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How “punishing evil and promoting good” promotes cooperation in social dilemma

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ABSTRACT

In this paper, based on the social dilemma model, we introduce the punishing evil and promoting good strategy, whose basic strategy is to cooperate, while to obtain benefits at the cost of payment. Specifically, when the person who punishes evil and promotes good plays a game with the cooperator, his contributions will bring the same benefits to both parties, reflecting the promoting good. However, when the person who punishes evil and promotes good plays a game with the defector, making the benefit of the defector loss and become his own benefit, reflecting the punishing evil. The results show that when only the role of punishing evil is considered, increasing punishment strength can easily make those punishing evil and promoting good and ordinary cooperators appear. But only increasing the punishment is not enough to weed out the defectors. When only the effect of promoting good is considered, under a smaller temptation to defection, the increase of the reward strength can transform the system from a full defection state to a full cooperation state. Whereas when the temptation is greater, the increase in the reward is not enough for the cooperator to survive and only defectors exist in the system at this time. In addition, there exist combinations of minimum values of punishment and reward that bring the system to a defector-free state. In conclusion, our study finds effective intervals for promoting the development of those who punish evil and promote good and ordinary cooperators, respectively.

1. Introduction

The evolutionary law of “survival of the fittest” reveals that intense competition among individuals is not conducive to the generation of cooperation behavior [1–3]. Nevertheless, cooperation exists widely in nature as well as in many areas of the real society such as the economy [4–7]. Evolutionary game theory provides a theoretical framework to explain the above contradictions [8–11]. It is hard to imagine a society without cooperation [12]. Therefore, it is necessary and important to explore in depth the emergence and maintenance of cooperation [13]. Nowak and May [14] pioneered spatial game by pointing out that cooperators are able to form stable clusters on specific spatial structures — the lattice networks. The traditional prisoner’s dilemma game (PDG), as one of the typical examples to describe social dilemmas [15–18], has been used by many scholars to analyze strategic choices of individuals in the face of conflicts. In addition, the study of the

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dynamical behavior of replication equations [19] in evolutionary game theory has attracted the attention of many scholars. In recent years, replication equation models have been widely used to study the evolution of cooperation [20–23].

To effectively promote cooperation, many mechanisms are considered, such as: reputation [24–26], aspiration [27–32], voluntary participation [33–37], migration [38], memory [39], etc. Among them, reward and punishment, as two effective ways to promote cooperation, have aroused widespread interest of scholars and made a large number of research achievements [40–45]. Wang et al. considered a delay-based reward mechanism, where individuals can choose larger but later available rewards or smaller but earlier available rewards in a cooperative round [46]. Fu et al. proposed a reward mechanism based on historical loyalty. When individuals maintain cooperation strategy for a period of time, they will receive additional rewards, and the rewards to the cooperators are shared equally by the defectors in their neighbors [47]. Wu et al. defined a kind of reward individuals designed to increase the income of cooperators in complex networks by paying the cost to impose social rewards on cooperators [48]. Szolnoki et al. introduced rewarding cooperators as well as rewarding defectors who invest in prosocial and antisocial pool respectively to reward players who are similar to themselves and found that antisocial rewards do not necessarily impede public cooperation in structured populations [49]. Gao et al. employed a democratic procedure in which cooperators simultaneously independently vote on whether to punish defectors [50]. Perc et al. established an adaptive punishment mechanism in which the remaining neighboring cooperators will punish the defector only when the defector successfully passes its strategy to a cooperator [51]. Nakamaru et al. introduced selfish punishers (a defector who punishes defectors) and analyzed the effect of selfish punishers and pure cooperators on the co-evolution of altruism and punishment [52]. Chen et al. proposed a new type of punishment — probabilistic sanction, in which cooperators are able to switch between contributing to the common pool and contributing to the common pool as well as punishing defectors in a probabilistic manner [53]. Gao et al. developed an analytical model based on two typical patterns — distributed peer punishment and centralized pool punishment, aiming to study the impact of punishment patterns on the role of punishment in cooperative evolution [54]. All of the above works studied the two mechanisms of reward and punishment separately. In Ref. [55] and Ref. [56], in the context of public goods game, the effect of the synergy of reward and punishment on the evolution of cooperation is discussed in two-strategy and four-strategy situations, respectively.

In real life, more good behaviors and less bad behaviors are the basic characteristics of a good social atmosphere. Encouraging good behaviors or suppressing bad behaviors, or encouraging good behaviors and suppressing bad behaviors at the same time, are the basic measures to form a good social atmosphere. These three measures have different focuses, involve different objects, and require different social costs. In a gaming environment, cooperation can be understood as good behaviors and defector can be understood as bad behaviors. Society encourages cooperation behavior and inhibits defection behavior. But the individual's behavior of cooperation or defection needs to be exposed by the exposer in order to be known by everyone. Therefore, there is such a class of cooperators, who can obtain the exposure qualification by paying a certain cost, so as to expose the strategies of their game objects, and then change the payoffs of their own and their game objects. Specifically, when he plays a game with the cooperator, due to cooperator being exposed and he himself is a cooperator, thus both are rewarded by society. When he plays a game with the defector, the defector is exposed, and the society will punish the defector and reward him for the act of exposure. This strategy objectively works to encourage good behaviors and inhibit bad behaviors. This paper calls it the "punishing evil and promoting good" strategy. In fact, punishing evil and promoting good in this paper is defined by social values and implemented by individuals, which is more relevant to the real situation. Corresponding to the above three measures, there are three different situations in the concrete embodiment of the strategy of punishing evil and promoting good. One is reflected only in the reward for cooperators, the other is reflected only in the punishment for defectors, and the third is reflected both in the reward for cooperators and in the punishment for defectors. Based on social dilemmas, this paper explores the effects of those who punish evil and promote good on the evolution of cooperation in an infinite well-mixed population under three different scenarios.

This paper is organized as follows. Section 2 introduces an evolutionary game model with the punishing evil and promoting good strategy. Section 3 presents the theoretical analysis as well as numerical simulations of the replication equations, respectively. Finally, Section 4 draws conclusions and summarizes the full text.

2. Model

In Ref. [14], Nowak discussed a simplified form of prisoner's dilemma (weak prisoner's dilemma) with the payoff matrix in Table 1 below. where b is the temptation to defection and its value range is $1 < b \leq 2$. On this basis, this paper puts forward a strategy of punishing evil and promoting good.

Table 1
Payoff matrix.

	C	D
C	1	0
D	b	0

Table 2
Payoff matrix.

	SC	*C*	*D*
SC	$1+\alpha_1-\frac{\gamma}{2}$	$1+\alpha_1-\gamma$	$\alpha_2-\gamma$
C	$1+\alpha_1$	1	0
D	$b-\alpha_2$	b	0

Suppose there are three strategies in the group: punishing evil and promoting good strategy (*SC*), ordinary cooperation strategy (*C*) and defection strategy (*D*). Based on the contribution of $\gamma(0 \le \gamma \le 1)$, when SC plays a game with the ordinary cooperator or the SC, each side will get the $\alpha_1(0 \le \alpha_1 \le 1)$ reward. And when two players who punish evil and promote good play a game, both sides will share the payment, that is, each individual pays $\frac{\gamma}{2}$. When SC plays a game with the defector, the reward for the SC comes from the punishment $\alpha_2(0 \le \alpha_2 \le 1)$ of the defector. The payoff matrix of this paper is shown in Table 2.

Here, α_1 is the reward value, and the larger the α_1, the greater the reward strength. α_2 is the punishment value, and the larger α_2 indicates that the punishment the greater the force. γ is the value of the cost paid by SC.

