.

The effect of exposure is reflected in the variation of reputation. Heat map of the fraction of ordinary cooperators, exposure cooperators and defectors on the $\delta - b$ parameter plane, and the phase diagram on the $\delta - b$ parameter plane is shown in Fig. 4.

In Fig. 4(a), it can be found that there is a δ region that keeps C at a better fraction when b is small, and outside this region the fraction of C decreases as both δ and b increase. In Fig. 4(d), the regions where C is existed are red and green. Comparing the junction between green and wathet reveals that the δ at which C is induced to emerge increases with b, meaning that for a larger temptation to defect, it requires a larger variation of reputation for the original cooperators to successfully hitch a ride. For exposure cooperators, in Fig. 4(b), the increase in δ favors the survival of EC, and as δ increases, EC is more likely to survive when the b is larger. In Fig. 3(d), EC is existed in all regions except mazarine. As b increases, the δ that can induce the emergence of EC decreases from 0.05 to 0.02, indicating that a smaller variation of reputation can induce the emergence of exposure cooperators under a higher temptation to defect. For defectors, in Fig. 4(c), D occupies a higher fraction when the δ is small and decreases monotonically as the δ increases. In Fig. 3(d), D is existed in all regions except red. As b increases, the δ corresponding to the disappearance of D gradually increases, meaning that a larger variation of reputation is needed to fully overcome defectors when the temptation to defect is greater.

The magnitude of the exposure cost directly affects the payoff of the exposure cooperators. Heat map of the fraction of ordinary cooperators, exposure cooperators and defectors on the $\theta - b$ parameter plane, and the phase diagram on the $\theta - b$ parameter plane is shown in Fig. 5.

It should be noted that in Fig. 5(a), when the value of b is small, there is a smaller θ interval that allows C to survive better. In Fig. 5(d), the regions where C is existed are red and green. When $b \leq 1.33$, the larger the value of b, the larger the θ corresponding to the disappearance of C. However, when $b > 1.33$, there are no ordinary cooperators regardless of the value of θ. For exposure cooperators, in Fig. 5(b), EC decreases as θ increases. In Fig. 5(d), The yellow region is when $\theta = 0$ and $b \leq 1.33$, the population is all EC. As b increases, the θ corresponding to the disappearance of EC increases, indicating that the higher exposure cost can also motivate the creation of exposure cooperators under the greater temptation to defect. This is because the greater temptation to defect will make the number of defectors increase, when the exposure cooperators have a greater opportunity to expose the defectors and thus gain a higher reputation. Their advantage in reputation can make up for a larger gap in payoff, and therefore can survive under the condition of greater exposure cost. For defectors, in Fig. 5(c), the fraction of D increases with increasing θ, and the smaller the value of b the faster the increase of D. In Fig. 5(d), when $b \leq 1.33$, the θ corresponding to the emergence of induced D decreases gradually with the increase of b. When $b > 1.33$, defectors can survive even if $\theta = 0$.

Finally, in order to understand the impact of the joint effect of exposure cost and the variation of reputation on the evolution of cooperation under different degrees of payoff attention, we take $b = 1.5$ and plot the heat map of the fraction of ordinary cooperators, exposure cooperators and defectors at $\alpha = 0.3$, 0.5 and 0.7 respectively in Fig. 6.

As shown in Fig. 6, when the variation of reputation increases to a certain value, continuing to increase the variation of reputation has little effect on the fraction of the three strategies. When $\alpha = 0.3$ is relatively small, it can be found from the first row that only a small δ is required to induce cooperation to flourish in the system, no matter what θ value is. Therefore, when the population pay more attention to reputation, the generation of cooperation is not dependent much on θ and δ. Even if the exposure cost is large, cooperators can still occupy the entire population. When $\alpha = 0.5$ is moderate, it can be found from the second row that the δ that can induce cooperation increases as θ increases. Three strategies can coexist when exposure cost is large. When $\alpha = 0.7$ is relatively large, it can be found from the third row that there is no cooperation when θ is large, no matter what the value of δ is.

