DSP2025 Homework 2   Due date: 12:00 noon, March 27, 2025

1. Consider an aperiodic signal $x(t)$ with its CFT being $X(jw)$, and a periodic signal $y(t)$ with fundamental frequency T_0 and Fourier coefficients c_k $(k \in Z)$. It is apparent that their product $z(t) = x(t)y(t)$ is an aperiodic signal. **Question**: Derive the CFT of $z(t)$.

2. Let $x(t) = \frac{2\sin(20\pi t)}{\pi t}$, $h(t) = \frac{5\sin(10\pi t)}{\pi t}$ be two sinc functions. Derive the convolution of them, $y(t) = x(t) * h(t)$. **Hint**: Using convolution theorem.

3. Let the DTFT of the following sequence be $R(e^{j\omega})$

$$r[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

(a) Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\frac{2\pi n}{M}\right], & 0 \leq n \leq M \\ 0, & otherwise \end{cases}$$

Sketch $w[n]$ and express $W(e^{j\omega})$, the DTFT of $w[n]$, in terms of $R(e^{j\omega})$, the DTFT of $r[n]$.

(b) Sketch the magnitude of $R(e^{j\omega})$ and $W(e^{j\omega})$ for the case when $M = 4$.

4. Suppose that a discrete-time signal $x[n]$ is given by the formula

$$x[n] = 2.2\cos(0.3\pi n - \frac{\pi}{3})$$

and that it was obtained by sampling a continuous-time signal

$$x(t) = 2.2\cos(2\pi f_0 t - \frac{\pi}{3})$$

at a sampling rate of f_s = 6000 samples/sec. Suppose the absolute value of f_0 is less than 8 kHz, i.e., $|f_0| < 8000$. What are the three values of f_0 that could have produced $x[n]$ under the sampling rate f_s = 6000?