

$$(a) u^{(1)} = (W^{(1)})^T x^{(0)} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1 \end{bmatrix}$$

$$\tanh(u^{(1)}) = \begin{bmatrix} \tanh(0.7) \\ \tanh(1) \end{bmatrix} \doteq \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix} \Rightarrow z^{(1)} = \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix}$$

$$u^{(2)} = (W^{(2)})^T z^{(1)} = \begin{bmatrix} 0.3 & 1 & -2 \\ 0.6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix} = \begin{bmatrix} -0.62 \\ -0.32 \end{bmatrix}$$

$$\tanh(u^{(2)}) = \begin{bmatrix} \tanh(-0.62) \\ \tanh(-0.32) \end{bmatrix} \doteq \begin{bmatrix} -0.55 \\ -0.31 \end{bmatrix} \Rightarrow z^{(2)} = \begin{bmatrix} -0.55 \\ -0.31 \end{bmatrix}$$

$$y^{(2)} = \tanh((W^{(3)})^T z^{(2)}) = \tanh(\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -0.55 \\ -0.31 \end{bmatrix}) = \tanh(-1.03) \doteq -0.77$$

$$(b) \delta^{(3)} = (y^{(2)} - t) h'(u^{(3)}) = (-0.77 - 1) (\tanh'(-1.03)) = (-1.77) (1 - \tanh^2(-1.03)) \doteq [-0.77]$$

$$\delta^{(2)} = \delta^{(3)} \cdot \begin{bmatrix} 2 \cdot \tanh'(-0.62) \\ 3 \cdot \tanh'(-0.32) \end{bmatrix} = (-0.77) \cdot \begin{bmatrix} 2 \cdot (1 - \tanh^2(-0.62)) \\ 3 \cdot (1 - \tanh^2(-0.32)) \end{bmatrix} \doteq \begin{bmatrix} -1 \\ -1.95 \end{bmatrix}$$

$$\delta^{(1)} = \begin{bmatrix} ((-1) \cdot 1 + (-1.95) \cdot 1) \cdot \tanh'(0.7) \\ ((-1) \cdot 2 + (-1.95) \cdot -2) \cdot \tanh'(1) \end{bmatrix} = \begin{bmatrix} (-2.95) \cdot (1 - \tanh^2(0.7)) \\ (5.9) \cdot (1 - \tanh^2(1)) \end{bmatrix} \doteq \begin{bmatrix} -1.89 \\ 2.49 \end{bmatrix}$$

$$(c) \frac{\partial E}{\partial W^{(1)}} = x^{(0)} (\delta^{(1)})^T = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} -1.89 & 2.49 \end{bmatrix} = \begin{bmatrix} -1.89 & 2.49 \\ -3.78 & 4.98 \end{bmatrix}$$

$$\frac{\partial E}{\partial W^{(2)}} = z^{(1)} (\delta^{(2)})^T = \begin{bmatrix} 0.6 \\ 0.76 \end{bmatrix} \begin{bmatrix} -1 & -1.95 \end{bmatrix} \doteq \begin{bmatrix} -0.6 & -1.17 \\ -0.76 & -1.48 \end{bmatrix}$$

$$\frac{\partial E}{\partial W^{(3)}} = z^{(2)} (\delta^{(3)})^T = \begin{bmatrix} -0.55 \\ -0.31 \end{bmatrix} \begin{bmatrix} -0.77 \end{bmatrix} \doteq \begin{bmatrix} -0.42 \\ 0.4 \end{bmatrix}$$

$$(d) W_{new}^{(1)} = W^{(1)} - \eta \cdot \frac{\partial E}{\partial W^{(1)}} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} - 0.5 \begin{bmatrix} -1.89 & 2.49 \\ -3.78 & 4.98 \end{bmatrix} \doteq \begin{bmatrix} 1.05 & -1.05 \\ 2.19 & -2.09 \end{bmatrix}$$

$$W_{new}^{(2)} = W^{(2)} - \eta \cdot \frac{\partial E}{\partial W^{(2)}} = \begin{bmatrix} 0.3 & 0.6 \\ 1 & 1 \end{bmatrix} - 0.5 \begin{bmatrix} -0.6 & -1.17 \\ -0.76 & -1.48 \end{bmatrix} \doteq \begin{bmatrix} 0.8 & 1.58 \\ 1.3 & 1.59 \end{bmatrix}$$

$$W_{new}^{(3)} = W^{(3)} - \eta \cdot \frac{\partial E}{\partial W^{(3)}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 0.5 \begin{bmatrix} -0.42 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 1.36 \\ 1.8 \\ 2.89 \end{bmatrix}$$

$$u^{(1)} = (W^{(1)})^T x^{(0)} = \begin{bmatrix} 1.05 & 2.19 \\ -1.05 & -2.09 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5.43 \\ -5.23 \end{bmatrix}$$

$$\tanh(u^{(1)}) = \begin{bmatrix} \tanh(5.43) \\ \tanh(-5.23) \end{bmatrix} \doteq \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow z^{(1)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$u^{(2)} = (W^{(2)})^T z^{(1)} = \begin{bmatrix} 0.8 & 1.3 & -1.62 \\ 1.58 & 1.59 & -1.26 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3.72 \\ 4.43 \end{bmatrix}$$

$$\tanh(u^{(2)}) = \begin{bmatrix} \tanh(3.72) \\ \tanh(4.43) \end{bmatrix} \doteq \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow z^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y^{(2)} = \tanh((W^{(3)})^T z^{(2)}) = \tanh(\begin{bmatrix} 1.36 & 1.8 & 2.89 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \tanh(6.05) \doteq 1$$