Final Project

2020 Fall

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Key point

Three-factor model. Quadratic optimization. Target beta portfolio

Backtest

Main process

1.Choose one period to do backtests using term structure:$S_m^n(\beta_T)$

2.Using m look back days to calculate the covariance with three-factor model. Using n look back days to calculate expected return.

3.Using optimize to get weights and invest

4.Rebalance every 5 days.

5.Change m n andβ_T .Check the performance and compared it with benchmark

The optimization problem

$$\begin{cases} \max \rho^T\omega - \lambda(\omega-\omega_P)^T\Sigma(\omega-\omega_P) \\ \beta^T\omega = \beta_T^M \\ e^T\omega = 1, \quad -2 \le \omega_i \le 2 \end{cases}$$

We need to simplify the problem:

$\rho^T\omega - \lambda(\omega-\omega_P)^T\Sigma(\omega-\omega_P) = \rho^T\omega - \lambda\omega^T\Sigma(\omega-\omega_P) + \lambda\omega_P^T\Sigma(\omega-\omega_P)$

$= \rho^T\omega - \lambda\omega^T\Sigma\omega + \lambda\omega^T\Sigma\omega_P + \lambda\omega_P^T\Sigma\omega - \lambda\omega_P^T\Sigma\omega_P$

$= \rho^T\omega - \lambda\omega^T\Sigma\omega + \lambda\omega_P^T\Sigma\omega + \lambda\omega_P^T\Sigma\omega - a \quad (\omega^T\Sigma\omega_P = \omega_P^T\Sigma\omega)$

$= (\rho^T\omega + 2\lambda\omega_P^T\Sigma)\omega - \lambda\omega^T\Sigma\omega - a = (\rho + 2\lambda\Sigma\omega_P)^T\omega - \lambda\omega^T\Sigma\omega - a$

Which equals to max $A^T\omega - \lambda\omega^T\Sigma\omega$ $(A = \rho + 2\lambda\Sigma\omega_P)$

In this project, I choose $\lambda = 0.5$

Subperiod

Before: First 360 days in data set

During: 361-510 (from the beginning of the subprime to the lowest point)

After:510-760 (1 year)

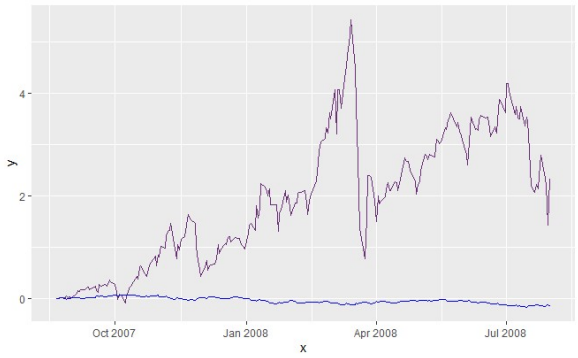
Backtest on all subperiod and whole period

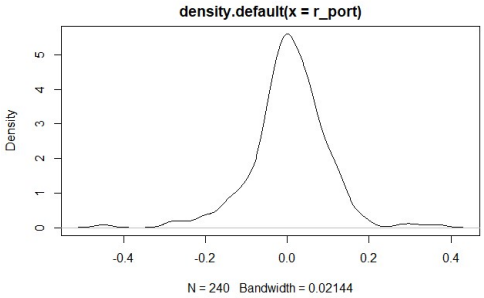
1.The period before subprime crisis:

(blue line is S&P)

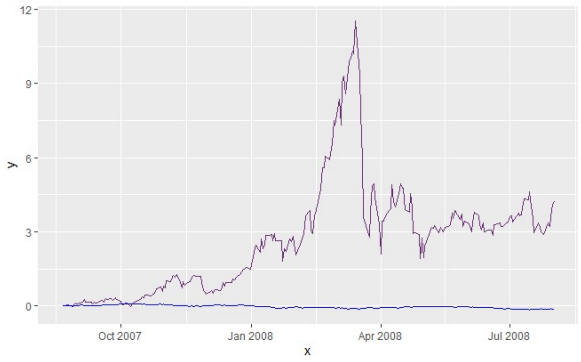
The first is cumulated return and the second is the distribution

