MAD 6406: HOMEWORK 3

Due: Friday, 09/25

Numbered problems are from Trefethen and Bau, Numerical Linear Algebra.

Starred problems (*) require the use of Matlab (or some other language, if you prefer).

(1) Trefethen & Bau, 3.5

(2) Let $A \in \mathbb{C}^{m\times n}$. Show
   (a) $\text{Col}\,(A) \perp \text{Null}\,(A^*)$.
   (b) Any $v \in \mathbb{C}^m$ orthogonal to $\text{Col}\,(A)$ is in $\text{Null}\,(A^*)$.
   (c) Each $v \in \mathbb{C}^m$ has a unique decomposition $v = v_R + v_N$ with $v_R \in \text{Col}\,(A)$ and $v_N \in \text{Null}\,(A^*)$.

(3) Show: $\text{Null}\,(A^*) = \text{Span}\,\{u_{r+1}, \ldots, u_m\}$, and $\text{Col}\,(A^*) = \text{Span}\,\{v_1, \ldots, v_r\}$.

(4)* In this problem you will estimate the 3-norm of a matrix and verify whether your approximation satsifies an analytic bound.
   (a) Write a function in Matlab (or your favorite language) to approximate the matrix 3-norm of a 10×10 matrix A. Use the command `A = 10*(rand(10,10)-0.5)` to generate a 10×10 matrix with entries between -5 and 5. You may use the Matlab command `norm(A*x,3)` to compute vector 3-norms of products Ax, if you wish.
   (b) Use timing commands (`tic` and `toc` if you are using Matlab) to time your routine. Also compute and time $\|A\|_\infty$ using the Matlab command `norm(A,inf)` (or equivalent, if you are using another language).
   (c) Prove an inequality of the form $\|A\|_\infty \leq \alpha \|A\|_3$, for some constant α. What is α?
   (d) Run your code ten times in a row and report back how many times the inequality was satisfied.

   For (a), you may either summarize your program in pseudocode, copy and paste it into your homework file, or turn it in (upload it to Canvas) as a separate file. For (b), you should briefly summarize in your written homework how the timings compared. Include (c) and your findings in (d) in your written homework as well.

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Date: September 16, 2020.