Oscillator Phase Noise: A 50-Year Review

David B. Leeson, *Life Fellow, IEEE*

(Invited Paper)

***Abstract*—Fifty years ago emerging requirements in oscillator applications led to the 1964 IEEE-NASA Symposium on Short-term Frequency Stability. Following that, IEEE Technical Committee 14.7 was established to unify time- and frequency-domain definitions of frequency stability. I had the good fortune to participate and contribute as a member of the symposium program committee and the IEEE committee. This paper is a personal retrospective of events that are said to have shaped our field: the 1964 Symposium, the 1966 *Proc. IEEE* special issue on frequency stability we edited (with comments on my oscillator-model paper), and our 1971 paper, "Characterization of Frequency Stability," written as a basis for IEEE STD 1139.**

***Index Terms*—1964 IEEE-NASA Symposium, frequency stability, history, phase noise, review, short-term stability**

I. Introduction

Oscillators, the sources of signals in electronic systems for time keeping, radio communications and radar, are characterized by frequency stability. I had the good fortune fifty years ago to join a new standards project and publish a brief paper defining oscillator phase noise [1] that has been said to be among those that "helped shape the field" [2]. Today the understanding of both noise-like and environmentally induced frequency instabilities in oscillators is rigorous and readily applied. Information on the subject is widely accessible. Some 20 000 citations appear in searches for "phase noise" (16 000) [3] and "frequency stability" (4000) [4].

Standards such as IEEE STD 1139 and CCIR Recommendation No. 686 offer concise definitions for the interpretation of measurements and optimization of performance [5]–[7]. The subject is well treated in published texts [8]–[12], handbooks [13], reports [14]–[17], journal papers [18]–[27], academic theses and online sites [28]–[31].

This advanced state of affairs has, of course, not always existed. Fifty years ago, differing applications had arisen in relative isolation, without sufficient interaction to consolidate basic concepts and terminology. The current beneficial outcome was formally initiated then by a unique IEEE-NASA symposium and an IEEE standards project intended to unify time- and frequency-domain definitions of frequency stability. I had the opportunity to participate in and contribute to those efforts. The present paper, an expansion of the author's 2015 IFCS presentation [32], offers my personal recollections and comments on a process beginning fifty years ago that ultimately led to the current understanding and standards.

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The author is with the Department of Electrical Engineering, Stanford University, Stanford, CA 94305 USA (e-mail: leeson@stanford.edu).

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Two generations ago was a time before personal computers, before integrated circuits, before the Internet and digital archiving, before email, before cell phones and before GPS. It was the height of the Cold War. The 1957 launch of Sputnik had initiated the space age, leading directly to the creation of NASA in 1958.

In the development of its Deep Space Network, NASA had a vital interest in aspects of frequency stability that were not being addressed sufficiently in traditional symposia or publications. This led it to initiate its own symposium, held with the IEEE in the late autumn of 1964, to bring together specialists in the field to integrate knowledge around the short-term regime.

NASA's immediate questions, and those of the community in general, were largely answered in the symposium and the ensuing IEEE Proceedings special issue and summary paper. This unique series of events fostered the current state of the discipline. Traditional symposia and journals included the additional issues in their agendas, so the need for a specialized meeting had been met and no more have been held.

Progress in many important areas has been promoted by a half-century's exchange of viewpoints and techniques, accelerated and expanded by the revolutions in semiconductors, computing and communications. Today's vigorous research and publication efforts continue to explore a host of emerging questions.

II. Phase Noise Fundamentals

A. The Two Fourier Domains

Today almost the entire phase-noise picture can be reduced to a few simple expressions and graphics. For the purposes of this history, only the most general points are outlined, as the reader may find in the literature as much detail as desired.

Disregarding amplitude variations, the output voltage of a stable oscillator is modeled

$$V(t) = V_0 \cos[2\pi \nu_0 t + \phi(t)] \quad (1)$$

where ν_0 is the nominal frequency and $\phi(t)$ is the instantaneous phase deviation from the nominal. The output of an ideal phase detector would be proportional to $\phi(t)$.

Drawing on the well-known Fourier-transform duality between time and frequency expressions, the noise fluctuations of the phase may be expressed in either frequency- or time-domain form, ideally with the ability to convert readily from one to the other. Note that a mathematical model is not identical to the physical reality it approximates ("The model

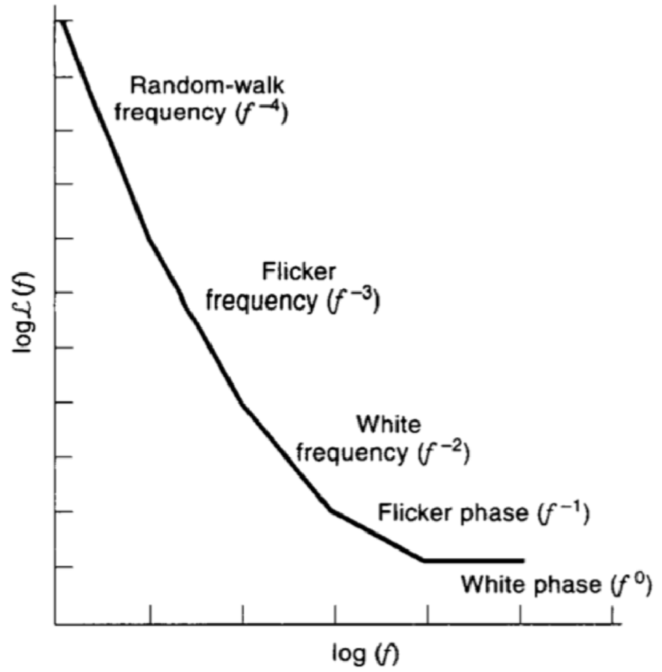

Fig. 1. Log-log plot of $\mathcal{L}(f) = S_\phi(f)/2$ vs. f, showing segments, reprinted from NIST Pub. 1286 [33]. Work of U.S. Govt. not subject to copyright.

is not the thing"), and practicality tempers mathematical rigor ("Experiment trumps theory").

B. Frequency Domain

The power spectral density (PSD) $S_\phi(f)$ of the phase $\phi(t)$ due to random noise appears in the frequency domain as a truncated power-law sum

$$S_\phi(f) = \Sigma b_\beta f^\beta \tag{2}$$

where f is the Fourier frequency and β ranges from $-4 \leq \beta \leq 0$. This is a continuous differentiable function that occurs in practice and is compatible with the inverse Fourier transform.

The frequency-domain section of IEEE STD 1139 [5] defines the preferred measure of phase noise as

$$\mathcal{L}(f) = S_\phi(f)/2. \tag{3}$$

$\mathcal{L}(f)$ is seen, for the common case $|\phi| \ll 1$ and AM $\ll$ FM, to be a useful approximation to the RF spectrum that would be observed on an ideal spectrum analyzer.

This frequency-domain definition is widely used in applications that have historically concentrated on the region of the RF spectrum conforming to the small-angle and minimal-AM assumptions. $\mathcal{L}(f)$ is commonly presented as in Fig. 1, the log-log plot of $\mathcal{L}(f)$ vs. f, which displays the power-law segments of Table I.

Individual segments, each due to a specific noise process, are seen to cross over with others of different slope, which then become dominant in their regime. It should be noted that this is a conceptual model, in that some segments may be masked or absent in a physical device.

In view of the equivalence between phase and frequency modulation, the PSD can be expressed in several equivalent forms. To avoid possible errors in frequency multiplication, the normalized frequency fluctuation $y(t) = (1/2\pi\nu_0)\,(d\phi/dt)$ leads to the similarly segmented but multiplier-invariant PSD

$$S_y(f) = (f^2/\nu_0^2)\ S_\phi(f). \tag{4}$$

Phase noise from power supply, frequency modulation inputs and vibration or acoustic exposure has long been recognized as a phenomenon that can potentially override quiescent noise performance [16], [17]. While this topic represents a central aspect of the author's own experience, and the literature is extensive, it is outside the scope of this paper.

C. Time Domain

The time-domain definition of stability over a time interval τ is a specific two-sample variance $\sigma_y^2(\tau)$ of normalized random frequency $y(t)$ known as the Allan Variance, for which a suitable model of physical noise processes presents as a truncated power-law sum of the form

$$\sigma_y^2(\tau) = \Sigma h_\mu \tau^\mu \tag{5}$$

where μ ranges from $-3 \leq \mu \leq 1$.

The Allan Variance can be derived from, and exists for, all spectral density power laws encountered in physical oscillators. This was recognized as the basis for direct conversion from frequency domain measurements to time domain. A modified form denoted mod $\sigma_y^2(\tau)$ is better suited for making explicit this relationship. This modified form is shown in Fig. 2, a log-log plot of mod $\sigma_y^2(\tau)$ vs. τ, whose segments correspond to the exponents of the PSD $S_\phi(f)$ [88].

