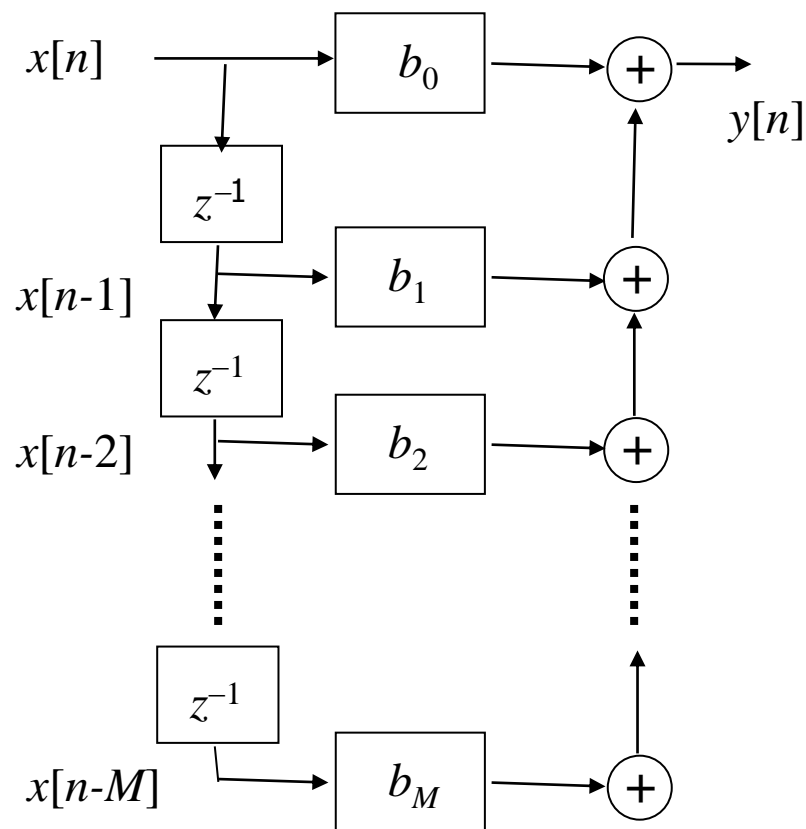# System Diagram of A Causal FIR System

- The block diagram of a causal FIR system can be represented by z-transforms:

