Search, Information, and Prices

Dirk Bergemann, Benjamin Brooks, Stephen Morris

Yale University, University of Chicago, Massachusetts Institute of Technology

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Introduction

Introduction

1. Economic theory predicts "**law of one price**".
2. Price dispersion is ubiquitous.
3. Imperfect competition: price count.

Model

1. Customer valuation $v > 0$ (deterministic).
2. n firms, indexed by $i \in N \triangleq \{1, \ldots, n\}$ (zero cost).
3. Customer receives price quotes from a subset K of those firms.
4. Price count $k = |K|$.
5. Let $\mu \in \Delta(N)$ denotes the ex-ante distribution of the price count.
6. Probability that the firms K are quoted given k is denoted $\nu(K \mid k)$.

Information Structure

1. An information structure (T, π): $t_K = (t_i)_{i \in K}$ and $\pi\,(dt_K \mid K)$.
2. Given an information structure, firms choose prices conditional on their signals:
$$F_i\,(x \mid t_i) \triangleq \Pr\,(p_i \geq x \mid t_i)\,.$$
3. The consumer will buy from one of the firms offering the lowest price:
$$K(p) \triangleq \underset{i \in N}{\arg\min} p_i.$$
4. Given the strategy profile $F = (F_1, \ldots, F_n)$, the expected revenue of firm i is
$$R_i(F) = \sum_{k=1}^{n} \mu(k) \sum_{K \subseteq N} \nu(K \mid k) \left(\int_{t \in T_K} \int_{p \in [0,]^{k^k}} p_i \frac{\mathbb{I}_{i \in K(p)}}{|K(p)|} F_K(dp \mid t) \pi(dt \mid K) \right)$$
5. The strategy profile F is a (Bayes-Nash) equilibrium if and only if $R_i(F) \geq R_i\,(F_i', F_{-i})$ for each i and strategy F_i'.

A Two-Firm Example

Full Information

1. $n = 2$ and $v = 1$.
2. Let μ denotes the probability that price count equals 1, so it equals 2 w.p. $1 - \mu$.
3. Suppose full information about the price count.
4. For monopolist market, the quoted firm sets price equals 1.
5. For competitive market, both firms charge the competitive price of 0.
6. The ex ante (upper cumulative) distribution of the sales price $S(x)$.
7. The price is at least 0 w.p. 1 and at least x for any $0 < x \leq 1$ w.p. $1/2$.

Full Information

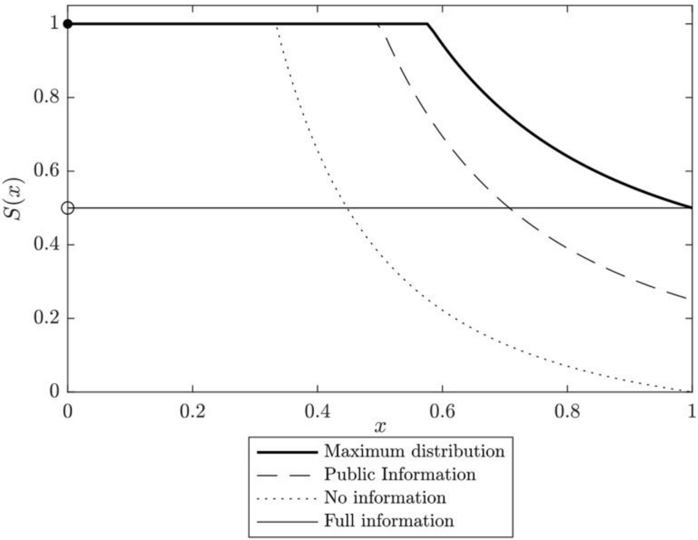

FIG. 3.—Sales price distributions with two firms, $v = 1$, $\mu = 1/2$.