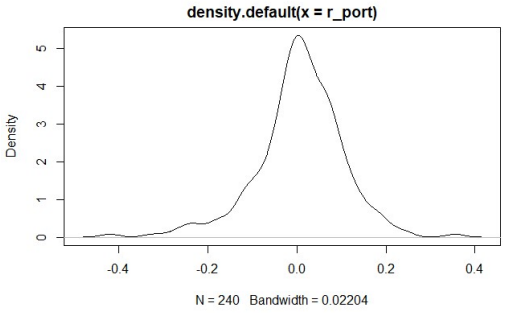
$$S_{30}^{30}(-1)$$



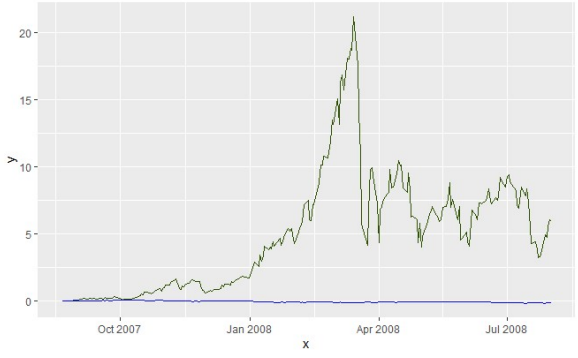


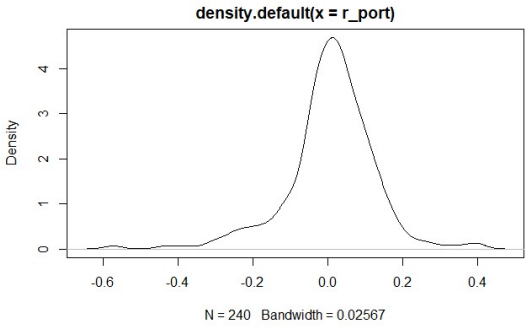
$$S_{80}^{30}(-1)$$



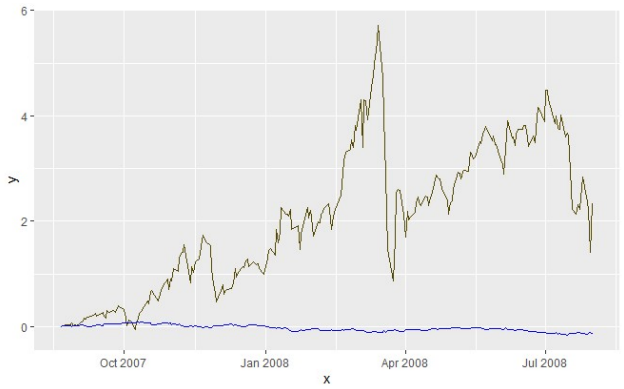


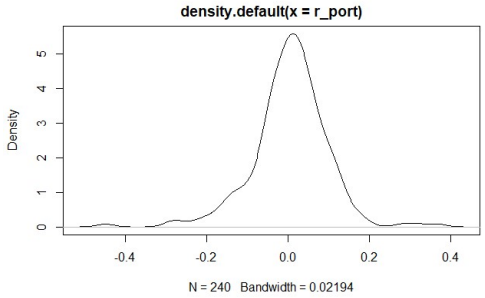
$$S_{120}^{30}(-1)$$



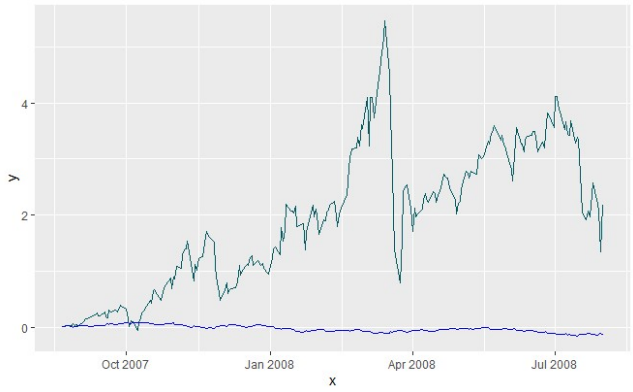


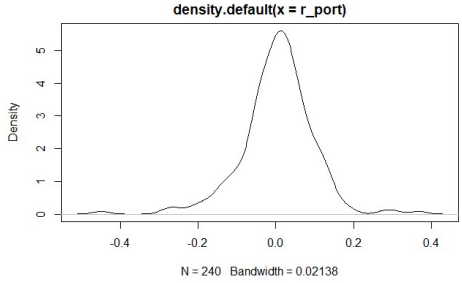
$S_{30}^{80}(-1)$



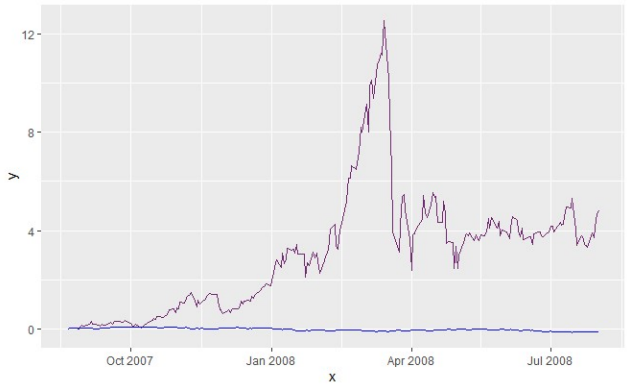


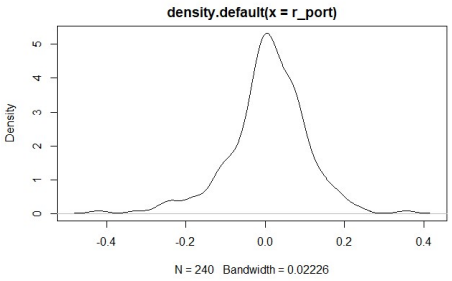
$S_{30}^{120}(-1)$





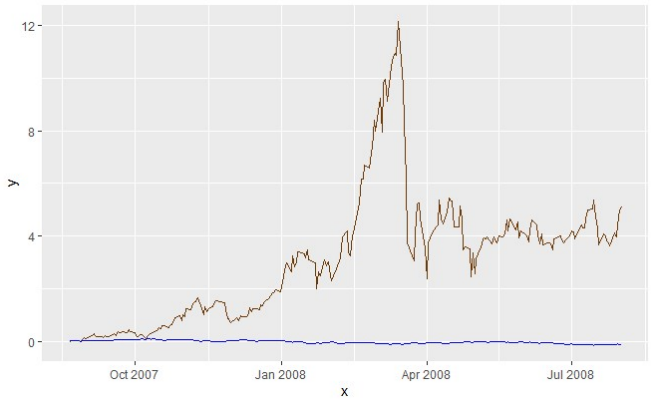
$S_{80}^{80}(-1)$

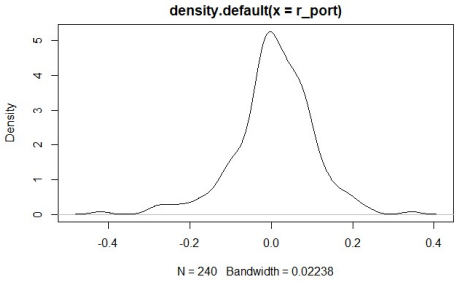




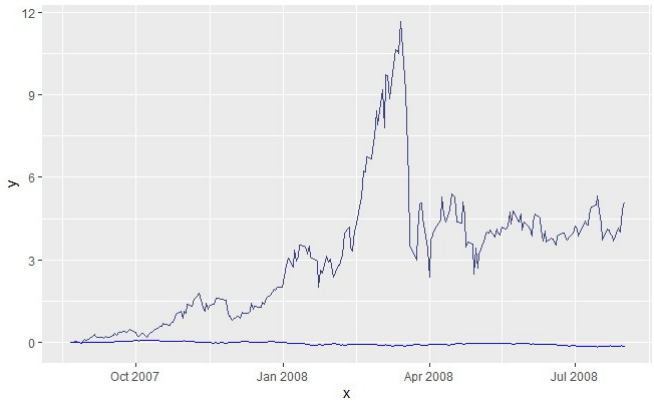
Then we choose the term structure $S_{80}^{80}(\beta)$ for different target beta.

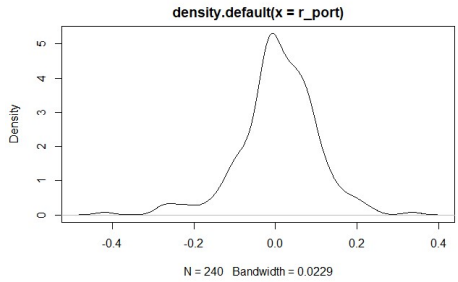
$S_{80}^{80}(-0.5)$



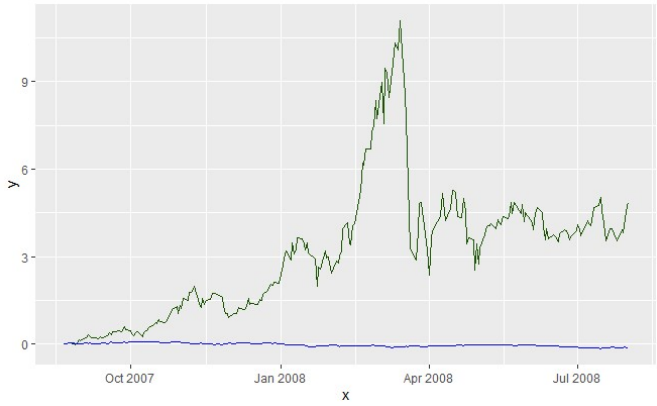


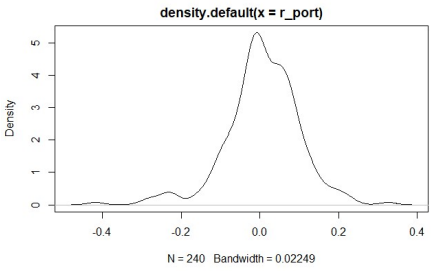
$S_{80}^{80}(0)$



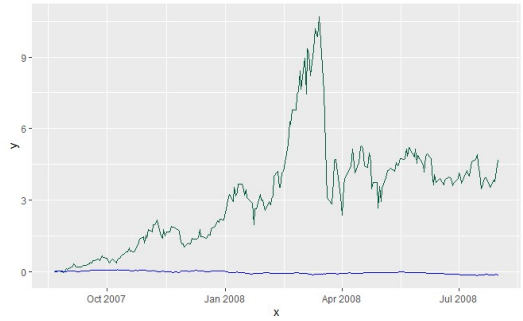


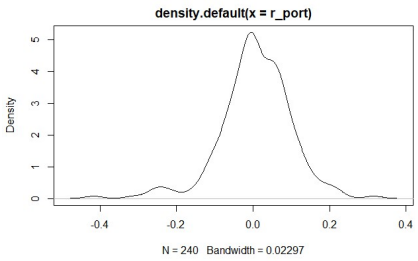
$S_{80}^{80}(0.5)$



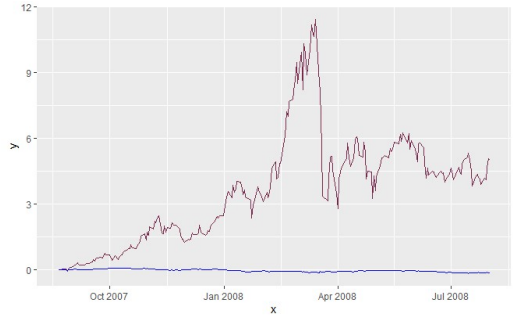


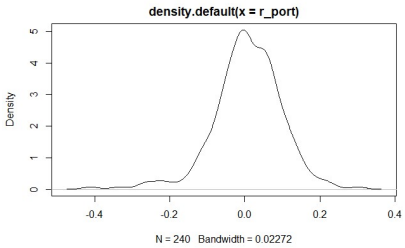
$S_{80}^{80}(1)$

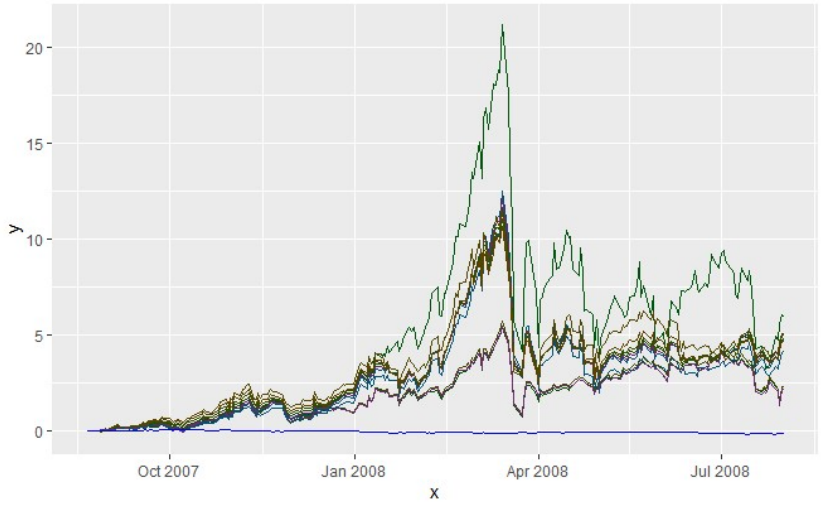




$S_{80}^{80}(1.5)$



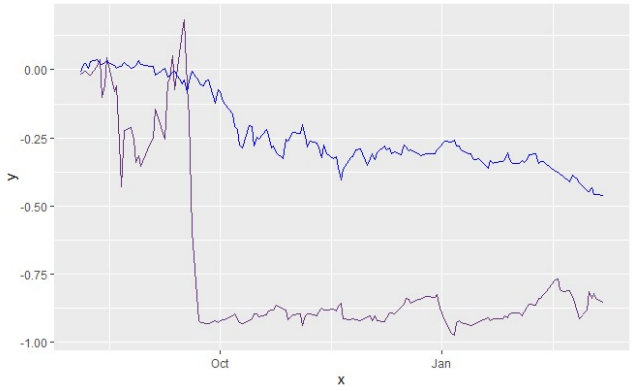


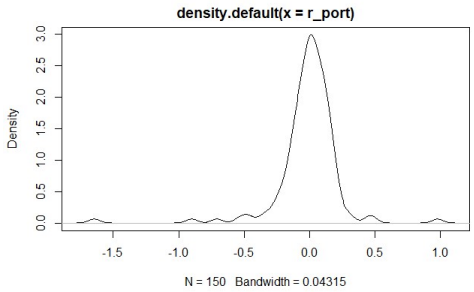


The distribution has negative skewness. It has long tail on the left side.