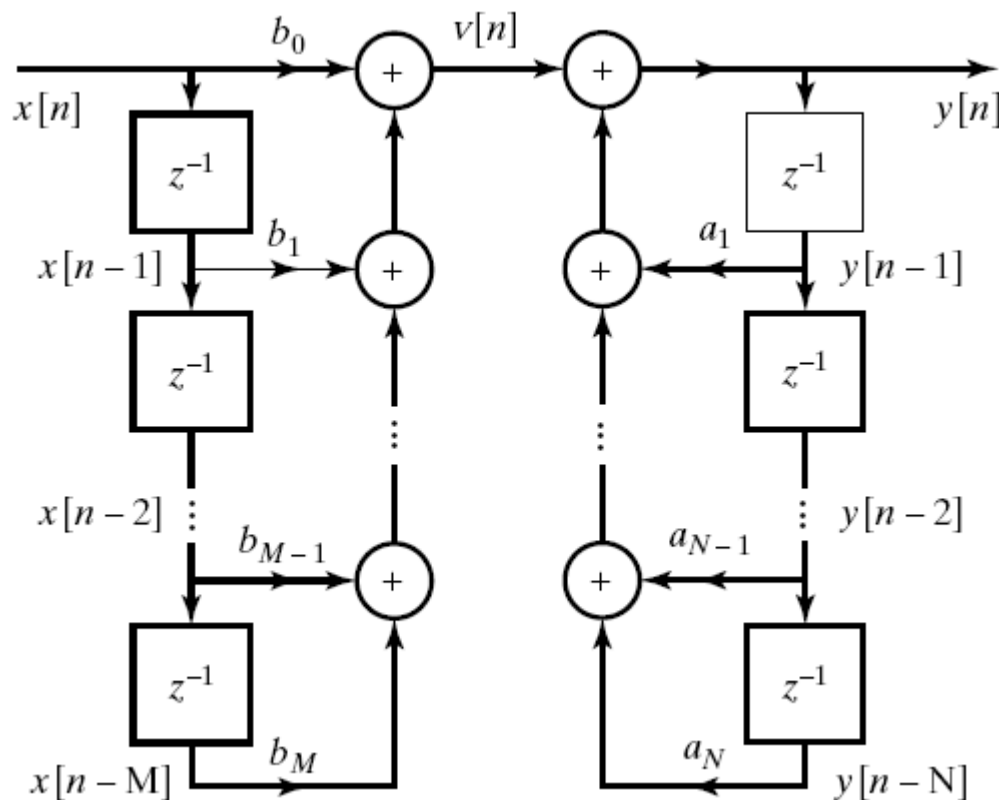$$y[n] = \sum_{m=0}^{M} b_m x[n-m]$$

Digital Filter Structures

- We know that there could be multiple ways to realize the system function of a difference equation.

- Z-transform is also useful in help characterize these solutions, and find an efficient implementation structure.

Digital Filter Structures (for IIR Filter)

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

$$v[n] = \sum_{k=0}^{M} (b_k) x[n-k]$$

$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + v[n]$$

Direct Form I implementation

On the z-domain

$$H(z) = H_2(z)H_1(z) = (\frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}})\left(\sum_{k-0}^{M} b_k z^{-k}\right)$$

or equivalently

$$V(z) = H_1(z)X(z) = \left(\sum_{k-0}^{M} b_k z^{-k}\right)X(z)$$

$$Y(z) = H_2(z)V(z) = (\frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}})V(z)$$

By changing the order of H_1 and H_2, onsider the equivalence on the z-domain:

$$H(z) = H_1(z)H_2(z)$$

where $\quad H_1(z) = \left(\sum_{k-0}^{M} b_k z^{-k} \right) \quad H_2(z) = (\dfrac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}})$

Let

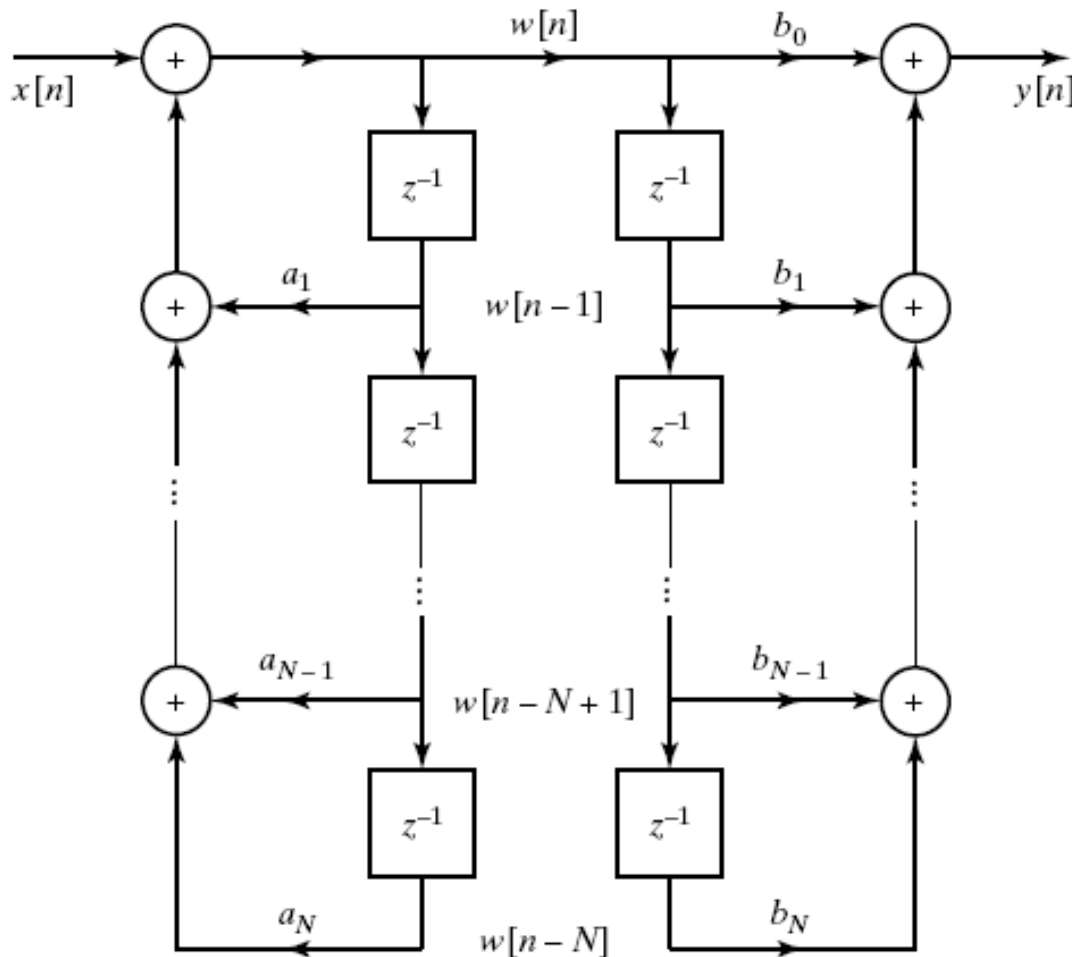$$W(z) = H_2(z)X(z) = (\dfrac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}})X(z)$$

