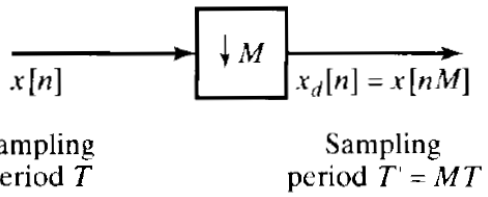
DSP2025 Homework 4 Due date: 12:00 noon, April. 17, 2025

1.




Which of the following signals can be downsampled by a factor of 2 using the system in the above without any loss of information? Explain your answer.

(a) $x[n] = \delta[n - n_0]$, for any $n_0$ is an integer.
(b) $x[n] = \cos(\pi n/4)$
(c) $x[n] = \cos(\pi n/4) + \cos(3\pi n/4)$

2. Suppose that three LTI systems are connected in cascade; i.e., the output of $S_1$ is the input of $S_2$, and the output of $S_2$ is the input of $S_3$. The three systems are specified as follows:
$S_1$: $y_1[n] = x_1[n] + x_1[n-1]$,
$S_2$: $y_2[n] = x_2[n] + 2x_2[n-1] - x_2[n-2]$,
$S_3$: $y_3[n] = x_3[n-1] + x_3[n-2]$,
where the input of $S_i$ is $x_i[n]$ and its output is $y_i[n]$.
(a) Consider the equivalent system that is a single operation from the input $x[n]$ (into $S_1$) to the output $y[n]$ (the output of $S_3$). Thus, $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$. Write down the **impulse response** of this system.
(b) Is this system **FIR** or **IIR**? Explain your answer

3. (a) Find the z-transform of the LTI system whose input and output satisfy the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2] \quad (1)$$

(b) Find the frequency response of the above LTI system.
(c) Find the impulse response of the LTI system defined by (1).