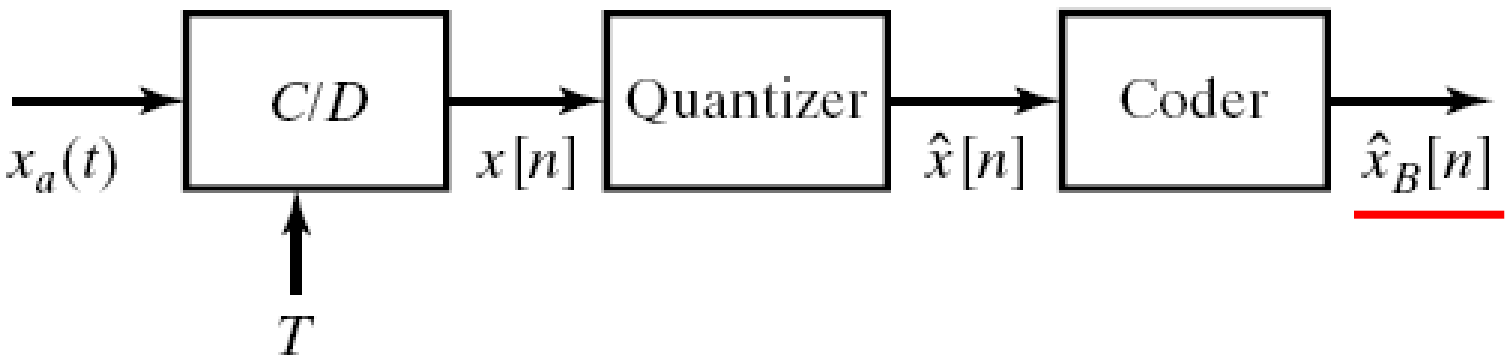
Effect of Quantizer (Quantization)

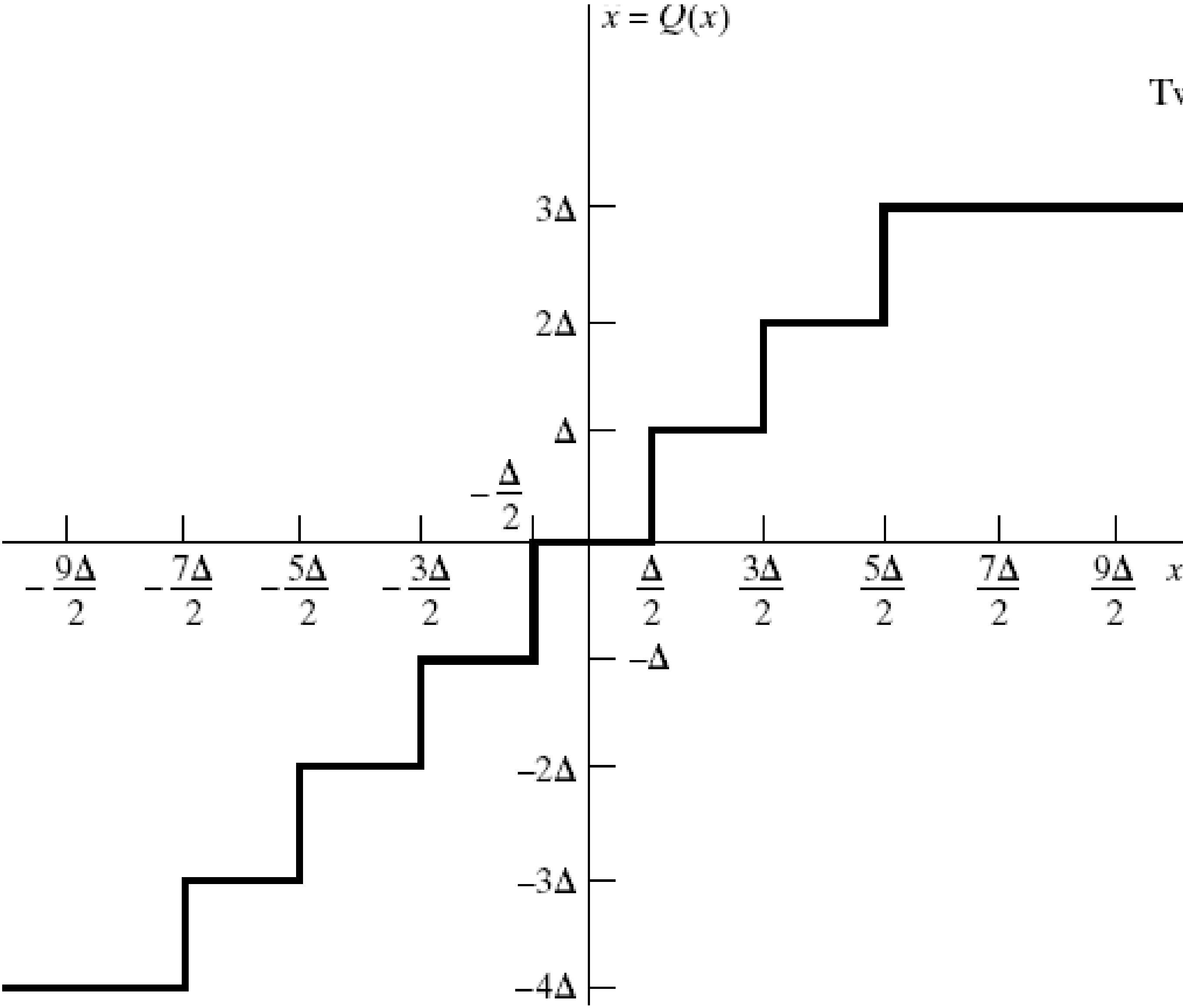
- The real-valued signal has to be stored as a code for digital processing. This step is called quantization.

