Screening In Vertical Oligopolies

Hector Chade, Jeroen Swinkels

Department of Economics, Arizona State University
Kellogg School of Management, Northwestern University

October 1, 2021

1 Introduction

2 Model

3 Main Theorem

4 Implications and Applications

5 Conclusion

Introduction

- Most markets consists of a small number of heterogeneous firms both compete for and screen their customers.
  - Saint Laurent handbag and artisans of Hermès and Coach.
  - Delta Airlines, Southwest below and a private jet firm.
- How does the oligopolistic screening changes the insights in the standard screening literature?

Model

- There is a unit measure of agents, N principals.
- Agents have type $\theta \in [0, 1]$, with cdf H and pdf h.
- The agent chooses an action $a \geq 0$.
- The value before transfers of action a to firm n is $\mathcal{V}^n(a)$.
- $\mathcal{V}^n(a)$ is strictly supermodular in (n, a).
- The value to the agent of type θ of action a is $\mathcal{V}(a, \theta)$.
- $\mathcal{V}(a, \theta) = \mathcal{U}(a) + a\theta$.
- Payoffs given action a and transfer t are $\mathcal{V}^n(a) + t$ and $\mathcal{V}(a, \theta) - t$.
- Define $V^n(a) = \mathcal{V}^n(a) + \mathcal{U}(a)$, so that $V^n(a) + a\theta$ is the match surplus between firm n and type θ with action a.

Model

- Let $v_*^n(\theta) = \max_a (V^n(a) + a\theta)$ be the most surplus firm n can offer type θ without losing money, and let $\alpha_*^n(\theta)$ be the associated maximizer.
- We assume that each firm n is relevant in that there is $\theta \in [0,1]$ such that

$$v_*^n(\theta) > \max_{n' \neq n} v_*^{n'}(\theta).$$

- There are consecutive open intervals $\left(a_e^{n-1}, a_e^n\right)$ of actions such that n is the most efficient firm at action a, where for $1 \leq n < N$, $V^n\left(a_e^n\right) = V^{n+1}\left(a_e^n\right)$, and where $a_e^0 = 0$ and $a_e^N = \infty$.
- Firms simultaneously offer menus of contracts, where firm n 's menu is a pair of functions $\alpha^n(\theta)$ the action required of an agent who chooses firm n and announces type θ, and $t^n(\theta)$ the transfer to that agent.
- Let v^n be the surplus function for an agent who takes the contract of firm n, given by

$$v^n(\theta) = \mathcal{V}\left(\alpha^n(\theta), \theta\right) - t^n(\theta) = \mathcal{U}\left(\alpha^n(\theta)\right) + \alpha^n(\theta)\theta - t^n(\theta).$$

Contract

- It is without loss that firms offer incentive-compatible menus.
- The incentive-compatibility requires the action schedule α^n is increasing and that for all θ
$$v^n(\theta) = v^n(0) + \int_0^\theta \alpha^n(\tau)d\tau.$$
- Firm n 's strategy set, S^n, is the set of incentive-compatible pairs $s^n = (\alpha^n, v^n)$.
- The joint strategy space is $S = \times_n S^n$ with typical element s.
- Let s^{-n} be a typical strategy profile for firms other than n.
- Firm n 's profit on a type- θ agent who takes action a and is given utility v_0 is
$$\pi^n(\theta, a, v_0) = V^n(a) + a\theta - v_0.$$
- For any n and for any menu (α, v) for n, we consolidate notation by writing as $\pi^n(\theta, \alpha(\theta), v(\theta))$.

Contract

- For any n and s^{-n}, define the scalar-valued function v^{-n} given by $v^{-n}(\theta) = \max_{n' \neq n} v^{n'}(\theta)$ as the most surplus offered by any of n 's competitors.
- Let a^{-n} be the associated scalar-valued action function so that (a^{-n}, v^{-n}) summarizes all competition revelent information.
- Define $\varphi^n(\theta, s)$ as the probability that n serves θ given s.
- Thus, $\varphi^n(\theta, s) = 0$ if $v^n(\theta) < v^{-n}(\theta)$ and $\varphi^n(\theta, s) = 1$ if $v^n(\theta) > v^{-n}(\theta)$.
- Define
$$\Pi^n(s) = \int \pi^n(\theta, \alpha^n, v^n)\, \varphi^n(\theta, s) h(\theta) d\theta,$$
as the profit to firm n given strategy profile s.
- The problem reduces to one among the firms, with strategy set S^n and payoff function Π^n for each n.
- Let $BR^n(s) = \arg\max_{s^n \in S^n} \Pi^n(s^n, s^{-n})$. Strategy profile s is a pure-strategy Nash equilibrium of $(S^n, \Pi^n)_{n=1}^N$ if for each $n, s^n \in BR^n(s)$.

