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The influence of own historical information and environmental historical information on the evolution of cooperation



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ABSTRACT

This article has studied the influence of own historical information and environmental historical information on the evolution of cooperation in the three common social dilemma games. The results show that only the past two-step information can effectively promote cooperation. In the prisoner's dilemma game, cooperation can be improved by using only the individual own information. In the stag hunt game, using only environmental information can promote cooperation. In the snowdrift game, when the parameter values are different, the results are different. The cooperation ratio can be maximized by using either its own information or environmental information only. This article provides a new way to promote and understand cooperation.

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1. Introduction

Cooperation behaviors are generally existed in the natural social system [1–4]. The purpose is that individuals can complete tasks that cannot be completed independently through cooperation, so as to obtain certain collective benefits. However, the cooperation may be accompanied by the sacrifice of personal interests. When the temptation of betrayal is greater, in the face of the conflict between collective interests and personal interests, it is difficult to choose cooperation as a rational individual, so it is important to find out the cause of cooperation and how to effectively promote cooperation. The evolutionary game theory provides a theoretical framework for the emergence and continued appearance of cooperation between rational individuals [5–8]. This includes kin selection, direct and indirect reciprocity, group selection, network reciprocity [9]. These methods can promote effective solutions for cooperation [10,11]. Nowak and May [12] proposed that using spatial lattice for cooperation and evolution attracted more attention. Since then, more factors have been taken into account in the spatial lattice, various mechanisms such as reward or punishment [13–16], individual mobility [17,18], reputation mechanism [19,20], and social diversity [21,22] are added to spatial reciprocity to explore whether cooperative behavior can be further promoted in game groups.

In recent years, along with the development of complex networks, the progress of network science has enriched the complement and identification of complex system models, and the spatial cooperation of structured groups has received extensive attention and research. Including small-world effect [23–25] and scale-free topology [26,27], network topology

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shows strong non-uniformity, and complex topology will further increase the proportion of cooperators in network population [28,26]. The dynamics of cooperation in complex networks are further revealed. Refs. [29] reviewed the latest progress in the field of cooperative dynamics in the evolution of complex networks.

In most group game models, memory mechanism has not received enough attention during the evolution of cooperation, and the interactive use of individual memory is fixed only once, that is, individuals will adjust their game strategies for the next moment according to the payoff of the previous moment. Therefore, it is very necessary for rational individuals to determine the actions of the next moment by evaluating the payoff information of the past period of time. [30-33] introduced memory mechanism into social dilemma game, discussed the influence of memory mechanism on cooperation evolution, and drew a series of conclusions. Considering the evaluation criteria of payoff information and the pros and cons of individual strategies in historical memory and the influence on the evolution results of the game model, Refs. [34-36] evaluated the memory mechanism in common games. Lu et al. [37] introduced the memory mechanism into the space prisoner's dilemma game(PDG), an individual will update his current strategy by taking the product of Fermi rule and a pre-factor as the probability, and this pre-factor is related to the proportion of the same strategy states within the individual's own memory length. The numerical simulation results show that the memory length has a great impact on the level of cooperation. Appropriate memory length is most conducive to the emergence and dynamic evolution of cooperation, Dong et al. [38] developed a memory-based stag hunt game(SHG), individuals will first choose the neighbors with the greatest cumulative benefit within the memory length as the learning object, and then decide whether to learn from the neighbors according to the Fermi rule. The results show that the memory length can promote the level of cooperation, and the larger the memory length, the more obvious the promotion effect. Shu et al. [39] based on the snowdrift game(SG), a new method based on memory mechanism is proposed. Each individual applies the given rules to compare its own historical benefits within the memory length, and takes the maximum benefits as virtual benefits to obtain the optimal strategy by comparing with its neighbors. Simulation results show that this method can effectively improve the level of cooperation in spatial structure. When the memory mechanism is introduced into the game group, we need to pay attention to the types of information stored in the memory length. Wang et al. [40] described the situation in which an individual adjusts the next moment of strategies according to the proportion of his own optimal strategies stored in memory. Ye et al. [41] considered the neighbor's strategy with a higher payoff than the average payoff within the memory length as the basis for its own strategy adjustment.