No Information

1. Suppose firms have no information about price count.
2. Firm i be a monopolist w.p. $\frac{\frac{\mu}{2}}{\frac{\mu}{2}+(1-\mu)} = \frac{\mu}{2-\mu}$ and competitive w.p. $\frac{2(1-\mu)}{2-\mu}$.
3. This model has a unique mixed-strategy equilibrium over $[\mu/(2-\mu), 1]$.
4. The equilibrium profit is independent of the price p_i, yield

$$\underbrace{\frac{\mu}{2-\mu} p_i}_{\text{price } p_i \text{ under k=1}} + \underbrace{\frac{2(1-\mu)}{2-\mu} F_j\left(p_i\right) p_i}_{\text{price } p_i \text{ under k=2}} = \underbrace{\frac{\mu}{2-\mu}}_{\text{price 1}}.$$

5. The equilibrium price strategy

$$F_i\left(p_i\right) = \frac{\mu\left(1-p_i\right)}{2(1-\mu) p_i}.$$

6. The resulting ex ante sales price distribution is

$$\begin{aligned} S(x) &= \frac{\mu}{2}\left(F_1(x)+F_2(x)\right)+(1-\mu) F_1(x) F_2(x) \\ &= \mu\left[\frac{\mu(1-x)}{2(1-\mu) x}\right]+(1-\mu)\left[\frac{\mu(1-x)}{2(1-\mu) x}\right]^2. \end{aligned}$$

No Information

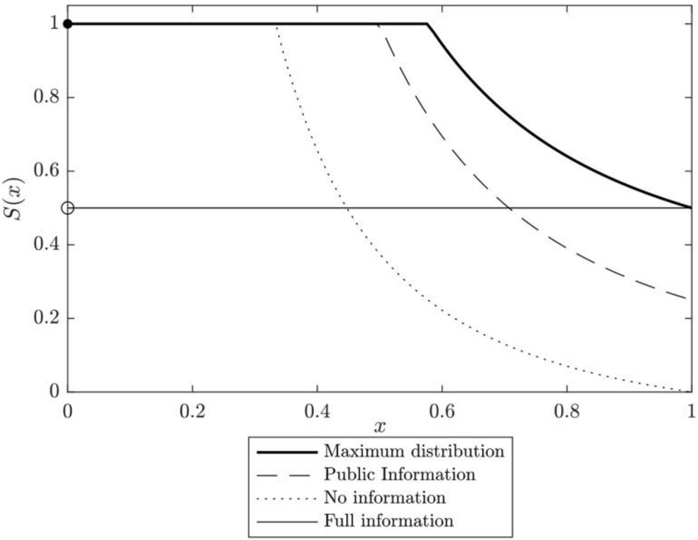

FIG. 3.—Sales price distributions with two firms, $v = 1$, $\mu = 1/2$.

Maximum Prices with Public Information

1. Consider the public information: we mean that **quoted** firms observe the same signal with probability 1.
2. Consider firms observe the same signal $t_i = t_j \in \{i, j\}$. If firm i is a monopolist, the signal is i; if the market is competitive, the signal is equally likely to be i or j.
3. If the public signal is i, both firms know that firm i is a monopolist with probability μ and that the market is competitive with probability $1 - \mu$.
4. In this case, the firms mix over the interval $[\mu, 1]$. Firm i 's strategy is
$$F_i(p_i) = \frac{\mu}{p_i},$$
which has a mass point of size μ on the monopoly price $p = 1$, while firm j follows strategy without a mass point
$$F_j(p_j) = \frac{\mu(1 - p_j)}{(1 - \mu)p_j}.$$
5. The resulting ex ante sales price distribution is
$$S(x) = \mu(F_i(x)) + (1 - \mu)F_i(x)F_j(x) = \left(\frac{\mu}{x}\right)^2,$$
for $x \in [\mu, 1]$, and $S(x) = 1$ for $x < \mu$. The expected sales price is $\mu(2 - \mu)$.