The first column corresponds to the original cooperators, and in Fig. 6(a), the fraction of C increases with θ, but increases and then

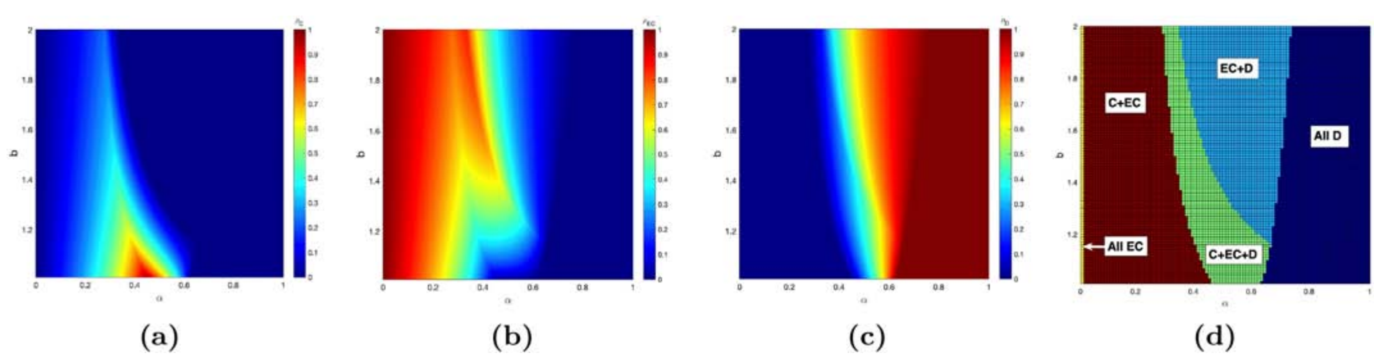

Fig. 3. Heat map of the fraction of ordinary cooperators (a), exposure cooperators (b) and defectors (c) and the phase diagram (d) on the $\alpha - b$ parameter plane. The regions in the phase diagram are indicated by colors, where yellow indicates all EC, red indicates C coexists with EC, green indicates three strategies coexist, wathet indicates EC coexists with D, and mazarine indicates all D. The fixed parameters are $\theta = 0.1$, $\delta = 0.1$.

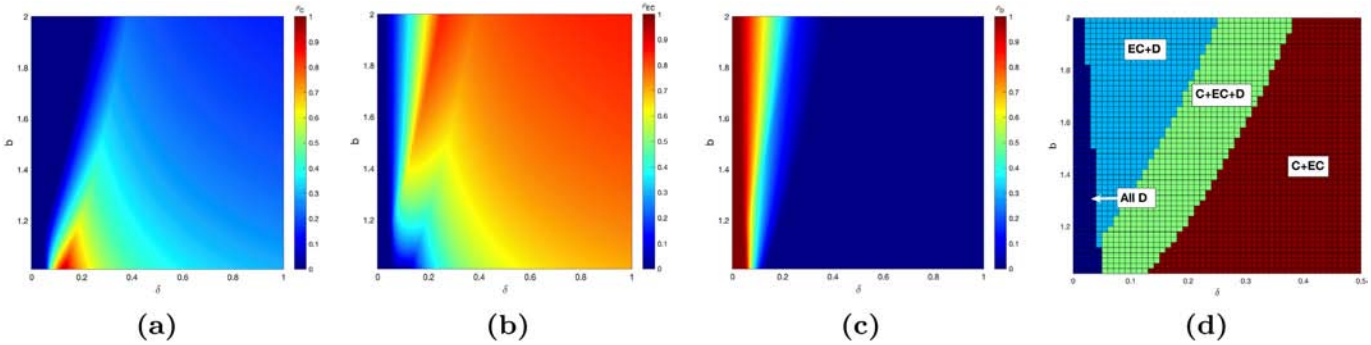

Fig. 4. Heat map of the fraction of ordinary cooperators (a), exposure cooperators (b) and defectors (c) and the phase diagram (d) on the $\delta - b$ parameter plane. The regions in the phase diagram are indicated by colors, where red indicates C coexists with EC, green indicates three strategies coexist, wathet indicates EC coexists with D, and mazarine indicates all D. The fixed parameters are $\alpha = 0.5$, $\theta = 0.1$.