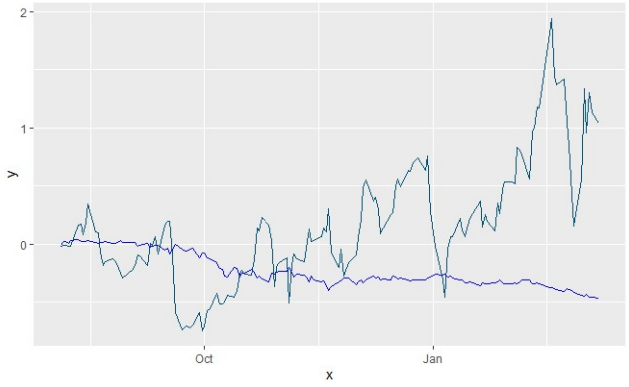
2. During the subprime crisis

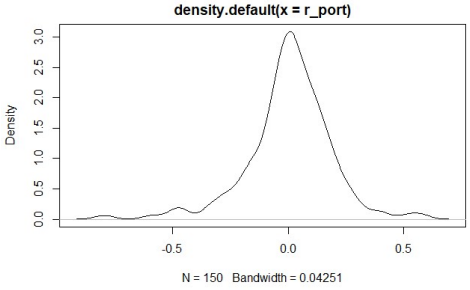
$S_{30}^{30}(-1)$



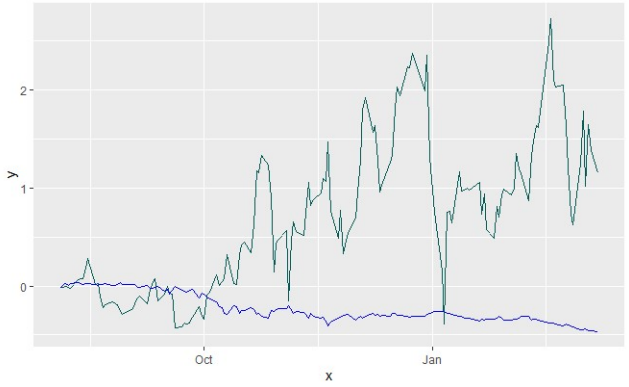


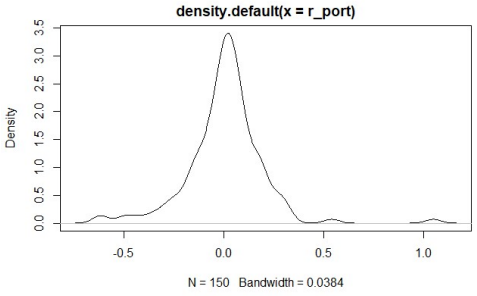
$S_{30}^{80}(-1)$





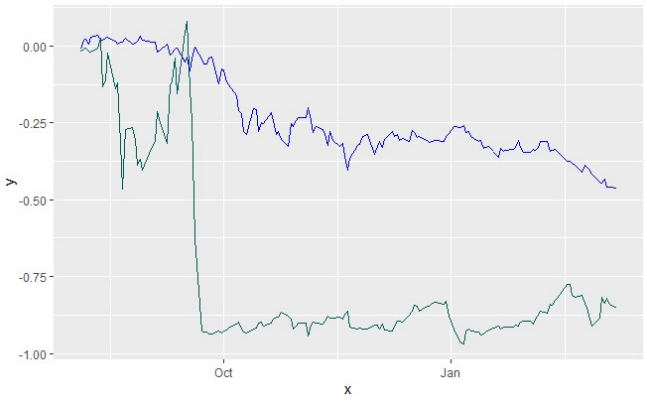
$S_{30}^{120}(-1)$

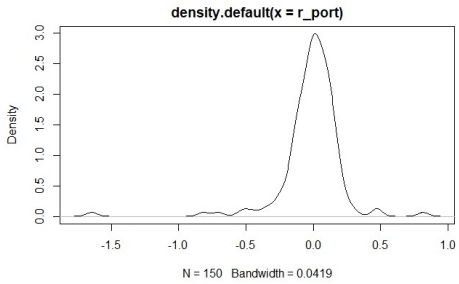




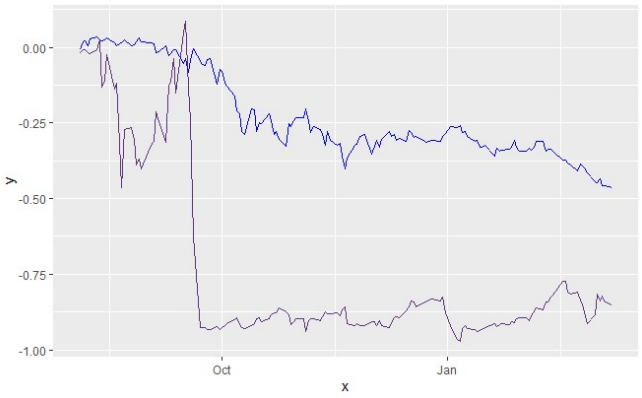
The shape will change when we change the length of the look back period for expected return.

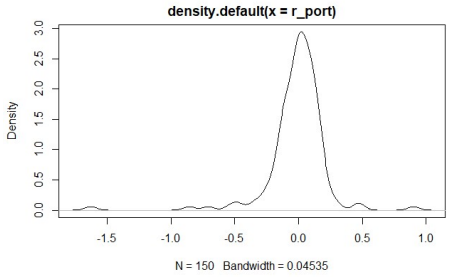
$S_{80}^{30}(-1)$



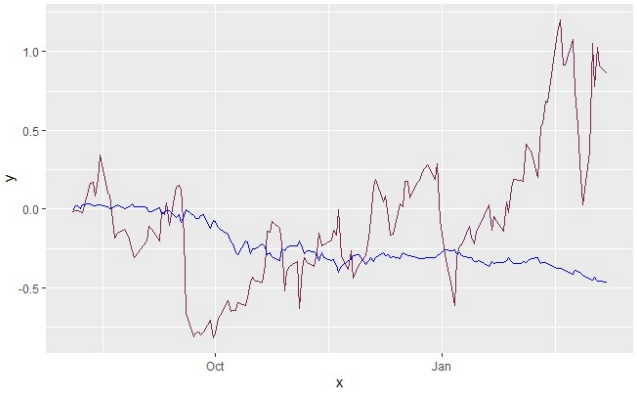


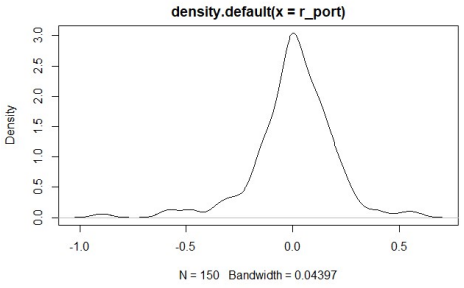
$S_{120}^{30}(-1)$





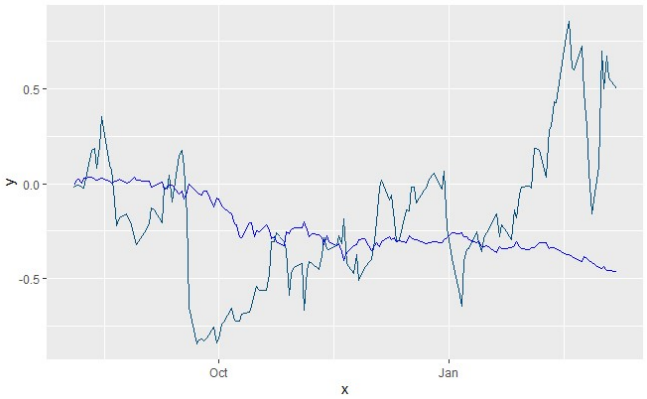
$S_{80}^{80}(-1)$

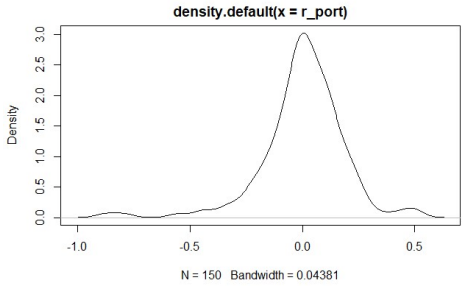




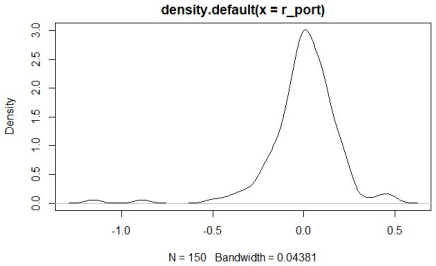
We still choose $S_{80}^{80}(\beta)$ as the term structure for different beta.