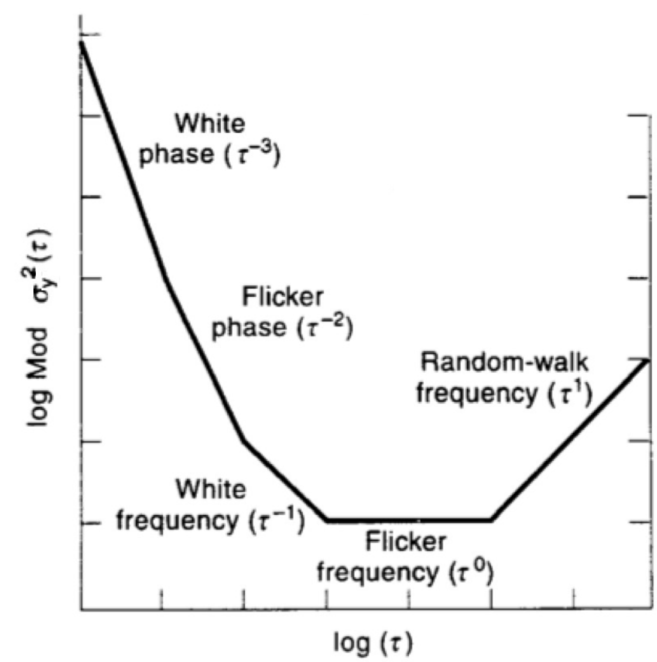

Fig. 2. Log-log plot of mod $\sigma_y^2(\tau)$ vs. τ, showing segments, reprinted from NIST Pub. 1286 [33]. Work of U.S. Govt. not subject to copyright.

TABLE I
NOISE PROCESSES WITH CORRESPONDING EXPONENT MAPPING

Noise Type	$S_\phi(f) = \Sigma b_\beta f^\beta$ Exponent β	mod $\sigma_y{}^2(\tau) = \Sigma h_\mu f^\mu$ Exponent μ
White phase	0	-3
Flicker phase	-1	-2
White frequency	-2	-1
Flicker frequency	-3	0
Random-walk frequency	-4	1

D. Conversion

A final point of this simplified introduction is the facility of conversion between $S_\phi(f)$ or $S_y(f)$ and $\sigma_y^2(\tau)$. The coefficients are related by known constants, so the conversion is directly accomplished. Although not all segments (noise types) may be explicitly observed for a given physical instance, each segment of $S_\phi(f)$ maps directly into a corresponding segment of mod $\sigma_y^2(\tau)$. It can be seen that over longer times, the low-frequency terms of phase PSD dominate. Note in particular that flicker of frequency sets a floor on the variance. As with other modern engineering and mathematical computations, computer applications are available to execute the process automatically.

Table I shows the mapping of the exponents between $S_\phi(f)$ and mod $\sigma_y^2(\tau)$. The conversion process is detailed widely in the literature [8], [9], as is the derivation of jitter from $S_\phi(f)$.

III. FREQUENCY STABILITY UP TO THE 1960'S

A. Frequency Stability Before 1940

The current enlightened state of affairs did not exist fifty years ago. Before 1940, physics and radio made use of frequency standards such as the WWV stations that employed the best quartz crystal techniques of the day. Instrumentation followed to provide portable metrology. During this period a key objective was to maintain sufficient stability to avoid interference in broadcasting and point-to-point radio links, a general problem that had plagued radio since its infancy. Users expected sufficient frequency stability to confine a signal to the ultimate receiver filter width that would exclude other signals and noise. Long-term aging and the effects of temperature received principal attention then. Frequency was measured by reference to the transmitted or portable standards, and was recorded as a data time series to determine stability.

B. Channelized Radio Systems in WWII

The need for large quantities of quartz crystals arose with the introduction of channelized radio communications for mobile and airborne warfare in WWII. A new objective arose: to be able to find and communicate with a correspondent on the same channel without manual tuning, over extended time and in the presence of more stringent environments. Since higher frequencies were now in use to offer additional spectrum and preclude interception via ionospheric skip, a greater degree of precision and stability was required. Measurement of RF spectrum concentrated largely on evaluation of modulation and distortion, or of adjacent-channel interference. A comprehensive effort identified and resolved problems of volume manufacturing and aging [34], [35].

C. Developments After WWII

After the war's end, promising new developments such as semiconductor devices, quantum-physics frequency and timekeeping devices, television, mobile radio communication, microwave Doppler radar and even space rocketry were ripe for development.

In particular, new navigation systems were implemented, great strides were made in quartz crystal and atomic oscillator technology, microwave Doppler radar was developed and fielded, and frequency modulation was put to use. These introductions exposed new gaps in the knowledge of frequency and time stability measures and techniques. During this time the issues of synchronizing color television receivers were also met and resolved, and linear particle accelerators achieved the phase stability necessary for their function.

1) Navigation Systems: Navigation and mapping have historically been implemented by means of a time reference to known points. Aircraft navigation and radar ranging are key examples from the period of interest. Over time, by applying advances in precision oscillators, substantial improvements were made in navigation systems such as LORAN.

2) Precision Quartz Crystals and Atomic Standards: Increasingly stringent requirements for long-term frequency stability led to extensive efforts to improve the performance of quartz crystals and oscillators. Major advances were made in the stability of quartz-crystal time and frequency standards. This was followed by the introduction of atomic frequency standards, which increased long-term stability by orders of magnitude.

The requirements for long-term frequency and time stability led naturally to the use of time-domain measures in this community. A principal focus was the establishment and measurement of stability over extended time, corresponding to gradual variations. Very short-term effects were generally considered to average out, and so were deemed less consequential. Frequency and time were represented as time series of data. This is seen in the terminology, literature and standards before the 1960's. This school of long-term time and frequency stability constituted what might be termed the "time-domain guild."

3) Doppler Radar: The WWII MIT Radiation Laboratory (the "Rad Lab") was created to develop microwave radar, which was required for airborne applications and to solve the "under the radar" problem of longer-wavelength radar [36], [37].[1] While other government laboratories were already active, the Rad Lab was founded in recognition that a new organization can be innovative because it is not bound by tradition (the later NASA mandate was similar).

The work of the Rad Lab, detailed in its monumental 28-volume series, introduced and initiated the development of stability techniques to a greater degree than may be

[1]The literature of WWII radar is vast. Among the most comprehensive references are [38]–[40].

recognized today. The topic "local oscillator noise" appears in a number of the volumes [41], and R. V. Pound's stabilization methods offered utility that persists to the present [42].

Radar range is limited by the masking of target signals by returns, termed clutter, from the much larger reflectors such as ground, sea, rain or even bird flocks. Doppler radar achieves subclutter visibility by using Doppler frequency to filter moving target returns from clutter. Instabilities of system oscillators result in spectral components that, when imposed on strong clutter returns, can mask targets. Hence there is a dynamic range limitation associated with the RF spectrum of short-term oscillator instabilities. Interest focused on the spectrum of oscillator instabilities in the range of ten's of kHz, since Doppler shift of aircraft is of that order at microwave frequencies. A related dynamic range limitation, now termed "near-far" noise masking of adjacent channels, has existed in multi-channel communications systems.

4) Frequency Division Multiplexed Telephony: In FM systems, the relationship between the modulating function and the RF spectrum was widely investigated, and residual FM oscillator noise at the output of frequency demodulators was identified as a contributor to overall system noise. The equivalence was noted between modulative and additive descriptions of the same spectrum phenomena.

This period saw the development of frequency division multiplex (FDM) systems for long-distance transmission of multiple voice channels. A carrier was frequency modulated with a composite noise-like signal composed of many individual single-sideband (SSB) channels.

Well-established terminology and standards such as EIA RS-250-1961 were unique to telephony [43]. The stability requirements of the baseband channel frequencies were stringent, but a greater problem for FM and FDM multiplex design was residual noise due to oscillator instability. Telephone microwave-radio engineers of the time were familiar both with the presence of oscillator FM noise and its representation as the output of a frequency discriminator.

In order to avoid the higher noise found at lower baseband frequencies, the Fourier frequency range of interest for multiplex began at 56–60 kHz, a comparable range to that of Doppler radar. A wider range of baseband frequencies was required for transmission of television signals over the same networks, but in general a system that met telephony requirements also met television requirements.

The Doppler-radar noise masking and FDM residual noise were identified with short-term random variation of oscillator frequency or phase. Thus Doppler radar and FDM telephony represented two related schools that together were the principal constituents of the "frequency-domain guild."

D. Frequency Stability at the Beginning of the 1960's

We see that by the 1960's, applications of stable oscillators fell into two broad classes. One, precision time and frequency standards and metrology, found the expression and measurement of frequency instabilities most natural in time-domain terms. The other, multi-signal systems such as Doppler radar and radio communications, with their dynamic range limitations due to spectrum, turned to frequency-domain definition and measurement as more applicable. Many systems were newly exposed to more stringent environments, as well.

TABLE II
REFERENCES TO "PHASE NOISE" OR "FREQUENCY NOISE"

Technology	Reference
Doppler radar	Hansen 1947 [57]-[60]
Linear particle accelerators	Hintz 1954 [61]
Television receiver color reference	Richman 1954 [62]
Deep-space phase lock systems	Jaffe, Rechtin 1955 [54]
Frequency-division multiplex	Bennett 1956 [63],[64]
Laser coherence and linewidth	Shimoda et al. 1956 [65]

The tools appropriate for each application had evolved along divergent paths. Long-term and time-domain tools were applied to time keeping and frequency standards, while short-term and frequency domain concepts were prominent in radar and telephony.

The annual Symposium on Frequency Control, sponsored for many years by the U.S. Army, and then by the IEEE, had served well as a common forum. Many frequency-domain papers of this period focused on the RF spectrum of the oscillator itself, or on linewidth, rather than the power spectrum of phase [44]–[51]. Predicted theoretical spectra were not in complete agreement with observed spectra [52].