decreases with δ. In Fig. 6(d), the larger the δ the higher the fraction of C, but as θ increases, the fraction of C first increases and then decreases. In Fig. 6(g), C is not existed when θ is sufficiently small or θ is large, and the fraction is low in the other regions where they can exist. From the second column, it can be found that the fraction of EC decreases with increasing θ and increases with increasing δ. In Fig. 6(b), EC is able to occupy a high fraction at large value of θ, while in Fig. 6(h) EC is only able to occupy a high fraction at small values of θ. From the third column, it can be found that the smaller the δ or the larger the θ is the more favorable to D. In Fig. 6(c), D can only survive in the region with a small δ, and in Fig. 6(f) and Fig. 6(i), the δ and θ regions where D survives gradually become larger, indicating that the greater payoff attention degree will make defectors survive with a larger variation of reputation and a smaller exposure cost.

4. Conclusion

This paper introduces a cooperation strategy with exposure behavior, which exposes the behavior of the individuals with whom interacted by paying a cost, thus causing a variation in their mutual reputation. Results show that five states emerge under different combinations of parameters, namely, all defection, all exposure cooperation, coexistence of exposure cooperation and ordinary cooperation, coexistence of exposure cooperation and defection, and coexistence of all three strategies. Exposure-based reputation mechanisms can enable original cooperators to hitch a ride and thus survive in the population. Specifically, there exists a smaller δ interval, a smaller θ interval, and a moderate α interval that allows the ordinary cooperators to occupy a high fraction.

Greater emphasis on reputation favors cooperators, greater emphasis on payoff favors defectors, and cooperators coexist with defectors when there is little difference in the emphasis on payoff and reputation. When groups pay equal emphasis on payoff and reputation, and the amount of exposure cost and variation of reputation are small, the three strategies have circular dominance with the increase of temptation to defect. There are four stages, first the ordinary cooperators prevail, followed by the defectors, then the exposure cooperators, and when the temptation to defect is large the defectors prevail again. In addition, we note that when the temptation to defect is greater, defectors can survive in the population that emphasize more on reputation, and exposure cooperators can survive in the population that emphasize more on payoff.

The variation of reputation has a facilitating effect on cooperation, the exposure cost has an inhibiting effect on cooperation. When the variation of reputation increases to a certain value, continuing to increase the variation of reputation has little effect on the fraction of the three strategies. When the exposure cost is large, cooperators can occupy the entire population when give smaller emphasis on payoff, the three strategies can coexist when give moderate emphasis on payoff, and cooperators cannot exist when give larger emphasis on payoff.

The purpose of the current model is to describe the impact of exposure-based reputation mechanisms on the evolution of cooperation. Future work will consider more forms of reputation (e.g., defectors with exposure behavior), and the impact of interactions between different reputation mechanisms on the evolution of cooperation. Exploring the spatial reciprocity of exposure-based reputation mechanisms in the complex network is also one of the future works.

