DSP2025 Homework 8 Due date: 12:00 noon, May. 29, 2025

1. Given two length-4 sequences $x[n] = [2, 0, 1, 0]$ and $y[n] = [1, -1, 0, 0]$, for n=0,1,2,3.
(a) (1%) Compute the length-4 circular convolution of $x[n]$ and $y[n]$.
(b) (2%) Find $X[k]$ and $Y[k]$, the DFT of $x[n]$ and $y[n]$, respectively.
(c) (1%) Find $Z[k] = X[k]Y[k]$, the multiplication of $X[k]$ and $Y[k]$.
(d) (1%) Find the inverse DFT of $Z[k]$.

2. (5%) Suppose that $v[n]$ is a real-number sequence of length $2N$. Let $g[n]$ and $h[n]$ be the even and odd parts of $v[n]$, respectively, i.e.,

$$g[n] = v[2n], \quad h[n] = v[2n+1], \quad 0 \le n < N.$$

Let $V[k]$ be the DFT of $v[n]$, then $V[k]$ can be computed via the DFT of $g[n]$ and $h[n]$ by the following equation:

$$V[k] = G[k \bmod N] + f[k]H[k \bmod N], \quad 0 \le k \le 2N - 1,$$

with $G[k]$ and $H[k]$ the DFT of $g[n]$ and $h[n]$, respectively. **Question**: what is $f[k]$?