Although the memory mechanism is applied in some game models, most of the game models use the memory effect only considering unilateral influence, either considering only their own historical information or the environmental historical information. We know that when the memory mechanism is introduced into the game model, the information of their own historical information and the environmental historical information will have an impact on the strategic choice of an individual at the next moment. Therefore, it is necessary to comprehensively consider the historical information of oneself and the historical information of the environment. Li et al. [42] comprehensively considered the individual's own historical payoffs, the payoffs of other neighbors, and the uncertain random factors related to opportunism, and analyzed the evolutionary prisoner's dilemma game models in BA network and ER network, but did not consider the neighbor's strategy information within the memory length. Therefore, the information of oneself and environment in the memory length should arouse our attention. Considering the optimal strategies of individuals and their neighbors within the memory length, it will have an impact on the strategy update of individuals at the next moment.

In most group game models, individuals use their own information and their neighbors' information in different ways within the memory length. In this paper, for their own historical information, we use the maximum payoff and the strategies adopted at that time. For environmental historical information, we use the maximum average payoff of neighbors and the strategies adopted by the neighbors at that time. On the basis of the current strategies and payoff, individuals determine the strategies for the next moment by combining the above information about themselves and the environment in history. This paper mainly considers the influence of memory length and the utilization degree of self and environment information on the evolution of cooperation.

This paper is organized as follows. Section 2 provides the game evolution model based on historical information. Section 3 presents and analyzes the evolution results. Finally, the main conclusions are given in Section 4.

2. Model

Suppose each individual has two strategies of cooperation (C) and defection(D), and the payoff matrix is as follows:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

Suppose the game is played on a $L \times L$ regular lattice with periodic boundary conditions, each point i has an individual, and the four neighbors of individual i are represented by Ω_i . In each round of the game, each player has only two strategies to choose from, either cooperation or defection, the definition of $S_i(t)$ is as follows:

$$S_i(t) = \begin{cases} 0, \text{ The strategy of individual i at time t is D} \\ 1, \text{ The strategy of individual i at time t is C} \end{cases}$$
 (2)

The individual is playing with all of his neighbors, $\Pi_i(t)$ is the payoff of individual i in moment t, it is calculated as follows:

$$\Pi_{i}(t) = \sum_{j \in \Omega_{i}} f_{ij}, \text{ where } f_{ij} = \begin{cases} S, & \text{if } S_{i} = 1, S_{j} = 0, \\ R, & \text{if } S_{i} = 1, S_{j} = 1, \\ T, & \text{if } S_{i} = 0, S_{j} = 1, \\ P, & \text{if } S_{i} = 0, S_{j} = 0 \end{cases}$$

$$(3)$$

When playing the game, individuals will remember their own and neighbors' strategies and corresponding payoffs within the historical length H, and individuals should comprehensively consider their own historical information and neighbor's historical information when updating the strategy. The specific rules are as follows:

(1) When $t \le H$, each individual i randomly selects an individual j from its neighbor Ω_i and uses the Fermi rule update strategy. The probability of learning j is as follows:

$$P_{j \to i} = \frac{1}{1 + \exp\left(\frac{\Pi_i(t) - \Pi_j(t)}{k}\right)} \tag{4}$$

where k describes the uncertainty introduced in the process of strategy adoption.

(2) When t > H, the individual strategy at the next moment is determined by his own factors on the one hand, and environmental factors on the other.

The action mode of the own factors is that the update of the strategy of individual i depends on the strategy of the highest payoff moment in the memory length H. Specifically, the individual chooses the strategy of the highest payoff moment in his memory with the probability of p_1 , and chooses the opposite strategy of this moment with the probability of $1 - p_1$, which $p_1 = \frac{1}{1 + \exp(\frac{\prod_i (t) - \prod_i (t_1)}{k})}$, t_1 is the moment with the highest payoff within the individual memory length H. $S_i(t_1)$ is the strategy of individual i at t_1 . The probability that individual i chooses cooperation at the next moment considering only his own factors is as follows:

$$G_i^i(t+1) = p_1 \cdot S_i(t_1) + (1-p_1) \cdot (1-S_i(t_1)) \tag{5}$$

The action of environmental factors is that the strategy update of individual i depends on the current strategy of the individual and the neighbors's strategy distribution at the time of the highest total payoff of the neighbors in the memory length H. Specifically, the individual uses the neighbors' information in his memory to determine his strategy at the next moment with the probability of p_2 , and keeps the current strategy unchanged with the probability of p_2 , which $p_2 = \frac{1}{\prod_i (t) - \prod_{i=1}^n (t)}$, t_2 is the moment when the average neighbors' payoffs the highest within the individual memory $p_2 = \frac{1}{1 + \exp(\frac{t_2}{k})}$.

