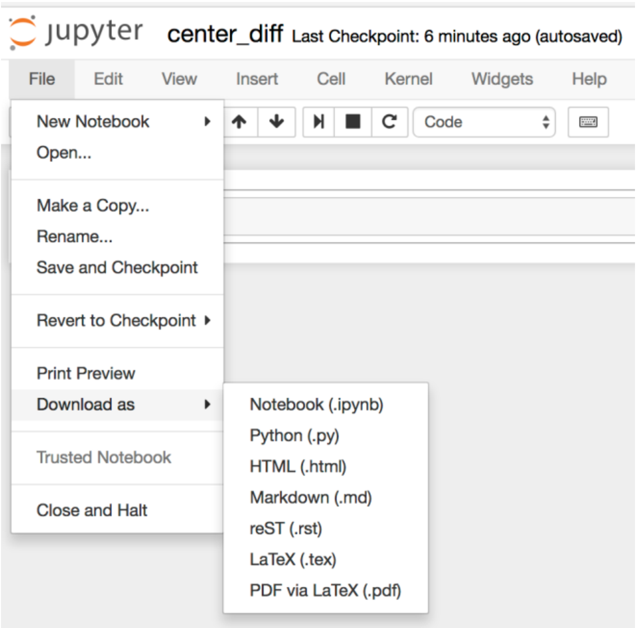
Homework 1
Concept, Derivation, Due 9:00, Tuesday, April 16, 2024

Late submission within 24 hours: score*0.9;
Late submission before post of solution: score*0.8 (the solution will usually be posted within a week); no late submission after the post of solution)

HW Submission Procedure (請仔細閱讀)

1. For **concept and derivation**, please write them in a professional format and submit a pdf file. Name your pdf file `YourID_HW1.pdf`, for example, `n96081494_HW1.pdf`

2. You should submit your Jupyter notebook and Python script (`*.py`, in Jupyter, click File, Download as, Python (`*.py`)).



3. Name a folder using your student id and HW number (e.g., n96081494_HW1), put the pdf and all the Jupyter notebooks and python scripts into the folder and zip the folder (e.g., n96081494_HW1.zip).

4. Submit your HW directly through the course website.

Total 120%

Concept and Derivation (Name your pdf file `YourID_HW1.pdf`, for example,`n96081494_HW1.pdf`)

1. (100%) Consider the simple network example with a single input $x = 2$ and a single output $y = 1$ shown in Figure 1 below.