E. First Instances of the Phrase "Phase Noise"

The emergence in the literature of the phrase "phase noise" or its close synonyms would be a measure of the interest then in oscillator phase or frequency noise. In answer to a query to me, "What is the first use of the phrase 'phase noise' in the literature?" a cursory search turned up publications from 1954–1956 and earlier, shown in Table II [53]. In particular, a notable early instance of the specific phrase "phase noise" appears in the 1955 phase-lock paper by R. Jaffe and E. Rechtin that led to NASA's deep-space communications [54] (see also [55]).

IV. NASA Deep Space Network Requirements

One of the key issues for NASA at the beginning of the space age was how to implement a control and data link to a distant planetary probe. After exploring the limits of all the variables, such as antenna size, transmitter power, receiver noise and onboard power, it was determined that a brute force approach would be orders of magnitude beyond possibility.

In the words of Eberhardt Rechtin of Caltech's Jet Propulsion Laboratory (JPL),

> We were confronted with the problem of communicating to the edge of the solar system. I was told by Nobel Prize winners that it would not be possible really to communicate to the edge of the solar system. If you could, you couldn't send back enough interesting information. You would have to have bandwidths of your receiving system wide enough to account for the Doppler shifts. If you did that, you had to send megawatts of power back from enormous antennas at the edge of the solar system.

TABLE III
NASA IN THE "NO MAN'S LAND" BETWEEN "GUILDS"

	Time domain		Both		Frequency domain	
<1960	Frequency & Time		"No man's land"		Radar & Telephony	
1964			NASA DSN			
>1970	Digital communications					
f Hz	10^{-6}	10^{-4}	10^{-2}	10^{0}	10^{2}	10^{4} 10^{6}

> "Wait a minute. We could track the Doppler." There is no reason we have to have a band width equivalent to the maximum spread ... If you could find the position and the velocity and the acceleration, the total number of bits that you were looking for in finding the carrier signal was very small. So we built phase lock receiving systems that had the equivalent bandwidth at ten gigahertz of ten cycles. [56]

This extreme narrow-band concept was at the root of the successful Deep Space Network (DSN) conceived and built by NASA to provide control uplinks and data downlinks, plus tracking of position, rate and acceleration.

Phase noise at baseband frequencies that had previously been ignored in prior applications was identified as a key limitation on the use of narrow phase lock loops to receive data [66] (see also [67] and [68]) But NASA also required superb long-term stability for long missions. With this combined requirement, NASA became the driving force to merge the knowledge of the two "guilds" to fill the "no man's land" depicted in Table III.

An additional impetus for this Fourier frequency range came from work in atomic frequency standards, where interest arose in "phase noise in the range of Fourier frequencies, f, of about 10^{-2} Hz to 10^{3} Hz, since most electronic servo systems in existing atomic frequency standards use modulation frequencies which fall within this range [69]."

V. STEPS TOWARD A STANDARD

A. IEEE-NASA Symposium on Short-Term Frequency Stability

The emergence of communications and ranging techniques in the space program created a need for understanding and advances in both time and frequency domain concepts, and a means to convert from one to the other. This created a new constituency whose unique issues were not fully addressed in existing forums, where they were only one of many categories.

When this point was raised early in the Apollo, satellite and planetary exploration programs, it became apparent to the several communities that they were experiencing the parable of the blind men and the elephant, each focusing on a different aspect of the whole picture, and that specific effort would be required to pool the independent reservoirs of knowledge.

Recall that this was the time of the highest tension of the Cold War, and of the surprise 1957 launch of Sputnik during the International Geophysical Year program. Sputnik marked the beginning of the Space Age, and it precipitated a crisis of confidence in the political and military leadership of the US [70]. Science and education quickly came to the fore of American attitudes and priorities [71].

NASA was created out of the national reaction to Sputnik and the failure of US military programs to launch a US satellite in response. JPL's Explorer proved to be the first US success, enhancing the standing of civilian organizations. NASA was created in the summer of 1958, and formally opened for business that October.

NASA was culturally inclined then toward independence and impatience, rather than tradition. It was imbued with a spirit that it had urgent work, and that other organizations would not be part of it. It was completely in character when NASA decided that, rather than work through any existing forum, it would create and fund its own meeting to get the answers to its specific needs in support of space exploration. If that helped advance the general state of the art, so much the better. The emphasis on "short-term" reflected the critical dependence of the DSN on the sub-audio Fourier regime, which was dominated by flicker noise and which had received less attention in the past.

1) The Symposium Program Committee: The first step of the program to craft a standard that would broadly define frequency stability was to understand and meld the frequency- and time-domain descriptions of phase instability to a degree that was mutually accepted, and that permitted analysis and optimization. To promote focused interchange in extension of NASA's own research, A. R. Chi of NASA Goddard Space Flight Center proposed a symposium to further these ends. Chi expressed his mandate, "The need for clarification in this field has been recognized for some time. [The symposium will offer] a unique opportunity for cross-fertilization of ideas [72]."

The urgent NASA interest in finding common terminology for oscillator and system specifications found fertile ground in the IEEE. The symposium concept gained the support of the chairman, J. H. Armstrong, and an influential member, R. A. Sykes, of the IEEE Technical Committee, Standards 14—Piezoelectric and Ferroelectric Crystals. Joint IEEE-NASA sponsorship was agreed upon, and a program committee was formed to plan and implement the IEEE-NASA Conference on Short-Term Frequency Stability, to be held at Goddard Space Flight Center on November 23 and 24, 1964.

It was at this time that I received an invitation to join this program committee through the efforts of a mentor and sponsor, W. K. Saunders of the Harry Diamond Laboratory, who was familiar with my prior publications and my work on pulse Doppler radar at Hughes Aircraft Co.

My early contact with oscillator noise had come as solid-state signal sources began to be applied to the airborne pulse Doppler radars that had been under development since the days of the MIT Radiation Laboratory [57, Sec. 5–11, pp. 150–159][2] (see also [58], [59, p. 292], and [60]). I had been initiated into the phase-noise requirements of these radar systems as we applied the nonlinear frequency multipliers of my graduate theses [73], [74].

[2]See Sec. 5–11 (CW Radar Systems) in Ch. 5 (Radar System Engineering), particularly p. 153.

TABLE IV
MEMBERS OF 1964 PROGRAM COMMITTEE, IEEE SUBCOMMITTEE 14.7 AND G-I&M TECHNICAL COMMITTEE ON FREQUENCY AND TIME

Name	Symposium Program Committee 1964 [79]	Proc. IEEE Editorial Committee 1966 [80]	Trans. I&M Paper Co-authors 1971 [109]	Organization
J. H. Armstrong	✓			IEEE/TC-14
E. J. Baghdady	✓	✓†		ADCOM
A. S. Bagley	✓			Hewlett Packard
J. A. Barnes	✓	✓	✓*	NBS Boulder Laboratories
K. M. Brown	✓			General Dynamics
A. R. Chi	✓*	✓*	✓	NASA/GSFC
M. H. C. Criswell	✓			US Navy/Bureau of Ships
L S. Cutler		✓	✓	Hewlett Packard
C. Friend	✓			USAF/WPAFB
J. G. Gregory	✓			NASA/MSFC
G. K. Gutwein	✓	✓		US Army/ERDL, US Army Electronics Command
D. J. Healey, III	✓	✓	✓	Westinghouse
D. B. Leeson	✓	✓	✓	Hughes, Applied Technology, California Microwave
T. E. McGunigal	✓	✓	✓	NASA/GSFC
J. A. Mullen		✓	✓	Raytheon
F. H. Reder	✓			US Army/ERDL
W. K. Saunders	✓	✓†		Harry Diamond Laboratory
C. L. Searle		✓		MIT
W. L. Smith		✓	✓	Bell Telephone Laboratories
H. P. Stratemeyer	✓			General Radio
R. L. Sydnor		✓	✓	Jet Propulsion Laboratory
R. F. C. Vessot	✓	✓	✓	Varian Associates
F. G. Vonbun	✓			NASA/GSFC
G. M. R. Winkler		✓	✓	US Army Electr. Command, US Naval Observatory

* Chair † Consultant

As part of this program, I had been assigned to encourage the adoption of the then-new ribbon-mount cold-weld quartz crystal mount. This fostered extensive personal contact with its originators at the Bell Telephone Laboratories, as well as the crystal manufacturers and additional members of the radar community [75]–[77].

The program committee attracted members from the full range of user and instrument communities. As we sat around the table together, we saw from our own interchanges that the cross-specialty symposium would be very useful for the exchange of viewpoints and techniques that would promote convergence. The sense of the group was that the independent use of frequency-domain and time-domain definitions stood in the way of development of a common standard. We hoped to find a common language to discuss frequency stability, one that could be understood by everyone in the discipline. The initial focus was on the short-term frequency stability regime, in which there were the greatest differences of viewpoint among the multiple user communities.

2) The 1964 Conference and its Proceedings: That conference, with some 350 attendees, was an opportunity for the cross-fertilization of ideas, and featured twenty-four papers on various aspects of generation, application and measurement of short-term frequency stability. Interim Proceedings appeared in December 1964 [78]. Formal Symposium Proceedings followed in May 1965 [79]. These gave us all an insight into the ripening questions and authoritative answers of that time.