Then

$$Y(z) = H_1(z)W(z) = \left(\sum_{k-0}^{M} b_k z^{-k} \right)W(z)$$

In the time domain,
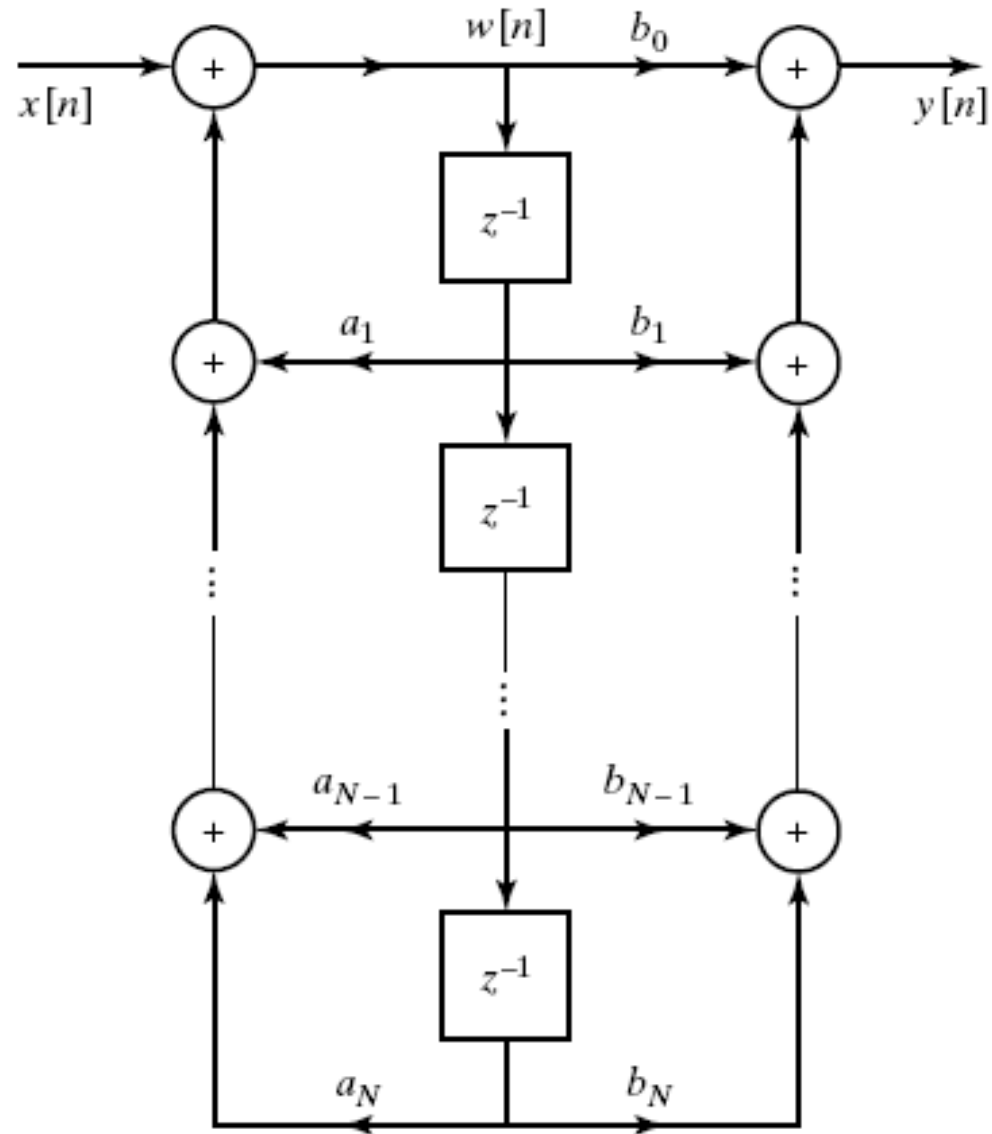$$w[n] = \sum_{k=1}^{N} a_k w[n-k] + x[n]$$

