MAD 6406: HOMEWORK 7

Due: Friday, October 23

Numbered problems are from Trefethen and Bau, Numerical Linear Algebra. Starred problems (*) require the use of Matlab (you can use another language if you prefer).

(1) Show that $A^\dagger = V_1 \hat{\Sigma}^{-1} U_1^*$, the (Moore-Penrose) pseudoinverse of A, satisfies the four Moore-Penrose conditions on matrix X:
   (a) $AXA = A$
   (b) $XAX = X$
   (c) $(AX)^* = AX$
   (d) $(XA)^* = XA$

(2) 12.1

(3) Show for full rank $A \in C^{m\times n}$ and $x \in \mathbb{C}^n$, $x \neq 0$, that

$$\sigma_1 \geq \frac{\|Ax\|_2}{\|x\|_2} \geq \sigma_n,$$

where σ_1 and σ_n are the largest and smallest singular values of A.

(4) Let Cond(A) be the condition number of matrix A. Show for nonsingular matrices A, B that $\mathrm{Cond}(AB) \leq \mathrm{Cond}(A)\,\mathrm{Cond}(B)$.

(5) (a) Give two matrices A, B for which

$$\mathrm{Cond}_2(AB) > \mathrm{Cond}_2(A)\,\mathrm{Cond}_2(B).$$

   (b) Give two matrices A, B (possibly the same ones you used above) that satisfy $\left\|(AB)^\dagger\right\| \neq \left\|B^\dagger A^\dagger\right\|$, where $A^\dagger$ is the pseudoinverse of A.

Not required, but if you are interested:

- Use Householder matrices to show $\det(I + x^T y) = 1 + xy^T$ for given vectors $x, y \in \mathbb{C}^n$.

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