In this paper, we consider to perform the above game in an infinite well-mixed population. Let the proportion of SC in the population be x, the proportion of C be y, and then the proportion of D be $z = 1-x-y$. Therefore the average payoffs for each of these three strategies are:

$$\begin{aligned} &\Pi_{SC} = x(1+\alpha_1-\tfrac{\gamma}{2}) + y(1+\alpha_1-\gamma) + z(\alpha_2-\gamma) \\ &\Pi_C = x(1+\alpha_1) + y \\ &\Pi_D = x(b-\alpha_2) + by \end{aligned} \tag{1}$$

In replication dynamics, the adjustment direction of the strategy is determined according to the payoff of the strategy and the average payoff of the population. For the individuals of class i(i=SC,C,D), if its average payoff exceeds the average payoff of the population, i.e., $\pi_i > \bar{\pi}$, then its proportion in the population will increase, otherwise it will decrease. From this, establish the replication equation as follows.

$$\begin{aligned} &\dot{x} = x(\Pi_{SC} - \bar{\Pi}) \\ &\dot{y} = y(\Pi_C - \bar{\Pi}) \\ &\dot{z} = z(\Pi_D - \bar{\Pi}) \end{aligned} \tag{2}$$

where $\bar{\Pi} = x\Pi_{SC} + y\Pi_C + z\Pi_D$.

Therefore, in the following analysis and discussion, we will revolve around the four parameters $\alpha_1, \alpha_2, \gamma$ and b.

3. Results

3.1. Equilibrium points and stability analysis

The dynamic replication Eq. (2) has six equilibria: $E_1 = (1,0,0), E_2 = (0,1,0), E_3 = (0,0,1), E_4 = \left(\frac{2\gamma-2\alpha_1}{\gamma-2\alpha_1}, \frac{-\gamma}{\gamma-2\alpha_1}, 0\right)$, $E_5 = \left(\frac{2\gamma-2\alpha_2}{2\alpha_1-2b+\gamma+2}, 0, \frac{-\gamma+2\alpha_2+2\alpha_1-2b+2}{2\alpha_1-2b+\gamma+2}\right)$ and $E_6 = \left(\frac{2(\gamma-\alpha_2)(b-1)}{2\alpha_1^2-2\alpha_1 b+\gamma b-2\alpha_2^2+2\alpha_1-\gamma}, \frac{2(\alpha_2-\gamma)(-\alpha_1+b-\alpha_2-1)}{2\alpha_1^2-2\alpha_1 b+\gamma b-2\alpha_2^2+2\alpha_1-\gamma}, \frac{2(\alpha_1-b-\gamma+\alpha_2+1)\alpha_1+\gamma(b-2\alpha_2-1)}{2\alpha_1^2-2\alpha_1 b+\gamma b-2\alpha_2^2+2\alpha_1-\gamma}\right)$.

After the analysis and proof of the stability of these equilibria (see the Appendix for details), the final evolution results of the system are as follows:

(1) There is only one strategy left in the system. When $\gamma = 0, b \le 1+\alpha_1+\alpha_2$, E_1 is the stable equilibrium point, i.e., when the system is stable, there exists a state in which all individuals choose the strategy of punishing evil and promoting good. When $\alpha_2 \le \gamma$, the equilibrium point E_3 is stable, that is, there is a state in which all individuals in the system adopt the defection strategy when stable. However, E_2 is an unstable equilibrium at any value, that is, ordinary cooperators cannot occupy the entire population alone.

(2) The system eventually has two strategies that coexist stably. When $\gamma < \alpha_1$, $2\alpha_1^2 - 2(b+\gamma-\alpha_2-1)\alpha_1 + \gamma(b-2\alpha_2-1) > 0$, the equilibrium point E_4 is stable, i.e., the punishing evil and promoting good strategy and ordinary cooperation strategy coexist stably. When $b > 1+\alpha_1+\alpha_2, \gamma < \alpha_2$, E_5 is the stable equilibrium point, i.e., the stable coexistence of the strategy of punishing evil and promoting good and defection strategy. It can be found that when the two strategies coexist, the punishing evil and promoting good strategy always exists.

(3) The three strategies coexist stably in the system. When $\left(-\frac{b}{2}+\alpha_1+\alpha_2+\frac{1}{2}\right)\gamma + (-\alpha_1+b-\alpha_2-1)\alpha_1 \ge 0, b \le 1+\alpha_1+\alpha_2, \gamma < \min\{\alpha_1,\alpha_2\}$ and $(-b+1)\gamma + 2\alpha_1 b - 2\alpha_1^2 + 2\alpha_2^2 - 2\alpha_1 > 0$, the equilibrium point E_6 is stable, that is, when the system is stable, there exists a state where three strategies coexist.

In a nutshell, there are five stable equilibrium states in the system and the cooperators survive for most of the parameter ranges, which indicates that those who punish evil and promote good can effectively promote cooperation.