$$y[n] = \sum_{k=0}^{M} b_k w[n-k]$$

We have the following equivalence for implementation:

We assume $M = N$ here

Note that the exactly the same signal, $w[k]$, is stored in the two chains of delay elements in the block diagram. The implementation can be further simplified as follows:
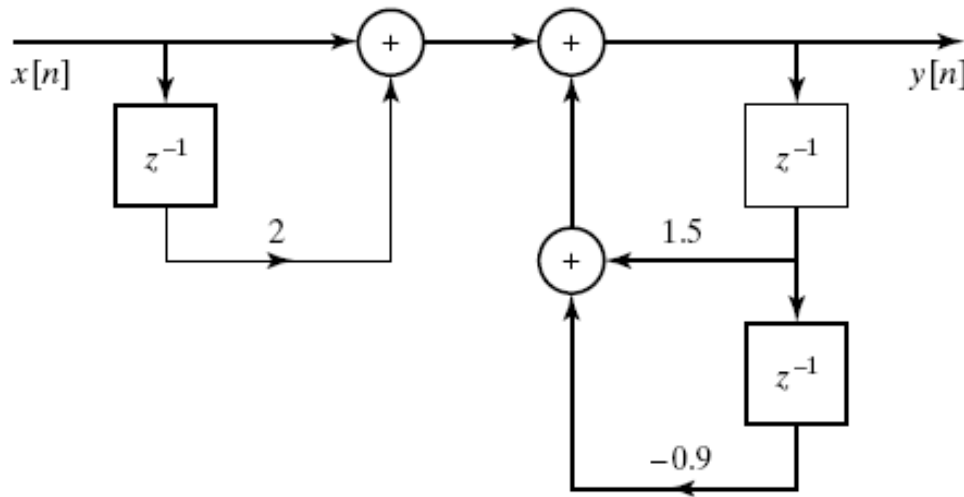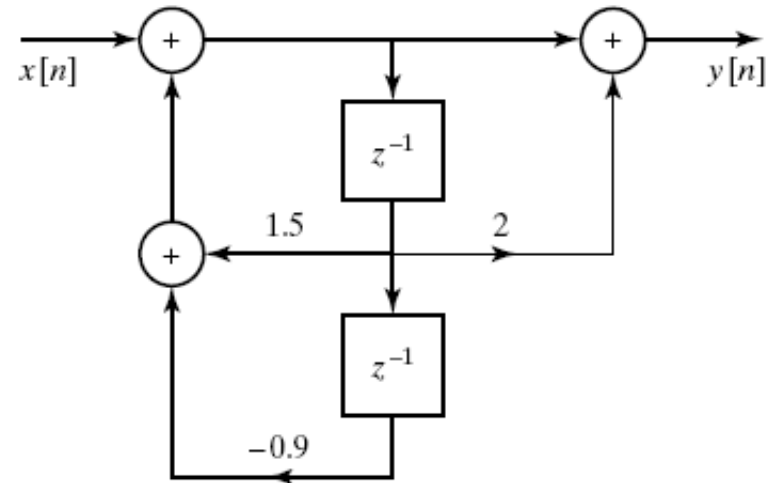
Direct Form II (or Canonic Direct Form) implementation