$$\hat{x}[n] = Q(x[n])$$



- The quantizer is a nonlinear system.
  - Typically, we apply two's complement code for representation.

Quantizer (Quantization)



Two's-complement code	Offset binary code
011	111
010	110
001	101
000	100
111	011
110	010
101	001
100	000

Quantizer (Quantization)

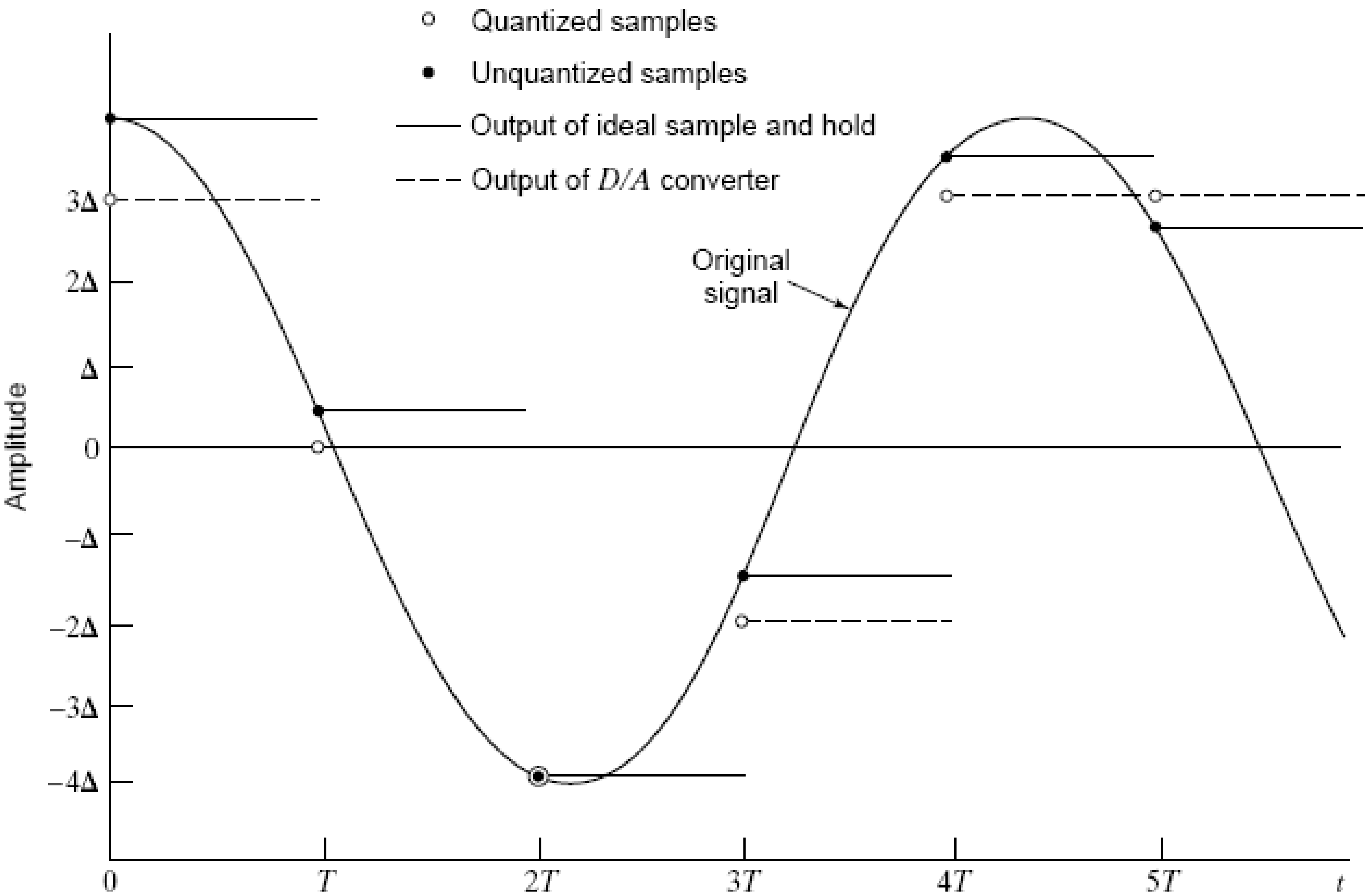
- In general, if we have a (B+1)-bit binary two's complement fraction of the form: $a_0 a_1 a_2 \ldots a_B$