Maximum Prices with Private Information

1. The expected price can be driven even higher with private signals.
2. The common support is a necessary feature of models with public information.
3. With private signals, however, we can similarly have a firm that thinks that it may be a monopolist always competing with a firm that knows that it is not, but with distinct supports.
4. The information structure is as follows. Each quoted firm i receives a signal $t_i \in \{1, 2\}$. This signal can be interpreted as conveying to firm i a lower bound on the total number of quoted firms-including firm i.
5. If only a single firm i is quoted, that is $K = \{i\}$, then only firm i receives a signal and $t_i = 1$ with probability one.
6. If both firms are quoted, that is, $K = \{1, 2\}$, then with probability $1 - 2\alpha$ both firms receive the signal $t_i = 2$, with probability α the signals $t_1 = 1$ and $t_2 = 2$, and with probability α the signals $t_1 = 2$ and $t_2 = 1$.

Maximum Prices with Private Information

1. The parameter $\alpha \in [0, 1/2)$ therefore controls the dispersion in the beliefs of the market participants.
2. If α is close to 0 , then the information structure is close to full information, and with high probability both of the firms learn that they are in a competitive environment.
3. If α is close to $1/2$, then the information structure is close to that we constructed with public signals, and with high probability exactly one firm learns that the environment is competitive.

Maximum Prices with Private Information

1. We now describe an equilibrium where the firm that has received signal $t_i = 1$ charges the monopoly price, $p_i = 1$, and the firm that receives the signal $t_i = 2$ mixes according to the upper cumulative distribution

$$F_i\left(p_i \mid t_i = 2\right) \triangleq F_i\left(p_i\right) = \frac{\alpha}{1-2\alpha}\frac{1-p_i}{p_i},$$

   with support $p_i \in [\alpha/(1-\alpha), 1]$.

2. We refer to the firm that receives the signal $t_i = 2$ as "**informed**," as such a firm knows the price count. Conversely, a firm that receives the signal $t_i = 1$ is "**uninformed**," as the firm is uncertain whether it is in a monopoly or a competitive environment.

Maximum Prices with Private Information

1. We claim that these strategies are an equilibrium if α is sufficiently small.
2. To see this, observe that the informed firm's profit from charging price p_i is generated by two events: with probability α the other firm observed signal $t_j = 1$, and with probability $1 - 2\alpha$ the other firm observed $t_j = 2$.
3. Interim expected profit from setting a price p_i in the support of F_j is therefore
$$\left(\frac{\alpha}{1-\alpha} + \frac{1-2\alpha}{1-\alpha} F_j\left(p_i\right)\right) p_i = \frac{\alpha}{1-\alpha},$$
so that firm i with signal $t_i = 2$ is indeed willing to randomize.
4. For the uninformed firm either is a monopolist w.p. $\mu/2$ or is in a competitive environment w.p. $(1-\mu)\alpha$.
5. We need to ensure that the uninformed firm receives a higher revenue from posting the monopoly price 1 rather than choosing a price $p_i \in [\alpha/(1-\alpha), 1)$, which reduces to the following inequality:
$$\frac{\mu/2}{\mu/2 + (1-\mu)\alpha} \geq \frac{\mu/2 + (1-\mu)\alpha F_j\left(p_i\right)}{\mu/2 + (1-\mu)\alpha} p_i .$$

Maximum Prices with Private Information

1. We can cancel terms and rearrange to obtain

$$\alpha \leq \alpha^* \triangleq \frac{1}{2}\frac{\sqrt{\mu(2-\mu)}-\mu}{1-\mu}.$$

2. It is straightforward to calculate the expected sales price and sales price distribution in this equilibrium. When a single firm is quoted, that firm receives signal 1, and the resulting sales price is 1. Thus, there is an atom of size μ on a sales price of 1. If two firms are quoted, then either one or two firms receive the signal 2, and they randomize according to F_i, given above. The sales price distribution is therefore

$$S(x) = \mu + (1-\mu)\frac{\alpha^2}{1-2\alpha}\left[2\frac{1-x}{x} + \left(\frac{1-x}{x}\right)^2\right]$$

for $x \in [\alpha/(1-\alpha), 1]$, and $S(x) = 1$ for $x < \alpha/(1-\alpha)$.

Maximum Prices with Private Information

1. Note that the sales price distribution is increasing in α for every x, that is, in the first-order stochastic dominance order. Thus, sales prices are increasing in the noise in firms' signals about whether the consumer is contested, and the sales price distribution is maximized at $\alpha = \alpha^*$.