$S_{80}^{80}(-0.5)$

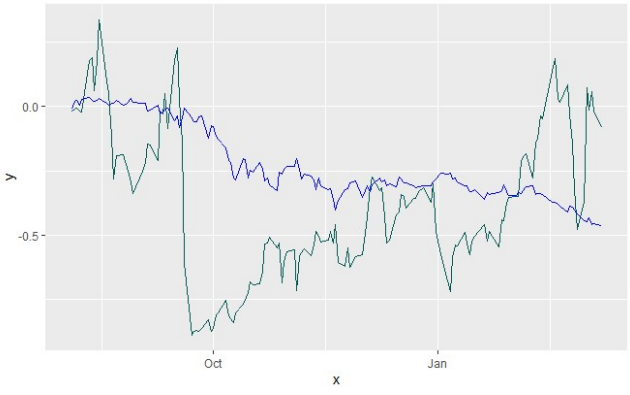


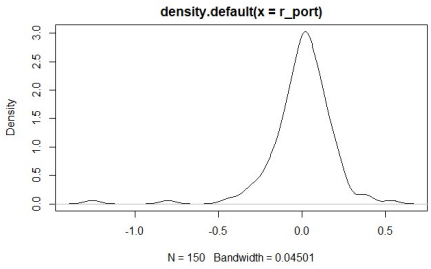


$S_{80}^{80}(0)$



$S_{80}^{80}(0.5)$





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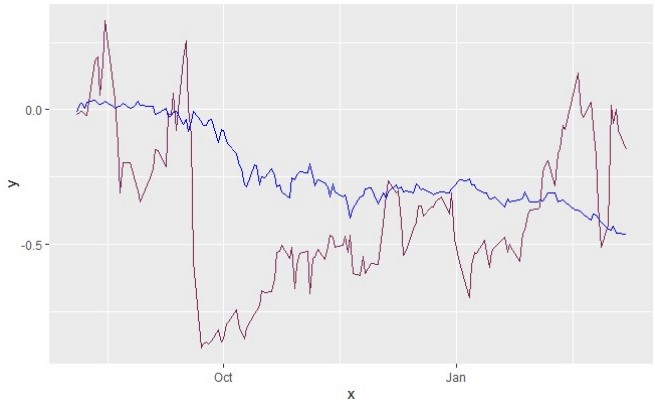
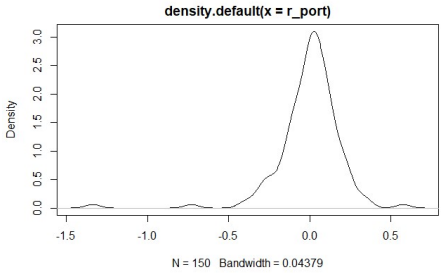
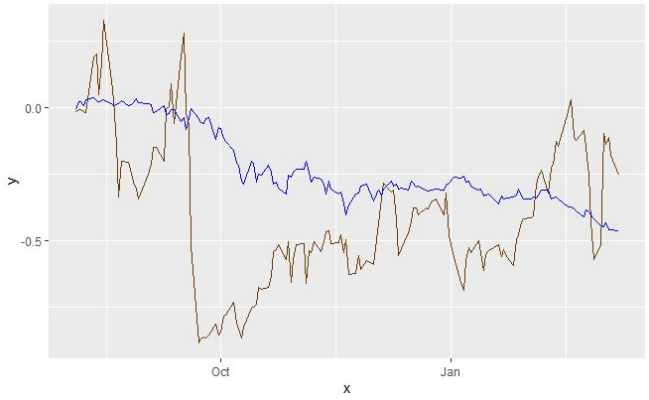
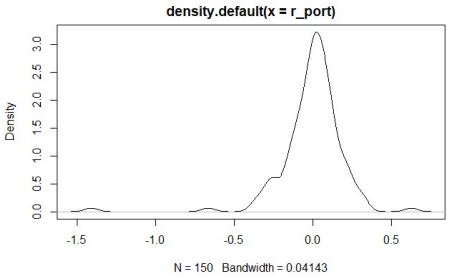
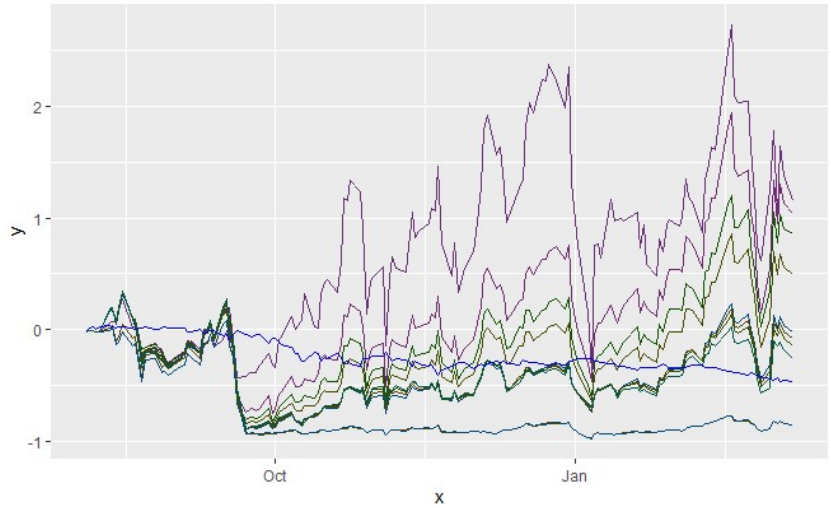




$S_{80}^{80}(1.5)$

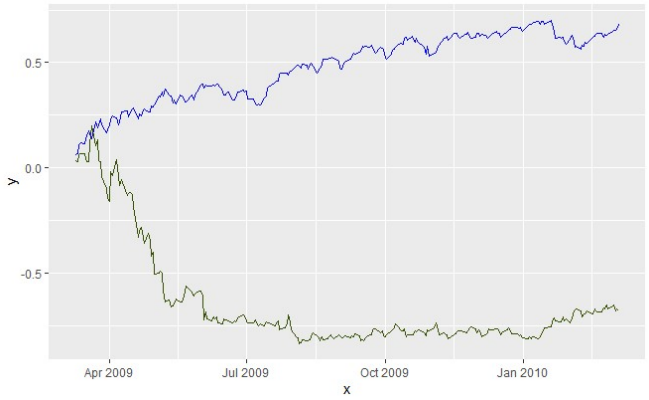


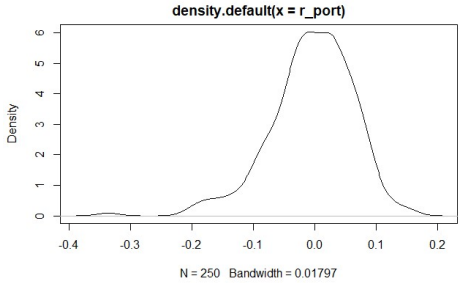




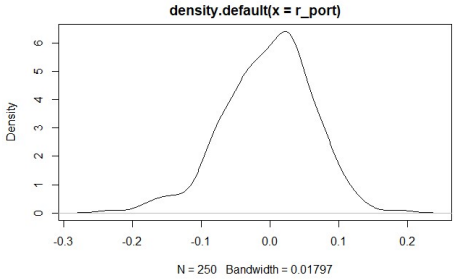
3.After the crisis (We use 1 years after the crisis)