then its value is $-a_0 2^0 + a_1 2^{-1} + a_2 2^{-2} + \ldots + a_B 2^{-B}$

- The step size of the quantizer is $\Delta = 2X_m / 2^{B+1} = X_m / 2^B$

where X_m is the full scale level of the A/D converter.

Example of quantization



Analysis of quantization errors

- Quantization error $e[n] = \hat{x}[n] - x[n]$
- In general, for a $(B+1)$-bit quantizer with step size Δ, the quantization error satisfies that

$$-\Delta/2 < e[n] \leq \Delta/2$$

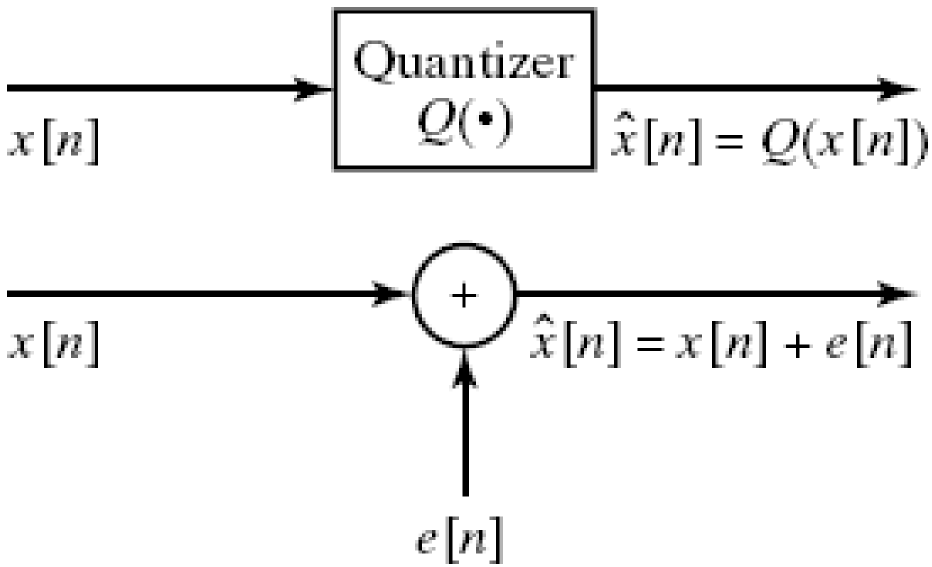
when

$$(-X_m - \Delta/2) < x[n] \leq (X_m - \Delta/2)$$

- If $x[n]$ is outside this range, then the quantization error is larger in magnitude than $\Delta/2$, and such samples are said to be clipped.

