0.1 problem 1

The issue at play is to find the least squares solution for some x, given some b that may or may not be in the column space of A. If we are sampling an overdetermined problem where b is not necessarily in the column space of A, then we solve the problem by minimizing a residual seen to be $||r||_2 = ||b - Ax||_2$. So in other words, we need to find the component of b that is in column space of A by orthogonal projection. Assuming r is the vector that takes us from some b to some y that is in the column space of A, we note that this value of r that satisfies this in addition to the minimization constraint introduced above is one that necessarily has to be perpendicular the column space of A, ie r is perpendicular to the column space of A. This is equivalent to saying $A^*r = 0$.

In summary, we are solving 2 equations simulateniously: one for some r such that $A^*r = 0$ and one $r = b - Ax$. Rearranging the ladder term, we see that these equations can be packaged neatly such that

$$\begin{pmatrix} I & A \\ A^* & 0 \end{pmatrix} \begin{pmatrix} r \\ x \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix} \tag{1}$$

0.2 problem 2

Line 1 computes the Singular value decomposition of a matrix A. Line 2 stores the singular values along the diagonal of a matrix and calls this S. Line 3 defines a tolerance value, which is equivalent to a multiplication of the largest singular value, multiplied by machine precision, multiplied by the maximum dimension of the matrix A. Line 4 returns determines the total number of elements in the diagonal of S that are larger than this tolerance. Line 5 then takes the reciporcal of the S, assuming that each of the elements along the diagonal are larger than the tolerance. The final line then calculates the pseudoinverse of a matrix A, using only the singular values that are considered stable with respect to some tolerance.

0.3 problem 3

From page 97 and 98 in the book, we recognize that the interval from $2^5 = 32$ to $2^6 = 64$ can be represented in double precision by

$$2^5, 2^5 + 2^5(2^{-52}), 2^5 + 2^5(2 \times 2^{-52}), \cdots, 2^6 \tag{2}$$

Thus, the number directly after 32 increases 32 by $2^5(2^{-52}) = 2^{-47}$.

Using the same train of thought, the interval 2^{53} to 2^{54} can be representing in a similar manner, with the number directly after 2^{53} being $2^{53}+2^{53}(2^{-52}) = 2^{53}+2$