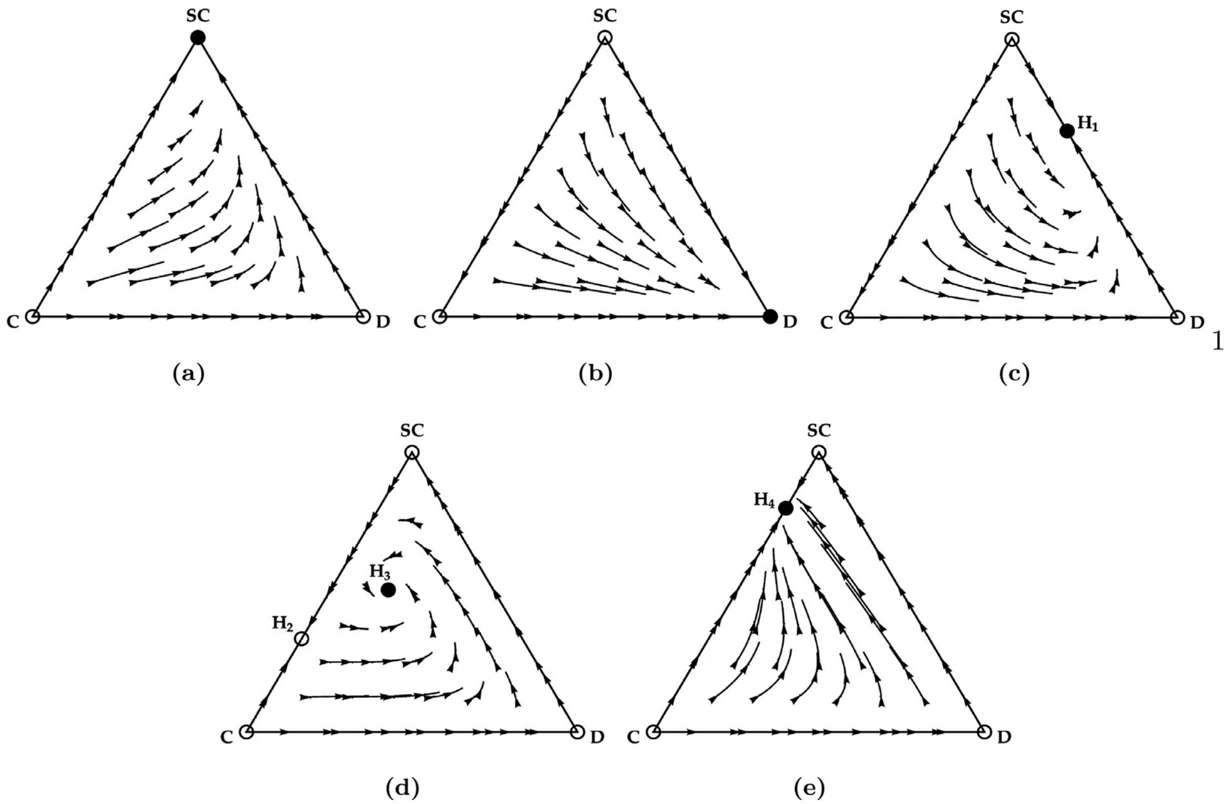

Fig. 1. The evolutionary trajectories of SC, C and D when $b = 1.4$. The simplex represents the state space $S_3 = \{(x, y, z) : x, y, z \geq 0, \textit{and } z + y + z = 1\}$, where x, y, z denote the frequency of SC, C, and D, respectively. The vertices SC, C, and D respectively correspond to the three equilibrium states in which all are SC($x = 1$), C($y = 1$), and D($z = 1$). There exist boundary equilibria H_1(c), H_2(d), H_4(e) on the SC-D edge and SC-C edge, respectively. In addition, there is an internal equilibria H_3(d). Arrows represent the direction of evolution. A hollow circle indicates an unstable equilibrium point, while a solid one indicates a stable equilibrium point. Other parameters: (a)$\gamma = 0, \alpha_1 = 0.2$,and $\alpha_2 = 0.2$;(b)$\gamma = 0.2, \alpha_1 = 0.2$,and $\alpha_2 = 0.2$;(c)$\gamma = 0.2, \alpha_1 = 0$,and $\alpha_2 = 0.4$;(d)$\gamma = 0.2, \alpha_1 = 0.25$,and $\alpha_2 = 0.4$;(e)$\gamma = 0.2, \alpha_1 = 0.6$,and $\alpha_2 = 0.4$.

3.2. Numerical simulations

In this paper, we study the effect of the punishing evil and promoting good strategy on the evolution of cooperation in an infinite well-mixed population. First, the evolution of any initial distribution of the three strategies' frequencies over time under the different combinations of parameters are simulated in the simplex S_3, and the results are shown in Fig. 1.

From Fig. 1, we can observe that starting from the different points inside the simplex S_3 (corresponding to the different initial distributions of the three strategies), the system can finally reach a stable state, and the evolution of the system is not affected by the initial distributions of the strategies. In addition, there are five stable equilibrium states for different parameter values.

The initial value of $(x(0), y(0), z(0)) = (0.25, 0.25, 0.5)$ is chosen for the following study. The variation of the proportion of SC, C and the two types of cooperators in the population along with α_2 when $\alpha_1 = 0, \gamma = 0.2, b \in \{1.4, 1.6, 1.8\}$ is shown in Fig. 2.

As can be seen from Fig. 2, for a fixed value of temptation, when the punishment is not strong enough to compensate for SC's payment, only defectors exist after the system stabilizes. SC emerges when the punishment strength is greater than the payment, and as punishment strength increases, the proportion of SC first increases and then decreases. When the proportion of SC reaches the maximum, a further increase in punishment strength makes ordinary cooperators appear, and the proportion of ordinary cooperators increases with the increase of punishment strength and the total number of both types of cooperators also increases. At this time, although the payoff of the ordinary cooperator in a single game does not increase, due to the punishment of the defectors by the SC, which makes the survival chance of the defector is reduced, and the survival chance of the ordinary cooperator is increased. In short, when only the role of punishing evil is considered, the stronger the punishment, the more favorable the cooperation. For SC, there is a moderate value of punishment for defectors, that maximizes the proportion of SC. For ordinary cooperators, along with the increasing of the temptation, the value of the punishment α_2 corresponding to making ordinary cooperators appear increases.

Then we explore the relationship between the fraction of SC, C, and both types of cooperators and the temptation to defection b at different α_2, and the results are shown in Fig. 3.

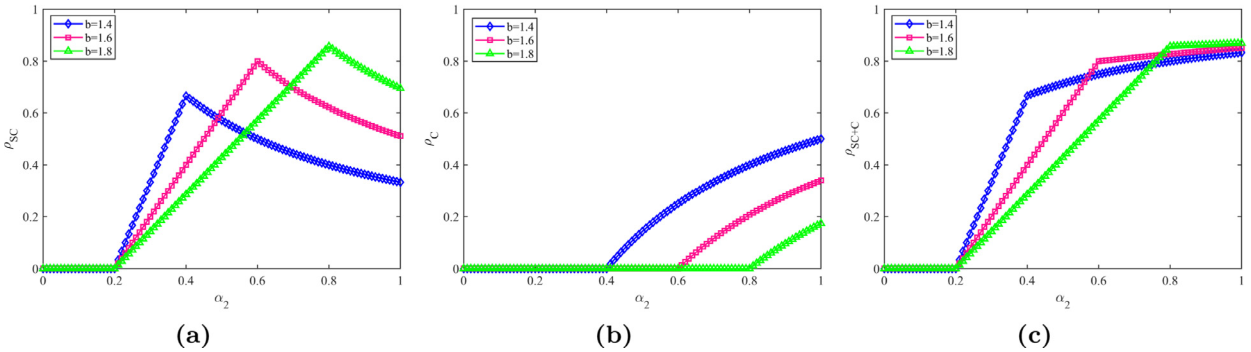

Fig. 2. $\alpha_1 = 0, \gamma = 0.2, b \in \{1.4, 1.6, 1.8\}$, the average proportion of SC (a), C (b) and the two types of cooperators (c) in equilibrium state as the function of $\alpha_2 \in [0, 1]$.

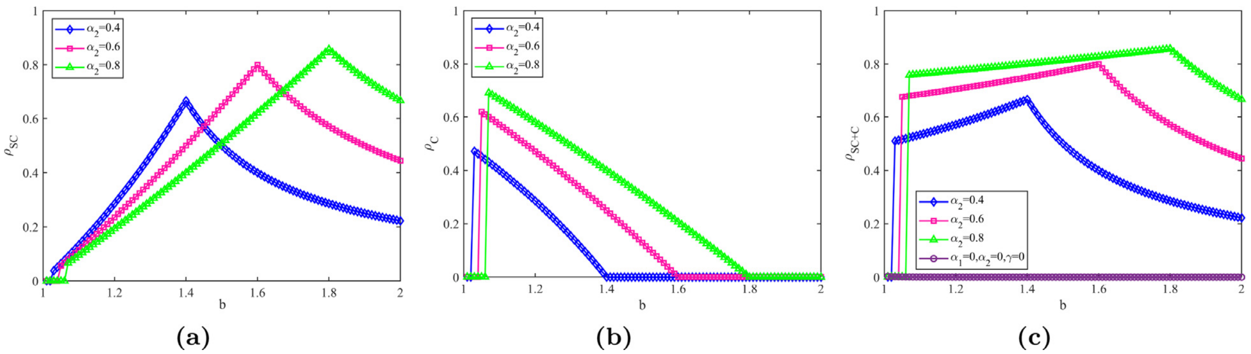

Fig. 3. $\alpha_1 = 0, \gamma = 0.2, \alpha_2 \in \{0.4, 0.6, 0.8\}$, the evolutionary results of the fraction stability value of SC (a), C (b) and the total cooperation (c) corresponding to the change of b.