The Main Theorem

Theorem (1)

Every pure-strategy Nash equilibrium with no extraneous offers has positive profits on each type served, no poaching, positive sorting, internal optimality, and optimal boundaries.

Positive Profits

- The positive profits condition (PP) is satisfied if for each n, the probability that n serves an agent on whom he strictly loses money is 0.
- The intuition is that by assumption for each n, there is an interval I such that $v_*^n(\theta) > v_*^{-n}(\theta)$ for all $\theta \in I$.
- If the firm-n offers surplus $v_*^n(\theta) - \varepsilon$, action $\alpha_*^n(\theta)$, the firm can at least earn ε on a positive-measure set of types.

No Poaching

- Fix an equilibrium and let $v^o(\cdot) = \max_n v^n(\cdot)$, with O mnemonic for "oligopoly," be the equilibrium surplus function.
- Let a^O be the associated action function. we take a^O to be right continuous for $\theta < 1$ and left continuous at 1.
- For any given a, let $V^{(2)}(a)$ be the second largest element of $\{V^n(a)\}_{n=1}^N$.
- The no-poaching condition (NP) holds if for all θ,

$$v^O(\theta) \geq \underbrace{V^{(2)}\left(a^O(\theta)\right) + a^O(\theta)\theta}_{\text{2nd efficient firm provides at action}} .$$

- Moving v^O across the inequality, NP says that the second most efficient firm would lose money by poaching θ at the current action.

$$0 \geq \underbrace{V^{(2)}\left(a^O(\theta)\right) + a^O(\theta)\theta - v^O(\theta)}_{\text{deviation profit}}.$$

No Poaching

- Rearranging the term, we have

$$\pi^n(\theta, \alpha^n, v^n) \leq \underbrace{V^n(\alpha^n(\theta)) - \max_{n' \neq n} V^{n'}(\alpha^n(\theta))}_{\text{Capability heterogeneity}}.$$

- The intuition is that if at least two firms can earn positive profit for some $\hat{\theta}$ with action $a^O(\hat{\theta})$ and $v^O(\hat{\theta})$.
- Then, those two firms will compete with each other and at least one of them will be better off. As a result, it is not an equilibrium.

No Poaching

- Suppose there exists $\hat{\theta}$, and two firms n' and n'' such that for $n \in \{n', n''\}$,

$$V^n\left(a^O(\hat{\theta})\right) + a^O(\hat{\theta})\hat{\theta} - v^O(\hat{\theta}) > 0.$$

- There exists $\rho > 0$, such that for all $\theta \in [\hat{\theta}, \hat{\theta} + \rho]$,

$$V^n\left(a^O(\theta)\right) + a^O(\theta)\theta - v^O(\theta) > \rho.$$

- Let $s^O = \left(a^o, v^O\right)$ and let $P^{O,n} = \left\{\theta \mid \pi^n\left(\theta, s^O\right) \geq 0\right\}$, and $\hat{s}^n(\varepsilon) = (\hat{\alpha}^n, \hat{v}^n + \varepsilon)$.
- Then, since $\hat{s}^n$ and s^O agree on $P^{O,n}$ and since v^O is the most anyone offers, $\varphi^n\left(\theta, (\hat{s}^n(\varepsilon), s^{-n})\right) = 1$ on $P^{O,n}$ for any $\varepsilon > 0$.