By using the direct form II implementation, the number of delay elements is reduced from $(M + N)$ to $\max(M, N)$.

Example:
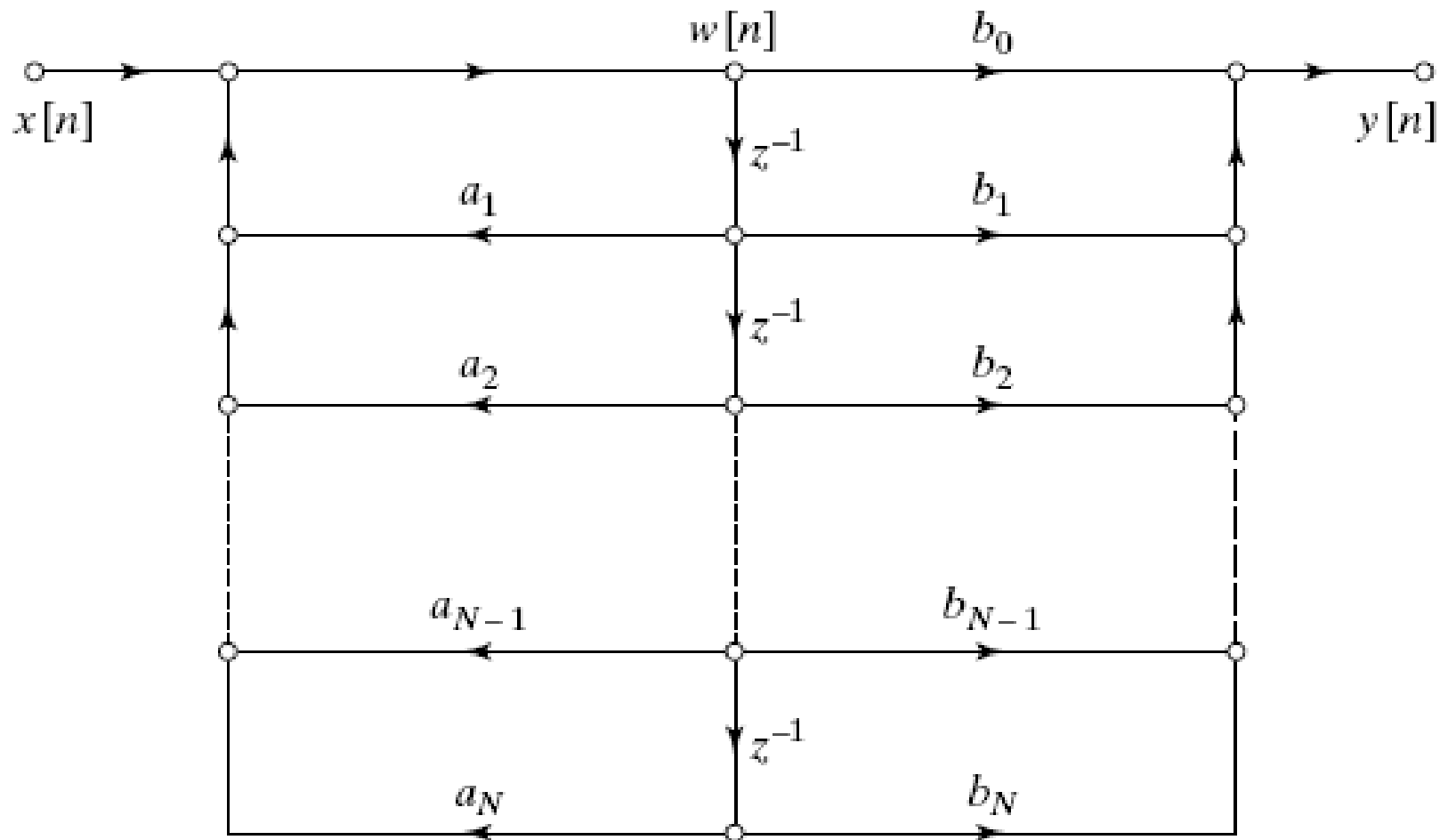$$H(z) = \frac{1 + 2z^{-1}}{1 - 1.5z^{-1} + 0.9z^{-2}}$$