length H. $\Pi_{\Omega_i}^*(t_2) = \frac{\sum_{j \in \Omega_i} \Pi_j(t_2)}{4}$ is the average neighbors' payoff at t_2 , $S_{\Omega_i}^*(t_2) = \frac{\sum_{j \in \Omega_i} S_j(t_2)}{4}$ is the average cooperation ratio of neighbors at t_2 . The probability of individual i choosing the cooperation strategy at the next moment considering only the environmental factors is as follows:

$$G_2^i(t+1) = p_2 \cdot S_{\Omega_i}^*(t_2) + (1-p_2) \cdot S_i(t)$$
(6)

Considering its own factor and environmental factor comprehensively, the calculation formula of the probability of individual *i* choosing cooperation at the next moment is as follows:

$$P_i(t+1) = q \cdot G_1^i(t+1) + (1-q) \cdot G_2^i(t+1) \tag{7}$$

where $q \in [0, 1]$ represents the weight of two factors when an individual performs policy update.

3. Results and discussion

Let L = 100, k = 0.1 and time step t = 1200. When R, S, T and P take different values, they correspond to different game models respectively. The prisoner's dilemma game, stag hunt game and snowdrift game are discussed below.

3.1. Prisoner's dilemma game

For the payoff matrix $({R\atop T} {S\atop P})$, it is prisoner's dilemma game when T>R>P>S and 2R>T+S. Accepting the idea suggested by Nowak and May [12], take $T=b(1\leq b\leq 2)$, S=P=0, R=1, and the payoff matrix is $[{1\atop b} {0\atop 0}]$, it is a weak prisoner's dilemma game. When b=1.5, q=0.5, $H\in\{2,3,4,5\}$, the corresponding cooperation evolution results are shown in Fig. 1.

As can be seen from Fig. 1, the proportion of cooperators in the system tends to be stable under corresponding parameters. The impact of parameter changes on stable values is discussed below. When q = 0.5, $H \in \{2, 3, 4, 5\}$, $b \in [1, 2]$, the evolution results are shown in Fig. 2.

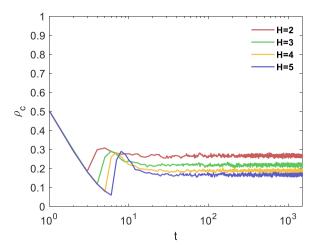


Fig. 1. Cooperation evolution results when $\mathbf{b} = 1.5$, $\mathbf{q} = 0.5$ and $\mathbf{H} \in \{2, 3, 4, 5\}$.

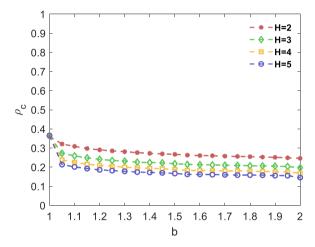


Fig. 2. Evolution of cooperator ratio ρ_c with betrayal temptation \boldsymbol{b} for $\boldsymbol{q}=0.5, \boldsymbol{H}\in\{2,3,4,5\}$.

As can be seen from Fig. 2, when own information and environmental information within the memory length are comprehensively used and their weights are the same, no matter what value b takes, there are cooperation strategies in the group. The increase of betrayal temptation b only makes the proportion of cooperators slightly decrease. In addition, memory length has an impact on the stable result of game evolution. The shorter the memory length, the higher the cooperation ratio, and the highest cooperation ratio is when H = 2. This indicates that comprehensive utilization of memory information is conducive to cooperation, and short-term memory is more conducive to cooperation.

When b = 1.5 and $H \in \{2, 3, 4, 5\}$, the evolution results of weight q on cooperation are shown in Fig. 3.

As can be seen from Fig. 3, for different H values, when q = 0, there is no cooperator in the game group, and the cooperation ratio increases with the increase of q. When q = 1, the cooperation ratio is the highest. In the Prisoner's dilemma game, each individual only considers his own factors and takes them as the basis for adjusting his strategy, which is more conducive to the emergence of cooperation. The shorter the memory length, the higher the proportion of cooperators, and this phenomenon will become more and more obvious with the increase of q. In other words, if individuals take their own factors as the basis for strategy update, the proportion of cooperators in the Prisoner's dilemma game will be higher, which also reflects that single use of their own historical information is more conducive to cooperation than comprehensive use of their own and environmental historical information.

In the prisoner's dilemma game, when H=2 and q=1, the cooperation ratio is the highest, while when q=0, the cooperation ratio is the lowest. Under this set of parameters, the comparison between the model in this paper and the results of the original prisoner's dilemma game is discussed, as shown in Fig. 4.