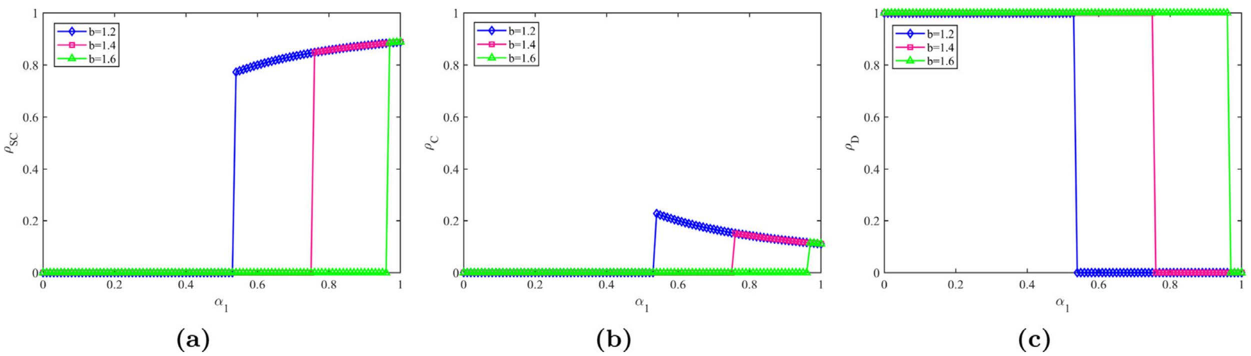

Fig. 4. $\alpha_2 = 0, \gamma = 0.2, b \in \{1.2, 1.4, 1.6\}$, the change in α_1 corresponds to the change in the stable value of the proportion of the three types of individuals.

From Fig. 3, it can be observed that when the temptation b is minimal, SC and the ordinary cooperator cannot survive in the system. When the temptation b increases to a certain level, SC and ordinary cooperator begin to appear, and as the value of b increases, the proportion of SC first increases and then decreases. When the proportion of SC reaches the maximum, ordinary cooperators disappear. On the whole, when both ordinary cooperators and SC appear, the cooperators' fraction ρ_{SC+C} first increases and then decreases with increasing temptation, and the larger the punishment, the system can maintain a higher cooperation level within a larger range of temptation to defection.

The effects of reward α_1 and temptation b on the evolution of the three strategies are discussed below when $\alpha_2 = 0$. When $\alpha_2 = 0, \gamma = 0.2, b \in \{1.2, 1.4, 1.6\}$, the trend of the average proportion of the three strategies relative to $\alpha_1 \in [0, 1]$ is shown in Fig. 4.

It can be seen from Fig. 4 that when only the role of promoting good is considered, for a fixed temptation value, if the reward strength is smaller, the defectors will take over the whole system. When the reward strength is greater than a certain value, all individuals in the system are transformed into cooperators, and the fraction of SC increases as the reward

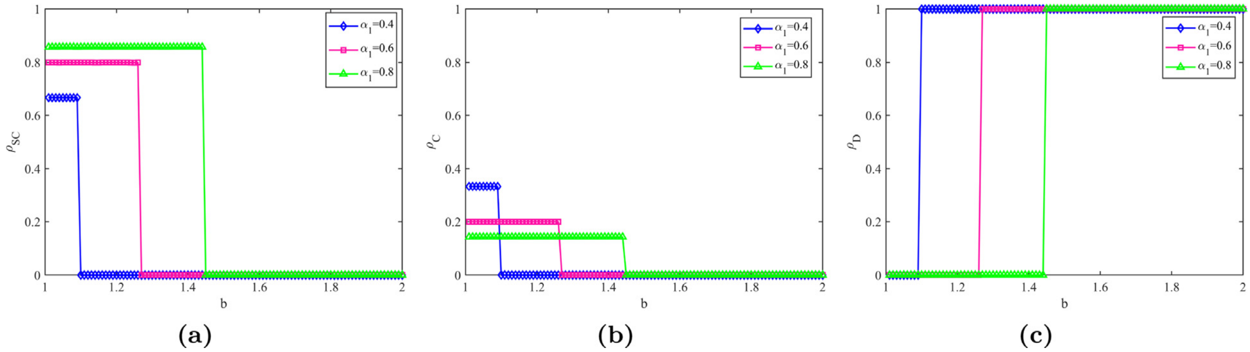

Fig. 5. $\alpha_2 = 0, \gamma = 0.2, \alpha_1 \in \{0.4, 0.6, 0.8\}$, the change results of the fraction of the three strategies along with b.

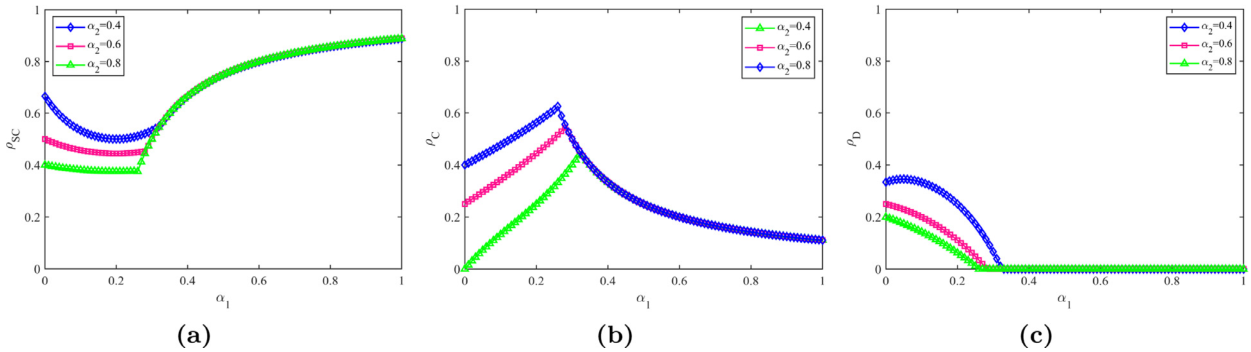

Fig. 6. $\gamma = 0.2, b = 1.4, \alpha_2 \in \{0.4, 0.6, 0.8\}$, the effect of $\alpha_1 \in [0, 1]$ on two types of cooperators and defectors.

strength increases and the fraction of ordinary cooperators decreases with increasing reward strength. This may be because when there are only cooperators in the population, SC can get rewards for playing with two types of cooperators, while the ordinary cooperators get rewarded only for playing with SC, which drives the average payoff of SC to be higher than the average payoff of the ordinary cooperators. In addition, with increasing temptation b, the reward value to make the defector disappear and the cooperator appear gradually increases. In general, when the system has no defectors, the greater the reward strength, the more conducive to the survival of SC.

When $\alpha_2 = 0, \gamma = 0.2$, the results of the number of SC, ordinary cooperators and defectors as a proportion in the population varies with temptation b for different α_1 are shown in Fig. 5.

In Fig. 5, the cooperation strategy cannot coexist with the defection strategy when the system is stable. When the temptation is smaller, there are no defectors in the system and the proportion of SC and ordinary cooperators does not vary with the temptation b, and the proportion of SC is higher than that of the ordinary cooperators. When the temptation value is greater, there are only defectors in the system. Furthermore, as the reward increases, the value of temptation b that makes the cooperators disappear and the defectors appear increases.

The following discusses when the actions of punishing evil and promoting good work simultaneously, the variation curves of the stable values of the three strategies' proportions with α_1, and the results are shown in Fig. 6.

