生醫電資所碩一
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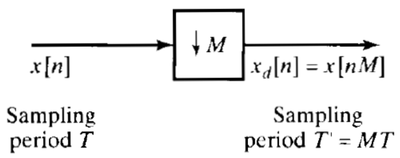
DSP2025 Homework 4 Due date: 12:00 noon, April. 17, 2025

1.

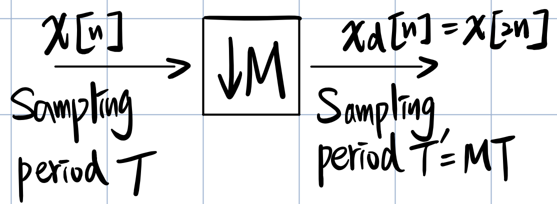


Which of the following signals can be downsampled by a factor of 2 using the system in the above without any loss of information? Explain your answer.

只保留偶數點 $x=[2n]$
頻率會放大2倍: $w'=2w$

(a) $x[n] = \delta[n - n_0]$, for any n_0 is an integer.
(b) $x[n] = \cos(\pi n/4)$
(c) $x[n] = \cos(\pi n/4) + \cos(3\pi n/4)$

<sol>



要避免 Aliasing
→ 要滿足 Nyquist條件.

Frequency domain $X(e^{jw}) \longrightarrow X_d(e^{jw}) = \frac{1}{2}\left[X(e^{jw/2}) + X(e^{j(w/2+\pi)})\right]$

$\Rightarrow X(e^{jw}) = 0$ for $|w| > \frac{\pi}{2}$

☆ DT signal：頻率範圍 $[-\pi, \pi]$
downsample by M：
頻譜會壓縮M倍，重疊M次 → 避免 Aliasing，訊號必須 bandlimited 到 $|w| \le \frac{\pi}{M}$

(a) $x[n] = \delta[n - n_0]$, for any n_0 is an integer.

∵ $x[n] = \delta[n-n_0]$ 是單一點為1，其餘為0的 unit impulse，出現在 n_0 位置
∴ 即使只保留偶數點(downsample)，只要 impulse 剛好出現在偶數點，
訊號就能完全被保留！→ 不會導致 Aliasing，訊號可重建 #

(b) $x[n] = \cos(\frac{n\pi}{4})$, $w = \frac{\pi}{4} \longrightarrow \frac{1}{2}e^{j\pi n/4} + \frac{1}{2}e^{-j\pi n/4}$, $w = \pm\frac{\pi}{4}$

downsampled by a factor of 2

$\longrightarrow X_d = \cos(\frac{n\pi}{2})$, $w' = 2 \times \frac{\pi}{4} = \frac{\pi}{2} \le \frac{\pi}{2}$

未超過 Nyquist
→ 不會 Aliasing
→ 訊號可重建 #

(c) $x[n]=\cos(\frac{\pi n}{4})+\cos(\frac{3\pi n}{4})$, $w_1=\frac{\pi}{4}$, $w_2=\frac{3\pi}{4}$

downsampled by a factor of 2.

$\longrightarrow w_1'=2\cdot\frac{\pi}{4}=\frac{\pi}{2}$, $w_2'=2\cdot\frac{3\pi}{4}=\frac{3\pi}{2}\equiv\frac{-\pi}{2}$

∵ 兩個不同頻率的波，Alias成同一個頻率 $\pm\frac{\pi}{2}$

∴ 頻譜產生 Aliasing $\longrightarrow$ 資訊會損失 ，#

2. Suppose that three LTI systems are connected in 串接 cascade; i.e., the output of S_1 is the input of S_2, and the output of S_2 is the input of S_3. The three systems are specified as follows:
S_1: $y_1[n] = x_1[n] + x_1[n-1]$,
S_2: $y_2[n] = x_2[n] + 2x_2[n-1] - x_2[n-2]$,
S_3: $y_3[n] = x_3[n-1] + x_3[n-2]$,
where the input of S_i is $x_i[n]$ and its output is $y_i[n]$.
(a) Consider the equivalent system that is a single operation from the input $x[n]$ (into S_1) to the output $y[n]$ (the output of S_3). Thus, $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$. Write down the **impulse response** of this system.
(b) Is this system **FIR** or **IIR**? Explain your answer