As can be seen from Fig. 4, when H = 2 and q = 1, the proportion of cooperators in this model is slightly lower than that in the original prisoner's dilemma game only when the temptation of betrayal b is small. With the increase of temptation of betrayal, the proportion of cooperation in this model is much higher than that in the original prisoner's dilemma game.

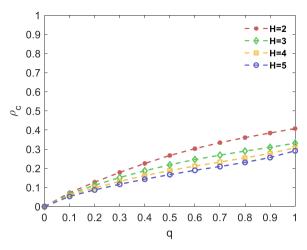


Fig. 3. Evolution of cooperator ratio ρ_c with weight q for betrayal temptation b = 1.5, $H \in \{2, 3, 4, 5\}$.

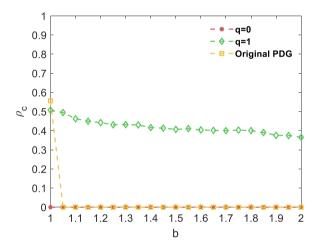


Fig. 4. Evolution of the ratio ρ_c of the cooperator in this model and the original Prisoner's dilemma game with betrayal temptation b when H=2 and q=0, q=1.

It can be seen that this model can effectively promote the production of cooperative behavior when H = 2, q = 1, and when q = 0, no matter what value b takes, the proportion of cooperators is always 0.

3.2. Stag hunt game

For the payoff matrix $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$, it is stag hunt game when R > T > P > S. Let T = r, S = -r, R = 1, P = 0, and the payoff matrix is $\begin{bmatrix} 1 & -r \\ r & 0 \end{bmatrix}$, it is the stag hunt game [43]. When r = 0.5, q = 0.5, $H \in \{2, 3, 4, 5\}$, the corresponding cooperation evolution results are shown in Fig. 5.

As can be seen from Fig. 5, under the given parameters, the proportion of cooperators in the game group tends to be stable with the further development of evolution, and the memory length has little influence on the stable value.

The effect of parameter r changes on the proportion of cooperators is discussed below. When q = 0.5, $H \in \{2, 3, 4, 5\}$, $r \in [0, 1]$, the cooperation evolution results are shown in Fig. 6.

As can be seen from Fig. 6, in the stag hunt game, when the memorized historical information of oneself and the historical information of the environment are comprehensively utilized, and the weight of the two is the same, the proportion of cooperators is high regardless of the value of r. In addition, the increase of parameter r has little effect on the proportion of cooperators. Only when r is large, the proportion of cooperators slightly decreases, and memory length has little effect on the proportion of cooperators.

When r = 0.5 and $H \in \{2, 3, 4, 5\}$, the influence of weight q on cooperation evolution is discussed, as shown in Fig. 7.

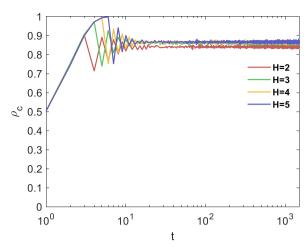


Fig. 5. Evolution results of cooperator ratio ρ_c when $r=0.5, q=0.5, H\in\{2,3,4,5\}$.

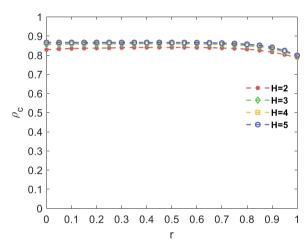


Fig. 6. Evolution results of cooperator ratio ρ_c with parameter r for q=0.5 and $H\in\{2,3,4,5\}$.

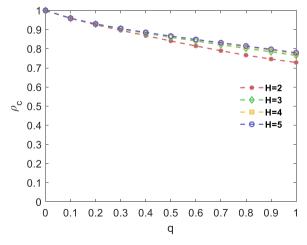


Fig. 7. Evolution of cooperator ratio ρ_c with weight q for parameter r=0.5, $H\in\{2,3,4,5\}$.

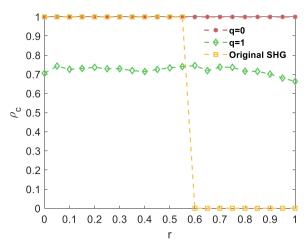


Fig. 8. Evolution of the ratio ρ_c of the cooperator in this model and the original stag hunt game with the parameter r when H=2 and q=0, q=1.