In Fig. 6, when the reward strength is smaller, three strategies coexist in the system, and the ratio of defectors generally shows a trend of decreasing as the reward increases. And the number of SC decreases as increasing reward strength, while the number of ordinary cooperators increases with the increase of reward strength. When the reward is smaller, ordinary cooperators will be rewarded for not paying the cost, thus indirectly crowding out SC. Moreover, the greater the punishment strength, the smaller the number of defectors and SC in the group, and the greater the number of ordinary cooperators. When the reward strength is larger, the defector disappears. And with the increase of reward strength, the proportion of SC increases, and the proportion of ordinary cooperators decreases. At this point, the change in punishment value α_2 could no longer have any effect on the proportion of the two types of cooperators in the group. This can help us to find the most reasonable interval to promote the growth of the number of SC or ordinary cooperators. when $\alpha_1 \in [0, 1], \alpha_2 \in [0, 1], \gamma = 0.2$, the change results of the fraction of the three strategies and the distribution of the population states at different values of b are shown in Fig. 7.

Fig. 7 (a)–(c) show that for smaller reward α_1, when the punishment α_2 reaches a certain value, SC begins to appear. And its proportion shows a non-monotonic change with increasing punishment. And for smaller punishment, when the reward

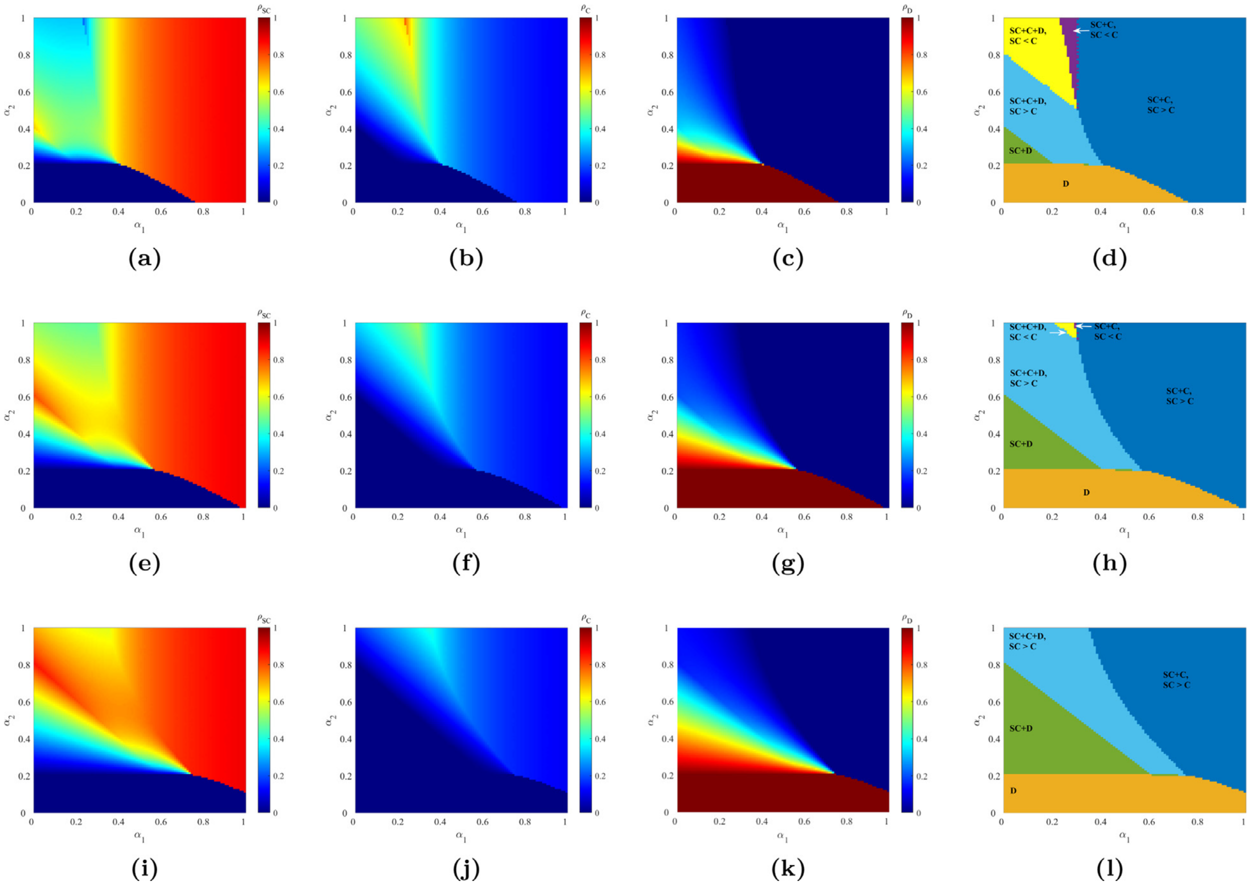

Fig. 7. Heat map of the proportion of the three strategies in the α_1_α_2 plane when $\gamma = 0.2$ and the distribution of the states. From top to bottom, each row corresponds to different temptation *b*, $b = 1.4, 1.6, 1.8$ respectively; from left to right, each column corresponds to the proportion of SC, C and D as well as the distribution of states.

exceeds a certain threshold, the system changes from a full defection state to a full cooperation state, and then increases the reward, SC increases, conversely, the number of ordinary cooperators decreases. Similar trends can also be observed in Fig. 7(e-g) and (i-k), the difference being that when the temptation is large ($b = 1.8$) (Fig. 7(i-k)), for smaller punishments, no matter what the value of α_1, there are only defectors in the system. Besides, looking at the Fig. 7(b)(f)(j), it can be seen that when the temptation and the reward are smaller and the punishment is larger, ordinary cooperators have an advantage in the population.

As can be seen from Fig. 7(d), in the interval $\alpha_2 \in [0, 0.2]$, the critical value $\alpha_1{}^*$ that makes cooperators appear and defectors disappear decreases with the increase of α_2. And when the system is in a full cooperation state, the proportion of SC is not less than that of ordinary cooperators. For the interval $\alpha_2 \in (0.2, 0.4]$, as α_1 increases, the ordinary cooperators appear first in the environment where SC and the defectors coexist and then the defector disappears, only two types of cooperators exist in the system. Interestingly, the fraction of ordinary cooperators is always smaller than SC in this interval, regardless of the shift in group state. For the interval $\alpha_2 \in (0.4, 1]$, the system stabilizes and transitions from a state where three strategies coexist to a state where two cooperation strategies coexist, where the larger α_2 is, the smaller the critical value $\alpha_1{}^*$ that makes the transition between the two states occur. The variation trends of population structure in Fig. 7(h,l) are approximately the same as those in Fig. 7 (d). Noteworthy, yellow and purple areas ($\rho_{SC} < \rho_C$) gradually shrink with the increase of *b*. Even when $b = 1.8$, the yellow and purple areas will no longer exist. In special, for smaller punishment strength, increasing the reward strength is not sufficient to make cooperation emerge under the greater temptation to defection. When the punishment strength is increased, even if the reward strength is weak, it can promote the emergence of cooperation. And if you want to achieve a state of full cooperation, you can't just rely on the punishment, you must be blessed with reward. It shows that punishment is more likely to induce cooperation, but if cooperation is to prevail, the combined effect of reward and punishment is required. In addition, there exist combinations of minimum values of punishment and reward strength that make the system without defector when the punishment and reward value reach any set of values in that combination.

4. Conclusion

This paper introduces the strategy of punishing evil and promoting good, which at the cost of payment to bring rewards for cooperation behavior and make the defector punished. The stability analysis of the equilibrium point shows that there are five stable equilibrium states: full defection, only those who punish evil and promote good, coexistence of those who punish evil and promote good and defectors, coexistence of those who punish evil and promote good and ordinary cooperators and the coexistence of three strategies. And both types of cooperators survive in the system for the vast majority of parameter values taken, indicating that the presence of the strategy of punishing evil and promoting good can avoid the extinction of cooperation in social dilemmas.