Perturbation

- We evaluate the profit gap of $\hat{s}^n$.
- We have

$$\Pi\left(\hat{s}^n(\varepsilon), s^{-n}\right) \geq \underbrace{-\varepsilon}_{\text{loss}} + \underbrace{\int_{P^{O,n}} \pi^n\left(\theta, s^O\right) h(\theta)d\theta}_{\text{gain}}.$$

$$\begin{aligned}\Pi\left(s^n, s^{-n}\right) &= \int \pi^n\left(\theta, s^n\right)\varphi^n(\theta, s)h(\theta)d\theta \\ &\leq \int_{PO,n} \pi^n\left(\theta, s^n\right)\varphi^n(\theta, s)h(\theta)d\theta \\ &= \int_{PO,n} \pi^n\left(\theta, s^O\right)\varphi^n(\theta, s)h(\theta)d\theta.\end{aligned}$$

Perturbation

- Combining two inequalities yields

$$\Pi\left(\hat{s}^n(\varepsilon), s^{-n}\right) - \Pi\left(s^n, s^{-n}\right) \geq -\varepsilon + \int_{PO,n} \pi^n\left(\theta, a^O, v^O\right)\left(1 - \varphi^n(\theta, s)\right) h(\theta) d\theta$$
$$\geq -\varepsilon + \rho \int_{[\hat{\theta}, \hat{\theta}+\delta]} \left(1 - \varphi^n(\theta, s)\right) h(\theta) d\theta$$

- Because ε is arbitrary, we have

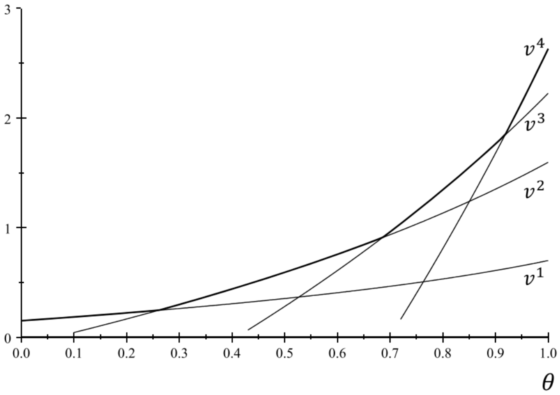
$$\Pi\left(\hat{s}^n(\varepsilon), s^{-n}\right) - \Pi\left(s^n, s^{-n}\right) \geq \rho \int_{[\hat{\theta}, \hat{\theta}+\delta]} \left(1 - \varphi^n(\theta, s)\right) h(\theta) d\theta.$$

- But at any given $\theta, \varphi^{n'}(\theta, s) + \varphi^{n''}(\theta, s) \leq 1$, and so the right-hand side (rhs) cannot be zero for both n' and n''. Hence, at least one of n' or n'' has a strictly profitable deviation.

Positive Sorting

- We say that quasi-positive sorting (QPS) holds for strategy profile s if four things are true if
  - First, for each firm n, there is a single nonempty interval (θ_l^n, θ_h^n) of agents such that firm n serves a full measure of the agents in that interval.
  - Second, these intervals are ordered, so that $\theta_h^n \leq \theta_l^{n+1}$ for all n.
  - Third, $\theta_l^1 = 0$, and $\theta_h^N = 1$.
  - Finally, if $\theta_h^n < \theta_l^{n+1}$, then for each type $\theta \in \left(\theta_h^n, \theta_l^{n+1}\right)$, both firms are offering action a_e^n and transferring all surplus, $V^n\left(a_e^n\right) + a_e^n\theta$ to the agent, so that each firm is winning half the time and profits are zero on these types.
- Say that an equilibrium has positive sorting (PS) if on (θ_l^n, θ_h^n), firm n serves each type with probability 1.
- Say that s has strictly positive sorting (SPS) if in addition $\theta_h^n = \theta_l^{n+1}$ for all $1 \leq n < N$, so that there are no intervals of ties.

Positive Sorting



Positive Sorting: Intuition

- To see the intuition, fix $\theta' > \theta$. By incentive compatibility, the equilibrium action of θ' is at least as high as that of θ. But V^n is strictly supermodular in n and a. Hence, if n sometimes serves θ' and $n' > n$ sometimes serves θ, then, by PP and NP, either n will want to always serve θ or n' will want to always serve θ', a contradiction.
- The idea of the proof: fix n and $n' > n$, let $\theta^{r'}_{\mathsf{inf}}$ be the infimum of the support of $\varphi^{n'}$, and let $\theta^{n}_{\mathsf{sup}}$ be the supremum of the support of φ^{n}.
- Assume $\theta^{n'}_{\inf} < \theta^{n}_{\sup}$. We know that by PP, we have $\pi^{n'}\left(\theta, \alpha^{n'}\ \ v^{n'}\right) \geq 0$ by PP and $\pi^{n}\left(\theta, \alpha^{n'}, v^{n'}\right) \leq 0$ by NP.

