1 Problem 1 - Book 9.1

1.1 part a

A plot of the approximated Legendre polynomials via QR decomposition can be seen in Figure 1.

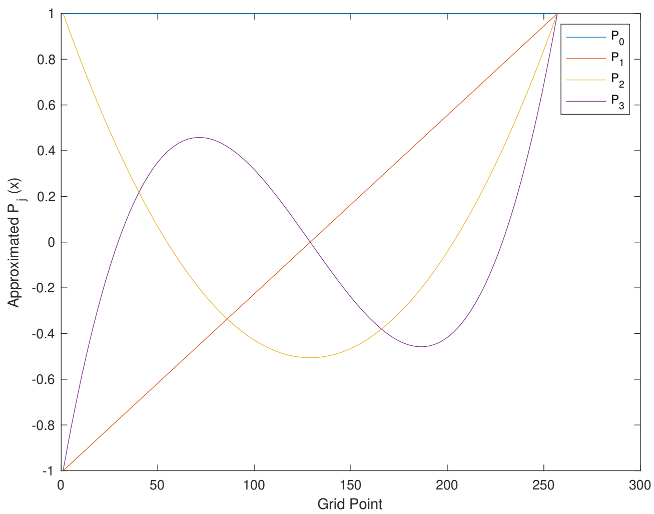

Figure 1: Problem 9.1, Part A: Plot of Approximated Legendre Polynomials as a function of grid point [1-257]

1.2 part b

The maximum error of the approximated P_j with respect to the real P_j was found to be 7.771561e-16, 2.220446e-16, 5.882353e-03, and 1.137999e-02 for P_0, P_1, P_2, and P_3 respectively. A plot of the error as a function of grid point can be seen in Figure 2.

1.3 part c

We started the study using $2^5 + 1 = 17$ grid points and ended at $2^{26} + 1 = 67108865$ grid points on the interval [-1,1]. Figure 3 shows a plot of the maximum error seen between the P_0, P_1, P_2, and P_3 cases for a given value of v in the grid spacing $\Delta x = 2^{-v}$. Furthermore, Figure 4 explicitly details the maximum error with respect to iterations in v. As can be seen, a grid spacing of 2^{-25} reduces the error to roughly 4.3e-8. I believe that at some

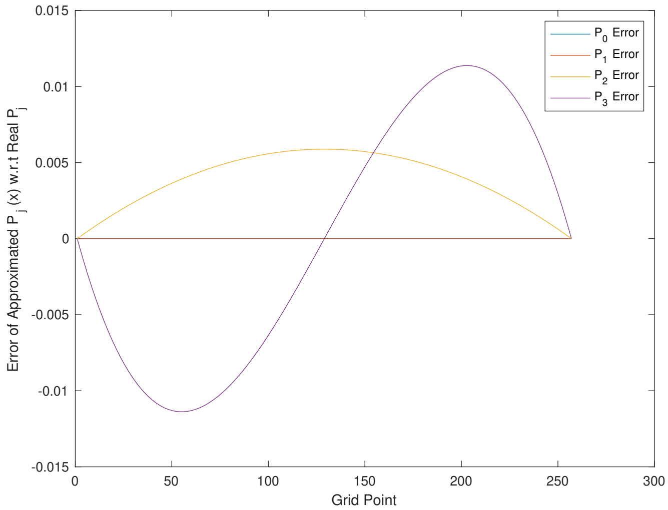

Figure 2: Problem 9.1, Part B: Plot of Error of Approximated Legendre Polynomials w.r.t real Legendre Polynomials as a function of grid point [1-257]

point beyond this, the linear relationship should level off as we begin to approach machine precision. I did not converge to this asymptote as I did not want to make my laptop laggy. But in my world view, an error of 4.3e-8 is pretty acceptable. Finally, Figure 5 shows how the log of the maximum error behaves for each Legendre polynomial as a function of increasing the grid density by a factor of 2. As can be seen, the error of the approximated P_0 and P_1 are in a noise regime, whereas the error of P_2 and P_3 linearly decrease to acceptable limits upon increasing grid.