Of particular interest to me now is the record of four panel discussions, led by prominent scientists and engineers of the period. The tension between rigorous theory and practical experiment came out often, as did the concern that adoption of a single standard would leave a portion of the community without the tools for its specific applications. Also evident was the full range of individual specialization and experience, and even of personality types.

Specific questions were raised about the uncertainties of the correspondence between near-carrier linewidth, the RF spectrum, and the underlying spectral density of phase or frequency. The conundrum of the nature of flicker noise, with its lack of convergence of integrals at zero frequency, received substantial questioning and discussion. Additionally, the subject arose whether higher-order effects or amplitude noise were adequately recognized, and it was asserted that experimental evidence supported the idea that these were not material in then-current applications.

One of my own curiosities on reviewing the Symposium proceedings from the present was to identify how and where certain key concepts were conclusively identified in the papers. For example, there was substantial discussion of power-law descriptions of PSD of phase and its relationship to RF spectrum, but my review finds no graphic that explicitly showed the segments. The issue of converting from frequency to time domain was explored, but not resolved at that early time. A number of authors reported flicker noise in amplifiers and other physical devices. There were several efforts to relate the output spectrum of an oscillator to the characteristics of the resonator feedback network and the active device, but the full connection between the amplifier PSD, resonator and amplifier parameters and the full output PSD remained to be clarified.

B. The Transition to IEEE Subcommittee 14.7

The symposium was considered a success by all concerned. The chairman's message in the Symposium Proceedings announced,

> Readers of these Proceedings will be pleased to know that continuation of work leading toward the development of standards in this area has been insured by the organization within the IEEE of a subcommittee under Technical Committee, Standards 14—Piezoelectric and Ferroelectric Crystals. The Technical Subcommittee, Standards 14.7—Frequency Stability will serve as a focal point for information in this field. The ultimate aim of the subcommittee will be an IEEE standard on the definition and measurement of both short-term and long-term frequency stability. [72]

As can be seen from Table IV, A. R. Chi continued as chairman, and many of the program committee members continued as members of the new IEEE subcommittee. The committee's first work was to be as editors. I recall the work of the subcommittee as a cordial continuation of the mutually respectful interchanges among all of us. It was a wonderful opportunity for me to learn from such a distinguished group.

C. 1966 Proceedings Special Issue on Frequency Stability

In order to consolidate the gains of the 1964 Symposium and promote further exchange of information, the members of Subcommittee 14.7 were invited to serve as the editorial committee of the February 1966 Special Issue on Frequency Stability of the Proceedings of the IEEE [80]. Our efforts were supported by the distinguished consultants listed in Table V. This issue attracted many submittals, including updated papers from the 1964 Symposium, with several by committee members who were also among the most active in the field.

We were most pleased to receive the paper by D. W. Allan that settled many problems with time series variances, and also introduced the "Rosetta Stone" to convert between time- and frequency-domain measures [81]. An anecdote regarding the discovery of the technique attends this classic paper. It is not unusual for a general mathematical process to be conceived well before its usefulness is recognized for a specific application.

Allan relates that the conversion ability was identified when his colleague J. A. Barnes called attention to a concise published mathematical table, relating variance and spectral density in power-law form, on page 43 of M. J. Lighthill, *Introduction to Fourier Analysis and Generalized Functions* [82, p. 43], [83]. This was recognized by the two young NBS engineers as directly applicable to the frequency stability problem.

The techniques of analyzing time series of data are, of course, of long-standing interest in many other fields, such

TABLE V

CONSULTANTS FOR FEBRUARY 1966 PROC IEEE [98]

Name	Organization
E. J. Baghdady	ADCOM
L. S. Beedle	US Naval Observatory
R. H. Dicke	Princeton University
E. A. Gerber	US Army Electronics Command
A. M. Harned	Office of Naval Materiel
W. Markowitz	US Naval Observatory
A. O. McCoubrey	Varian Associates
R. N. Pash	US Air Force
J. M. Richardson	National Bureau of Standards
N. G. Roman	NASA
W. K. Saunders	Harry Diamond Laboratory
R. A. Sykes	Bell Telephone Laboratories
R. F. Tiner	US Army Materiel Command

as weather and economics [84]. Subsequent progress included improving the relationship between variance and spectrum by means of the modified Allan Variance shown in Fig. 2 above [9], [85]–[88].

By that time it had become accepted that spectral density of phase or frequency, rather than RF spectrum or linewidth, was the more fundamental frequency-domain measure of short-term stability. PSD could be directly related both to the time-domain definition that became identified as the Allan Variance, and it determined the RF spectrum. IEEE STD 1139 now applies the small-angle limitation in reverse, such that the RF spectrum is *defined* as half the PSD of phase *except* where the small-angle condition is not met.

In preparation for the present writing, I revisited the special issue, again curious to find when and where key points became clear. Although there were many instances of power-law spectra, I still found no full example of the now-accepted multi-segment PSD of phase. Papers that dealt with determining oscillator output PSD from input PSD were restricted to a subset of the overall question. Questions remained regarding flicker noise, nonlinearities, the interrelation of PSD of phase and RF spectrum, and AM noise.

A number of papers dealt with flicker noise, including one that specifically mentioned flicker noise in resonators. Papers on oscillator-multipliers suggested a choice of higher oscillator frequency because of the multiplication of modulation index. The radar community, responding to its typical vibration environment, was in fact finding success with the ribbon-mounted quartz crystal developed at Bell Labs [75].

D. Model of Feedback Oscillator Phase Noise Spectrum

In our final deliberations to settle the contents of the special issue, it seemed to me that we had not received a paper on frequency-domain techniques that presented the same clarity as the Allan paper on time-domain issues. I thought I could see a way to create such a paper.

The committee, in reviewing the submitted papers (many authored by the members themselves), had continued illuminating discussions of how to merge, or at least conform, the differing time-domain and frequency-domain viewpoints. We had settled on variance and spectrum of phase as our favored definitions of short-term stability. With our press date almost upon us, I raised the point that it seemed possible not only to make a clear statement of the origin and character of such spectra, but also to make a concise example through the expected performance of the quartz crystal oscillators used in radars.

It seemed to me that enough was known at that point to assemble a model that used the power-law forms of PSD, with graphics to provide additional clarity. The input and output PSD could be related by a transfer function embodying the key parameters of the active element and resonator.

By that time in our editing process, the formal papers section of the journal was essentially completed. So the only opening was space held for last-minute correspondence, which was limited to two pages. Under that limit, I was challenged to produce a concise paper modeling the phase noise spectrum of a quasi-linear oscillator in terms of resonator frequency and Q, steady-state signal power P and device noise FkT.

1) The White-Noise Model: In the 1964 and 1966 papers I had coauthored, I had previously proposed a simple model for the white noise spectrum regime outside the resonator bandwidth.

> Short-term instabilities in the oscillator are related to the signal-to-noise ratio at the input of the oscillator amplifier. For modulation rates higher than the oscillator feedback loop bandwidth, an approximate calculation of signal-to-noise ratio can be made. If the signal level at the oscillator input is −10 dbm and the thermal noise level including a 4-db noise figure is −140 dbm/kc, the best expected signal-to-noise ratio is 130 db referred to 1 kc/sec bandwidth far from the center frequency.[3] In practice, this limit is not achieved; and noise level increases rapidly as the carrier is approached. The role of drive level in determining signal-to-noise ratio is fairly clear; the highest drive level consistent with long-term stability requirements appears to give the best signal-to-noise ratio. The effect of noise enhancement by multiplication prompts the choice of a high oscillator frequency for a Doppler radar—this generally has been upheld in practice. [89, p. 6], [90, p. 246]

This calculation of FkT/P_s, along with the comments about the frequency behavior of the spectrum and the effect of multiplication, prefigured the oscillator phase transfer function in the two regimes, above and below the feedback network bandwidth, of the 1966 phase noise model. A full transfer function would combine this regime with the existing representations of noise performance that were limited to the frequency regime within the resonator bandwidth [43]–[50].

2) The Nonlinearity Question: The Penfield paper selected for our special issue showed that, subject only to conditions that were typically met in oscillators, small AM and PM noise in a nonlinear circuit driven by a periodic input could be treated as strictly linear and stochastic, and thus could be described in terms of spectral densities [91]. Penfield and I had been graduate students together at MIT in 1959, sharing a thesis advisor, C. L. Searle. Searle was also a member of our editorial committee and co-author with L. S. Cutler of another highly regarded paper our editorial committee had accepted [92].

My own publications on nonlinear circuits had been initiated with Prof. Searle's encouragement at MIT. I sensed that a simple quasi-linear model of phase noise as a small perturbation of the oscillator steady-state signal, even in a nonlinear oscillator, could have broad applicability.

3) The Resulting Paper: That was the origin of my 1966 paper on the oscillator noise model [1]. Looking back, I am very satisfied with what I was able to shoehorn into the two pages. That short paper was a last-minute effort, submitted after helpful discussions with colleagues on December 29, barely a month before our publication date. IEEE Proceedings correspondence was not archived then, so for a number of years the paper remained obscure except to insiders. During that same period I was fully occupied with founding and managing a new company, so the paper led something of a life of its own [93]–[96]. I am pleased to find its continuing utility has raised it now to the most cited paper in the "phase noise" category [97].

