MAD 6406: HOMEWORK 8

Due: Friday, October 30

Numbered problems are from Trefethen and Bau, Numerical Linear Algebra. Starred problems (*) require the use of Matlab (you can use another language if you prefer).

(1) Let $x \in \mathbb{R}^n$ and let H be a Householder matrix such that $Hx = \|x\|_2 e_1$. Let $G_{1,2}, \ldots, G_{1,n}$, be Givens rotation matrices such that $Gx = G_{1,n} \cdots G_{1,2} x = \|x_2\| e_1$. True or False: $G = H$? EXPLAIN.

(2) Suppose A is $n \times n$ and nonsingular, and exact data b and solution x satisfy $Ax = b$. Suppose data pertubation Δb and solution perturbation Δx further satisfy $A(x + \Delta x) = (b + \Delta b)$. Show

$$\frac{\|\Delta x\|}{\|x\|} \leq \mathrm{Cond}(A) \frac{\|\Delta b\|}{\|b\|}.$$

(3) Let $\|\cdot\|$ be a subordinate (induced) matrix norm. If A is $n \times n$ invertible and and E is $n \times n$ with $\|A^{-1}\| \, \|E\| < 1$, then show
   (a) $A + E$ is nonsingular
   (b)

$$\left\|(A+E)^{-1}\right\| \leq \frac{\|A^{-1}\|}{1 - \|A^{-1}\| \, \|E\|}.$$

(4)* Consider, for even positive integers n, the ratio

$$R(n) = \frac{n(n-2)(n-4)\cdots 2}{(n-1)(n-3)(n-5)\cdots 1}$$

It holds that $R(100) \approx 12.5645$, and $R(400) \approx 25.0820$. Compute $R(4000000)$. Explain your method.

(5)* Compute

$$\sum_{k=0}^{1000} \frac{\cosh(k)}{1 + \sinh(k)}.$$

Explain your method. (You may find it useful that e^x is computed in Matlab by the command `exp(x)`).

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Date: October 22, 2020.