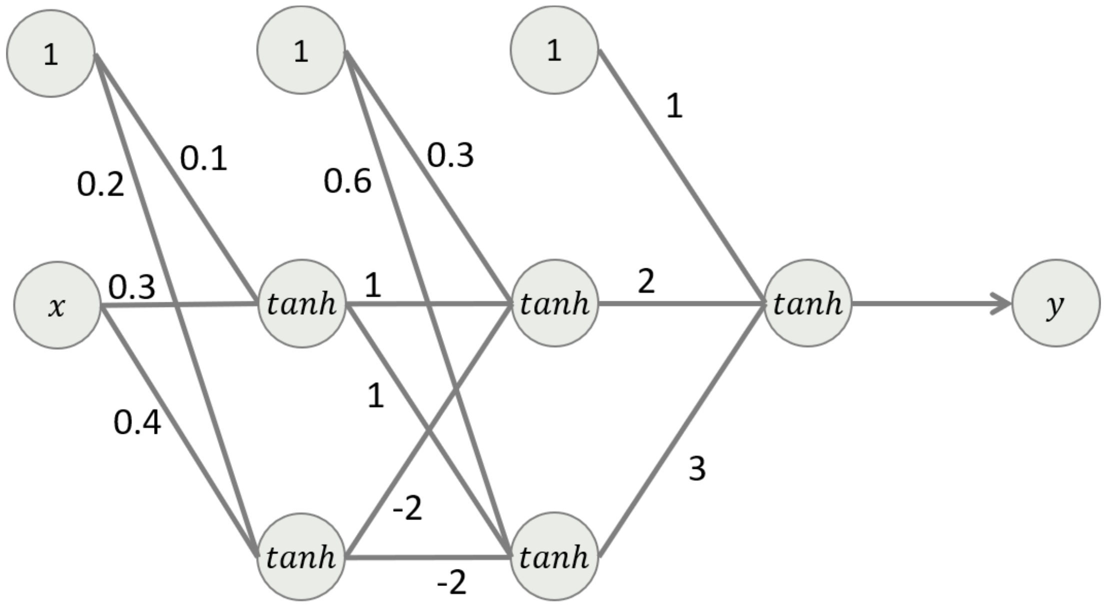


Figure 1

The weight matrices are:

$$\mathbf{W}^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}; \ \mathbf{W}^{(2)} = \begin{bmatrix} 0.3 & 0.6 \\ 1 & 1 \\ -2 & -2 \end{bmatrix}; \ \mathbf{W}^{(3)} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and the summation of weighted nodes for layer 1 can be expressed as $\mathbf{u}^{(1)} = (\mathbf{W}^{(1)})^T \mathbf{x}^{(0)}$; you can perform similar operation for other layers.

(a) (20%) Derive and compute $\mathbf{u}^{(1)}$, $\mathbf{z}^{(1)}$, $\mathbf{u}^{(2)}$, $\mathbf{z}^{(2)}$, and $\mathbf{y}^{(3)}$.

$$\mathrm{u}^{(1)} = w^{(1)^T} x_0 = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1 \end{bmatrix}$$

$$\mathrm{z}^{(1)} = \begin{bmatrix} 1 \\ tanh(0.7) \\ tanh(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.6044 \\ 0.7616 \end{bmatrix}$$

$$\mathrm{u}^{(2)} = \begin{bmatrix} 0.3 & 1 & -2 \\ 0.6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0.6044 \\ 0.7616 \end{bmatrix} = \begin{bmatrix} -0.6188 \\ -0.3188 \end{bmatrix}$$

$$\mathrm{z}^{(2)} = \begin{bmatrix} 1 \\ tanh(-0.6188) \\ tanh(-0.3188) \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5503 \\ -0.3084 \end{bmatrix}$$

$$y^{(3)} = tanh(u^{(3)}) = tanh\left(\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1\\-0.5503\\-0.3084\end{bmatrix}\right) = tanh(-1.0258) = -0.7722$$

(b) (35%) Using the half of the sum square as our error function, derive and compute $\boldsymbol{\delta}^{(3)}$, $\boldsymbol{\delta}^{(2)}$, $\boldsymbol{\delta}^{(1)}$.

$$\delta^{(3)} = \frac{\partial E_n}{\partial u^{(3)}} = (y - t) * g'(u^{(3)}) = (-0.7722 - 1) * (1 - tanh^2(-1.0258) = -0.7154$$

$$\delta^{(2)} = \begin{bmatrix}(1 - tanh^2(-0.6188)) * (-0.7154) * 2\\(1 - tanh^2(-0.3188)) * (-0.7154) * 3\end{bmatrix} = \begin{bmatrix}-0.9975\\-1.9420\end{bmatrix}$$

$$\delta^{(1)} = \begin{bmatrix}((1 - tanh^2(0.7)) * ((-0.9975) * 1 + (-1.9420) * 1)\\((1 - tanh^2(1)) * ((-0.9975) * (-2) + (-1.9420) * (-2))\end{bmatrix} = \begin{bmatrix}-1.8658\\2.4690\end{bmatrix}$$

(c) (20%) Compute $\frac{\partial E_n}{\partial \mathbf{W}^{(1)}}$, $\frac{\partial E_n}{\partial \mathbf{W}^{(2)}}$, $\frac{\partial E_n}{\partial \mathbf{W}^{(3)}}$.