Much later, novel requirements and solutions would arise from the emergence of the integrated circuits and digital techniques that would completely reshape what was possible in electronics, and would require enhanced thinking. In the intervening fifty years there have been advances in the clarity with which the concepts could be expressed, and the original model has been extended to new frequency-determining and active elements. Many of the questions about nonlinearity, RF spectrum and flicker noise have been resolved through experiment, physical argument or mathematical rigor. Building on the foundation of the past, many promises of greatly improved frequency stability have been realized.

VI. Derivation of the Phase Noise Model

The 1966 simple model of phase-noise [1] will be referred to here as the "model paper," and all quotations in this section are taken from it unless otherwise noted. At the time, the familiar example of a VHF overtone crystal oscillator was the basis for the model paper, with the expectation that the result would have more general applicability.

The intent of the model paper was stated:

> A basic requirement on an oscillator noise model is that it show clearly the relationship of the spectrum of the phase $S_\phi(\omega_m)$ to the known or expected noise and signal levels and resonator characteristics of the oscillator. A simple picture can be constructed using a model of a linear feedback oscillator. Minor corrections to the results are necessary to account for nonlinear effects which must be present in a physical oscillator.

[3] Note that "Hertz" as the unit of frequency was formally adopted in 1965, so many papers from this period used the prior nomenclature. Also, at the time the common abbreviation for decibel was db rather than dB.

A. Frequency-Domain Stability Measures

In the derivation of the phase-noise model, I focused on $S_\phi(\omega_m)$ and $S_{\dot\phi}$ (ω_m), along with the spectral density of Δf_{rms} used in FDM telephony. Here ω_m was taken as the Fourier radian frequency associated with the noise-like variations in $\phi(t)$. The RF spectrum was denoted as $G(\omega\text{-}\omega_o) = S_\phi(\omega_m)/2$, subject to the limitations that $\langle\phi^2\rangle \ll 1$ and AM $\ll$ FM.

The choice of PSD in the model paper was described:

> Consider a stable oscillator whose measurable output can be expressed as $v(t) = A\cos[\omega_0 t + \phi(t)]$. It is common to treat $\phi(t)$ as a zero-mean stationary process describing deviations of the phase from the ideal. The frequency domain information about phase or frequency variations is contained in the 'power' spectral density $S_{\dot\phi}$ (ω_m) [sic] of the phase $\phi(t)$ or, alternatively, in the 'power' spectral density $S_\phi(\omega_\mathrm{m})$ [sic] of the frequency $\dot\phi$.

Note the two typographical errors: The symbols for PSD, S_ϕ and $S_{\dot\phi}$, were interchanged in typesetting, although fortunately the meaning could be determined from the context.[4]

The evolution to $S_y(f)$ as the preferred stability measure has been noted above, as has the redefinition of $\mathcal{L}(f)$ to represent a more tractable approximation to the RF spectrum. The script $\mathcal{L}(f)$ formerly was used to denote the RF spectrum, but because RF spectrum is not unambiguously related to $S_\phi(f)$ if the limitations are exceeded, the symbol $\mathcal{L}(f)$ was later redefined in IEEE Std 1139-1999 [98].[5] Failure to differentiate between the old and new definitions can lead to confusion.

The model paper did not address linewidth, nor does that concept appear in the eventual IEEE standards. By that time it was accepted that linewidth was not suitable as a definition because it predominantly represents the lowest frequency components of noise (even in the absence of flicker noise) and is not uniquely predictive of the full spectrum or variance.

B. The Internal Noise Model of Amplifier Input Phase

The PSD of the input phase uncertainty referred to the input terminal of the sustaining active element was characterized:

> For a physical oscillator the spectrum $S_{\Delta\theta}(\omega)$ of the input phase uncertainty $\Delta\theta(t)$ is expected to have two principal components. One component is due to phase uncertainties resulting from additive white noise at frequencies around the oscillator frequency, as well as noise at other frequencies mixed into the pass band of interest by nonlinearities. The second component is due to parameter variations at video frequencies which affect the phase.

So the model paper took $S_{\Delta\theta}(\omega)$ to have two components, (a) additive white noise around the oscillator frequency and (b) modulative noise, "found typically to have a power spectral density varying inversely with frequency," flicker $1/f$ noise. Other noise types (e.g., random walk) were not considered in this analysis.

The spectral density of input phase due to additive white noise was known from modulation theory to be the ratio of noise power to signal power. The modulative component of input phase spectrum was seen to result from parameter variations that modulate the internal phase at Fourier or baseband rates. This $1/f$ modulation impresses its effect on the oscillator signal without any appeal to nonlinearity, and is independent of signal amplitude. Flicker noise has been the subject of substantial subsequent inquiry, as discussed in a section VIII-B *Flicker Noise* below.

Summing the two noise components, the "total power spectral density of oscillator input phase errors is of the form $S_{\Delta\theta}(\omega_m) = \alpha/\omega_m + \beta$," where α is a constant determined by the magnitude of $1/f$ variations and $\beta = FkT_o/P_\mathrm{s}$ for one-sided spectra (noting that, then, $T \approx T_o$ and correcting an errant factor of 2). Here F was an effective noise figure, the already modest noise figure F of those days raised by "corrections necessary to account for nonlinear effects." This issue was expanded in a later section of the model paper, NONLINEAR EFFECTS.

C. Power Spectral Density of Output Phase

The key intent of the model was:

> A basic requirement on an oscillator noise model is that it show clearly the relationship of the output spectral density of phase $S_\phi(\omega_\mathrm{m})$ to the known or expected noise and signal levels and resonator characteristics of the oscillator,

These would be defined by F, P_s, Q and ω_o.

1) The Phase Transfer Function: To deduce from physical reasoning the transfer function from input phase spectrum to output phase spectrum, the paper considered the following logic:

> Assume a single resonator feedback network of fractional bandwidth $2B/\omega_o = 1/Q$, where Q is the operating, or loaded, quality factor. For small phase variations at video rates which fall within the feedback half-bandwidth $\omega_0/2Q$, a phase error at the oscillator input due to noise or parameter variations results in a frequency error determined by the phase-frequency slope of the feedback network, $\Delta\theta = 2Q\Delta\dot\phi/\omega_0$. Thus for modulation rates *less* than the half-bandwidth of the feedback loop, the spectrum of the *frequency* $S_{\dot\phi}$ (ω) is identical (with a scale factor) to the spectrum of the uncertainty of the oscillator input phase $S_{\Delta\theta}$ (ω_m).
>
> For modulation rates *large* compared to the feedback bandwidth. . .the power spectral density of the output

[4]As a note, the model paper used a dot over the symbol ϕ to denote the time derivative ("Newton's notation"). Without notice to the author, the publication staff interchanged S_ϕ and $S_{\dot\phi}$ in the first paragraph and equation of that paper. This problem also plagued other authors in our issue, and apparently continues to the present. Recalling the frustration of chasing dots in the galley proof, the author applauds the modern use of ν or Ω for frequency.

[5]The RF spectrum, which before the publication of IEEE Std 1139-1999 was denoted by $\mathcal{L}(f)$ (or $G(\omega\text{-}\omega_o)$ in the model paper), is not always unambiguously related to $S_\phi(f)$. The symbol $\mathcal{L}(f)$ was redefined to be just $\mathcal{L}(f) = S_\phi(f)/2$, avoiding the problem that the RF spectrum at high deviation is not correctly determined from the underlying $S_\phi(f)$ because the "definition breaks down when the mean square phase deviation $<\phi^2> =$ the integral of $S_\phi(f)$ from f to ∞, exceeds about 0.1 rad[ian]." [98, p. 6]

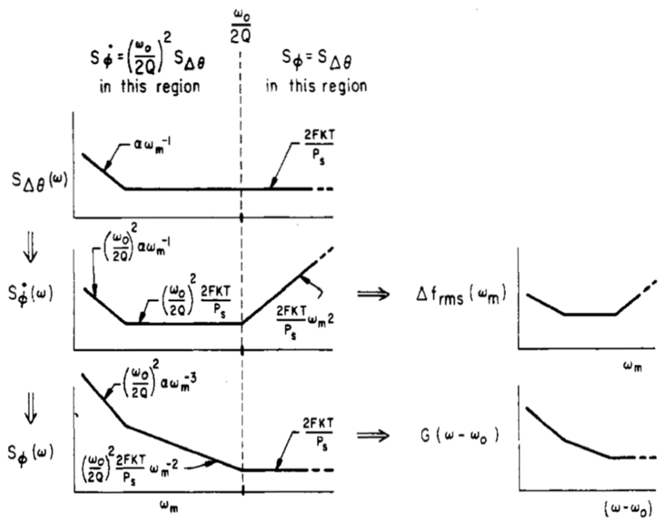

Fig. 3. Graphical derivation of output $S_\phi(\omega_m)$ from $S_{\Delta\theta}(\omega_m)|H(\omega)|^2$, reprinted from the model paper [1].

phase is identical to the spectrum of the oscillator input phase uncertainty $S_{\Delta\theta}(\omega_m)$.

Hence, interchanging $S_\phi(\omega)$ and $S_{\dot\phi}(\omega)$ to correct the typographical reversal as published, "to find $S_\phi(\omega)$ or $S_{\dot\phi}(\omega)$ we use the fact that

$$\text{for } \omega_m < \omega_o/2Q,\; S_{\dot\phi}(\omega) = (\omega_o/2Q)^2 S_{\Delta\theta}(\omega_m)$$
$$\omega_m > \omega_o/2Q,\; S_\phi(\omega) = S_{\Delta\theta}(\omega_m)."$$

Combining the two regimes leads to the relationship of the output phase to input phase uncertainty,

"A suitable composite expression is

$$S_\phi(\omega_m) = S_{\Delta\theta}(\omega)[1 + (\omega_o/2Q\omega_m)^2]."$$

From this, the phase transfer function

$$|H(\omega)|^2 = S_\phi(\omega)/S_{\Delta\theta}(\omega) = [1 + (\omega_o/2Q\omega_m)^2] \quad (6)$$

can be seen to represent a linear phase servo [99].