In the numerical simulation, we fixed the payment cost and analyzed the results of different values of the three parameters of temptation to defection, reward and punishment from the perspective of the effect of punishing evil and promoting good. The results show that, as in previous studies, either increasing the reward or the punishment significantly promoted cooperation. However, the promotion effect of the two on cooperation is different from other papers. Specifically, when only punishment is considered, increasing punishment strength alone is not enough to weed out defectors. When only reward is considered, an increase in reward strength at a smaller temptation to defection can shift the system from a full defection state to a full cooperation state. Whereas when the temptation to defection is greater, the increase in reward strength is not enough to keep the cooperator alive, and only defectors exists in the system at that time. In other words, when only reward is considered, cooperators and defectors can never coexist. When considered simultaneously, there exists combinations of minimum values of punishment and reward that bring the system to a full cooperation state.

Moreover, for the ordinary cooperators, when the temptation to defection and the reward are smaller, but the punishment is larger, the ordinary cooperators has the advantage and its proportion reaches a higher level. For those who punish evil and promote good, when the reward is smaller, those who punish evil and promote good appear after the punishment reaches a certain level, and with the increase of punishment, the proportion of those who punish evil and promote good first increases and then decreases. This is because the moderate punishment limits the defectors to a certain extent, but the defectors don't disappeared and still pose a threat to ordinary cooperators. However, when rewards are higher, with a smaller temptation to defection, the proportion of those who punish evil and promote good proportion is unaffected by changes in punishment strength and remains at a higher level.

In conclusion, the presence of those who punish evil and promote good can help cooperators resist the invasion of defectors and facilitate the emergence and maintenance of cooperation. But, the effective intervals for promoting the two cooperation strategies are not the same, which provides a new perspective for understanding reward and punishment. Future work will consider the impact of environmental factors on system dynamics, and the impact of punishing evil and promoting good behavior on the evolution of cooperation in structured networks or more complex networks.

Appendix A

Substituting the $z = 1 - x - y$ into the replication Eq. (2), we get:

$$\begin{aligned}\dot{x} = \quad & f(x,y) = \left(b - \alpha_1 - \tfrac{\gamma}{2} - 1\right)x^3 + \left(\alpha_1 - b + \tfrac{3\gamma}{2} - \alpha_2 + 1\right)x^2 + 2(b - \alpha_1 - 1)x^2y \\ & +(-b + \alpha_1 - \alpha_2 + 1)xy + (b-1)xy^2 + (\alpha_2 - \gamma)x \\ \dot{y} = \quad & g(x,y) = (b-1)y^3 - (b-1)y^2 + 2(b - \alpha_1 - 1)y^2x + (-b + \alpha_1 + \gamma + 1)xy \\ & +\left(b - \alpha_1 - \tfrac{\gamma}{2} - 1\right)x^2y \end{aligned} \tag{3}$$

Calculating $\dot{x} = 0$ as well as $\dot{y} = 0$, we can obtain a total of 6 equilibrium points for this system of differential equations, including three vertex equilibrium points: $(1,0,0), (0,1,0), (0,0,1)$ and two boundary equilibrium points: $\left(\frac{2\gamma-2a}{\gamma-2a}, \frac{-\gamma}{\gamma-2a}, 0\right), \left(\frac{2\gamma-2\alpha_1}{2\alpha-2b+\gamma+2}, 0, \frac{-\gamma+2\alpha_1+2\alpha-2b+2}{2\alpha-2b+\gamma+2}\right)$ and one internal equilibrium point: $\left(\frac{2(\gamma-\alpha_2)(b-1)}{2\alpha_1{}^2-2\alpha_1 b+\gamma b-2\alpha_2{}^2+2\alpha_1-\gamma}, \frac{2(\alpha_2-\gamma)(-\alpha_1+b-\alpha_2-1)}{2\alpha_1{}^2-2\alpha_1 b+\gamma b-2\alpha_2{}^2+2\alpha_1-\gamma}, \frac{2(\alpha_1-b-\gamma+\alpha_2+1)\alpha_1+\gamma(b-2\alpha_2-1)}{2\alpha_1{}^2-2\alpha_1 b+\gamma b-2\alpha_2{}^2+2\alpha_1-\gamma}\right)$.

According to the Eq. (3), the corresponding *Jacobian* matrix is as follows:

$$J = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} \tfrac{\partial f(x,y)}{\partial x} = \quad & 3\left(b - \alpha_1 - \tfrac{\gamma}{2} - 1\right)x^2 + 2\left(\alpha_1 - b + \tfrac{3\gamma}{2} - \alpha_2 + 1\right)x + 4(b - \alpha_1 - 1)xy \\ & +(-b + \alpha_1 - \alpha_2 + 1)y + (b-1)y^2 + \alpha_2 - \gamma \\ \tfrac{\partial f(x,y)}{\partial y} = \quad & 2(b - \alpha_1 - 1)x^2 + 2(b-1)xy + (-b + \alpha_1 - \alpha_2 + 1)x \\ \tfrac{\partial g(x,y)}{\partial x} = \quad & 2(b - \alpha_1 - 1)y^2 + (-b + \alpha_1 + \gamma + 1)y + 2\left(b - \alpha_1 - \tfrac{\gamma}{2} - 1\right)xy \\ \tfrac{\partial g(x,y)}{\partial y} = \quad & 3(b-1)y^2 - 2(b-1)y + 4(b - \alpha_1 - 1)xy + (-b + \alpha_1 + \gamma + 1)x \\ & +\left(b - \alpha_1 - \tfrac{\gamma}{2} - 1\right)x^2 \end{aligned} \tag{5}$$

The following tests the stability of the above equilibrium points according to the *Jacobian* matrix. Here, we assume that (x_0, y_0, z_0) is the equilibrium point of the system, and

