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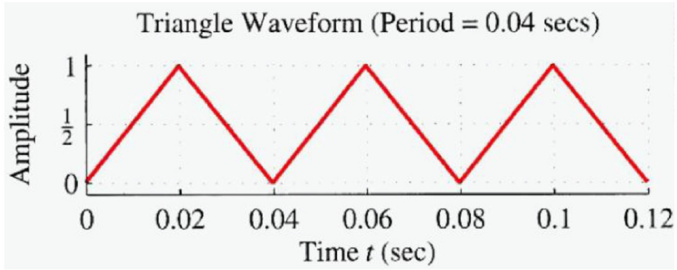
DSP2025 Homework 1　　　Due date: 12:00 noon, March 13, 2025

1. Find the **DC component** of the following periodical signal with the period 0.04.



hint: write the signal $x(t)$ in a single period $[0,\ T_0]$, where $T_0 = 0.04$. Then use the Fourier series integral formula to find the Fourier-series coefficients.

Ans:

$$x(t) = \begin{cases} 50t, & 0 \le t \le 0.02 \\ 2-50t, & 0.02 \le t \le 0.04 \end{cases}$$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

(1) $\int_0^{0.02} 50t\,dt = \left[\frac{50}{2}t^2\right]_0^{0.02} = \frac{50}{2}\times(0.02)^2 = 0.01$

(2) $\int_{0.02}^{0.04}(2-50t)\,dt = \int_{0.02}^{0.04} 2\,dt - \int_{0.02}^{0.04} 50t\,dt$

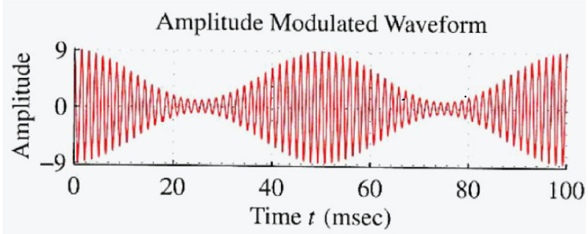
$= 0.04 - 0.03 = 0.01$

$$a_0 = \frac{1}{0.04}(0.01+0.01) = 0.5$$

2. The amplitude-modulation (AM) signal is a product the form,

$$x(t) = v(t)cos(2\pi f_c t)\ .$$

Consider the case where $v(t) = 5 + 4cos(40\pi t)$, and the carrier frequency $f_c = 700Hz$. The time-domain of the signal $x(t)$ is shown as



Question: Find and draw the spectrum of $x(t)$ in terms of Continuous Fourier Transform

Ans: $X\cos A\cos B = \frac{X}{2}(\cos(A+B)+\cos(A-B))$

$$x(t) = v(t)\cos(2\pi f_c t)$$

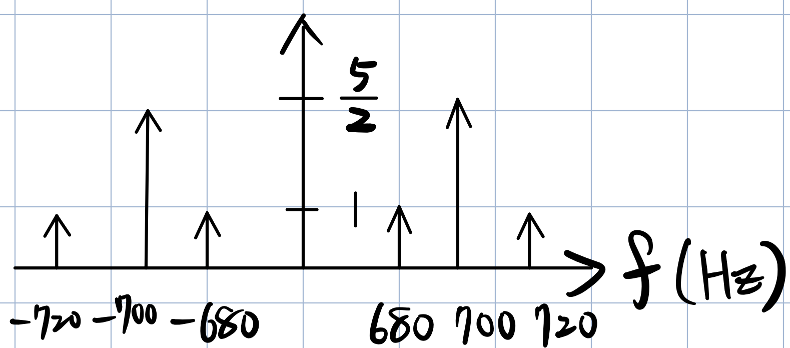
$$= [5+4\cos(40\pi t)]\cos(2\pi 700 t)$$

$$= 5\cos(2\pi 700t) + \underline{4\cos(40\pi t)\cos(2\pi 700 t)}$$

$$= 5\cos(2\pi 700 t) + 4\times\frac{1}{2}[(\cos 2\pi(700+20)t) + \cos(2\pi(700-20)t)$$

$$= 5\cos(2\pi 700t) + 2\cos(2\pi 720t) + 2\cos(2\pi 680 t)$$

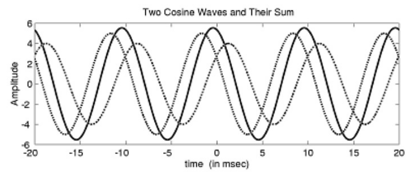
$$= \frac{5}{2}e^{j2\pi(700)t} + \frac{5}{2}e^{-j2\pi(700)t} + e^{j2\pi(720)t} + e^{-j2\pi(720)t} + e^{j2\pi(680)t} + e^{-j2\pi(680)}$$



3. Does the following statement hold?
"Sum of sinusoids of equal frequencies is still a sinusoid of the same frequency."
Show your reason.
(Here we assume that zero signal $x(t) = 0\ \forall t$ can be explained as a sinusoid of any frequency).

ADD SINUSOIDS

- Sum Sinusoid has SAME Frequency



Ans:

Suppose two sinusoid has same frequency

$$x_1(t) = A\sin(\omega t+\alpha) = \frac{A}{2j}\left(e^{j(\omega t+\alpha)} - e^{-j(\omega t+\alpha)}\right)$$

$$x_2(t) = B\sin(\omega t+\beta) = \frac{B}{2j}\left(e^{j(\omega t+\beta)} - e^{-j(\omega t+\beta)}\right)$$

Goal: pf $x_1(t)+x_2(t)$ is ω frequency sinusoid

$$\left(Ae^{j\alpha}e^{j\omega t} - Ae^{j\alpha}e^{-j\omega t}\right) + \left(Be^{j\beta}e^{j\omega t} - Be^{j\beta}e^{-j\omega t}\right)$$

$$= \left(Ae^{j\alpha} + Be^{j\beta}\right)e^{j\omega t} - \left(Ae^{j\alpha} + Be^{-j\beta}\right)e^{-j\omega t}$$

Let $Ce^{j\psi} = Ae^{j\alpha} + Be^{j\beta}$

$$C = \sqrt{(A\cos\alpha + B\cos\beta)^2 + (A\sin\alpha + B\sin\beta)^2}$$

$$\psi = \tan^{-1}\left(\frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}\right)$$

$\therefore A\sin(\omega t+\alpha) + B\sin(\omega t+\beta) = C\sin(\omega t+\psi)$ #

4. Derive that the following is a continuous Fourier transform pair $(a > 0)$, where $u(t)$ is the unit step function.

Time-Domain		***Frequency-Domain***
$e^{-at}u(t)$	$\xleftrightarrow{\mathcal{F}}$	$\dfrac{1}{a+j\omega}$

Ans:

PF: $\mathcal{F}\{e^{-at}u(t)\} = \dfrac{1}{a+j\omega}$ $a>0$,

$$= \int_{-\infty}^{\infty} e^{-j\omega t} e^{-at} u(t)\, dt$$

u(t) : Unit Step Function

表示在e^{-at}在$t \geq 0$定義為非零

在$t<0$為零.

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \times 1\, dt$$

$$= \frac{-1}{a+j\omega}\left(e^{-(a+j\omega)t}\Big|_0^{\infty}\right)$$

$$= \frac{-1}{a+j\omega}(0-1) = \frac{1}{a+j\omega}$$

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