The PSD of output phase was then shown to be just the product of the input spectrum and the transfer function, so

$$\begin{aligned} S_\phi(\omega_m) &= S_{\Delta\theta}(\omega_m)|H(\omega)|^2 \\ &= [\alpha/\omega_m + FkT/P_s][1 + (\omega_0/2Q\omega_m)^2]. \end{aligned} \quad (7)$$

This yields an asymptotic log-log model for $S_\phi(\omega)$ in the graphic construction of Fig. 3, reproduced from the model paper. The feedback bandwidth and the breakpoint of the flicker segment are identified.

The example given was the case in which *1/f* effects predominate only for frequencies that are small compared with the feedback loop bandwidth. A text comment noted that if flicker predominates, the breakpoints would be interchanged.

It was noted that output PSD could be modified by subsequent band limiting filtering and by the noise of following amplifiers. Also, the potential was identified for coupling the signal directly from the resonator to filter the white noise component.

This model has been extended by others to include oscillators with delay-lines, filters or multiple resonators, characterized by the feedback-network group delay $\tau = \Delta\theta/\Delta\omega$ determined from the feedback-network phase slope. The effect of frequency deviations resulting from integration of phase deviations within the feedback bandwidth is common to quasi-linear oscillators [96].

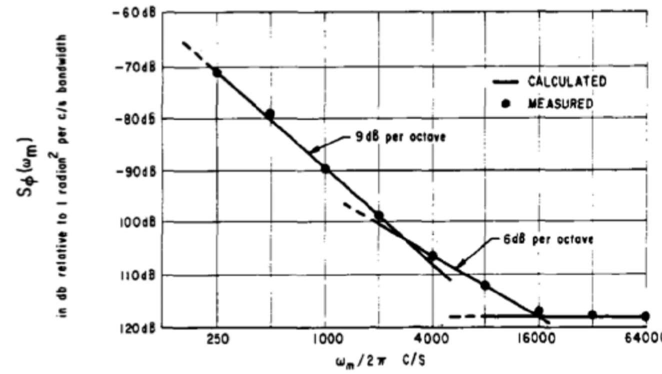

Fig. 4. Output PSD explicitly showing multiple power law segments, reprinted from the model paper [1].

2) The Fourier Frequency Range of Interest: Space systems and Doppler radar were of particular interest to the author. Space data links used narrow bandwidth, and so the low Fourier frequencies of the flicker segment were seen as critical. Radar and telephony requirements then ranged up to 100 kHz.

3) Output PSD Experimental Verification: A measurement was presented to validate the theoretical model. The model and data are compared in Fig. 4, reproduced from the original 1966 paper. The agreement was reassuring.

A key point is that this figure, which appeared in the original model paper, showed explicitly the range of power-law segments of oscillator PSD. This may be among the first instances of this now-familiar graphic form.

Measurements of this type were historically made with microwave discriminators of the Marsh and Wiltshire type [100]. This particular measurement was made using the test set described by Grauling and Healey [101]. Great advances have been made in the application of modern digital cross-correlation techniques to produce measurements that are much more sensitive and accurate [102].

4) Choice of Oscillator Frequency for Multiplication: Frequency multipliers were known to increase modulation index by a factor equal to the multiplication ratio N, so PSD is increased by N^2. My work was at 10 GHz; for a given output frequency, effective choice of oscillator frequency was vital.

The graphical construction in Fig. 5, alluded to but not included in the model paper, confirms that under the reason-

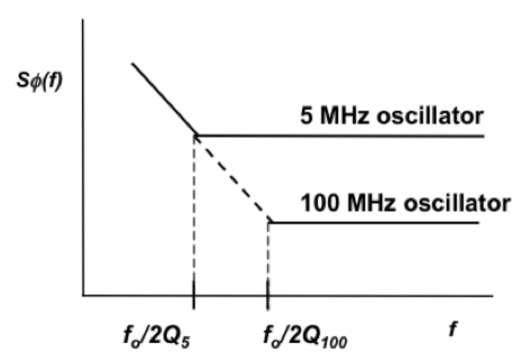

Fig. 5. Graphical construction for choice of VHF oscillator for multiplier.

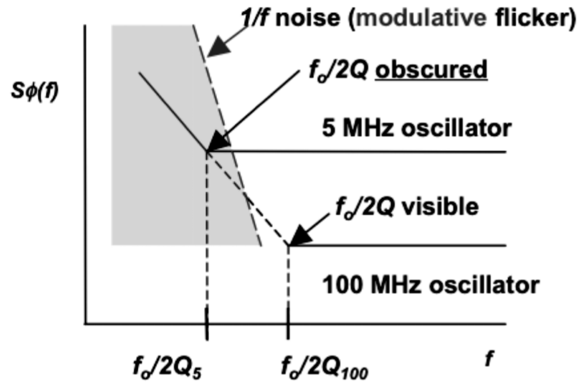

Fig. 6. $1/f$ noise obscures $f_o/2Q$ for high-Q or low-power oscillator.

able assumptions $Q \propto 1/f_o$ and FkT/P_s is constant, a higher oscillator frequency yields lower noise for Fourier frequencies above the resonator bandwidth. From comparisons such as this, it was also noted that the most favorable PSD segments of oscillators could be combined by use of phase lock loops in synthesizers, a technique now widely practiced.

5) Obscuration of White Noise by Flicker: The model paper noted that, "For a high Q oscillator, $1/f$ effects in $S_{\Delta\theta}$ can predominate out to a modulation rate exceeding ($\omega_0/2Q$)." In that case the slope transition at the resonator half bandwidth is obscured, as shown in Fig. 6. Because flicker modulation does not vary with signal power as both the f^0 and f^{-2} segments do, it can be more likely to obscure the resonator parameters in low power oscillators.

This construction reveals why resonator flicker noise was first observed on 5 MHz crystals, rather than on VHF overtone crystals. Note that, inasmuch as its slope is greater than the white-noise component in the same Fourier frequency range, once flicker noise dominates it will continue to do so to very low frequencies. Hence the white-noise segments will be obscured in this case. Also, insofar as flicker modulation in amplifiers is modulative rather than additive noise process, it is cumulative in cascaded stages, rather than being reduced by gain as seen in the Friis noise equation [103].

6) Nonlinear and Resonance Effects on Noise Figure: The small-signal noise figure F was a known parameter of the bipolar junction transistors (BJTs) used so long ago. Considering mixer noise figures, I felt that the increase due to noise mixed from harmonics of the oscillator frequency could be estimated, and the additional magnitude would not be dominant. This is shown schematically in Fig. 7. To adjust the noise figure F to reflect the presence of oscillator nonlinearity, I estimated an added 4 dB above the published small-signal value "to account for nonlinear mixing of noise at third harmonic and higher frequencies."

It has been suggested that the noise figure F was merely a fitting factor that cannot be determined independently from characteristics of the oscillator or active element. However, working with known nonlinear mixer theory it seemed possible to me to estimate a reasonable large-signal value from modeled or published small-signal specifications. I was familiar with strongly nonlinear circuits, having published circuit analyses of nonlinear reactive frequency multipliers in 1959 (harmonic

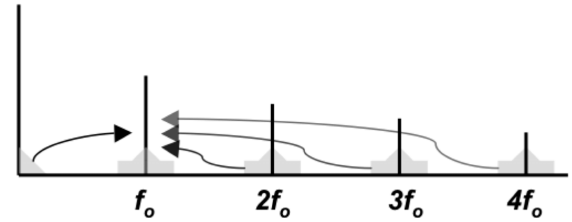

Fig. 7. Nonlinear mixing of noise from harmonics, plus $1/f$ modulation.

balance) [73] and 1962 (linear time varying) [74]. Recall that VHF transistor noise figures have improved greatly since 1965, so nonlinear effects may be more observable now.

Today, modeling software permits direct prediction of large-signal noise figure. The affect on device noise of frequency selective source impedance is now recognized. To improve estimation of the large-signal noise figure from small-signal data, Hajimiri and Lee calculated the harmonic conversion coefficients. They also pointed out, "It is critical to note that the current-to-phase transfer function is practically linear even though the active elements may have strongly nonlinear voltage-current behavior" [104, p. 182].

More recent references are supportive of the relatively modest impact of nonlinearity on BJT noise figure [105], [106, Fig. 6, p. 1514] (see also [107]), [108]. Measurements from phase noise show an increase from 5–6 dB above small-signal F to 9 dB large-signal F for example BJTs. These more contemporary analyses confirm the estimate of large-signal noise figure in weakly nonlinear oscillators.

VII. "Characterization of Frequency Stability"

A. Background for a Standard

After the papers in the 1966 Special Issue had been read and taken into account by the frequency stability community, it was felt that sufficient progress had been made that we could distill the accumulated literature and our collective viewpoints into a paper that could serve as "technical background for an eventual IEEE standard definition [109]."