$0 \le x_0, y_0, z_0 \le 1, x_0 + y_0 + z_0 = 1.$

1. $(x_0, y_0, z_0) = (1, 0, 0)$
   $|J| = \frac{\gamma(\gamma - 2\alpha_1 - 2\alpha + 2b - 2)}{4}$, $trJ = b - \alpha + \gamma - 1 - \alpha_1$.
   - When $\gamma > 0$, $\gamma - 2\alpha_2 - 2\alpha_1 + 2b - 2 < 0$, the equilibrium point is unstable due to $|J| < 0$.
   - When $\gamma > 0$ and $\gamma - 2\alpha_2 - 2\alpha_1 + 2b - 2 \ge 0$, $trJ > 0$, indicating that this equilibrium point is unstable.
   - When $\gamma = 0, b \le 1 + \alpha_1 + \alpha_2$, the eigenvalues of the above matrix are $\lambda_1 = -1 - \alpha_1 - \alpha_2 + b$ and $\lambda_2 = \frac{\gamma}{2}$, then $\lambda_1 \le 0$, $\lambda_2 = 0$, and λ_2 corresponds to the first-order Jordan block, so the equilibrium point is stable.
2. $(x_0, y_0, z_0) = (0, 1, 0)$
   $|J| = (\alpha_1 - \gamma)(b - 1)$, $trJ = -1 + b + \alpha_1 - \gamma$.
   - When $\gamma > \alpha_1$, since $|J| < 0$, $(0, 1, 0)$ is unstable.
   - When $\gamma \le \alpha_1$, $trJ > 0$, so the equilibrium point is unstable.
3. $(x_0, y_0, z_0) = (0, 0, 1)$
   $|J| = 0$, $trJ = \alpha_1 - \gamma$.
   - When $\alpha_2 > \gamma$, since $trJ > 0$, $(0, 0, 1)$ is unstable.
   - When $\alpha_2 < \gamma$, The eigenvalues of the above matrix are $\lambda_1 = \alpha_2 - \gamma$ and $\lambda_2 = 0$, since $\lambda_1 < 0$ and $\lambda_2 = 0$ corresponds to the first-order Jordan block, indicating that the equilibrium point is stable.
4. $(x_0, y_0, z_0) = \left(\frac{2\gamma - 2\alpha_1}{\gamma - 2\alpha_1}, \frac{-\gamma}{\gamma - 2\alpha_1}, 0\right)$
   $|J| = \frac{2\left(-\alpha_1{}^2 + (b + \gamma - \alpha_2 - 1)\alpha_1 - \frac{\gamma(b - 2\alpha_2 - 1)}{2}\right)\gamma(\gamma - \alpha_1)}{(-\gamma + 2\alpha_1)^2}$, $trJ = \frac{-2\alpha_1{}^2 + (2b + \gamma - 2\alpha_2 - 2)\alpha_1 - \gamma(b - \gamma - 2\alpha_2 - 1)}{-\gamma + 2\alpha_1}$.
   - When $\gamma > 0$ and $\left(-\alpha_1{}^2 + (b + \gamma - \alpha_2 - 1)\alpha_1 - \frac{\gamma(b - 2\alpha_2 - 1)}{2}\right)(\gamma - \alpha_1) < 0$, $|J| < 0$, indicating that this equilibrium point is unstable.
   - When $\gamma < \alpha_1$ and $2\alpha_1{}^2 - 2(b + \gamma - \alpha_2 - 1)\alpha_1 + \gamma(b - 2\alpha_2 - 1) > 0$, $|J| > 0$, $trJ < 0$, indicating that $\left(\frac{2\gamma - 2\alpha_1}{\gamma - 2\alpha_1}, \frac{-\gamma}{\gamma - 2\alpha_1}, 0\right)$ is stable.
5. $(x_0, y_0, z_0) = \left(\frac{2\gamma - 2\alpha_2}{2\alpha_1 - 2b + \gamma + 2}, 0, \frac{-\gamma + 2\alpha_2 + 2\alpha_1 - 2b + 2}{2\alpha_1 - 2b + \gamma + 2}\right)$
   $|J| = \frac{4(-\alpha_2 + \gamma)^2\left(\frac{\gamma}{2} - \alpha_2 - \alpha_1 + b - 1\right)(-\alpha_1 + b - \alpha_2 - 1)}{(2b - 2\alpha_1 - \gamma - 2)^2}$, $trJ = \frac{(-\alpha_2 + \gamma)(\gamma + 4b - 4\alpha_1 - 4\alpha_2 - 4)}{2b - 2\alpha_1 - \gamma - 2}$
   - When $-\frac{\gamma}{2} < b - \alpha_1 - \alpha_2 - 1 < 0$, $|J| < 0$, indicating that this equilibrium point is unstable.
   - When $b - \alpha_1 - \alpha_2 - 1 < -\frac{\gamma}{2}, \gamma > \alpha_2$ indicating that the equilibrium point is unstable.
   - When $b > 1 + \alpha_1 + \alpha_2$ and $\gamma < \alpha_2$, $|J| > 0$, $trJ < 0$, indicating this equilibrium point is stable.
6. $(x_0, y_0, z_0) = \left(\frac{2(\gamma - \alpha_2)(b - 1)}{2\alpha_1{}^2 - 2\alpha_1 b + \gamma b - 2\alpha_2{}^2 + 2\alpha_1 - \gamma}, \frac{2(\alpha_2 - \gamma)(-\alpha_1 + b - \alpha_2 - 1)}{2\alpha_1{}^2 - 2\alpha_1 b + \gamma b - 2\alpha_2{}^2 + 2\alpha_1 - \gamma}, \frac{2(\alpha_1 - b - \gamma + \alpha_2 + 1)\alpha_1 + \gamma(b - 2\alpha_2 - 1)}{2\alpha_1{}^2 - 2\alpha_1 b + \gamma b - 2\alpha_2{}^2 + 2\alpha_1 - \gamma}\right)$
   $|J| = -\frac{4(-\alpha_2 + \gamma)^2\left(\left(-\frac{b}{2} + \alpha_1 + \alpha_2 + \frac{1}{2}\right)\gamma + (-\alpha_1 + b - \alpha_2 - 1)\alpha_1\right)(-\alpha_1 + b - \alpha_2 - 1)(b - 1)}{(-2\alpha_1{}^2 + 2\alpha_1 b - \gamma b + 2\alpha_2{}^2 - 2\alpha_1 + \gamma)^2}$, $trJ = \frac{\gamma(-\alpha_1 + \gamma)(b - 1)}{(-b + 1)\gamma + 2\alpha_1 b - 2\alpha_1{}^2 + 2\alpha_2{}^2 - 2\alpha_1}$.
   - When $\left(\left(-\frac{b}{2} + \alpha_1 + \alpha_2 + \frac{1}{2}\right)\gamma + (-\alpha_1 + b - \alpha_2 - 1)\alpha_1\right)(-\alpha_1 + b - \alpha_2 - 1) > 0$, $|J| < 0$, indicating that this equilibrium point is unstable.
   - This equilibrium point is stable if $\left(-\frac{b}{2} + \alpha_1 + \alpha_2 + \frac{1}{2}\right)\gamma + (-\alpha_1 + b - \alpha_2 - 1)\alpha_1 \ge 0, b \le 1 + \alpha_1 + \alpha_2, \gamma < \min\{\alpha_1, \alpha_2\}$ and $(-b + 1)\gamma + 2\alpha_1 b - 2\alpha_1{}^2 + 2\alpha_2{}^2 - 2\alpha_1 > 0$.

References

[1] C. Darwin, The origin of species by means of natural selection, or, the preservation of favoured races in the struggle for life, H.Milford: Oxford University Press, 1859.
[2] M.A. Nowak, Evolutionary Dynamics: Exploring the Equation of Life, Harvard University Press, 2006.
[3] Z. Wang, C.T. Bauch, S. Bhattacharyya, Statistical physics of vaccination, Phys. Rep. 664 (2016) 1–113.
[4] H. Ohtsuki, C. Hauert, E. Lieberman, et al., A simple rule for the evolution of cooperation on graphs and social networks, Nature 441 (7092) (2006) 502–505.
[5] J.M. Smith, Evolution and the theory of games, Cambridge university press, 1982.
[6] E. Fehr, U. Fischbacher, The nature of human altruism, Nature 425 (6960) (2003) 785–791.
[7] Q. Luo, L. Liu, X. Chen, Evolutionary dynamics of cooperation in the n-person stag hunt game, Physica D 424 (2021) 132943.
[8] J. Hofbauer, K. Sigmund, Evolutionary games and population dynamics, Cambridge university press, 1998.
[9] M. Perc, J.J. Jordan, D.G. Rand, et al., Statistical physics of human cooperation, Phys. Rep. 687 (2017) 1–51.
[10] J. Tanimoto, Fundamentals of evolutionary game theory and its applications 2015, Springer Japan, 2015.
[11] R. Axelrod, W.D. Hamilton, The evolution of cooperation, Science 211 (1981) 1390–1396.
[12] Y. Dong, S. Sun, C. Xia, et al., Second-order reputation promotes cooperation in the spatial prisoner's dilemma game, IEEE Access 7 (2019) 82532–82540.
[13] Z.H. Deng, Z.R. Wang, H.B. Wang, et al., Impact of informers on the evolution of cooperation in prisoner's dilemma game, Chaos Solitons Fractals 149 (2021) 111069.
[14] M.A. Nowak, R.M. May, Evolutionary games and spatial chaos, Nature 359 (6398) (1992) 8269.
[15] Z.R. Wang, Z.H. Deng, H.B. Wang, et al., Moderate irrational sentiment-driven fitness can promote cooperation in the prisoner's dilemma game, Physica A 584 (2021) 126379.
[16] K. Huang, Y. Zhang, Y. Li, et al., Effects of external forcing on evolutionary games in complex networks, Chaos 28 (9) (2018) 093108.
[17] Z. Wang, M. Perc, Aspiring to the fittest and promotion of cooperation in the prisoner's dilemma game, Phys. Rev. E 82 (2010) 021115.