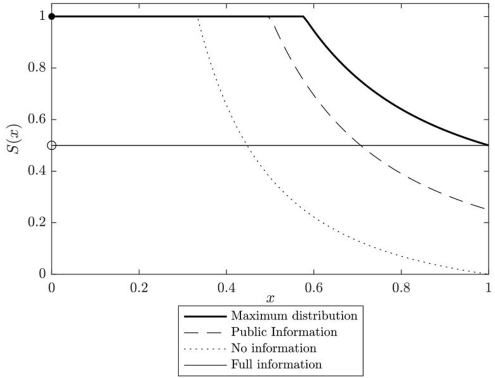

FIG. 3.—Sales price distributions with two firms, $v = 1$, $\mu = 1/2$.

Bounds on Equilibrium Sales Prices

Maximal Sales Price Distribution

1. Let us define a decreasing sequence of cutoff prices:

$$v = x_0 = x_1 > x_2 > \cdots > x_n$$

where for $k > 1$,

$$x_k \triangleq v \left(\prod_{m=1}^{k} \left(\frac{Q_{m-1}}{Q_m} \right)^{(m-1)/m} \right),$$

where

$$Q_m \triangleq \sum_{l=1}^{m} l\mu(l),$$

for $m > 0$ and $Q_0 = 1$. We also define, for each $k \geq 1$, an upper cumulative distribution $\bar{S}(\cdot \mid k)$ whose support is $[x_k, x_{k-1}]$. In particular, $\bar{S}(\cdot \mid 1)$ puts probability one on v, and for $k > 1$,

$$\bar{S}(x \mid k) \triangleq \frac{(x_k/x)^{k/(k-1)} - (x_k/x_{k-1})^{k/(k-1)}}{1 - (x_k/x_{k-1})^{k/(k-1)}}$$

for $x \in [x_k, x_{k-1}]$, $\bar{S}(x \mid k) = 1$ for $x < x_k$, and $\bar{S}(x) = 0$ for $x > x_{k-1}$.

Maximal Sales Price Distribution

1. We then define $\bar{S}(x)$ according to

$$\bar{S}(x) \triangleq \sum_{l=1}^{n} \mu(l)\bar{S}(x \mid l)$$

   or equivalently,

$$\bar{S}(x) = \mu(k)\bar{S}(x \mid k) + \sum_{m=1}^{k-1} \mu(m)$$

   when $x \in [x_k, x_{k-1}]$.

2. Finally, given sales price distributions $S(\cdot)$ and $S'(\cdot)$, we say that S **first-order stochastically dominates** S' if $S(x) \geq S'(x)$ for all x.

Maximal Sales Price Distribution

Theorem (First-order stochastic dominance)

Fix a price count distribution μ. In any information structure $\{T, \pi\}$ and equilibrium F consistent with μ, the distribution of sales prices must be first-order stochastically dominated by $\bar{S}$, given by above equations. Moreover, there exists an information structure and equilibrium consistent with μ for which $\bar{S}$ is the equilibrium sales price distribution.

Corollary (Maximum producer surplus and minimum consumer surplus)

Maximum producer surplus across all information structures and equilibria consistent with the price count distribution μ is $\bar{R} = \int_{x=0}^{v} x\bar{S}(dx)$. Minimum consumer surplus across all information structures and equilibria consistent with the price count distribution μ is $v - \bar{R}$.

Proof of Theorem 1

1. The proof of theorem 1 is divided into propositions $1-3$, the formal proofs of which are in the appendix.
2. Proposition 1 presents an integral inequality that any equilibrium sales price distribution must satisfy.
3. Proposition 2 shows that any equilibrium sales price distribution that satisfies this inequality must be first-order stochastically dominated by $\bar{S}$.
4. Finally, proposition 3 constructs an information structure and equilibrium for which the equilibrium sales price distribution is precisely $\bar{S}$.

