Домашнее задание 2

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Вариант 1

Задача 1

$$a_n = \frac{3n-2}{2n-1}, a = \frac{3}{2}$$

Доказать:

$$\lim_{n\to\infty} a_n = a$$

Analyze:

$$\left|\frac{3n-2}{2n-1} - \frac{3}{2}\right| = \frac{1}{4n-2} \leq \frac{1}{n}(n \geq 2) \tag{1}$$

therefore, for any $\epsilon > 0$, if $\frac{1}{n} < \epsilon$, we get:

$$\left|\frac{3n-2}{2n-1} - \frac{3}{2}\right| < \epsilon \tag{2}$$

it mean that when $n > \frac{1}{\epsilon}$,(2) is workable ,and because (1) is holds under the condition $n \geq 2$, so:

$$N = max\left\{2, \frac{1}{\epsilon}\right\} \tag{3}$$

Prove: $\forall \epsilon > 0, \exists N = max\left\{2, \frac{1}{\epsilon}\right\}$, according to the analysis, when n>N, the formula (2) is workable.

Задача 2

Задание 2(a)

$$\lim_{x \to -1} \frac{(x^3 - 2x - 1)(x+1)}{x^4 + 4x^2 - 5}$$

$$solve: \lim_{x \to -1} \frac{(x^3 - 2x - 1)(x+1)}{x^4 + 4x^2 - 5} = \lim_{x \to -1} \frac{(x^3 - 2x - 1)(x+1)}{(x^3 - x^2 + 5x - 5)(x+1)} =$$
$$\lim_{x \to -1} \frac{x^3 - 2x - 1}{x^3 - x^2 + 5x - 5} = -\frac{0}{12} = 0$$

Ответ:

$$\lim_{x \to -1} \frac{(x^3 - 2x - 1)(x+1)}{x^4 + 4x^2 - 5} = 0$$

Задание 2(b)

$$\lim_{x \to +\infty} \frac{x^2 + \sqrt{16x^4 - x\sqrt{x}}}{3x^2 + 1}$$

$$solve: \lim_{x \to +\infty} \frac{x^2 + \sqrt{16x^4 - x\sqrt{x}}}{3x^2 + 1} = \lim_{x \to +\infty} \frac{x^2(1 + \sqrt{16 - \frac{1}{\sqrt{x^5}}})}{x^2(3 + \frac{1}{x^2})} =$$
$$= \lim_{x \to +\infty} \frac{1 + \sqrt{16 - \frac{1}{\sqrt{x^5}}}}{3 + \frac{1}{x^2}} = \frac{1+4}{3} = \frac{5}{3}$$

Ответ:

$$\lim_{x \to +\infty} \frac{x^2 + \sqrt{16x^4 - x\sqrt{x}}}{3x^2 + 1} = \frac{5}{3}$$

Задание 2(c)

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$$

Solve:

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\lim_{x\to 4}\frac{2\sqrt{x-2}}{\sqrt{2x+1}}=\frac{2\sqrt{2}}{3}(L'Hospital'sRule)$$

Answer:

$$:\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$$

Задание 2(d)

$$\lim_{x\to 8}\left(\frac{2x-7}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-2}}$$

Solve: Замена $x-8=t, x=t+8$ so when $x\to 8, t\to 0$.

$$\lim_{x\to 8}\left(\frac{2x-7}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-2}}=\lim_{t\to 0}\left(\frac{2t+9}{t+9}\right)^{\frac{1}{\sqrt[3]{t+8}-2}}=\lim_{t\to 0}\left(1+\frac{t}{t+9}\right)^{\frac{1}{\sqrt[3]{t+8}-2}}=$$
$$=\lim_{t\to 0}\left(1+\frac{t}{t+9}\right)^{\frac{t+9}{t}\cdot\frac{t}{t+9}\cdot\frac{1}{\sqrt[3]{t+8}-2}}=\lim_{t\to 0}\left(1+\frac{t}{t+9}\right)^{\frac{t+9}{t}\cdot\frac{t}{t+9}\cdot\frac{\sqrt[3]{(t+8)^2}+2\sqrt[3]{t+8}+4}{t}}=$$
$$=e^{\lim_{t\to 0}\frac{\sqrt[3]{(t+8)^2}+2\sqrt[3]{t+8}+4}{t+9}}=e^{\frac{4}{3}}$$

Answer:

$$\lim_{x\to 8}\left(\frac{2x-7}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-2}}=e^{\frac{4}{3}}$$

Задание2 (e)

$$\lim_{x\to 0}\left(\frac{x^2 sin2x}{\arctan 3x^3}\right)^{\frac{x+2}{x+1}}$$

Solve:

$$\lim_{x\to 0}\left(\frac{x^2 sin2x}{\arctan 3x^3}\right)^{\frac{x+2}{x+1}}=\quad\lim_{x\to 0}\left(\frac{2x^3}{3x^3}\right)^{\frac{x+2}{x+1}}=\frac{4}{9}$$

Answer:

$$\lim_{x\to 0}\left(\frac{x^2 sin2x}{\arctan 3x^3}\right)^{\frac{x+2}{x+1}}=\frac{4}{9}$$

Задание 2(f)

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan(3x)}{\tan x}$$

Solve: Замена $x - \frac{\pi}{2} = t, t = x + \frac{\pi}{2}$, so when $x \to \frac{\pi}{2}, t \to 0$.

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan(3x)}{\tan x} = \lim_{t \to 0} \frac{\tan(3t + \frac{3\pi}{2})}{\tan(t + \frac{\pi}{2})} = \lim_{t \to 0} \frac{-\cot(3t)}{-\cot t} = \lim_{t \to 0} \frac{\tan t}{\tan(3t)} = \frac{1}{3}$$

Answer:

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan(3x)}{\tan x} = \frac{1}{3}$$

Задача 3

$$f(x) = 2^x - 8, g(x) = \ln \frac{x}{3}, x \to 3$$

1)

$$a) \lim_{x \to 3} 2^x - 8 = 0, \quad b) \lim_{x \to 3} \ln \frac{x}{3} = 0$$

поэтому f и g являются бесконечно малыми

2)

3)

$$\lim_{x \to 3} \frac{f(x)}{g(x)} = \lim_{x \to 3} \frac{2^x - 8}{\ln \frac{x}{3}} = \lim_{x \to 3} \ln 2 \cdot 2^x \cdot x = 24 \ln 2 = C$$

Поэтому f и g являются бесконечно малыми того же порядка когда $x \to 3$.

Задача 4

$$f(x) = \begin{cases} 2^{\frac{1}{x}}, & x < 1 \\ \frac{\sqrt{x+3}}{2-x}, & x \geq 1 \end{cases}$$

когда x=2 и x=0, функция не определена, и :

$$\lim_{x\to 0^-} f(x) = 0, \lim_{x\to 0^+} f(x) = +\infty, \lim_{x\to 2^-} f(x) = +\infty, \lim_{x\to 2^+} f(x) = -\infty.$$

поэтому точки разрыва функции $x_1 = 0, x_2 = 2$.