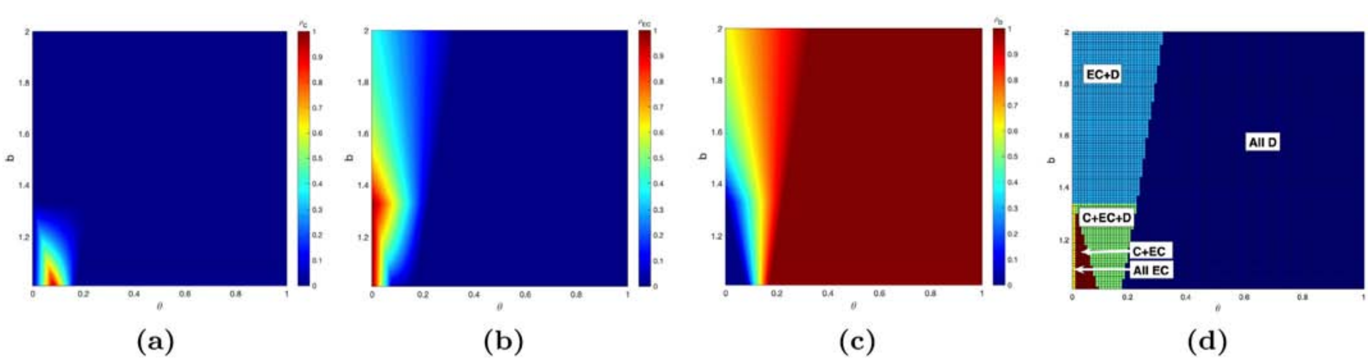

Fig. 5. Heat map of the fraction of ordinary cooperators (a), exposure cooperators (b) and defectors (c) and the phase diagram (d) on the $\theta - b$ parameter plane. The regions in the phase diagram are indicated by colors, where yellow indicates all EC, red indicates C coexists with EC, green indicates three strategies coexist, wathet indicates EC coexists with D, and mazarine indicates all D. The fixed parameters are $\alpha = 0.5$, $\delta = 0.1$.