Analysis of quantization errors

- Analyzing the quantization by introducing an additional error source and linearizing the system:



- The model is equivalent to quantization if we know $e[n]$.

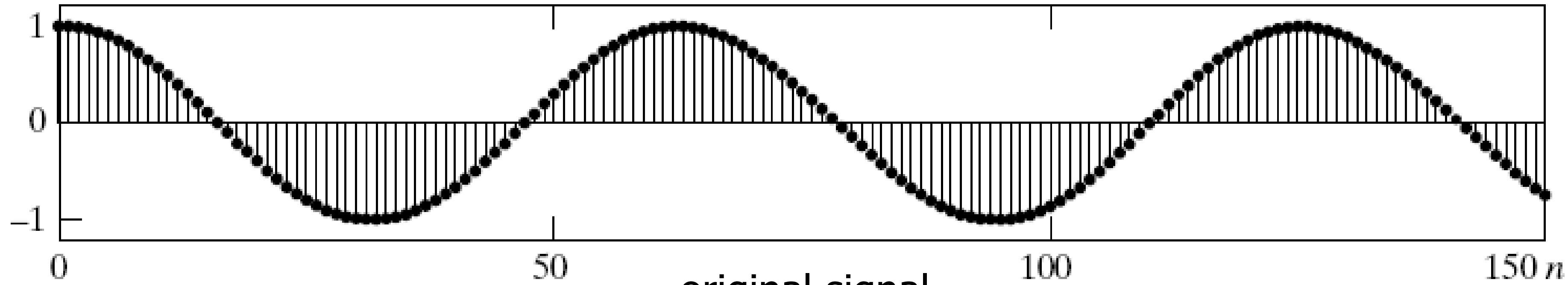
Assumptions about $e[n]$

- $e[n]$ is a sample sequence of a stationary random process.
- $e[n]$ is uncorrelated with the sequence $x[n]$.
- The random variables of the error process $e[n]$ are uncorrelated; i.e., the error is a white-noise process.
- The probability distribution of the error process is uniform over the range of quantization error, and is **not clipped**.

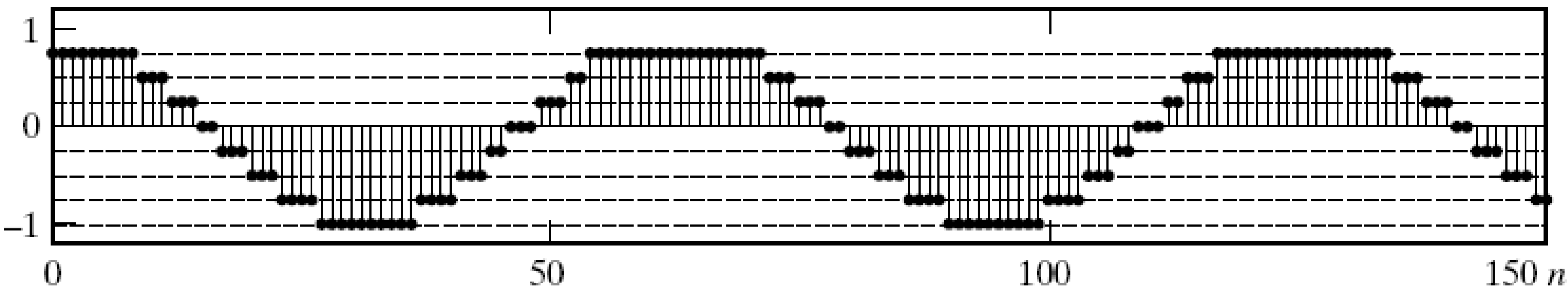
Assumptions about $e[n]$

- The assumptions would not be justified.
- However, when the signal is a complicated signal (such as speech or music), the assumptions are more realistic.
  - Experiments have shown that, when the signal becomes more complicated, the measured correlation between the signal and the quantization error decreases, and the error also becomes uncorrelated.