$S_{30}^{30}(-1)$

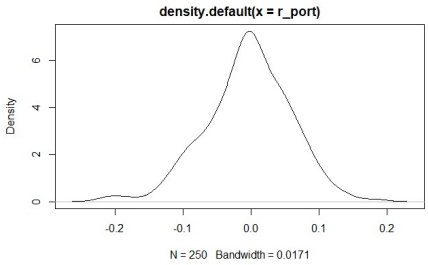




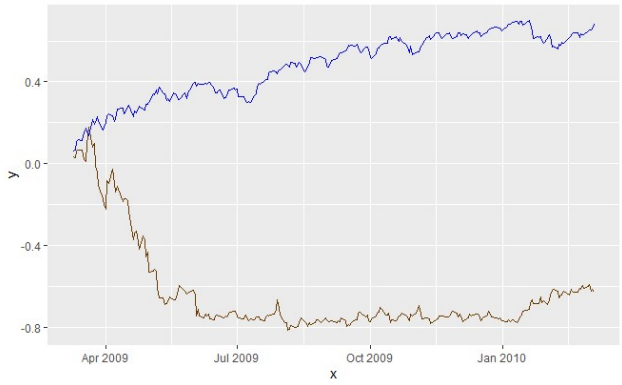
$S_{30}^{80}(-1)$

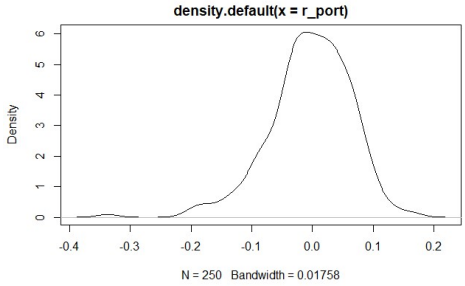


$S_{30}^{120}(-1)$

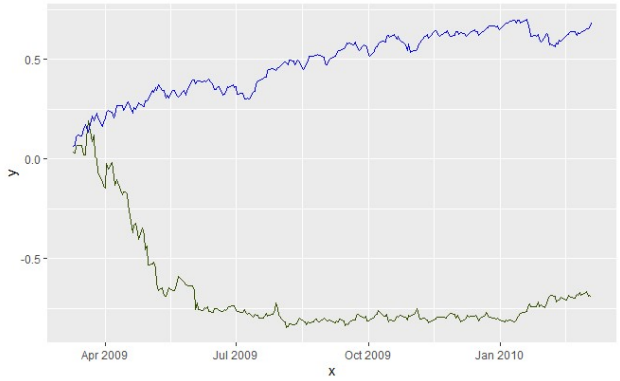


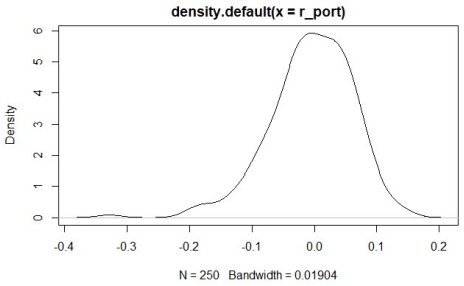
$S_{80}^{30}(-1)$





$S_{120}^{30}(-1)$





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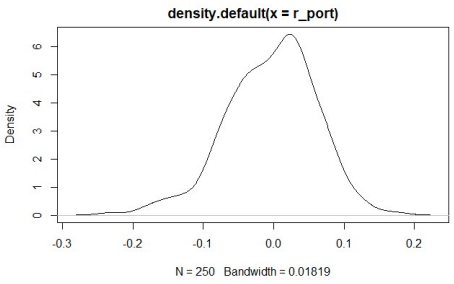
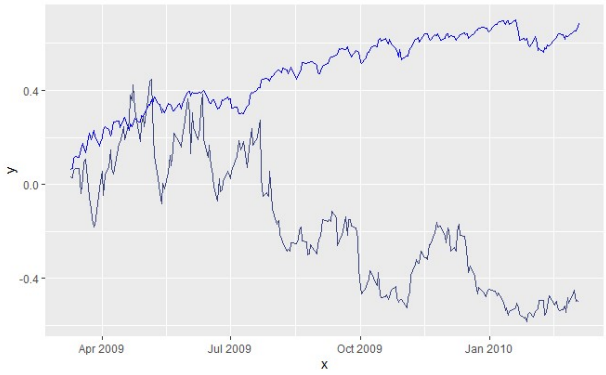
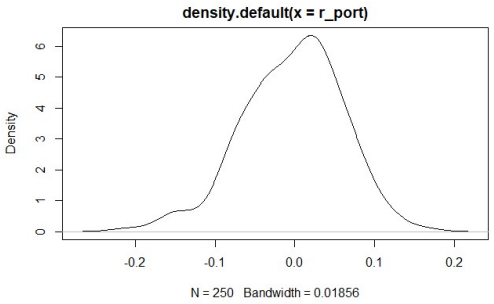


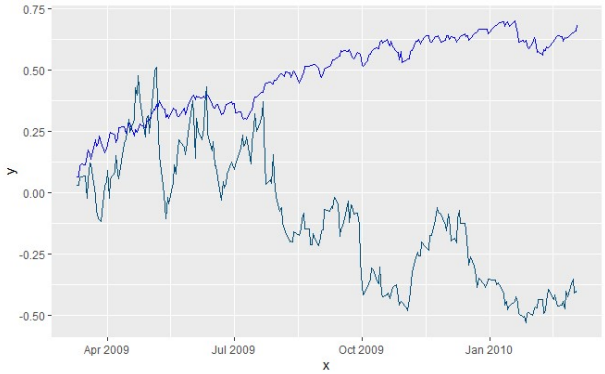
We still choose $S_{80}^{80}(\beta)$ as the term structure for different beta.

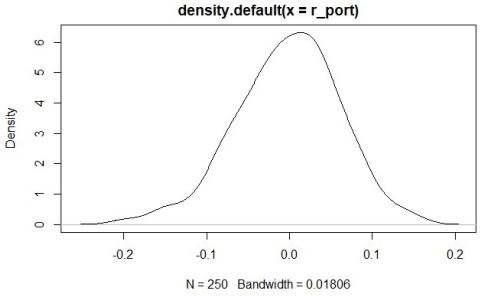
$S_{80}^{80}(-0.5)$



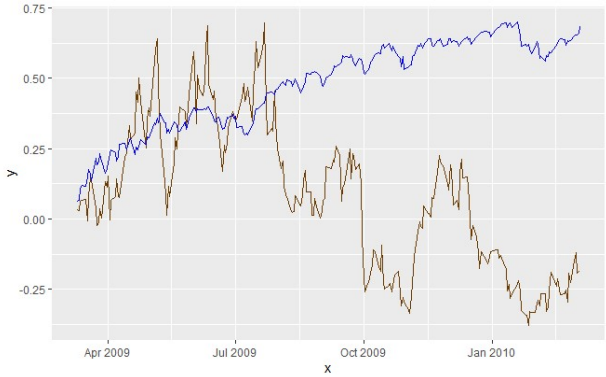


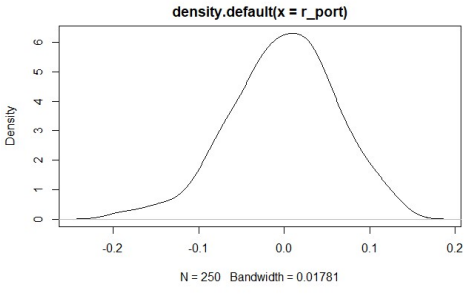
$S_{80}^{80}(0)$



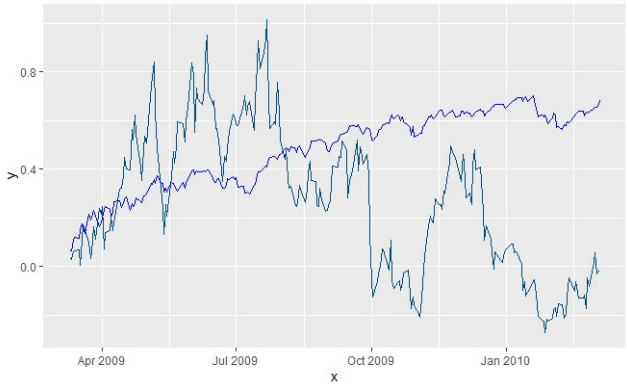


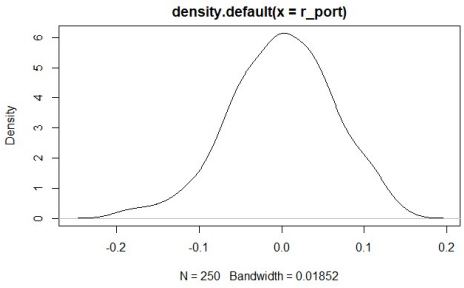
$S_{80}^{80}(0.5)$



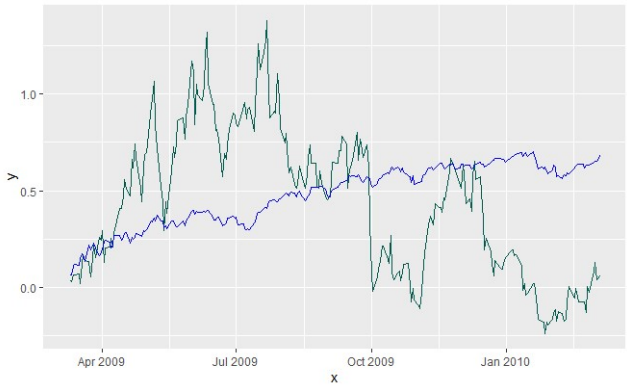


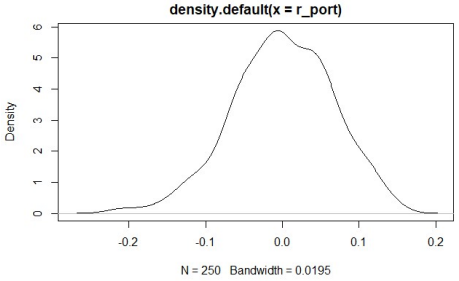
$S_{80}^{80}(1)$

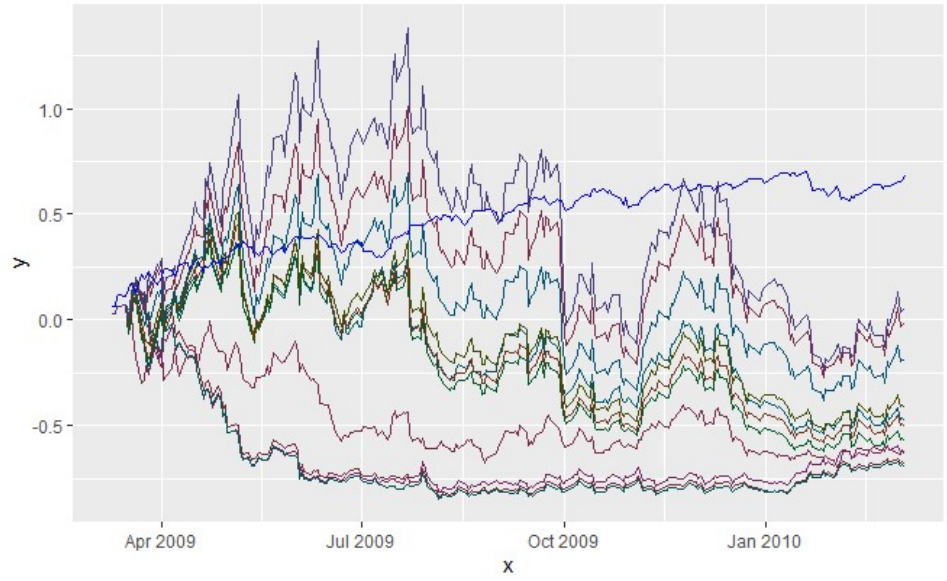




$S_{80}^{80}(1.5)$



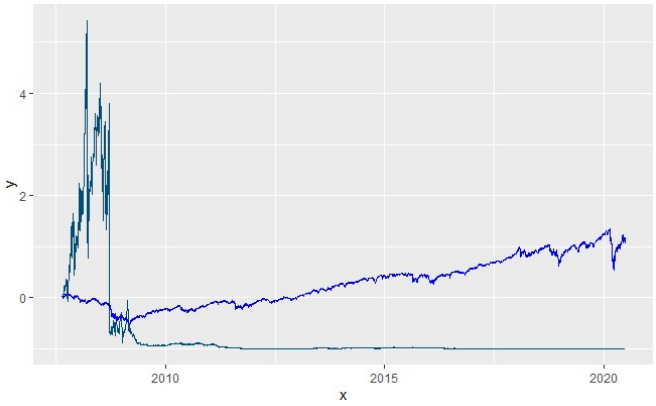


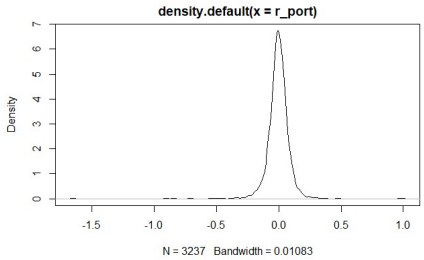


The kurtosis is very small.