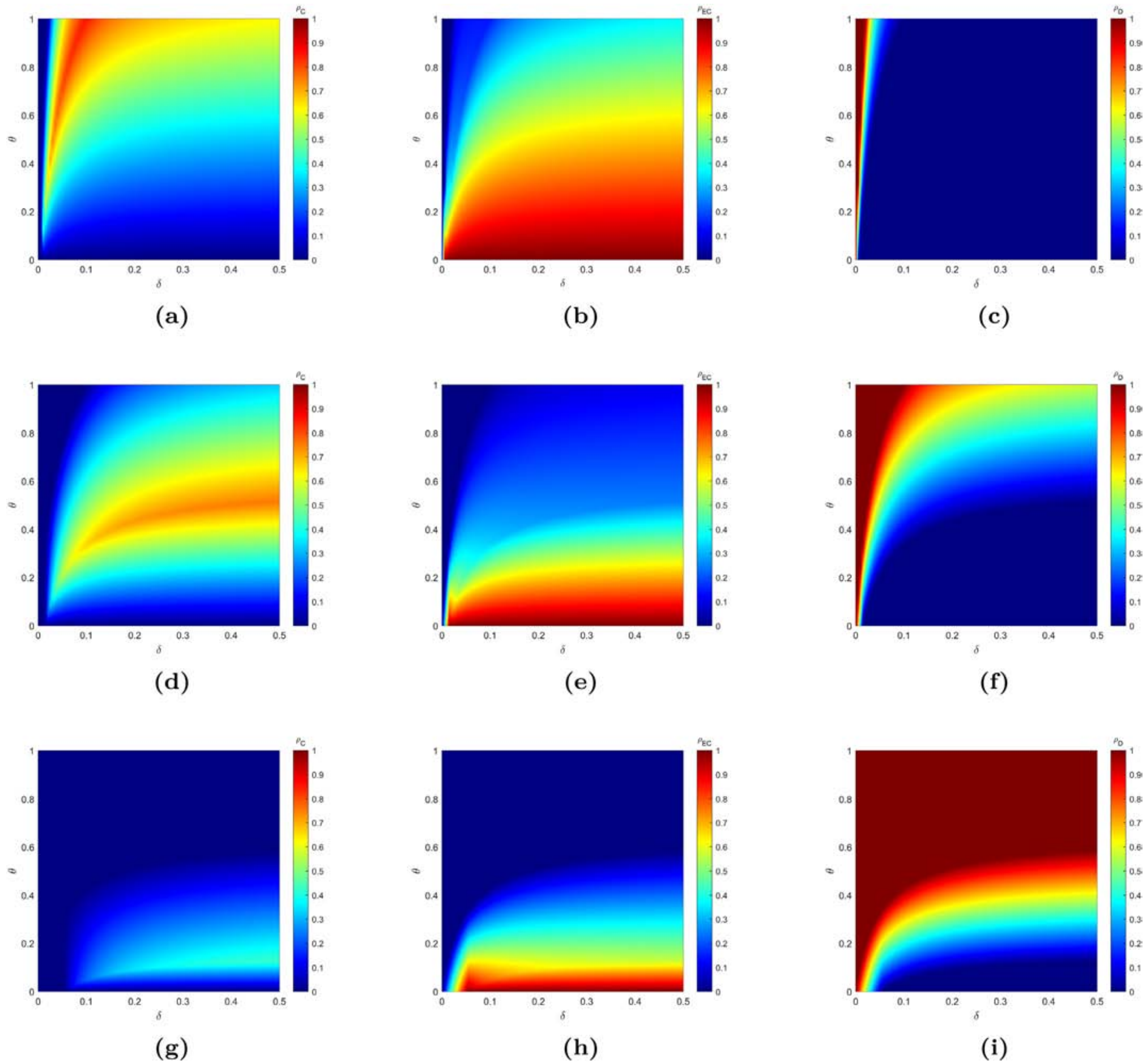

Fig. 6. Heat map of the fraction of different strategies on the $\delta - \theta$ parameter plane. From left to right, the subfigures indicate the fraction of original cooperators, exposure cooperators and defectors, respectively. From top to bottom, the value of α equals to 0.3, 0.5, and 0.7, respectively. The fixed parameter is $b = 1.5$.

CRediT authorship contribution statement

Wenqiang Zhu: Conceptualization, Methodology, Software, Writing – original draft. **Qiuhui Pan:** Conceptualization, Methodology, Supervision. **Mingfeng He:** Conceptualization, Methodology, Writing – review & editing.

Declaration of competing interest

No conflict of interest exits in the submission of this manuscript, and manuscript is approved by all authors for publication. I would like to declare on behalf of my co-author that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the manuscript that is enclosed.

Appendix A

Let $\psi_1 = \alpha(b-1), \psi_2 = \frac{(1-\alpha)b\delta}{1+2\delta}$, because of $z = 1 - x - y$, we obtain:

$$\begin{aligned} \dot{x} &= f(x,y) = \psi_1 x^3 + 2(\alpha b - \psi_2)x^2 y + (\psi_1 - \psi_2)xy^2 - \psi_1 x^2 - (\psi_1 - \psi_2)xy \\ \dot{y} &= g(x,y) = (\psi_1 + \psi_2)y^3 + (\psi_1 - \psi_2)xy^2 + \psi_1 x^2 y + (\alpha\theta - \psi_1 - \psi_2)y^2 - (\psi_1 + \psi_2)xy \\ &\quad + (2\psi_2 - \alpha\theta)y \end{aligned} \tag{5}$$

Let $\varphi_1 = \delta b - \left[\left(\left(\mathrm{b} + \frac{1}{2}\theta\right)\delta + \theta\right]\alpha$, $\varphi_2 = [(3b - 2)\delta + b - 1]\alpha - \delta b$, $\varphi_3 = [(5b + 2\theta - 2)\delta + b + \theta - 1]\alpha - 3\delta b$, $\varphi_4 = b\delta(1 - \alpha)$, $\varphi_5 = \alpha\theta(1 + 2\delta)$, $\varphi_6 = [(b + 2)\delta - b + 1]\alpha - 3\delta b$, The system has six equilibria: (1,0,0), (0,1,0), (0,0,1), $\left(0, \frac{2\varphi_1}{\varphi_2}, \frac{\varphi_3}{\varphi_2}\right)$,$\left(\frac{\varphi_5}{\varphi_4}, \frac{\varphi_4 - \varphi_5}{\varphi_4}, 0\right)$and

$\left(\frac{2\varphi_1(\varphi_5-\varphi_3)}{\varphi_4\varphi_6}, \frac{2\varphi_1\varphi_5(b-1)}{\varphi_4\varphi_6\theta}, \frac{-(\varphi_3+2\varphi_5)}{\varphi_6}\right)$. According to the Jacobian matrix to examine the stability of these equilibria:

$$J = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{bmatrix} \tag{6}$$

where

$$\begin{aligned} \frac{\partial f(x,y)}{\partial x} &= 3\psi_1 x^2 + 4(\alpha b - \psi_2)y + (\psi_1 - \psi_2)y^2 - 2\psi_1 x - (\psi_1 - \psi_2)y \\ \frac{\partial f(x,y)}{\partial y} &= 2(\alpha b - \psi_2)x^2 + 2(\psi_1 - \psi_2)xy - (\psi_1 - \psi_2)x \\ \frac{\partial g(x,y)}{\partial x} &= 2(\psi_1 - \psi_2)y^2 + 2\psi_1 xy - (\psi_1 + \psi_2)y \\ \frac{\partial g(x,y)}{\partial y} &= 3(\psi_1 + \psi_2)y^2 + 2(\psi_1 - \psi_2)xy + 2(\alpha\theta - \psi_1 - \psi_2)y + \psi_1 x^2 \\ &\quad -(\psi_1 + \psi_2)x + (2\psi_2 - \alpha\theta) \end{aligned} \tag{7}$$