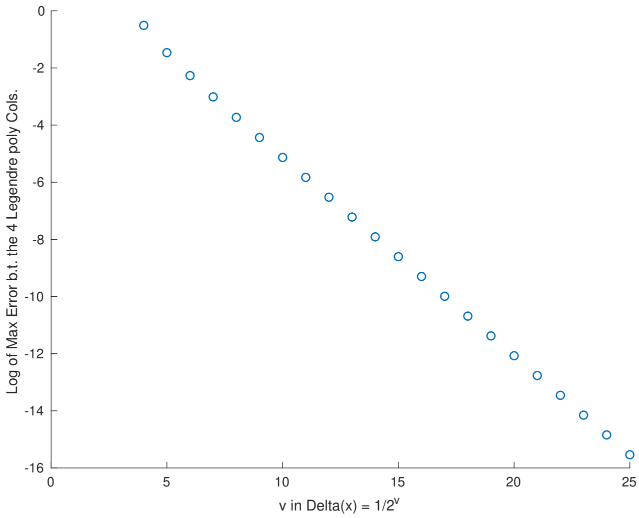

Figure 3: Problem 9.1, Part C: Plot of the Log of the Error of Approximated Legendre Polynomials w.r.t real Legendre Polynomials as Increasing the Grid Point density by a factor or 2

```
iteration: 4 and max. error 9.716403e-02
iteration: 5 and max. error 4.676081e-02
iteration: 6 and max. error 2.297080e-02
iteration: 7 and max. error 1.137999e-02
iteration: 8 and max. error 5.663974e-03
iteration: 9 and max. error 2.825523e-03
iteration: 10 and max. error 1.411154e-03
iteration: 11 and max. error 7.051748e-04
iteration: 12 and max. error 3.524870e-04
iteration: 13 and max. error 1.762184e-04
iteration: 14 and max. error 8.810293e-05
iteration: 15 and max. error 4.404990e-05
iteration: 16 and max. error 2.202456e-05
iteration: 17 and max. error 1.101218e-05
iteration: 18 and max. error 5.506066e-06
iteration: 19 and max. error 2.753027e-06
iteration: 20 and max. error 1.376512e-06
iteration: 21 and max. error 6.882555e-07
iteration: 22 and max. error 3.441277e-07
iteration: 23 and max. error 1.720638e-07
iteration: 24 and max. error 8.603190e-08
iteration: 25 and max. error 4.301595e-08
```

Figure 4: Problem 9.1, Part C: Maximum error associated with iteration v.

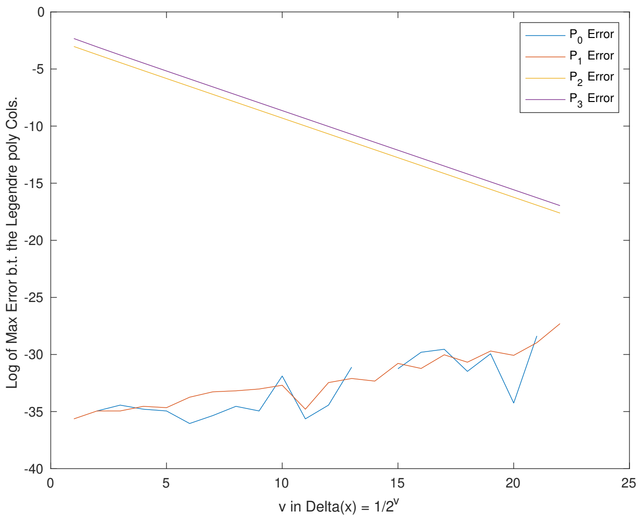

Figure 5: Problem 9.1, Part C: Maximum Error associated with each Approximated Legendre Polynomials w.r.t real Legendre Polynomials as Increasing the Grid Point density by a factor or 2

2 Problem 2

The errors associated with Experiment 1 are shown in Figure 6, while the reconstructed image and associated error for Experiment 2 are shown in Figures 7 and 8 respectively. As can be seen in the appreciably small errors, implementing the QR factorization via the Householder reflections is a viable strategy. I apologize if the code for this section feels hacked. I feel the same way.

```
maximum orthogonality error: 2.10595e-15
average orthogonality error: 3.0907e-16
maximum normalization error: 2.66454e-15
average normalization error: 5.58072e-16
```

Figure 6: Exp. 1 errors

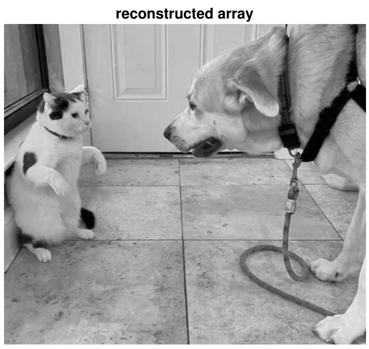

Figure 7: Exp. 2 reconstructed picture

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maximum orthogonality error: 3.48332e-15
average orthogonality error: 3.53266e-16
maximum normalization error: 6.66134e-15
average normalization error: 1.20228e-15
```

Figure 8: Exp. 2 errors