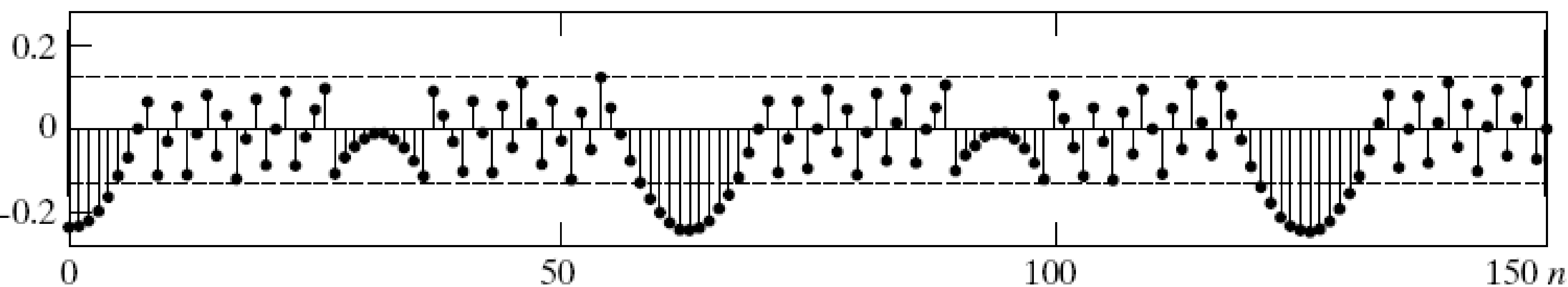
Example of quantization error



original signal

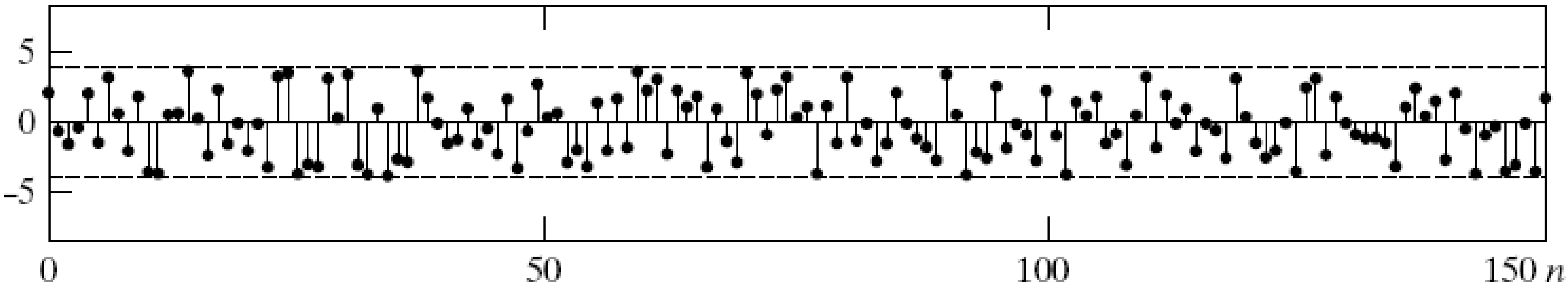


3-bit quantization result



3-bit quantization error

Example of quantization error



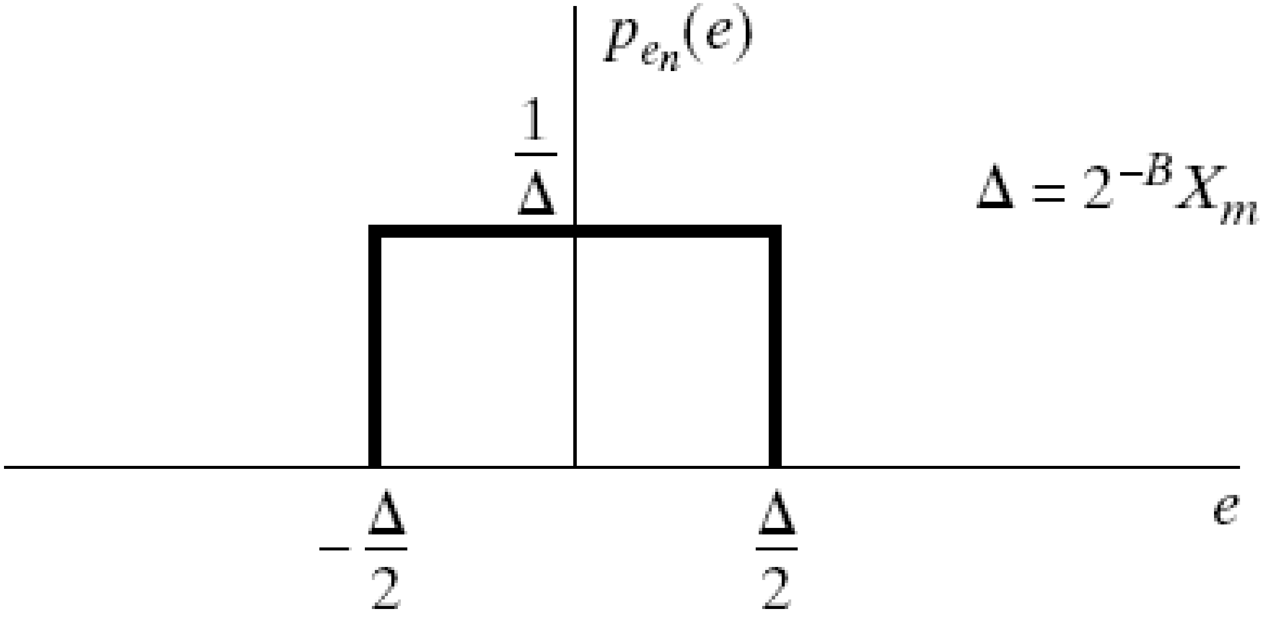
8-bit quantization error

- In a heuristic sense, the assumptions of the statistical model appear to be valid if the signal is sufficiently complex and the quantization steps are sufficiently small, so that the amplitude of the signal is likely to traverse many quantization steps from sample to sample.

Quantization error analysis

$$-\Delta/2 < e[n] \le \Delta/2$$

- Assume the probability density function of $e[n]$ is a uniform distribution:



Quantization error analysis

- The mean value of $e[n]$ is zero, and its variance is

$$\boxed{\sigma_e^2 = \int_{-\Delta/2}^{\Delta/2} e^2 \frac{1}{\Delta} de = \frac{\Delta^2}{12}}$$

- Since $\boxed{\Delta = \frac{X_m}{2^B}}$

For a $(B+1)$-bit quantizer with full-scale value X_m, the noise variance, or power, is

$$\sigma_e^2 = \frac{2^{-2B} X_m^2}{12}$$

Signal to Noise Ratio (SNR)

- In signal processing, SNR is defined as

$$\mathrm{SNR} = 20log_{10} \frac{\text{signal power}}{\text{noise power}}$$

$$= 10log_{10} \frac{\text{signal power}^2}{\text{noise power}^2}$$

Quantization error analysis

- A common measure of the amount of degradation of a signal by additive noise is the signal-to-noise ratio (SNR), defined as the ratio of signal variance (power) to noise variance. Expressed in decibels (dB), the SNR of a (B+1)-bit quantizer is

$$SNR = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) = 10\log_{10}\left(\frac{12\cdot 2^{2B}\sigma_x^2}{X_m^2}\right)$$

$$= 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

- Hence, the SNR increases approximately 6dB for each bit added to the word length of the quantized samples.

Quantization error analysis

- The equation can be further simplified for analysis. For example, if the signal amplitude has a Gaussian distribution, only 0.064 percent of the samples would have an amplitude greater than $4\sigma_x$.
- Thus to avoid clipping the peaks of the signal (as is assumed in our statistical model), we might set the gain of filters and amplifiers preceding the A/D converter so that $\sigma_x = X_m/4$. Using this value of σ_x gives $SNR \approx 6B - 1.25dB$
- For example, obtaining a SNR about 90-96 dB in high-quality music recording and playback requires 16-bit quantization.
  - But it should be remembered that **such performance is obtained only if the input signal is carefully matched to the full-scale of the A/D converter** (i.e., without clipping).