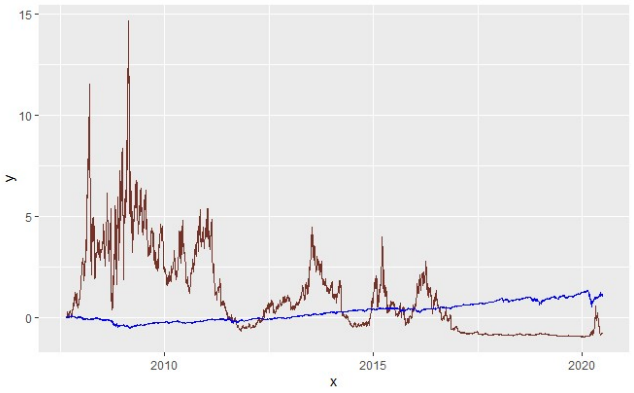
4. Whole period

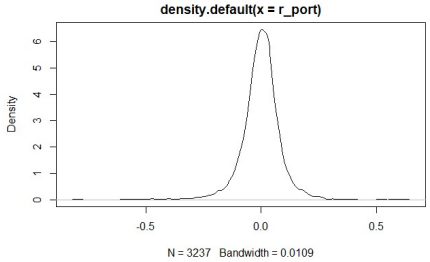
$S_{30}^{30}(-1)$



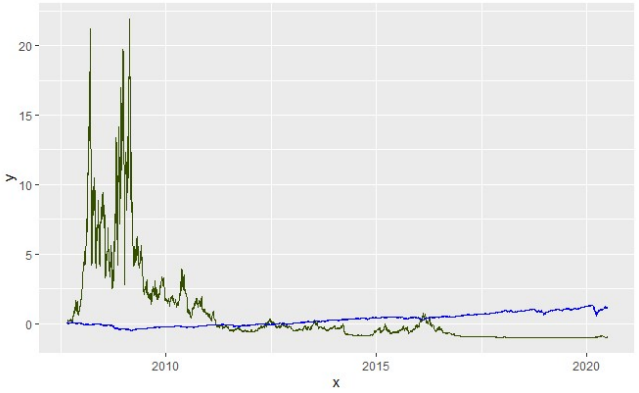


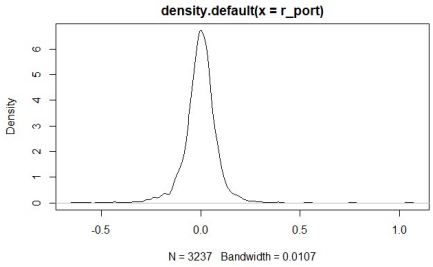
$S_{30}^{80}(-1)$



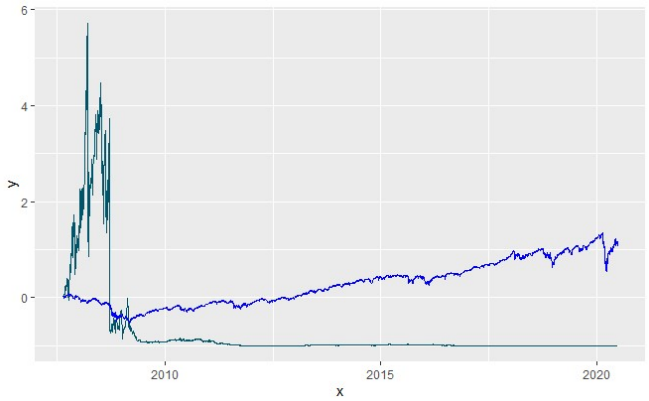


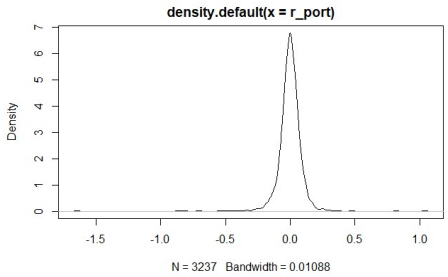
$S_{30}^{120}(-1)$



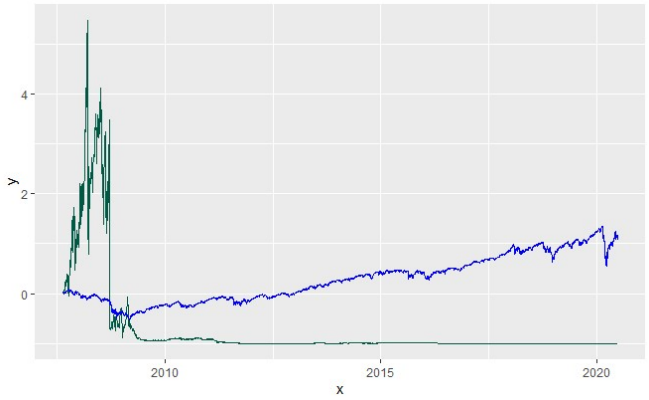


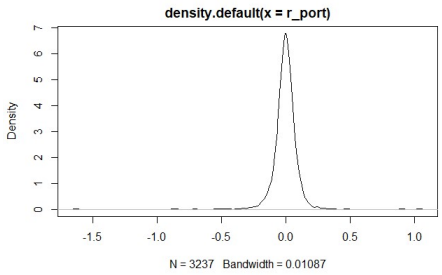
$S_{80}^{30}(-1)$



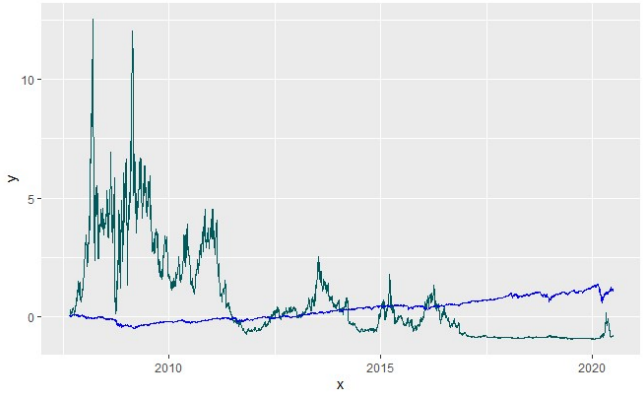


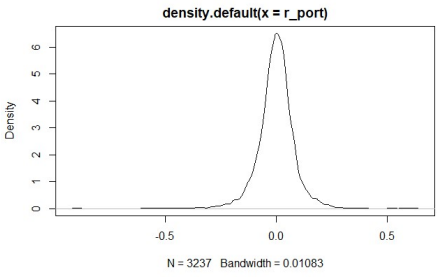
$S_{120}^{30}(-1)$





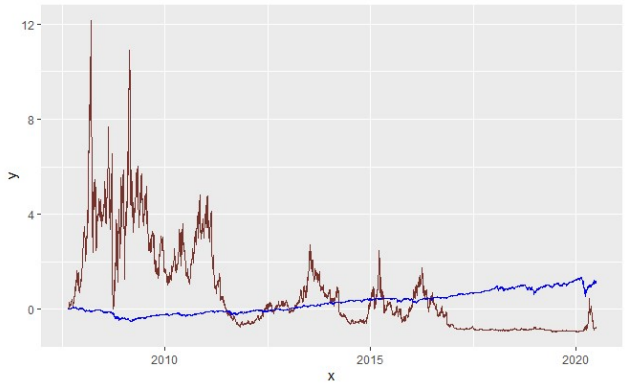
$S_{80}^{80}(-1)$

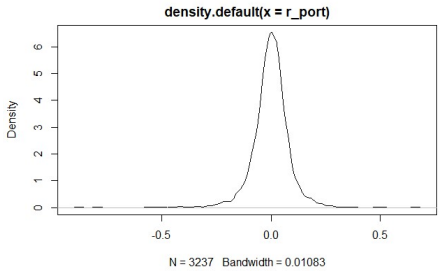




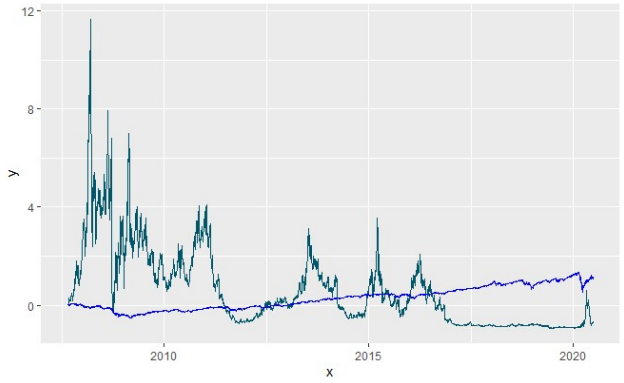
We still choose $S_{80}^{80}(\beta)$ as the term structure for different beta.