Positive Sorting: Intuition

- Hence, for any $\varepsilon \in \left(0, \left(\theta^{n}_{\text{sup}} - \theta^{n'}_{\text{inf}}\right)/2\right)$, there is $\theta_1 \in \left[\theta^{n'}_{\inf}, \theta^{n'}_{\inf} + \varepsilon\right]$, where $\varphi^{n'}(\theta_1) > 0$ and

$$\pi^{n'}\left(\theta_1, \alpha^{n'}, v^{n'}\right) \geq 0 \geq \pi^{n}\left(\theta_1, \alpha^{n'}, v^{n'}\right)$$

and, similarly, there is $\theta_2 \in \left[\theta^{n}_{\text{sup}} - \varepsilon, \theta^{n}_{\text{sup}}\right]$, where $\varphi^{n}(\theta_2) > 0$ and

$$\pi^{n}\left(\theta_2, \alpha^{n}, v^{n}\right) \geq 0 \geq \pi^{n'}\left(\theta_2, \alpha^{n}, v^{n}\right)$$

- By incentive compatibility, since $\theta_2 > \theta_1$, and since $\varphi^{n'}(\theta_1) > 0$ and $\varphi^{n}(\theta_2) > 0$, it must be that $\alpha^{n}(\theta_2) \geq \alpha^{n'}(\theta_1)$.
- Adding two inequalities, and cancelling common terms yields

$$V^{n'}\left(\alpha^{n'}(\theta_1)\right) + V^{n}\left(\alpha^{n}(\theta_2)\right) \geq V^{n}\left(\alpha^{n'}(\theta_1)\right) + V^{n'}\left(\alpha^{n}(\theta_2)\right)$$

- By the strictly supermodularity of $V^{n}(a)$, we have $\alpha^{n'}(\theta_1) = \alpha^{n}(\theta_2) \equiv \tilde{a}$.

No Extraneous Offers

- We say that an equilibrium has no extraneous offers (NEO) if each function α^n is continuous everywhere and takes on values that fall in $\left[a_e^{n-1}, a_e^n\right]$.
- We show that under NEO, any equilibrium has PS.

Internal Optimality

- Each firm will distort the action schedule so as to reduce information rents on its interior types.
- Fix n and for $\kappa \in [0,1]$, define γ^n by

$$\pi_a^n\left(\theta, \gamma^n(\theta,\kappa), v^n(\theta)\right) = \frac{\kappa - H(\theta)}{h(\theta)}.$$

- Strategy profile s satisfies internal optimality (IO) if for each n, there is $\kappa^n \in \left[H\left(\theta_l^n\right), H\left(\theta_h^n\right)\right]$, where $\kappa^1 = 0$ and $\kappa^N = 1$, such that $\alpha^n(\cdot) = \gamma^n\left(\cdot, \kappa^n\right)$ on $[\theta_l^n, \theta_h^n]$.
- By IO, there is a type $\theta_0^n\left(\kappa^n\right) \in [\theta_l^n, \theta_h^n]$ satisfying $H\left(\theta_0^n\left(\kappa^n\right)\right) = \kappa^n$.
- We can see that actions are **distorted downward** below θ_0^n, **distorted upward** above θ_0^n, and are efficient at θ_0^n.
- The intuition is that downward distortion followed by the upward distortion maintains the surplus (outside option) for the boundary types and lowers the information rents of interior types.