<sol> (a) $y_1[n]=x_1[n]+x_1[n-1]$

$\xrightarrow[\text{response}]{\text{impulse}} h_1[n]=\delta[n]+\delta[n-1]$

$y_2[n]=x_2[n]+2x_2[n-1]-x_2[n-2]$

$\xrightarrow[\text{response}]{\text{impulse}} h_2[n]=\delta[n]+2\delta[n-1]-\delta[n-2]$

$y_3[n]=x_3[n-1]+x_3[n-2]$

$\xrightarrow[\text{response}]{\text{impulse}} h_3[n]=\delta[n-1]+\delta[n-2]$

$\Rightarrow h[n]=h_1[n]*h_2[n]*h_3[n]$ ☆ $(\delta[n]+\delta[n-1])*f[n]=f[n]+f[n-1]$

$h_{12}[n]=h_1[n]*h_2[n]=(\delta[n]+\delta[n-1])*(\delta[n]+2\delta[n-1]-\delta[n-2])$

$=h_2[n]+h_2[n-1]=\delta[n]+2\delta[n-1]-\delta[n-2]+\delta[n-1]+2\delta[n-2]-\delta[n-3]$

$=\delta[n]+(2+1)\delta[n-1]+(-1+2)\delta[n-2]-\delta[n-3]$

$=\delta[n]+3\delta[n-1]+\delta[n-2]-\delta[n-3]$

$$\rightarrow h[n] = h_{12}[n] * h_3[n]$$

$$= (\delta[n] + 3\delta[n-1] + \delta[n-2] - \delta[n-3]) * (\delta[n-1] + \delta[n-2])$$

$$= h_{12}[n-1] + h_{12}[n-2]$$

$$\Rightarrow h[n] = \delta[n-1] + 4\delta[n-2] + 4\delta[n-3] + \delta[n-5] \quad \#$$

(b)

$\because$ $h[n] = \delta[n-1] + 4\delta[n-2] + 4\delta[n-3] + \delta[n-5]$

只有有限個非零點 $(n = 1, 2, 3, 5)$,

$\therefore$ the impulse response has finite length and contains no recursive (feedback) terms

the system is an FIR system #

3. (a) Find the z-transform of the LTI system whose input and output satisfy the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2] \quad (1)$$

(b) Find the frequency response of the above LTI system.

(c) Find the impulse response of the LTI system defined by (1).

<sol> (a) Let $Z\{y[n]\} = Y(z)$, $Z\{x[n]\} = X(z)$

代入(1)

$$\rightarrow Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

$$\rightarrow (1 - \frac{1}{2}z^{-1})Y(z) = (1 + 2z^{-1} + z^{-2})X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{1}{2}z^{-1}} \quad \#$$

(b)

$Z = e^{j\omega}$ 代入,

$$H(e^{j\omega}) = \frac{1+2e^{-j\omega}+e^{-j2\omega}}{1-\frac{1}{2}e^{-j\omega}}$$ #

(c)

h[n] is the impulse response of system

When input $x[n]=\delta[n]$, output $y[n]=h[n]$

Let $x[n]=\delta[n]$ 代入 (1)

$$h[n]-\frac{1}{2}h[n-1]=\delta[n]+2\delta[n-1]+\delta[n-2]$$

and Assuming that system is a causal system ($h[n]=0$ for $n<0$)

$n=0$, $h[0]-\frac{1}{2}h[-1]=\underline{\overset{1}{\delta[0]}}+2\delta[-1]+\delta[-2] \Rightarrow h[0]=1$

$n=1$, $h[1]-\frac{1}{2}h[0]=\delta[1]+2\delta[0]+\delta[-1] \Rightarrow h[1]=2.5$

$\Rightarrow h[2]=2.25$

⋮

$\Rightarrow h[3]=1.125$

$\Rightarrow h[4]=0.5625$

⋮

$h[n]=\{1, 2.5, 2.25, 1.125, 0.5625, \dots\}$

∵ system have recursive (feedback)

h[n] is infinite length (has always been $\frac{1}{2}$ proportional decay)

∴ This system is IIR (Infinite Impulse Response) and not FIR #.