Proposition 1

Proposition (Upper bound on sales price distribution)

In any equilibrium, the sales price distributions must satisfy, for all $x \in [0, v]$,

$$x \sum_{k=1}^{n} \mu(k) k S(x \mid k) \leq \int_{y=x}^{v} y S(dy).$$

Proof of Proposition 1

1. Consider a particular deviation: uniform price cut to x.
2. Consider the case where there is zero ex ante probability that x is the sales price; that is, $S(\cdot)$ does not have an atom at x.
3. Suppose that we first select a firm at random, and then the selected firm deviates by setting a price of x whenever they would have set a price greater than x in equilibrium.
4. The deviator's surplus:

$$\sum_{k=1}^{n} \mu(k) \sum_{K \subseteq N} \nu(K \mid k) \int_{t \in T_K} \int_{p \in [0,v]^K} \Bigg(\underbrace{x \mathbb{I}_{\min p \geq x} \mathbb{I}_{i \in K}}_{\text{sales price is x}} + \underbrace{\min p \mathbb{I}_{\min p < x} \frac{\mathbb{I}_{i \in K(p)}}{|K(p)|}}_{\text{sales price unchanged}} \Bigg) F_K(dp \mid t) \pi(dt \mid K).$$

5. The deviator's surplus

$$\text{deviator's surplus} \leq \sum_{k=1}^{n} \mu(k) \int_{x=0}^{v} x S_i(dx \mid k).$$

Proof of Proposition 1

1. Summing the deviation surplus across i, we obtain

$$\sum_{k=1}^{n}\mu(k)\sum_{K\subseteq N}\nu(K\mid k)\int_{t\in T_K}\int_{p\in[0,v]^{\kappa}}\left(kx\mathbb{I}_{\min p\geq x}+\mathbb{I}_{\min p<x}\right)F_K(dp\mid t)\pi(dt\mid K)$$
$$=\sum_{k=1}^{n}\mu(k)\left(kx\int_{t\in T_K}\int_{p\in[x,v]^{\kappa}}\sum_{K\subseteq N}\nu(K\mid k)F_K(dp\mid t)\pi(dt\mid K)\right.$$
$$\left.+\int_{t\in T_K}\int_{p\in[0,v]^{\kappa}}\min p\mathbb{I}_{\min p<x}\sum_{K\subseteq N}\nu(K\mid k)F_K(dp\mid t)\pi(dt\mid K)\right)$$
$$=\sum_{k=1}^{n}\mu(k)\left(kxS(x\mid k)+\int_{y=0}^{x}y\int_{t\in T_K}\int_{\{p\mid\min p=y\}}\sum_{K\subseteq N}\nu(K\mid k)F_K(dp\mid t)\pi(dt\mid K)\right)$$
$$=\sum_{k=1}^{n}\mu(k)\left(kxS(x\mid k)+\int_{y=0}^{x}yS(dy\mid k)\right).$$

2. This total surplus must less than the sum of the firm's equilibrium revenues, $\int_{y=x}^{v}yS(dy)$.

Proposition 2

Proposition (First-order stochastic dominance)

If the ex ante sales price distribution S deters uniform price cuts, then $\bar{S}$ first-order stochastically dominates S.

- The three main steps in proving this proposition are as follows.
- First, we argue that when maximizing the ex ante sales price distribution, it is without loss to consider distributions that have ordered supports, meaning that the supports of $S(\cdot \mid k)$ are intervals of the form $[y_k, y_{k-1}]$, where $\{y_k\}_{k=1}^n$ is an increasing sequence.
- Second, we argue that it is without loss to consider distributions for which equality in Proposition 1 holds as an equality. If not, it is possible to push up the sales price distribution everywhere, while still satisfying inequality.
- Third, we show that the ordered-supports property, together with inequality as an equality, reduce to a first-order differential equation whose unique solution is the distribution $\bar{S}$.