Internal Optimality

- To see the functional form of γ^n, fix boundary points θ_l and θ_h for firm n, and let $\mathcal{P}(\theta_l, \theta_h)$ be the following problem for firm n (per our convention, we omit the superscript n for simplicity):

$$\begin{array}{ll} \max_{(\alpha,v)} & \int_{\theta_l}^{\theta_h} \pi(\theta,\alpha,v)h(\theta)d\theta \\ \text{s.t.} & v(\theta_l) \geq v^{-n}(\theta_l) \\ & v(\theta_h) \geq v^{-n}(\theta_h) \\ & v(\theta) = v(0) + \int_0^{\theta} \alpha(\tau)d\tau \quad \text{for all } \theta \end{array}$$

- We relax the problem and drop the monotonicity of α, ignore the outside option except at θ_h and θ_l.

Internal Optimality

- Let

$$\iota\left(\theta_l, \theta_h, \kappa\right) = \underbrace{v^{-n}\left(\theta_h\right) - v^{-n}\left(\theta_l\right)}_{\text{increment of outside option}} - \underbrace{\int_{\theta_l}^{\theta_h} \gamma(\theta, \kappa) d\theta}_{\text{rent increment}}.$$

- Let

$$\tilde{\kappa}\left(\theta_l, \theta_h\right) = \arg \min_{\kappa \in [H(\theta_l), H(\theta_h)]} \left|\iota\left(\theta_l, \theta_h, \kappa\right)\right|.$$

Internal Optimality

Lemma (1)

Relaxed Problem: Problem $\mathcal{P}(\theta_l, \theta_h)$ has a solution $\tilde{s}(\theta_l, \theta_h) = (\tilde{\alpha}, \tilde{v}).On$ $(\theta_l, \theta_h), \tilde{\alpha}$ is uniquely defined and equal to $\gamma(\cdot, \tilde{\kappa}(\theta_l, \theta_h))$. If $\tilde{\kappa}(\theta_l, \theta_h) > H(\theta_l)$, then $\tilde{v}(\theta_l) = v^{-n}(\theta_l)$, and if $\tilde{\kappa}(\theta_l, \theta_h) < H(\theta_h)$, then $\tilde{v}(\theta_h) = v^{-n}(\theta_h)$.

- Look at the shadow value of increasing the surplus of type θ_h, holding fixed surplus of type θ_l.
- Increasing the action at any given interior type θ.
- The marginal impact is

$$\eta = \underbrace{-\pi_a(\theta, \tilde{\alpha}, \tilde{v})h(\theta)}_{\text{type around } \theta} + \underbrace{H(\theta_h) - H(\theta)}_{\text{type between } \theta \text{ and } \theta_h} .$$

- At type θ_0, $\pi_a = 0$, and $H(\theta_0) = \kappa$, we have $\eta = H(\theta_h) - \kappa$.

Optimal Boundaries

- Strategy profile s satisfies the optimal boundary condition (OB) if for $\theta = \theta_l^n$ and $\theta = \theta_h^n$,

$$\pi^n\left(\theta, \alpha^n, v^n\right) + \pi_a^n\left(\theta, \alpha^n, v^n\right)\left(a^{-n}(\theta) - \alpha^n(\theta)\right) = 0.$$

- Increasing action of type near θ_h a little, it incurs direct benefit $\pi_a\left(\theta_h, \alpha, v\right) h\left(\theta_h\right)$, but raises $v\left(\theta_h\right)$.
- As $v\left(\theta_h\right)$ is raised, θ_h increases at rate $1/\left(a^{-n}\left(\theta_h\right) - \alpha\left(\theta_h\right)\right)$.
- Hence, the profits from the new types served are

$$\pi\left(\theta_h, \alpha, v\right) h\left(\theta_h\right) / \left(a^{-n}\left(\theta_h\right) - \alpha\left(\theta_h\right)\right).$$

Optimal Boundaries

Lemma (2)

Profit Single-Peaked: For any $(\alpha, v) \in S^n$,

$$\frac{d}{d\theta}\pi(\theta, \alpha, v) = \pi_a(\theta, \alpha, v)\alpha_\theta(\theta)$$

If $\alpha = \gamma(\cdot, H(\theta_0))$, *then* $\pi(\cdot, \alpha, v)$ *is strictly single-peaked with peak at* θ_0

- Customers in the middle of the participation range find neither of the alternative firms very attractive, and so are the easiest from whom to extract rents.

Sufficiency and Existence

Definition (1)

Stacking is satisfied if for all $n < N, \gamma^{n+1}(\cdot, 1) > \gamma^{n}(\cdot, 0)$.