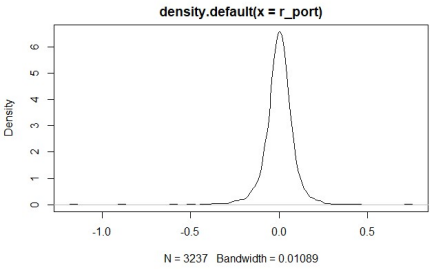
$S_{80}^{80}(-0.5)$



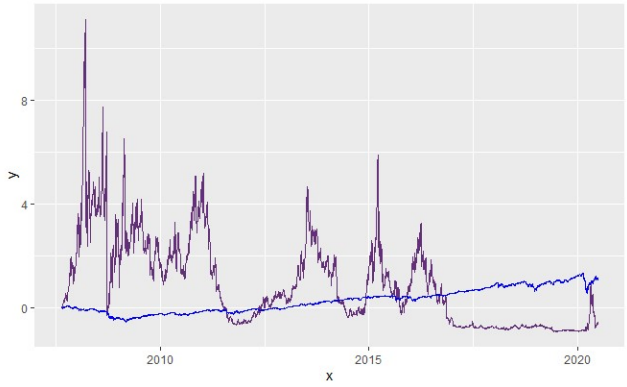


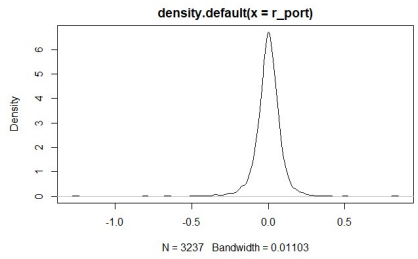
$S_{80}^{80}(0)$



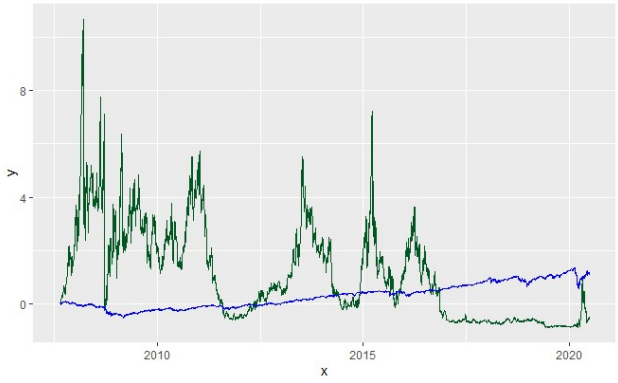


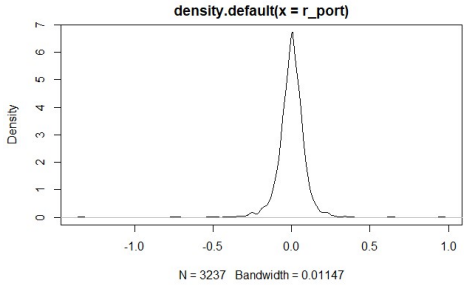
$S_{80}^{80}(0.5)$



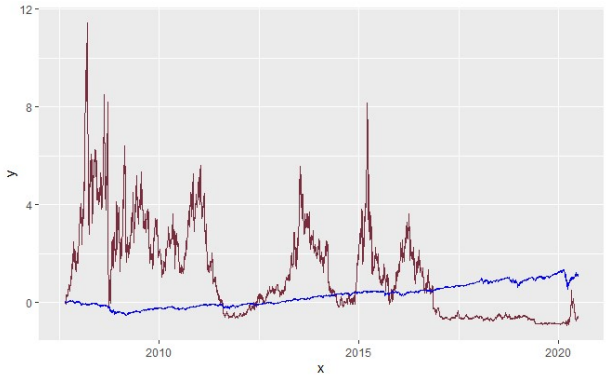


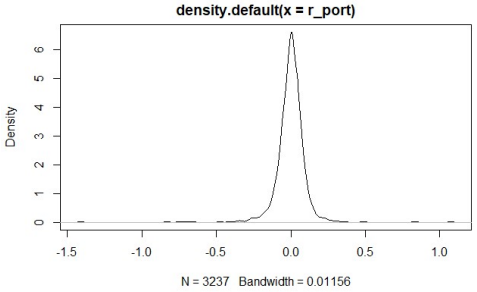
$S_{80}^{80}(1)$

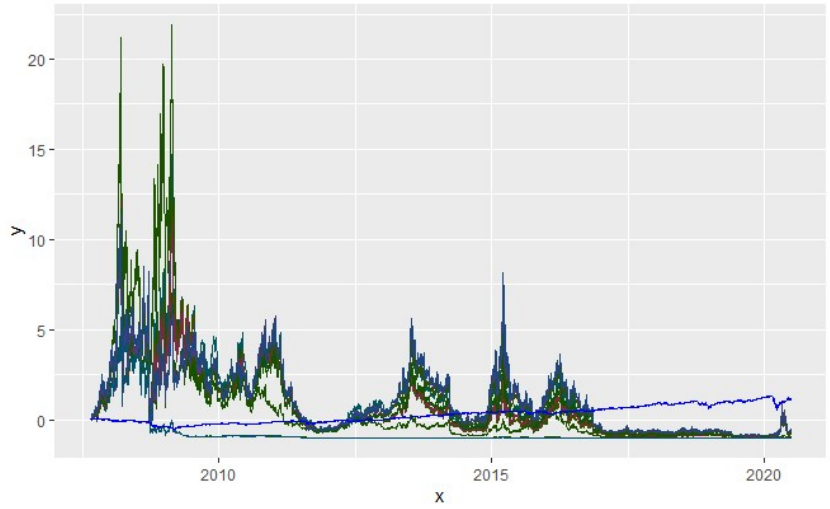




$S_{80}^{80}(1.5)$







The kurtosis is very big when it compared to other periods. There still many peaks. It even have 2000% return.

Performance comparison(Before)

Before crisis	$S_{30}^{30}(-1)$	$S_{80}^{30}(-1)$	$S_{120}^{30}(-1)$	$S_{30}^{80}(-1)$	$S_{30}^{120}(-1)$	$S_{80}^{80}(-1)$	
Cumulated return	234.64%	420.85%	591.02%	235.14%	217.78%	481.68%	
Mean	125.82%	171.91%	201.35%	125.98%	120.44%	183.41%	
Volatility	1.4714	1.5560	1.8901	1.4691	1.4599	1.5564	
Skewness	-0.0283	-0.0369	-0.0479	-0.0296	-0.0292	-0.0380	
Kurtosis	0.0144	0.0090	0.0145	0.0147	0.0144	0.0088	
Sharpe ratio	0.0536	0.0694	0.0670	0.0538	0.0517	0.0741	
MDD	-62.33%	-69.94%	-71.16%	-62.21%	-62.26%	-69.85%	
Modified VaR	-0.1525	-0.1659	-0.2038	-0.1525	-0.1517	-0.1660	
Modified CVaR	-0.2642	-0.2652	-0.3642	-0.2665	-0.2637	-0.2647	

	$S_{80}^{80}(-0.5)$	$S_{80}^{80}(0)$	$S_{80}^{80}(0.5)$	$S_{80}^{80}(1)$	$S_{80}^{80}(1.5)$	S&P
Cumulated return	509.46%	511.78%	480.73%	469.03%	505.43%	-12.81%
Mean	188.27%	188.67%	183.24%	181.12%	187.58%	-14.28%
Volatility	1.5283	1.4999	1.4843	1.4702	1.4559	0.2042
Skewness	-0.0376	-0.0382	-0.0395	-0.0419	-0.0451	0.0025
Kurtosis	0.0092	0.0097	0.0100	0.0104	0.0105	0.0016
Sharpe ratio	0.0775	0.0791	0.0776	0.0774	0.0810	-0.8656
MDD	-68.44%	-66.56%	-64.92%	-64.60%	-63.97%	-7.87%
Modified VaR	-0.1623	-0.1591	-0.1580	-0.1572	-0.1565	-0.0215
Modified CVaR	-0.2611	-0.2587	-0.2586	-0.2595	-0.2589	-0.0276

For the period before the subprime crisis, the strategy works very well compared to the benchmark S&P 500.

m for covariance, n for expected return

$S_m^n(\beta_T)$	Mean return	Sharp ratio	volatility
Increase n	No influence	No influence	No influence
Increase m	Increase	Increase	Increase
Increase beta	No influence	No influence	Decrease(little)

In this period, short term doesn't work well. It gives us little information about the past.

We can choose $S_{80}^{80}(0)$ as our strategy for this period.