Stability analysis is carried out according to the determinant and the trace of matrix J:

1. For point (1,0,0), $|J| = \frac{\alpha(b-1)\{\delta b - \alpha[(b+2\theta)\delta+\theta]\}}{(1+2\delta)b^2}$, tr $J = \frac{\delta b + \alpha[(b-2-2\theta)\delta - 1 + b - \theta]}{(1+2\delta)b}$

- When $\alpha = 0$, $|J| = 0$, tr $J > 0$.
- When $\alpha > 0$, and $\delta b - \alpha[(b + 2\theta)\delta + \theta] \geq 0$, $|J| \geq 0$, tr $J > 0$.
- When $\alpha > 0$, and $\delta b - \alpha[(b + 2\theta)\delta + \theta] < 0$, $|J| < 0$.

Therefore, at any parameter value, the equilibrium is unstable.

2. For point (0,1,0), $|J| = \frac{\alpha\theta\{[(5b+2\theta-2)\delta+b+\theta-1]\alpha-3\delta b\}}{(1+2\delta)b^2}$, tr $J = \frac{[(5b-2+4\theta)\delta+b-1+2\theta]\alpha-3\delta b}{(1+2\delta)b}$

- When $\alpha = 0$, $|J| = 0$, $trJ < 0$.
- When $\alpha > 0$, and $\theta = 0$ and $[(5b - 2)\delta + b - 1]\alpha \leq 3\delta b$, $|J| = 0$, tr $J \leq 0$.
- When $\alpha > 0$, and $\theta = 0$ and $[(5b - 2)\delta + b - 1]\alpha > 3\delta b$, $|J| = 0$, tr $J > 0$.
- When $\alpha > 0$, and $\theta > 0$ and $[(5b + 2\theta - 2)\delta + b + \theta - 1]\alpha - 3\delta b \geq 0$, $|J| \geq 0$, tr $J > 0$.
- When $\alpha > 0$, and $\theta > 0$ and $[(5b + 2\theta - 2)\delta + b + \theta - 1]\alpha - 3\delta b < 0$, $|J| < 0$.

Therefore, when $\alpha = 0$ or $\theta = 0$ and $[(5b - 2)\delta + b - 1]\alpha \leq 3\delta b$, the equilibrium is stable.

3. For point (0,0,1), $|J| = 0$, $J = \frac{[2b-(2b+2\theta)\alpha]\delta - \alpha\theta}{(1+2\delta)b}$

- When$2(1 - \alpha)b\delta > \alpha\theta(1 + 2\delta)$, tr $J > 0$.
- When $2(1 - \alpha)b\delta \leq \alpha\theta(1 + 2\delta)$, tr $J \leq 0$.

Therefore, when $2(1 - \alpha)b\delta \leq \alpha\theta(1 + 2\delta)$, the equilibrium is stable.

4. For point $\left(0, \frac{2\varphi_1}{\varphi_2}, \frac{\varphi_3}{\varphi_2}\right)$, $|J| = \frac{\{[2(b+\theta)\delta+\theta]\alpha-2b\delta\}^2\{[(5b-2)\delta+b-1]\alpha-3b\delta\}\{[(5b+2\theta-2)\delta+b+\theta-1]\alpha-3b\delta\}}{(1+2\delta)^2(3ab\delta+\alpha b-2\alpha\delta-b\delta-\alpha)^2 b^2}$, tr $J = \frac{4\left[\left[(5b+\theta-2)\delta+b+\frac{\theta}{2}-1\right]\alpha-3b\delta\right\}\left[(b+\theta)\delta+\frac{\theta}{2}\right]\alpha-b\delta\right\}}{(1+2\delta)[(3b-2)\delta+b-1)\alpha-b\delta]b}$