In the days before the Internet and email, the writing of this paper by such an extended group involved numerous discussions and correspondence among the ten authors identified in Table IV. At the time it was determined that the group would prepare a paper to underlie development of a new standard, J. A. Barnes became the chairman of the committee. The members maintained a respectful, collegial interaction throughout our writing. Everyone involved struck me as earnest, wanting only to succeed and indifferent to who got the credit. The process was a genuinely satisfying experience of the very best kind.

Later, the committee would be transferred to the IEEE Group on Instrumentation and Measurement. From today's remove, our progress can be glimpsed in examples from my own archived records of correspondence from that time.

B. Editorial Committee Correspondence

In April 1968, Barnes sent us a brief letter on his view of principles for a definition of frequency stability. His comments at that time identified some of the remaining difficulties to be resolved:

> The measurement of frequency and fluctuations in frequency has received such great attention for many years that at first thought it is surprising that the concept of frequency stability does not have an accepted definition. At least part of the reason has been that some uses are most readily described in the frequency domain and other uses in the time domain.
>
> Often questions of covariance stationarity cannot be answered and often conventional statistical measures (such as variance) do not appear to converge toward a value as the number of samples is increased …Thus, in the most general cases, one must specify both frequency and time domain performances. [110]

Also, J. A. Mullen mailed a memorandum with many pages of bibliography for use in the forthcoming paper [111]. Then in June 1968, W. L. Smith mailed an expanded version for the "proposed 'Measurement in the Frequency Domain' section" to D. J. Healey and the other participants in our radar subset that was especially interested in frequency-domain measures [112]. Noting the dates of these letters, I was reminded that from April to June of 1968, I was riveted by the founding and funding my new company, which then claimed my full attention for the next twenty-five years.

By 1969 we were receiving reviews from our members and their associates. An extensive "Review of Specification and Measurement of Frequency Stability" arrived from Barnes' colleague D. Halford [113]. This included the outline of a lecture and laboratory demonstration on February 18–20, 1969. Halford's outline reflected progress to date. He detailed simplifying conditions and the relationship between specification and measurement, including restricting the focus to measurement of fluctuations in high-quality oscillators for which AM levels are small. He also listed the ongoing formal attempts to clarify and define stability, including the IEEE Subcommittee, and presented a thorough view of frequency- and time-domain measures that were coming to the fore.

His comments reveal the direction things were headed toward the ultimate selection of a limited range of favored measures. In the frequency-domain category he lists many different options for spectral density specification, including S_ν, S_ϕ and $\mathcal{L}$. From the viewpoint of one working with time, he put forward S_x as a "good measure." In the time domain, he listed several forms of standard deviation and variance, concluding with a recommendation "the Allan variance is a good and useful time domain measure."

By February 1970, Barnes had integrated all the various inputs, suggestions and proposals into a single draft manuscript [114]. In his words,

> The enclosed manuscript is presented for your comments. It has been revised several times during the past year by the members of the Subcommittee of Frequency Stability and, I think, represents the majority opinions of the subcommittee.

In May 1970, G. M. R. Winkler responded with a draft revision of the proposed "Standards & Procedures for the Specification of Frequency [115]." His cover letter referred to the existing draft by Barnes and the Appendix by L. S. Cutler. He commented he would expect "our 'Real SHORT TIMERS' to improve" the section on the applications depending upon spectrum characteristics. As one of the "short timers," I shared and discussed with the committee members the draft of my own paper on short-term stable microwave sources that would appear in print in June 1970 [99].

For a time-domain standard, Winkler expressed his preference for "standard deviation and not the variance" because "confidence in our estimates [would be] expressed in the same dimension as the estimate itself." In his draft, he was careful to discriminate between formal statistical functions and the estimates that would apply to actual physical situations.

For a frequency-domain definition, Winkler favored S_y and S_x. In a section referring to pitfalls, he noted that spectral density data should be band-limited, which would happen automatically at the low frequency end by the length of the measurement. He noted the need for "spectral purity" in, for example, radar applications.

C. *"Characterization of Frequency Stability," 1971*

Despite the complexity of responding to all viewpoints, we deemed the final result to be a useful step forward, and it was sent off for publication. The paper appeared in 1971 (later also published as part of an NBS technical report) [109].

On reading it at the time, I recall feeling that the preponderance of emphasis still remained on the mathematics of time-domain definitions. However, on revisiting it now in preparation for this history, I am struck by how well the paper balanced the needs of all the author communities, closing the gaps between time- and frequency-domain concepts and leaving no specialty ignored or unrepresented.

The published paper, referred to as "Barnes *et al.* 1971," reflected the previous decade's efforts to clarify and integrate the formerly dispersed knowledge of frequency instabilities:

> . . . only recently have noise models been presented that both adequately describe performance and allow a translation between the time and frequency domains. Indeed, only recently has it been recognized that there can be a wide discrepancy between commonly used time domain measures themselves.
>
> This paper attempts to present (as concisely as practical) adequate, self-consistent definitions of frequency stability . . . an important part of this paper (perhaps the most important part) deals with translations among the suggested definitions of frequency stability. [109, pp. 106–107]

The paper also hinted at the authors' general inclination toward practicality rather than rigorous theory only, noting that "the utility of statistics is in the formation of idealized models that *reasonably* describe significant observables."

Appendix I, in large part initiated by L. S. Cutler, suggested that integrals converged in band-limited spectra:

> In practice, the high frequency cutoff f_h is always present either in the device being measure or in the measuring equipment itself. The low frequency

TABLE VI
MEMBERS OF IEEE STD 1139 STANDARDS COORDINATING COMMITTEES

Name	STD 1139 -1988 [117]	STD 1139 -1999 [98]	STD 1139 -2008 [5]	Organization
D. W. Allan	✓			NBS Boulder Labs
J. C. Camparo		✓	✓	Aerospace Corp.
L. S. Cutler		✓		Hewlett Packard
C. R. Ekstrom			✓	US Naval Observatory
E. S. Ferre-Pikal		✓†	✓*	NIST Boulder Labs
C. A. Greenhall			✓	Jet Propulsion Lab
H. Hellwig	✓*			Nat'l Bureau of Stds.
P. Kartaschoff	✓			Swiss PTT
L. Maleki		✓	✓	Jet Propulsion Lab
W. J. Riley		✓	✓	E. G. & G.
S. R. Stein		✓		Timing Solutions Corp.
V. Reinhardt			✓	Raytheon
C. Thomas		✓		BIPM France
J. Vanier	✓			Nat'l Res. Council, Canada
J. Vig	✓	✓*	✓†	US Army Res. Lab.
F. L. Walls		✓	✓	NIST Boulder Labs
G. M. R. Winkler	✓			US Naval Observatory
J. D. White		✓	✓	US Naval Res. Lab.
N. F. Yannoni	✓			Rome Air Devel. Cntr.

* Chair † Vice Chair

cutoff f_l may be taken to be much smaller than the reciprocal of the longest time of interest.

In summary, in recognition that integral convergence and frequency multiplication were significant, the band-limited and multiplier-invariant form $S_y(f)$ was proposed as the frequency-domain measure of stability. The time-domain measure proposed was Allan's $\sigma_y^2(\tau)$. It took an additional decade for Barnes *et al.* 1971 to be reduced to a standard, but the knowledge was put to work almost immediately by the frequency stability community. This paper ultimately became the most cited work on the subject "frequency stability [116]."

Following this effort, the resulting standard IEEE STD 1139 was released in three subsequent versions [5], [98], [117]. With responsibilities in a new company, I was not able to be part of that process. The members of the working groups, whose considerable contributions have contributed to the current state of advancement, are listed in Table VI.

D. The Central Role of J. A. Barnes

In conclusion of this section, I wish to pay tribute to the late J. A. Barnes (1933–2002) for his key roles in editing Barnes *et al.* 1971, and in advancing the techniques of time and frequency in general. Jim Barnes was an extremely capable technologist with a gracious and understanding personality. He left no stone unturned in his central role in our editing and publication effort.

Barnes graduated from the University of Colorado at Boulder in 1956 with a B.S. in engineering physics, and began summer work at the National Bureau of Standards (NBS), now the National Institute of Standards and Technology (NIST). In 1958, he received his M.S. degree from Stanford University and became a fulltime employee at NBS. He received a Ph.D. in physics in 1966 from the University of Colorado, and an MBA from the University of Denver.

Jim became the first Chief of the Time and Frequency Division when it was created in 1967, and set the direction for this division during his 15 years of leadership. Barnes was the mentor and inspiration of a generation of NBS technologists. Characteristically, he never sought recognition for Allan's work, instead allowing his protégé to receive the full credit. Barnes left NBS in 1982 to join Austron, Inc. He passed away in 2002 [118], [119].

E. The Eventual Outcome

The work of the separate "guilds" went on as before, but they had proven willing to share their viewpoints to synthesize what NASA and others were looking for. Our meetings were largely devoted to each specialty appreciating why the other needed what they used, and how it fit the bigger picture. Then the applicable part of what was learned was put to work.

For example, the timekeeping specialists benefited from the new attention to the impact of flicker noise and the short-term regime to expand the range of stability measures and realize advances in overall stability. The radar community continued its focus on efforts to reduce the noise in the Fourier range of interest, ultimately exploring new types of resonators to its benefit. Telephony retained its focus on its own specialized terminology and techniques until digital communications and mobile wireless emerged. In time, advances in long-term stability, especially in atomic frequency standards, enabled development of such diverse systems as synthetic-aperture and planetary radars, cellular telephony, and GPS.