$$\frac{\partial E_n}{\partial W^{(1)}} = x^{(0)}\left(\delta^{(1)}\right)^T = \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}-1.8658 & 2.4690\end{bmatrix} = \begin{bmatrix}-1.8658 & 2.4690\\-3.7316 & 4.9380\end{bmatrix}$$

$$\frac{\partial E_n}{\partial W^{(2)}} = \begin{bmatrix}1\\0.6044\\0.7616\end{bmatrix}\begin{bmatrix}-0.9975 & -1.9420\end{bmatrix} = \begin{bmatrix}-0.9975 & -1.9420\\-0.6029 & -1.1737\\-0.7597 & -1.478\end{bmatrix}$$

$$\frac{\partial E_n}{\partial W^{(3)}} = \begin{bmatrix}1\\-0.5503\\-0.3084\end{bmatrix}\begin{bmatrix}-0.7154\end{bmatrix} = \begin{bmatrix}-0.7154\\0.3937\\0.2206\end{bmatrix}$$

(d) (25%) Update the weight matrices using learning rate $\eta = 0.5$, repeat the forward propagation and compute $\mathbf{u}^{(1)}$, $\mathbf{z}^{(1)}$, $\mathbf{u}^{(2)}$, $\mathbf{z}^{(2)}$, and $\mathbf{y}^{(3)}$.

$$w^{(1)'} = w^{(1)} - \eta\frac{\partial E_n}{\partial w^{(1)}} = \begin{bmatrix}0.1 & 0.2\\0.3 & 0.4\end{bmatrix} - 0.5\begin{bmatrix}-1.8658 & 2.4690\\-3.7316 & 4.9380\end{bmatrix} = \begin{bmatrix}1.0328 & -1.0345\\2.1658 & -2.0690\end{bmatrix}$$

$$w^{(2)'} = \begin{bmatrix}0.3 & 0.6\\1 & 1\\-2 & -2\end{bmatrix} - 0.5\begin{bmatrix}-0.9975 & -1.9420\\-0.6029 & -1.1737\\-0.7597 & -1.478\end{bmatrix} = \begin{bmatrix}0.7988 & 1.5710\\1.3014 & 1.5869\\-1.6202 & -1.2605\end{bmatrix}$$

$$w^{(3)'} = \begin{bmatrix}1\\2\\3\end{bmatrix} - 0.5\begin{bmatrix}-0.7154\\0.3937\\0.2206\end{bmatrix} = \begin{bmatrix}1.3577\\1.8032\\2.8897\end{bmatrix}$$

$$u^{(1)'} = w^{(1)'^T}x_0 = \begin{bmatrix}1.0328 & 2.1658\\-1.0345 & -2.0690\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}5.3645\\-5.1725\end{bmatrix}$$

$$z^{(1)'} = \begin{bmatrix}1\\tanh(5.3645)\\tanh(-5.1725)\end{bmatrix} = \begin{bmatrix}1\\1\\-0.9999\end{bmatrix}$$

$$u^{(2)'} = \begin{bmatrix}0.7988 & 1.3014 & -1.6202\\1.5710 & 1.5869 & -1.2605\end{bmatrix}\begin{bmatrix}1\\1\\-0.9999\end{bmatrix} = \begin{bmatrix}3.7202\\4.4183\end{bmatrix}$$

$$z^{(2)} = \begin{bmatrix} 1 \\ tanh(3.7202) \\ tanh(4.4183) \end{bmatrix} = \begin{bmatrix} 1 \\ 0.9988 \\ 0.9997 \end{bmatrix}$$

$$u^{(3)'} = [1.3577 \quad 1.8032 \quad 2.8897] \begin{bmatrix} 1 \\ 0.9988 \\ 0.9997 \end{bmatrix} = [6.0476]$$

$$y^{(3)} = tanh\left(u^{(3)'}\right) = tanh(6.0476) = 0.999$$