As can be seen from Fig. 7, no matter what value H is, when q=0, all players in the game group are cooperators, and the proportion of cooperators decreases with the increase of q. When q=1, the proportion of cooperators is the lowest. That is to say, in the stag hunt game, when each individual only considers the environmental factors within the memory length and uses them as the basis for individual adjustment strategies, the group can be full of cooperators. In addition, individual only need to consider the environmental factors of the past two time steps as the basis of strategy update, so that the system is full of cooperators. This also reflects that in the stag hunt game, the single use of environmental information is more conducive to cooperation, and short-term memory is better for cooperation promotion.

When H = 2 and q = 0, q = 1, the comparison of cooperative evolution results between this model and the original stag hunt game is shown in Fig. 8.

As can be seen from Fig. 8, when H=2 and q=0, q=1, the proportion of cooperators in the model is less affected by the parameter r. When q=0, all player in the system are cooperators, compared with q=1, the proportion of cooperators in the system will decrease slightly. When the parameter r is small, the proportion of cooperators in the model is slightly lower than that in the original stag hunt game when q=1, while when the parameter r is large, all the cooperators in the original stag hunt game are defectors, and the model in this paper can effectively promote the generation of cooperative behavior at this time.

3.3. Snowdrift game

For the payoff matrix $\binom{R}{T} \binom{S}{P}$, it is snowdrift game when T > R > S > P. Let R = 1, P = 0, T = 1 + r, S = 1 - r, and the payoff matrix is $\begin{bmatrix} 1 & 1-r \\ 1+r & 0 \end{bmatrix}$, it is the snowdrift game [44]. When r = 0.5, q = 0.5, $H \in \{2, 3, 4, 5\}$, the corresponding cooperation evolution results are shown in Fig. 9.

As can be seen from Fig. 9, the proportion of cooperators tends to be stable over time, and memory length has little influence on the stable value.

The effect of parameter r changes on the proportion of cooperators is discussed below. When q = 0.5, $H \in \{2, 3, 4, 5\}$, $r \in [0, 1]$, the cooperation evolution results are shown in Fig. 10.

As can be seen from Fig. 10, comprehensive use of self and environmental information within the memory length with the same weight can enable the existence of cooperation strategy in the group. No matter what the value of memory length H is, the proportion of cooperators will show a downward trend with the increase of parameter r, and the influence of different memory lengths on results will become more and more obvious with the increase of r. When the parameter r is small, the proportion of cooperators corresponding to different memory lengths is roughly the same. With the increase of parameter r, the smaller the memory length, the higher the cooperation proportion.

The influence of weight q on cooperation evolution is further discussed. When $r \in \{0.2, 0.5\}$, $H \in \{2, 3, 4, 5\}$, $q \in [0, 1]$, the cooperation evolution results are shown in Fig. 11.

As can be seen from Fig. 11(a), when r = 0.2, no matter what value H is, the proportion of cooperators decreases with the increase of q, that is, when q = 0, the proportion of cooperators is the highest. It is noted that when the memory length H = 2, all the game groups are cooperators. In other words, if the game individual takes the highest average payoff of the neighbors in the past two time steps and the corresponding strategy as the basis for their own strategy update, all the players in the group can be cooperators. As can be seen in Fig. 11(b), when r = 0.5, no matter what value H is, the proportion of cooperators increases with the increase of q, that is, the proportion of cooperators is the highest when q = 1,

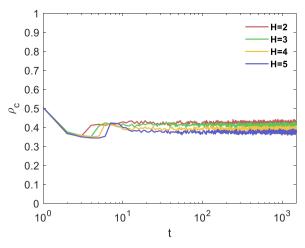


Fig. 9. Evolution of cooperator ratio ρ_c for parameter $\mathbf{r} = 0.5$, $\mathbf{q} = 0.5$, $\mathbf{H} = \{2, 3, 4, 5\}$.

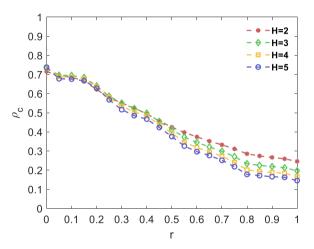


Fig. 10. Evolution of cooperator ratio ρ_c with parameter r for q = 0.5, $H \in \{2, 3, 4, 5\}$.

and it is still the highest when H=2. In other words, when r=0.5, players only need to use their own information in the past two time steps as the basis for strategy update, so that the cooperation ratio in the game group can be at a high level. It can be seen from Fig. 11 that short-term memory is more conducive to cooperation, and it is easier to produce cooperation when only one factor is taken into account in strategy update, and which factor should be taken into account depends on different values of r.