Performance comparison(During)

During crisis	$S_{30}^{30}(-1)$	$S_{30}^{80}(-1)$	$S_{30}^{120}(-1)$	$S_{80}^{30}(-1)$	$S_{120}^{30}(-1)$	$S_{80}^{80}(-1)$
Cumulated return	-82.24%	103.59%	115.28%	-85.15%	-85.20%	85.97%
Mean	-319.17%	118.49%	127.80%	-317.81%	-318.42%	103.41%
Volatility	3.7225	2.9211	3.0306	3.6100	3.6571	2.9964
Skewness	-0.1471	-0.0445	0.0278	-0.1686	-0.1541	-0.0604
Kurtosis	0.0699	0.0123	0.0257	0.0737	0.0695	0.0165
Sharpe ratio	-0.0543	0.0256	0.0266	-0.0557	-0.0551	0.0218
MDD	-93.81%	-78.62%	-79.06%	-93.99%	-93.74%	-83.96%
Modified VaR	-0.4472	-0.3218	-0.2597	-0.4444	-0.4450	-0.3388
Modified CVaR	-1.0000	-0.5452	-0.2752	-1.0000	-1.0000	-0.6175
	$S_{80}^{80}(-0.5)$	$S_{80}^{80}(0)$	$S_{80}^{80}(0.5)$	$S_{80}^{80}(1)$	$S_{80}^{80}(1.5)$	**S&P**
Cumulated return	49.79%	-1.58%	-8.05%	-14.79%	-25.44%	-46.32%
Mean	67.34%	-2.65%	-13.99%	-26.67%	-48.92%	-103.69%
Volatility	2.9707	3.0856	3.0593	3.0634	3.1042	0.5291
Skewness	-0.0760	-0.1250	-0.1411	-0.1544	-0.1619	0.0051
Kurtosis	0.0197	0.0397	0.0502	0.0605	0.0691	0.0046
Sharpe ratio	0.0143	-0.0006	-0.0029	-0.0056	-0.0100	-0.2037
MDD	-86.42%	-90.56%	-89.99%	-89.26%	-88.49%	-25.88%
Modified VaR	-0.3456	-0.3761	-0.3732	-0.3721	-0.3740	-0.0575
Modified CVaR	-0.6526	-0.8743	-0.9522	-1.0000	-1.0000	-0.0756

$S_m^n(\beta_T)$	Mean return	Sharp ratio	volatility
Increase n	Increase	Increase	Decrease
Increase m	No influence	No influence	No influence
Increase beta	Decrease	Decrease	No influence

The short term works terrible. The drawdown is too big. But the term structure with bigger n works well compared to S&P 500.

During the subprime crisis, the information about the covariance doesn't matter at all. But the information about the expected return helps us a lot. It even has positive cumulated return. The beta changes the result too. But we need it to be negative.

So we may want a long look back period for expected return. We can choose $S_{80}^{80}(-1)$

Performance comparison(After)

After crisis	$S_{30}^{30}(-1)$	$S_{30}^{80}(-1)$	$S_{30}^{120}(-1)$	$S_{80}^{30}(-1)$	$S_{120}^{30}(-1)$	$S_{80}^{80}(-1)$
Cumulated return	-68.28%	-47.42%	-63.29%	-62.80%	-69.57%	-56.86%
Mean	-114.82%	-64.28%	-100.23%	-98.88%	-118.98%	-84.06%
Volatility	1.0734	1.0110	0.9912	1.0666	1.0588	1.0028
Skewness	-0.0542	-0.0250	-0.0201	-0.0519	-0.0509	-0.0270
Kurtosis	0.0082	0.0021	0.0024	0.0085	0.0074	0.0020
Sharpe ratio	-0.0677	-0.0402	-0.0640	-0.0587	-0.0711	-0.0530
MDD	-48.65%	-40.14%	-40.55%	-50.02%	-49.29%	-40.53%
Modified VaR	-0.1288	-0.1138	-0.1117	-0.1267	-0.1267	-0.1143
Modified CVaR	-0.1934	-0.1522	-0.1496	-0.1925	-0.1874	-0.1518
	$S_{80}^{80}(-0.5)$	$S_{80}^{80}(0)$	$S_{80}^{80}(0.5)$	$S_{80}^{80}(1)$	$S_{80}^{80}(1.5)$	S&P
Cumulated return	-49.95%	-40.59%	-18.60%	-1.61%	5.96%	68.31%
Mean	-69.21%	-52.07%	-20.57%	-1.62%	5.79%	52.07%
Volatility	0.9885	0.9889	0.9803	0.9998	1.0337	0.2256
Skewness	-0.0233	-0.0198	-0.0210	-0.0186	-0.0187	0.0265
Kurtosis	0.0012	0.0009	0.0005	0.0004	0.0004	0.0113
Sharpe ratio	-0.0443	-0.0333	-0.0133	-0.0010	0.0035	0.1239
MDD	-38.33%	-36.28%	-37.12%	-38.27%	-41.55%	-6.49%
Modified VaR	-0.1114	-0.1099	-0.1081	-0.1089	-0.1123	-0.0188
Modified CVaR	-0.1454	-0.1426	-0.1394	-0.1401	-0.1446	-0.0242

$S_m^n(\beta_T)$	Mean return	Sharp ratio	volatility
Increase n	Mid term is the best	Mid term is the best	No influence
Increase m	Mid term is the best	Mid term is the best	No influence
Increase beta	Increase	Increase	0 is the best

The result is interesting. In this period, the look back period may cover some days of the subprime crisis. So it doesn't follow the market. On the contrary, it goes down. It tells us that the look back period gives us wrong information. But we still want choose the best one, so the mid-term is the best choice. But there are ways to make profit, which is having a positive beta bigger than 1.

All these choices can't outperform the S&P 500. The market can make a profit of 68.31%.

Performance comparison(Whole)

Whole period	$S_{30}^{30}(-1)$	$S_{30}^{80}(-1)$	$S_{30}^{120}(-1)$	$S_{80}^{30}(-1)$	$S_{120}^{30}(-1)$	$S_{80}^{80}(-1)$
Cumulated return	-99.99%	-80.72%	-98.20%	-99.99%	-99.99%	-86.03%
Mean	-73.17%	-12.71%	-31.03%	-71.52%	-75.66%	-15.20%
Volatility	1.4286	1.3418	1.3694	1.4175	1.4220	1.3494
Skewness	-0.1325	-0.0406	0.0022	-0.1396	-0.1320	-0.0494
Kurtosis	0.2033	0.0344	0.0575	0.2022	0.1995	0.0404
Sharpe ratio	-0.0325	-0.0061	-0.0145	-0.0320	-0.0338	-0.0072
MDD	-93.81%	-79.38%	-79.27%	-93.99%	-93.74%	-83.96%
Modified VaR	-0.1052	-0.1402	-0.1177	-0.1069	-0.1064	-0.1415
Modified CVaR	-0.1052	-0.2974	-0.1177	-0.1069	-0.1064	-0.3207
	$S_{80}^{80}(-0.5)$	$S_{80}^{80}(0)$	$S_{80}^{80}(0.5)$	$S_{80}^{80}(1)$	$S_{80}^{80}(1.5)$	**S&P**
Cumulated return	-80.32%	-75.18%	-64.51%	-61.85%	-65.65%	111.22%
Mean	-12.55%	-10.76%	-8.00%	-7.44%	-8.25%	5.77%
Volatility	1.3381	1.3464	1.3533	1.3790	1.4238	0.2119
Skewness	-0.0550	-0.0838	-0.0861	-0.0851	-0.0829	-0.0338
Kurtosis	0.0446	0.0767	0.0911	0.1100	0.1341	0.0515
Sharpe ratio	-0.0061	-0.0052	-0.0039	-0.0035	-0.0038	-0.1740
MDD	-86.42%	-90.60%	-89.99%	-89.26%	-88.49%	-25.88%
Modified VaR	-0.1404	-0.1368	-0.1319	-0.1257	-0.1181	-0.0203
Modified CVaR	-0.3308	-0.3530	-0.2853	-0.1257	-0.1181	-0.0385

$S_m^n(\beta_T)$	Mean return	Sharp ratio	volatility
Increase n	Mid term is the best	Mid term is the best	Mid term is the best
Increase m	No influence	No influence	No influence
Increase beta	Increase(No influence if bigger than1)	Increase(No influence if bigger than1)	No influence

This is a very long period. The final cumulated return is meaningless. But the look back period for expected return is very important. The long-term of it gives us many chances to win big.

The best one is $S_{80}^{80}(1)$ if we take long position.

If we take short position of the portfolio, we can make big profit.

Conclusion.

1.No matter what period it is, the volatility is very big. The reason is that the optimization solver can't have a good solution of weights. Most of the weights lie on the Boundary (-2 and 2). So $\omega_i \omega_j$ can be very big. Also, it makes the absolute value of investment on each equity is also very big. Just like $200 short and $200 long. It is kind of like leverage. I think the volatility shown in table is multiplied.

2.Sometimes the covariance matrix is not positive definite(Some equities have negative beta), the optimization solver cannot solve the problem. We need a function called `nearPD()` to find the nearest positive definite matrix. I think it's also one of the reason that the volatility is so big.

3.When we look at different subperiod, the strategy is different. What we focus on is different. For one subperiod, we may like to concern more about the covariance from past. But for another, it has no influence when we change the length of look back period.

4.Change the look back period for expected return will change the shape of line of the portfolio value. But change another look back period or beta will not change the shape.

5. Negative return doesn't mean that we could not make any profit. We can take short position as long as we know the return would be negative. When we create a portfolio, we should not just look at the end, we should set a line when we get enough profit or lose and then quit. Just like the performance on the whole period, if we take long position of the portfolio, we cannot hold it to the end which make us lose all the money. As there are many peaks, what we should do is to make profit form it.

6. This portfolio can be used to combine with some stocks to kill the volatility