- When $[(5b - 2)\delta + b - 1]\alpha \geq 3b\delta$, and $\left[(b + \theta)\delta + \frac{\theta}{2}\right]\alpha \leq b\delta$, $|J| \geq 0$, tr $J \leq 0$.
- When $[(5b - 2)\delta + b - 1]\alpha \geq 3b\delta$, and $\left[(b + \theta)\delta + \frac{\theta}{2}\right]\alpha > b\delta$, $|J| \geq 0$, tr $J < 0$.
- When $[(5b - 2)\delta + b - 1]\alpha < 3b\delta$, and $[(5b + 2\theta - 2)\delta + b + \theta - 1]\alpha \leq 3b\delta$, $|J| \geq 0$, $J > 0$.
- When $[(5b - 2)\delta + b - 1]\alpha < 3b\delta$, and $[(5b + 2\theta - 2)\delta + b + \theta - 1]\alpha > 3b\delta$, $|J| < 0$.

Therefore, when $\left[3(b + \theta)\delta + \frac{3}{2}\theta\right]\alpha \leq 3b\delta \leq [(5b-2)\delta + b-1]\alpha$, the equilibrium is stable.

5. For point $\left(\frac{\varphi_5}{\varphi_4}, \frac{\varphi_4-\varphi_5}{\varphi_4}, 0\right)$, $|J| = \frac{\alpha\theta\{[(5b+6\theta-2)\delta+b+3\theta-1]\alpha-3b\delta\}\{[(b+2\theta)\delta+\theta]\alpha-b\delta\}}{(1+2\delta)(1-\alpha)b^3\delta}$, tr $J = \frac{\{[2(5b+\theta-2)\delta+2b+\theta-2]\alpha-3b\delta\}\{[2(b+\theta)\delta+\theta]\alpha-2b\delta\}}{(1+2\delta)(1-\alpha)b^2\delta}$

- When $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha - 3b\delta \leq 0$, $|J| \geq 0$, $J \leq 0$.
- When $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha - 3b\delta > 0$, and $[(b + 2\theta)\delta + \theta]\alpha - b\delta < 0$, $|J| < 0$.
- When $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha - 3b\delta > 0$, and $[(b + 2\theta)\delta + \theta]\alpha - b\delta \geq 0$, $|J| \geq 0$, tr $J > 0$.

Therefore, when $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha \leq 3b\delta$, the equilibrium is stable.

6. For point $\left(\frac{2\varphi_1(\varphi_5-\varphi_3)}{\varphi_4\varphi_6}, \frac{2\varphi_1\varphi_5(b-1)}{\varphi_4\varphi_6\theta}, \frac{-(\varphi_3+2\varphi_5)}{\varphi_6}\right)$,

$|J| = \frac{2\alpha(b-1)\{[2(b+\theta)\delta+\theta]\alpha-2b\delta\}^2\{[(5b+6\theta-2)\delta+b+3\theta-1]\alpha-3b\delta\}\{[(5b-2)\delta+b-1]\alpha-3b\delta\}}{(1+2\delta)b^3\delta(\alpha-1)\{[(b+2)\delta-b+1]\alpha-3b\delta\}^2}$,
tr $J = \frac{\alpha^2\theta(b-1)(1+2\delta)\{[2(b+\theta)\delta+\theta]\alpha-2b\delta\}}{b^2\delta(\alpha-1)\{[(b+2)\delta-b+1)\alpha-3b\delta\}}$

- When $[(5b - 2)\delta + b - 1]\alpha - 3b\delta > 0$, $|J| < 0$.
- When $[(5b - 2)\delta + b - 1]\alpha - 3b\delta \leq 0$, and $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha - 3b\delta < 0$, $|J| < 0$.
- When $[(5b - 2)\delta + b - 1]\alpha - 3b\delta \leq 0$, and $[(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha - 3b\delta \geq 0$, $|J| \geq 0$, tr $J \leq 0$.

Therefore, when $[(5b - 2)\delta + b - 1]\alpha \leq 3b\delta \leq [(5b + 6\theta - 2)\delta + b + 3\theta - 1]\alpha$, the equilibrium is stable.

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