When H = 2, $q \in \{0, 1\}$, $r \in [0, 1]$, the comparison result of cooperation evolution between this model and the original snowdrift game is shown in Fig. 12.

It can be seen from Fig. 12 that under the background of snowdrift game, the curve when q=0 and the curve when q=1 have an intersection point, denoted as (r^*,c^*) . When parameter $r< r^*$, each individual uses the environmental factor in the memory of the past two steps as the basis for their own strategy adjustment, which can make the proportion of cooperators in the game group at a higher level, even when r is small, the system will eventually be all cooperators. When the parameter $r>r^*$, when individual use own factor in the past two steps of memory as the basis for strategy adjustment, cooperative strategies can exist. In addition, the curve when q=1 also has an intersection point with the results of the original snowdrift game model, which is recorded as (r^{**},c^{**}) , when the parameter r is small, H=2 and q=0 in this model and the original snowdrift game model can form a fully cooperative game group. When the parameter r increases slightly, but $r< r^{**}$, the proportion of cooperators in the original snowdrift game is slightly higher than the result of this model. If the parameter r continues to increase to $r> r^{**}$, when H=2 and q=1 in this model, the proportion of cooperators is higher than the proportion of cooperators in the original snowdrift game. When the parameter r is large, our model can promote cooperation, while when the parameter r is small, the proportion of cooperators in this model is only slightly lower than the result of the original snowdrift game in a very small interval.

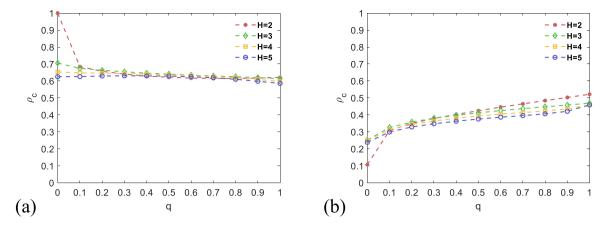


Fig. 11. When $\mathbf{r} \in \{0.2, 0.5\}$, $\mathbf{H} \in \{2, 3, 4, 5\}$, the evolution of the cooperator ratio ρ_c with the weight \mathbf{q} , (a) $\mathbf{r} = 0.2$,(b) $\mathbf{r} = 0.5$.

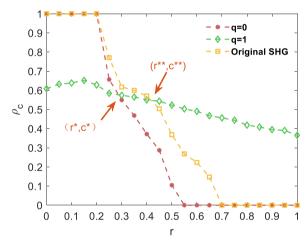


Fig. 12. When H = 2 and q = 0, q = 1, evolution of the ratio ρ_c of the cooperator in this model and the original snowdrift game with the parameter r.

4. Conclusion

This paper proposes a strategy updating method based on historical information, in which individuals can remember their own information and environmental information for a period of time, and can comprehensively use the two types of information for strategy updating. On the basis of this strategy updating method, we discuss the cooperative evolution in three models: prisoner's dilemma game, stag hunt game and snowdrift game. In the prisoner's dilemma game, when individuals use their own historical information to a high degree, cooperation can be promoted. Especially when individuals completely use their own information as the basis for strategy updating, despite the great temptation to betray, there are still a certain proportion of cooperators in the group. In stag hunt game, cooperation is more likely to occur when individuals use environmental information to a high degree. When individuals only use environmental historical information to update strategies, no matter what the value of parameter r is, the system can finally achieve full cooperation. In snowdrift game, when the parameter r is small, the higher the individual utilization of environmental historical information is, the more conducive to cooperation. The proportion of cooperators in this model is only slightly lower than the result of the original snowdrift game in a very small interval. When the parameter r is large, the higher the utilization rate of players' own historical information, the more conducive to cooperation, and significantly higher than the proportion of cooperators in the original snowdrift game.

The above simulation results show that the use of single information is more conducive to cooperation. Although own information and environmental information are all information that can be memorized by individuals, their comprehensive use is not better for promoting cooperation. For different game models, different payoff matrices, and different types of historical information, the promotion effect of cooperation in the game is also different. In addition, in the three game models, the increase of memory step size will not significantly improve the cooperation proportion. Individuals only need to remember two steps of historical information to make the proportion of cooperators reach a higher level, which reflects that short-term memory contributes more to cooperation. The model and results of this paper, to some extent, give a possible

reason for cooperation. Our work is helpful to understand the impact of historical memory on group cooperation in games, and further discuss the impact of memory on cooperation in complex networks and other spatial structures.

Data Availability

No data was used for the research described in the article.

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