

2. From network 1 in Figure 2a, we can give:

$$\vec{a}^{(1)} = w^{(1)} \vec{a}^{(0)} + \vec{b}^{(1)} \quad (1)$$

$$\vec{a}^{(2)} = w^{(2)} \vec{a}^{(1)} + \vec{b}^{(2)} \quad (2)$$

$$\vec{a}^{(3)} = w^{(3)} \vec{a}^{(2)} + \vec{b}^{(3)} \quad (3)$$

For the network 2 in Figure 2b:

If network 1 and network 2 are said to be equivalent,  $a^{(0)}$  is the input of network 2 and  $a^{(3)}$  is the output of network 2

Then, we can give:

$$\vec{a}^{(3)} = \tilde{w} \vec{a}^{(0)} + \tilde{b} \quad (4)$$

From Eq. (1) (2) (3), we can give:

$$\begin{aligned} \vec{a}^{(3)} &= w^{(3)} \vec{a}^{(2)} + \vec{b}^{(3)} \\ &= w^{(3)} (w^{(2)} \vec{a}^{(1)} + \vec{b}^{(2)}) + \vec{b}^{(3)} \\ &= w^{(3)} [w^{(2)} (w^{(1)} \vec{a}^{(0)} + \vec{b}^{(1)}) + \vec{b}^{(2)}] + \vec{b}^{(3)} \\ &= w^{(3)} w^{(2)} w^{(1)} \vec{a}^{(0)} + w^{(3)} w^{(2)} \vec{b}^{(1)} + w^{(3)} \vec{b}^{(2)} + \vec{b}^{(3)} \end{aligned}$$

contrast with Eq. (4), we can get:

$$\tilde{w} = w^{(3)} w^{(2)} w^{(1)}$$

$$\tilde{b} = w^{(3)} w^{(2)} \vec{b}^{(1)} + w^{(3)} \vec{b}^{(2)} + \vec{b}^{(3)}$$