MAD 6406: HOMEWORK 12

Do, but please do not turn in

Numbered problems are from Trefethen and Bau, Numerical Linear Algebra. Starred problems (*) require the use of Matlab (you can use another language if you prefer).

(1) Let $A \in \mathbb{C}^{n\times n}$ be a diagonalizable matrix with basis of eigenvectors $\{v_i\}_{i=1}^n$, where each v_i has unit length.

(a) Show that applying a power iteration to initial vector x_0, it holds that

$$x_k = \frac{A^k x_0}{\|A^k x_0\|_2}.$$

(b) Assuming that $c_1, \lambda_1 \neq 0$, show that

$$\left\| \sum_{i=2}^{n} \frac{c_i}{c_1} \left(\frac{\lambda_i}{\lambda_1}\right)^k v_i \right\| \leq \left(\sum_{i=2}^{n} \left(\frac{c_i}{c_1}\right)^2 \left(\frac{\lambda_i}{\lambda_1}\right)^{2k} \right)^{1/2}$$

(2) Run the m-file `powerm.m`. It uses the Power method to compute the largest eigenvalue-eigenvector pair of a matrix A. Run it with $A = A3$, $A = A5$, $A = A9$, $A = A17$ and $A = A63$. How does the number of iterations to converge to residual tolerance 10^{-7} change as the size of A is increased? Explain.

(3) Run the m-file `invpower.m`. It uses the Inverse Power method (with shift). Now compute the largest eigenvalue of A by first running 10 iterations of the Power method to compute an approximation to the eigenvalue, then use that value as σ to start the inverse power method. How does the number of iterations change as the size of the matrix is increased? Does it converge to the same eigenvalue? Explain.

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