Ultimately the knowledge was expanded to be applied to the needs of the then new digital world and its semiconductor integration. But the synthesis of how to characterize frequency stability arose out of the cooperation among us that was fostered by NASA and the IEEE, resulting in a symposium, publications and ultimately a standard that defined the field.

VIII. PERSISTENCE OF THE PHASE NOISE MODEL

A. Areas of Questions and Progress

Over time, questions in the following areas have been raised about the limits of applicability of the simple model:

1) Flicker noise in devices and resonators
2) Obscuration of white noise by flicker noise
3) Noise Integral Convergence
4) Near-carrier limits of conversion from PSD to RF
5) Effect of AM noise
6) Frequency selective source impedances
7) Nonlinear effects on noise
8) Alternative frequency-determining elements

Many of these concerns were treated in the original paper, or have been found not to have practical impact. Investigation of others has resulted in expansion of the body of knowledge.

B. Flicker Noise

Flicker noise appears in many different physical systems [120]. The nature and effect of flicker noise has been the subject of substantial attention in subsequent years. Flicker noise is seen as setting a floor on the Allan Variance of an oscillator.

1) Device Flicker Noise: Since *1/f* variations and nonlinearity compromised the achievable PSD, it was suggested in the model paper that automatic-gain-control (AGC) oscillators with large-area high-power transistors could provide simultaneous improvements in flicker and nonlinear effects. Within a few years, the reduction of flicker noise by negative feedback and choice of active element resulted in substantial improvement in oscillator stability in a relatively short time [121]. It has been found that bipolar devices continue to be superior to field-effect alternatives in this aspect.

2) Resonator Flicker Noise: Flicker noise in the resonator itself had been suggested by Cutler and Searle in their 1966 paper [92]. Subsequent investigations have confirmed the significance of flicker in resonators [122]–[128]. As can been seen from Fig. 6, this effect would be much greater in high-Q HF oscillators, as opposed to lower-Q VHF overtone oscillators [103]. It could also be masked by the higher device noise levels of the time, especially at VHF frequencies. As a result, I disregarded this noise source in the simple model. Resonator flicker noise has come to be seen as important, and the model has since been extended to include it [129].

3) Flicker Noise Integral Convergence: A question raised early in the process of defining a standard was the infinity at zero frequency for PSD rising as $1/f^{\beta}$. Under this flicker model, the spectral density of phase grows without limit as the Fourier frequency tends to zero. This has been termed the "infrared catastrophe" by allusion to the ultraviolet catastrophe of pre-quantum radiation physics. As noted above, in the appendix of Barnes *et al.* 1971 [109], convergence was treated by appeal to limited bandwidth. It was suggested that finite bandwidth and measurement time create the equivalent of a bandpass filter that acts to truncate the PSD [20]. More recent papers rigorously confirm earlier conclusions that the convergence issue is not a hindrance to the application of the flicker model [130], [131].

4) Linewidth with Large Modulation Index: The RF spectrum of a signal that is modulated with white or colored noise is, in the low frequency limit where the small-angle assumption is not valid, quite different from the modulating spectrum. This aspect of the RF spectrum has been widely studied over the years. Papers before the 1960s modeled the RF spectrum of an oscillator frequency modulated by noise with components down to zero frequency. In FDM telephony, this was a common measurement approach in which a noise signal was used to represent a large number of multiplexed channels [51], [132]–[136].

Mathematical theory took many years to characterize rigorously the results of these direct measurements. The limiting RF spectrum had been identified as Lorentzian or Gaussian [137], [138]. Note also the curiously named spectrum function, the "Witch of Agnesi" [139].

C. AM Noise

The effect of AM noise, including AM-PM conversion, remains a concern that must be considered. Oscillators generally meet the criterion AM $\ll$ PM, and experiment has shown it not to be a primary issue in many systems of interest. By modulation theory, equal RF sidebands confirm that one or the other form dominates. From experience, in an oscillator with limiting or frequency multiplication, this has typically been phase noise.

As phase-noise levels have been reduced, the potential has risen that AM noise would become significant by comparison. Thus, this is an important issue to observe and test.

One area in which AM noise has proven significant is in the interpretation of measurements using phase or frequency discriminators that have internal levels of AM noise comparable with the PM noise being measured.

Assuming a common source, the correlation between AM and PM noise in an oscillator has been proposed as a means of reducing PM by measuring AM (a relatively simpler technology) and using the result as a feedback signal [140].

D. Strong Nonlinearity and New Models

The original phase noise model was suited to the VHF crystal oscillators of that period, and has stood the test of time as an accessible introduction to the definition and sources of oscillator phase noise. But in the years since then there has been a complete revolution in electronics technology, the beginning of which was marked by the famous 1965 Moore's law paper [141].

1) Impact of Moore's Law and Progress in Electronics: Since that time revolutionary advances in the following areas are well recognized:

(a) Semiconductor integration has made possible systems that were once beyond reach, including powerful personal computers and handheld mobile devices. The dramatic economic advantages of integration have led to the dominance of certain oscillator types, among them low-Q or strongly nonlinear oscillators that have commanded much additional study.

(b) Enabled by vastly increased computing capability, modern software techniques make it possible to analyze oscillator circuits to a new degree. This has made obsolete some of the hard-won closed-form descriptions, and has served to erase the boundary between linear and nonlinear circuits. A quote from earlier times regarding the value of a practical approximation is a measure of the burden of calculations then: "An exact calculation of the error is a formidable mathematical exercise, with potential results of little practical interest [142]." Class dismissed! Now, complex new analyses can be adapted to software form for facile use.

(c) New forms of acoustic and electronic resonators have made possible the design and construction of oscillators at frequencies, or for environments, not possible for quartz crystals. In particular, new forms of resonators make possible miniaturization and integration with semiconductors to provide both low noise and distinct economic advantages.

Oscillator phase noise performance has greatly improved with advances in resonator and sustaining stage technologies. Noise floors of VHF crystal oscillators have improved from $\mathcal{L}(f) = -160$dBc/Hz to -180dBc/Hz. Sustaining stage equivalent open loop *1/f* noise $\mathcal{L}(f = 1\text{Hz}) = -145$dBc/Hz is now routinely obtained at VHF and low UHF frequencies [143], [144].

Advances in photonics and precision clocks have made for many orders of magnitude improvement in both long-term and short-term stability. In particular, low noise photonic sources can be divided or mixed down to the microwave region to provide unprecedented levels of stability. 10-GHz sapphire dielectric-resonator oscillators achieve $\mathcal{L}(f = 1\text{Hz}) = -50\text{dBc/Hz}$ at 10 GHz, while optical comb signal generators achieve the remarkable level of $\mathcal{L}(f = 1\text{Hz}) = -100\text{dBc/Hz}$ at similar frequencies [145].

2) Models for Strongly Nonlinear Oscillators: In time, strongly nonlinear oscillators that arose from the proliferation of semiconductor integration required quite different fundamentally nonlinear analyses, as seen in more recent elegant papers [146]–[150], in one case by close colleagues at Stanford [104, p. 182]. I recall with some pleasure a young Stanford graduate student coming up to me, after my lecture on phase noise around 1996, to inquire if I thought there was still work to be done in the field. I hope my encouragement then to Ali Hajimiri made some contribution to his subsequent success as the Executive Officer for Electrical Engineering at Caltech.

E. Conclusion

These more rigorous nonlinear models have shown the way to many advances. However, measurements of high-Q oscillators continue to confirm the persistent utility of the simple model, as well [151], [152].

The fruits of the efforts of a half-century ago have been realized in the many successes of government, academic and industrial enterprises, including my own company. This is evident in the range of subjects in current symposia and journals. Modern systems such as cellular telephones are made possible through developments in the field of phase noise.

I recently enjoyed the live video of the moment of carrier acquisition for the deep-space link from NASA's New Horizons mission to Pluto. This was a personally satisfying example of the significance of phase-noise advances over the past fifty years.

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David B. Leeson (M'57–SM'64–F'88–LF'03) received the B.S. degree in engineering from the California Institute of Technology, Pasadena, CA, USA, in 1958, the S.M. degree in electrical engineering from the Massachusetts Institute of Technology, Cambridge, MA, USA, in 1959, and the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA, USA, in 1962.

Since 1994, he has been a Consulting Professor with the Department of Electrical Engineering at Stanford. From 1968 to 1993, he was the founding Chairman and CEO of California Microwave, Inc., a company formed to reduce phase-noise theory to practice. Prior to that he was an engineer with Applied Technology, Inc. and Hughes Aircraft Company. He is the author of journal papers on oscillator phase noise and nonlinear circuits that introduced the eponymous model, equation and effect that bear his name. He is also the author of the book *Physical Design of Yagi Antennas*. His interests include oscillator phase noise, microwave communications, antennas, entrepreneurism, and history. His avocations have included amateur radio and automobile racing.

Dr. Leeson received the IEEE W. G. Cady Award (UFFC) in 2001. He was a member of IEEE Subcommittee 14.7 and of G-I&M Technical Committee on Frequency and Time. He was also a member of IEEE Committee 802.11 for Wireless Local Area Network Standards, and of the IEEE G-MTT Administrative Committee. Additionally, he was a Director of the American Electronics Association, a Governor of the Electronics Industry Association, and the founding Chairman of the Wireless Information Networks Forum (WINForum).