- Stacking holds if firms are sufficiently differentiated.

Theorem (2)

Sufficiency and Existence: Assume stacking. Then any strategy profile satisfying PS, IO, and OB is equivalent to a Nash equilibrium, and a Nash equilibrium exists.

Who Does Incomplete Information Help or Hurt?

- A monopolist is better off, since, compared to incomplete information, it can undo any inefficiency and then extract all the surplus, leaving all types worse off.
- In oligopoly, there is another effect: competition under complete information increases the agents' outside option relative to incomplete information.
- The rationale is that with incomplete information, the firm may not want to attract some new types of agents, because it may reduce the profit from the existing agents. This effect is missing under the complete information.

Who Does Incomplete Information Help or Hurt?

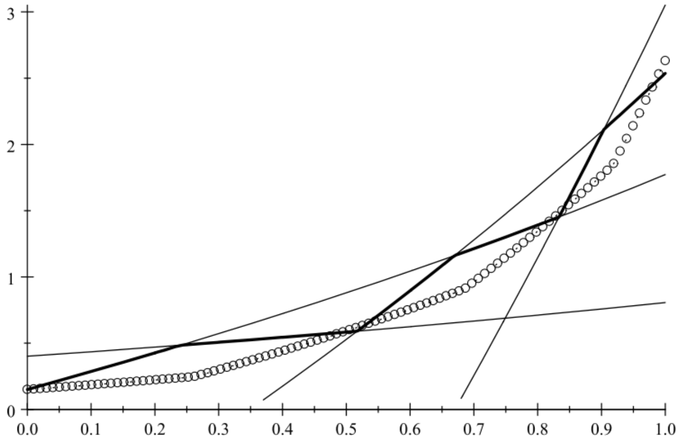
- For $1 \leq n < N$, let θ_*^n be the boundary point between the intervals of types where n and $n+1$ are the most efficient firms to serve θ; that is, $v_*^n(\theta_*^n) = v_*^{n+1}(\theta_*^n)$.
- Let θ^n be the boundary point between n and $n+1$ in the equilibrium under incomplete information.

Theorem (3)

Welfare: Under stacking, the following statements hold:

1. For each $1 \leq n \leq N-1$, an interval of types containing θ_*^n and θ^n is strictly better off under complete information.
2. If any type served by firm n under incomplete information is strictly better off than under complete information, then there is a single subinterval of $\left[\theta_*^{n-1}, \theta_*^n\right]$ of such types.
3. All types may be strictly better off under complete information.

Who Does Incomplete Information Help or Hurt?



1. In this example, incomplete information is preferred by types on a right neighborhood of $\theta = 0$, on an interval around the point where $v_*^1 = v_*^3$ and similarly where $v_*^2 = v_*^4$, and on a left neighborhood of $\theta = 1$.

The Competitive Limit

- Consider a setting where firms can enter at a fixed cost $F > 0$ and then choose freely from a set of potential technologies parameterized by $z \in [0, \bar{z}]$.
- Let $V(a, z) + a\theta$ be the match surplus from action a, technology z, and type θ.
- In a pure-strategy equilibrium with endogenous entry, the N extant firms each earn at least F, but no new entrant can do so. Note that the most type θ could possibly hope for is

$$v_*(\theta) = \max_{a,z}(V(a, z) + \theta a).$$

Theorem (4)

Limit Efficiency: There is $\rho \in (0, \infty)$ such that in any pure-strategy equilibrium with endogenous entry and NEO, $1/\left(\rho F^{1/3}\right) \leq N \leq \left(\rho/F^{1/3}\right) + 2$. The profit per type, π, and the difference between what each type θ earns and $v_*(\theta)$ are each of order $1/N^2$.

A Multi-Technology Monopoly

- Let a single firm M control technologies $n_l, \ldots, n_h$.
- For any given type, firm M can choose which technology to use, but cannot combine aspects of the different technologies into a hybrid technology.
- Assume first that M is a monopolist facing a convex outside option $\bar{u}$.
- Define $\bar{V}$ be the concave envelope of $\max\{V^{n_l}, \ldots, V^{n_h}\}$.
- They will show that M acts as if it had technology $\bar{V}$, and so we can apply all of what we already know about a single firm.