Step 1

1. When maximizing $S(x)$, it is without loss to restrict attention to $\{S(\cdot \mid k)\}_{k=1}^{n}$ that satisfy the following ordered-supports property:

$$S(y \mid k) < 1 \Rightarrow S\left(y \mid k'\right) = 0 \quad \forall k' > k.$$

2. Indeed, given any $\{S(\cdot \mid k)\}$ and associated ex ante distribution $S(\cdot)$, we can define a new $\{\tilde{S}(\cdot \mid k)\}$ with the same ex ante distribution, but where there is negative assortative matching between k and x.
3. For conditional distributions satisfying ordered supports, the inequality reduces to

$$S(y \mid k) \leq \max\left\{0, \frac{1}{\mu(k)(k-1)}\left(\frac{1}{y}\int_{z=y}^{v} S(z)dz - \sum_{m=1}^{k-1}\mu(m)(m-1)\right)\right\}.$$

Step 2

1. Among distributions with the ordered-supports property, the inequality should hold as equality whenever $S(y \mid k) < 1$.

$$S(y \mid k) = \max\left\{0, \frac{1}{\mu(k)(k-1)}\left(\frac{1}{y}\int_{z=y}^{v} S(z)dz - \sum_{m=1}^{k-1}\mu(m)(m-1)\right)\right\}. \tag{A1}$$

2. Otherwise, we can define $\tilde{S}(y \mid k)$ to be

$$\tilde{S}(y \mid k) = \max\left\{0, \min\left\{1, \frac{1}{\mu(k)(k-1)}\left(\frac{1}{y}\int_{z=y}^{v} S(z)dz + \sum_{m=1}^{k-1}\mu(m)(m-1)\right)\right\}\right.$$

3. Because $\tilde{S}(y \mid k) \geq S(y \mid k)$, ex ante price $\tilde{S}$ must also be higher.

Step 3

1. We show that the ordered-supports property and condition (A1) holding as an equality uniquely define the distributions $\{\bar{S}(\cdot \mid k)\}$.
2. It is immediate that $S(y \mid k)$ will have a support that is an interval $[y_k, y_{k-1}]$, with $y_0 = y_1 = v$, and it is strictly increasing on its support.
3. Now, suppose inductively that we have defined $S(y \mid m)$ and y_m for $m < k$. Then $S(y)$ must satisfy the boundary conditions $S(y_m) = \sum_{l=1}^{m} \mu(l)$ for all $m < k$. On $[y_k, y_{k-1}]$, (A1) holds as an equality, and moreover

$$S(y) = \mu(k)S(y \mid k) + \sum_{m=1}^{k-1} \mu(m).$$

4. As a result, (A1) with equality rearranges to

$$y(k-1)S(y) - \int_{z=y}^{v} S(z)dz = y \sum_{m=1}^{k-1} \mu(m)(k-m).$$

5. This reduces to a differential equation together with boundary condition yields $\bar{S}$.

Comparative Static in Price Count Distribution μ

Proposition (Price count and equilibrium sales price distribution)

Let μ and μ' be price count distributions, with corresponding maximal sales price distributions $\bar{S}$ and $\bar{S}'$. If μ' first-order stochastically dominates μ, then $\bar{S}$ first-order stochastically dominates $\bar{S}'$.

Corollary (Competitive limit)

Among all price count distributions with probability $\mu(1)$ of a price count of 1 , a tight upper bound on the expected sales price is $v[\mu(1)(2-\mu(1))]^{1/2}$. Marginal maximal revenue with respect to the probability of being a monopolist is $(1-v)/\ [\mu(1)(2-\mu(1))]^{1/2}$, which is unbounded at $\mu(1)=0$, where the market is fully competitive.

Minimum Expected Price and No Information

Proposition (Minimum Expected Price)

The minimum expected sales price across all information structures and equilibria is $v\mu(1)$. The maximum consumer surplus across all information structures and equilibria is $v - \mu(1)v$. Minimum revenue and maximum consumer surplus are attained under full information and no information.

Endogenizing the Price Count

Feedback versus No Feedback

1. We have thus far studied equilibrium sales price distributions holding the price count distribution fixed.
2. An important question is whether our bounds still apply when we endogenize price counts, for example, with a dynamic model of consumer search.
3. Considering various ways of endogenizing price counts that have been proposed in the literature: with feedback versus no feedback.
4. In particular, there is no feedback from realized prices to price counts and to other firms' prices.
5. When there is no feedback, our analysis in the previous section immediately applies to whatever price count distribution is realized in equilibrium.
6. There are other models, however, that exhibit feedback, meaning that a firm's realized price can directly affect price counts and/or other firms' prices.