[18] Y.S. Chen, H.X. Yang, W.Z. Guo, Aspiration-induced dormancy promotes cooperation in the spatial prisoner's dilemma game, Phys. A 469 (2017) 625–630.
[19] W. Schnabl, P.F. Stadler, C. Forst, et al., Full characterization of a strange attractor:chaotic dynamics in low-dimensional replicator systems, Phys. D 48 (1) (1991) 65–90.
[20] F. Zhang, C. Hui, Eco-evolutionary feedback and the invasion of cooperation in prisoner's dilemma games, PLoS One 6 (11) (2011) e27523.
[21] D. Toupo, S.H. Strogatz, Nonlinear dynamics of the rock-paper-scissors game with mutations, Phys. Rev. E 91 (5) (2015) 052907.
[22] X.Q. Yang, F. Zhang, W.X. Wang, Predation promotes cooperation in prisoner's dilemma games, Physica A 514 (2019) 20–24.
[23] K.D. Jaegher, High thresholds encouraging the evolution of cooperation in threshold public-good games, Sci. Rep. 1 (2020) 5863.
[24] Z. Wang, L. Wang, Z.Y. Yin, et al., Inferring reputation promotes the evolution of cooperation in spatial social dilemma games, PLoS One 7 (7) (2012) e40218.
[25] J. Quan, C. Tang, X. Wang, Reputation-based discount effect in imitation on the evolution of cooperation in spatial public goods games, Physica A 563 (2021) 125488.
[26] W. Zhu, Q. Pan, M. He, Exposure-based reputation mechanism promotes the evolution of cooperation, Chaos Solitons Fractals 160 (2022) 112205.
[27] M.W. Macy, A. Flache, Learning dynamics in social dilemmas, Proc. Natl. Acad. Sci. 99 (2002) 7229–7236.
[28] X. Han, X. Zhao, H. Xia, Evolution of cooperation through aspiration-based adjustment of interaction range in spatial prisoner's dilemma game, Appl. Math. Comput. 393 (2021) 125746.
[29] L. Zhou, B. Wu, J.M. Du, et al., Aspiration dynamics generate robust predictions in heterogeneous populations, Nat. Commun. 12 (1) (2021) 1–9.
[30] J.M. Du, Redistribution promotes cooperation in spatial public goods games under aspiration dynamics, Appl. Math. Comput. 363 (2019) 124629.
[31] J.M. Du, B. Wu, P.M. Altrock, et al., Aspiration dynamics of multi-player games in finite populations, J. R. Soc. Interface 11 (94) (2014) 20140077.
[32] J.M. Du, Aspiration dynamics in structured population acts as if in a well-mixed one, Sci. Rep. 5 (2015) 1–7.
[33] C. Hauert, S.D. Monte, J. Hofbauer, et al., Volunteering as red queen mechanism for cooperation in public goods games, Science 296 (5570) (2002) 1129–1132.
[34] J.M. Du, L.X. Tang, Evolution of global contribution in multi-level threshold public goods games with insurance compensation, J. Stat. Mech. Theory Exp. 2018 (1) (2018) 013403.
[35] J.M. Du, Z. Wu, Evolutionary dynamics of cooperation in dynamic networked systems with active striving mechanism, Appl. Math. Comput. 430 (2022) 127295.
[36] B. Wu, F. Fu, L. Wang, Imperfect vaccine aggravates the long-standing dilemma of voluntary vaccination, PLoS One 6 (6) (2011) e20577.
[37] G. Szabo, C. Hauert, Phase transitions and volunteering in spatial public goods games, Phys. Rev. Lett. 89 (11) (2002) 118101.
[38] D. Helbing, W. Yu, The outbreak of cooperation among success-driven individuals under noisy conditions, Proc. Natl. Acad. Sci. 106 (10) (2009) 3680–3685.
[39] Y. Deng, J. Zhang, Memory-based prisoner's dilemma game with history optimal strategy learning promotes cooperation on interdependent networks, Appl. Math. Comput. 390 (2021) 125675.
[40] A. Szolnoki, M. Perc, Reward and cooperation in the spatial public goods game, EPL 92 (2010) 38003.
[41] D.G. Rand, A. Dreber, T. Ellingsen, et al., Positive interactions promote public cooperation, Science 325 (5945) (2009) 1272–1275.
[42] A. Szolnoki, G. Szabo, L. Czako, Competition of individual and institutional punishments in spatial public goods games, Phys. Rev. E 84 (4) (2011) 046106.
[43] X. Chen, A. Szolnoki, Punishment and inspection for governing the commons in a feedback-evolving game, PLoS Comput. Biol. 14 (7) (2018) e1006347.
[44] A. Szolnoki, M. Perc, Second-order free-riding on antisocial punishment restores the effectiveness of prosocial punishment, Phys. Rev. X 7 (4) (2017) 041027.
[45] A. Szolnoki, X. Chen, Alliance formation with exclusion in the spatial public goods game, Phys. Rev. E 95 (5) (2017) 052316.
[46] X.W. Wang, S. Nie, L.L. Jiang, et al., Role of delay-based reward in the spatial cooperation, Physica A 465 (2017) 153–158.
[47] M. Fu, J. Wang, L. Cheng, et al., Promotion of cooperation with loyalty-based reward in the spatial prisoner's dilemma game, Physica A 580 (2021) 125672.
[48] Y. Wu, S. Chang, Z. Zhang, et al., Impact of social reward on the evolution of the cooperation behavior in complex networks, Sci. Rep. 7 (1) (2017) 1–9.
[49] A. Szolnoki, M. Perc, Antisocial pool rewarding does not deter public cooperation, Proc. R. Soc. B Biol. Sci. 282 (1816) (2015) 20151975.
[50] S. Gao, J. Du, J. Liang, Evolution of cooperation under punishment, Phys. Rev. E 101 (6) (2020) 062419.
[51] M. Perc, A. Szolnoki, Self-organization of punishment in structured populations. new journal of physics, N. J. Phys. 14 (4) (2012) 043013.
[52] M. Nakamaru, Y. Iwasa, The coevolution of altruism and punishment:role of the selfish punisher. journal of theoretical biology, Front. Phys. 240 (3) (2006) 475–488.
[53] X. Chen, A. Szolnoki, M. Perc, Probabilistic sharing solves the problem of costly punishment, N. J. Phys. 16 (8) (2014) 083016.
[54] S. Gao, T. Wu, S. Nie, et al., Promote or hinder? the role of punishment in the emergence of cooperation, J. Theor. Biol. 386 (2015) 69–77.
[55] X. Chen, T. Sasaki, A. Brannstrom, et al., First carrot, then stick: how the adaptive hybridization of incentives promotes cooperation, J. R. Soc. Interface 12 (102) (2015) 20140935.
[56] R. Cong, K. Li, L. Wang, et al., Cooperation induced by wise incentive allocation in spontaneous institution, EPL 115 (3) (2016) 38002.