A Multi-Technology Monopoly

- Since $\bar{V}$ will have a linear segment as it moves from each interval $[\underline{a}^n, \bar{a}^n]$ to the next,

$$\gamma^M(\theta, \kappa) = \max \left\{ a \mid \underbrace{\bar{V}_a(a) + \theta}_{\pi_a} = \frac{\kappa - H(\theta)}{h(\theta)} \right\}.$$

- The optimal boundaries can be shown satisfies

$$\pi^n\left(\theta^{M,n}\right) - \pi^{n+1}\left(\theta^{M,n}\right) + \frac{\kappa - H\left(\theta^{M,n}\right)}{h\left(\theta^{M,n}\right)}\left(\underline{a}^{n+1} - \bar{a}^n\right) = 0$$

A Multi-Technology Monopoly

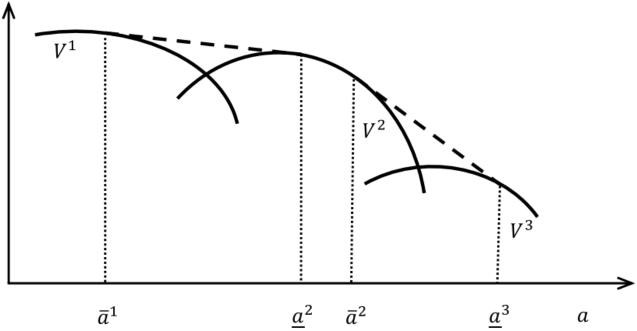


FIGURE 3.—A multi-technology firm. The firm controls three technologies, V^1, V^2, and V^3. Below $\bar{a}^1$ $\bar{V} = V^1$, on $(\underline{a}^2, \bar{a}^2)$, we have $\bar{V} = V^2$, and above $\underline{a}^3$, we have $\bar{V} = V^3$. The dotted lines complete the concave envelope of the technologies.

Oligopoly versus Monopoly With a Fixed Market Size

- We compare the outcome of the monopolist firm M with a setting where firms $n_l, \ldots, n_h$, with the associated technologies, competeoligopolistically given a convex outside option $\bar{u}$.

Theorem (5)

Fixed Span: Let $[\theta_l, \theta_h]$ be the set of types served in oligopoly by firms $n_l, \ldots, n_h$ facing outside option $\bar{u}$. If forced to serve exactly $[\theta_l, \theta_h]$, then firm M will choose κ in $(\kappa^{n_l}, \kappa^{n_h})$. All types in (θ_l, θ_h) are strictly worse off, with an interval of low types receiving a strictly lower action than before and an interval of high types receiving a strictly higher action than before.

- M offers less surplus to every interior type. Thus, to protect consumers or workers after a merger, it is not enough to require the merged firm to serve the same set of types, since it will reoptimize its rent extraction so as to hurt them all.

Oligopoly versus Monopoly With Endogenous Market Size

- The firm M may not serve all of $[\theta_l, \theta_h]$.
- In oligopoly, then each firm, knowing that it would lose types at each end, was indifferent about decreasing the surplus by a small constant.
- But then the merged firm—which no longer suffers the loss of types at interior boundaries—strictly prefers to do so.

Theorem (6)

Endogenous Span: Let M optimally serve $\left[\theta_l^M, \theta_h^M\right]$. If $0 < \theta_l$, then $\theta_l < \theta_l^M$, and if $\theta_h < 1$, then $\theta_h^M < \theta_h$. All types in (θ_l, θ_h) are strictly worse off compared to when M is forced to serve $[\theta_l, \theta_h]$, and so, a fortiori, are strictly worse off compared to oligopoly.

- Thus, the merged firm is both harder on the types it keeps and, except perhaps at the endpoints 0 and 1, strictly shrinks the set of types served.

Conclusion

- We extend the ubiquitous principal–agent problem to a vertical oligopoly.
- We derive the equilibrium sorting, distortions, and gaps in quality/effort across firms. Under enough firm heterogeneity, a simple set of conditions is sufficient for a strategy profile to be an equilibrium, and an equilibrium exists.
- Contrary to monopoly, complete information can help the agents. We examine the model's competitive limit and the effect of mergers.

Thanks!