Feedback versus No Feedback

1. The bound violates for some models: for example, a two-firm, full-information Stackelberg game.
2. The bound violates for model of sequential consumer search: in which a consumer iteratively solicits price quotes and, after observing the quoted price, decides whether to purchase or continue searching.
3. The reason is that price cuts tend to make consumers stop searching sooner, so that a deviating firm faces less competition relative to the benchmark with fixed price counts.
4. Hence, constraint will still hold in equilibrium, and the rest of our bounding argument goes through.

A Model of Sequential Search

1. The consumer has value v for a single unit and a type $\theta \in \Theta$ with distribution $\eta \in \Delta\Theta$.
2. The consumer chooses a number $k \geq 1$ of firms to search. If a consumer searches k firms, then they pay a cost $c(k, \theta)$.
3. If the consumer purchases at price p after visiting k firms, their payoff is $v - p - c(k, \theta)$. The payoff to the firm that makes the sale is p, and other firms' payoffs are zero.
4. Each firm has a set of signals T_i. Conditional on θ and ξ, there is a joint distribution over signals denoted by $\pi(t \mid \theta, \xi)$.
5. The strategy of firm i is a pricing kernel:

$$F_i : T_i \to \Delta([0, v]).$$

6. To summarize, the parameters of the sequential search model are $\{\Theta, \eta, c, T, \pi\}$.

Extending the Upper Bound to Sequential Search

Theorem (Sequential search and upper bound)

Fix a price count distribution $\mu \in \Delta(\{1, .., n\})$. For any sequential search model $\{\Theta, \eta, c, T, \pi\}$ and equilibrium (F, σ) such that the equilibrium price count distribution is μ, the induced sales price distribution is first-order stochastically dominated by $\bar{S}$. Moreover, there exists a sequential search model and equilibrium that induce μ, and the equilibrium sales price distribution is $\bar{S}$.

1. We simply set $\theta = \{1, \dots, N\}$, $\eta(\theta) = \mu(k)$, and

$$c(k, \theta) = \begin{cases} 0 & \text{if } k = \theta, \\ v + k & \text{otherwise.} \end{cases}$$

2. With this model, it is a strictly dominant strategy for the consumer of type θ to search θ firms. As a result, the equilibrium price count distribution must be μ, regardless of firms' strategies. In addition, the information is again given by $\bar{T}_i = \{1, \dots, n\}$ and

$$\pi(t \mid \theta, \xi) = \bar{\pi}(t || \{i \mid \xi(i) \leq \theta\} |).$$

Further Topics

1. Beyond a Symmetric Quote Distribution: We argue here that our construction continues to yield an upper bound on the sales price distribution even if quotation probabilities differ across firms, although the bound may no longer be tight.
2. Connection to First-Price Auctions: The pricing game that we analyze here is strategically related to a first-price auction where each bidder has either a low or a high value for a good and each bidder knows their private value but is uncertain about the values of the other bidders.
3. In the equilibrium of the first-price auction, low-value bidders will always bid the low value, and high-value bidders follow mixed strategies that depend on their beliefs about the number of other bidders with high values.
4. The pricing game studied in this paper can be viewed as a procurement auction, where bidders quote prices at which they are willing to sell and the auctioneer buys at the lowest price. Quoted firms are analogous to low-cost bidders, while non-quoted firms are like high-cost bidders.

Conclusion

Conclusion

1. The work takes the price count as a primitive, and from it they derive a tight upper bound on the equilibrium distribution of sales prices.
2. The bound holds across a rich family of models that endogenize the price count and for all common-prior beliefs that firms might have about the price count.
3. An important direction for future research is to further relax our modeling assumptions by allowing for more complicated forms of feedback from prices to price counts and partial observability of prices by other firms. Also, homogenous goods and have symmetric and publicly known costs of production.