Direct form I implementation
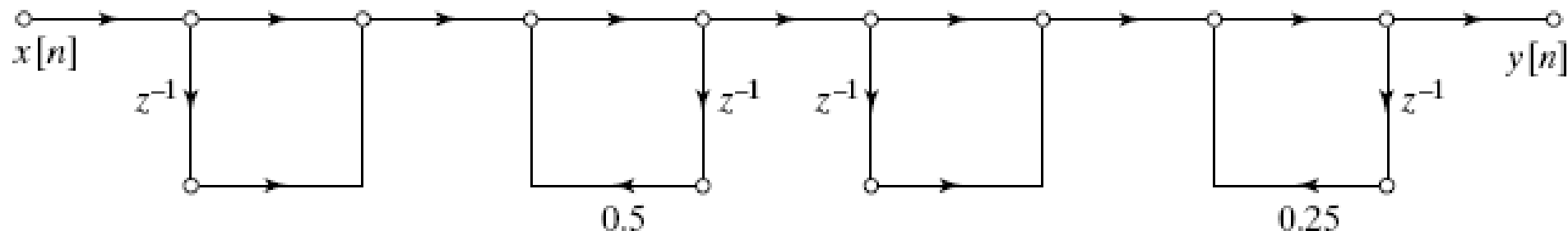
Direct form II implementation

Representing by signal-flow graph

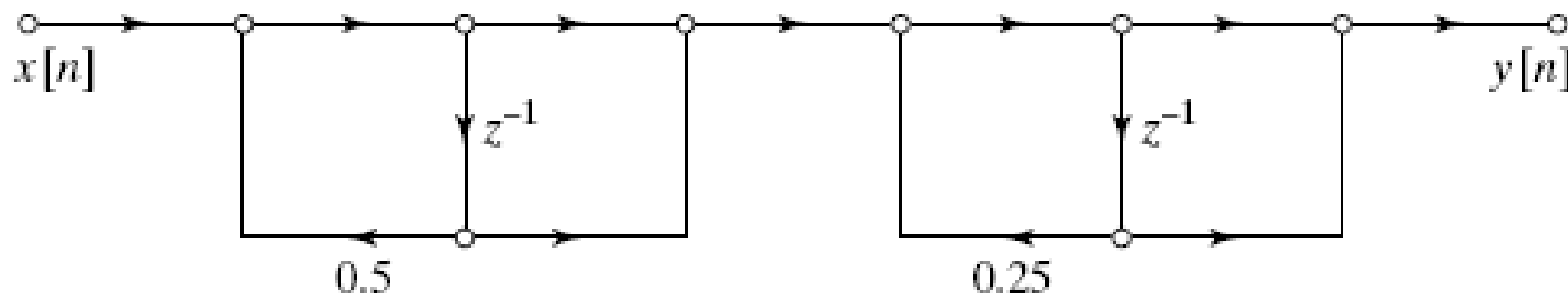Example: the signal-flow graph of direct form II.

Example:

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} = \frac{(1 + z^{-1})(1 + z^{-1})}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}$$

Cascade structure: direct form I implementation

Cascade structure: direct form II implementation