Pattern Recognition Homework1

楊憲閔 613410047

1.

$g_1(\mathbf{x}) = p(\mathbf{x}|\omega_1)P(\omega_1)$

$g_2(\mathbf{x}) = p(\mathbf{x}|\omega_2)P(\omega_2)$

$P(\omega_1) = 0.6$

$P(\omega_2) = 0.4$

$p(\mathbf{x}|\omega_1) = \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{(x-m)^2}{2\sigma^2}] = \frac{1}{2\sqrt{2\pi}} exp[-\frac{(x+1)^2}{8}]$

$p(\mathbf{x}|\omega_2) = \frac{1}{\sqrt{2\pi}\sigma} exp[-\frac{(x-m)^2}{2\sigma^2}] = \frac{1}{\sqrt{3}\sqrt{2\pi}} exp[-\frac{(x-2)^2}{6}]$

$Let\ g_1(\mathbf{x}) = g_2(\mathbf{x})$

$\Rightarrow \frac{1}{2\sqrt{2\pi}} exp[-\frac{(x+1)^2}{8}] * 0.6 = \frac{1}{\sqrt{3}\sqrt{2\pi}} exp[-\frac{(x-2)^2}{6}] * 0.4$

$\Rightarrow \frac{3\sqrt{3}}{4} = exp[\frac{(x+1)^2}{8} - \frac{(x-2)^2}{6}]$

$\Rightarrow \frac{3\sqrt{3}}{4} = exp[\frac{6(x+1)^2 - 8(x-2)^2}{48}]$

$\Rightarrow ln\frac{3\sqrt{3}}{4} = \frac{6(x+1)^2 - 8(x-2)^2}{48}$

$\Rightarrow ln\frac{3\sqrt{3}}{4} = \frac{-x^2 + 22x - 26}{24}$

$\Rightarrow x \fallingdotseq 0.91432\ or\ 21.08568$

Ans : $R_1 = (\infty, 0.91432]\ \cup\ [21.08568, \infty)\ ,\ R_2 = [0.91432, 21.08568]$

2.

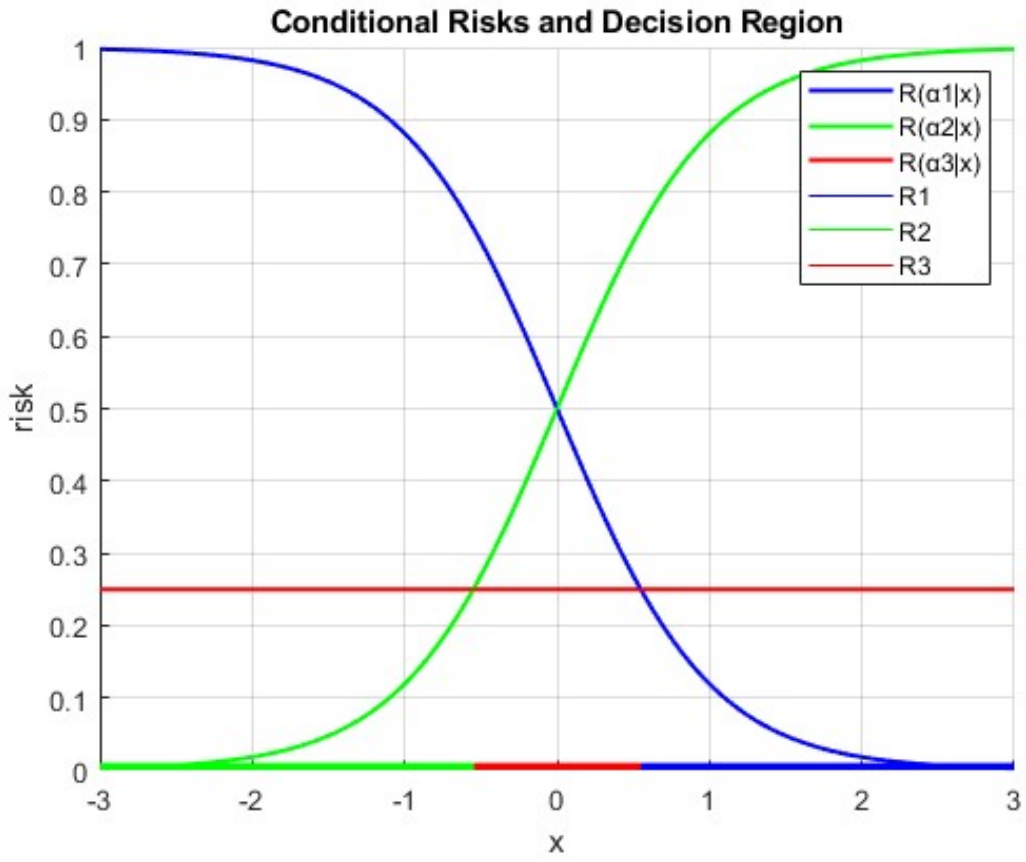

Figure 1: Conditional Risks and Decision Region

3.

(a) ω_1 mean vector $= \begin{bmatrix} -0.4400 \\ -1.7490 \end{bmatrix}$

ω_2 mean vector $= \begin{bmatrix} -0.5430 \\ -0.7620 \end{bmatrix}$

ω_3 mean vector $= \begin{bmatrix} 3.8830 \\ 1.3760 \end{bmatrix}$

ω_1 covariance vector $= \begin{bmatrix} 12.9425 & 6.9258 \\ 6.9258 & 13.1608 \end{bmatrix}$

ω_2 covariance vector $= \begin{bmatrix} 33.1464 & 8.9828 \\ 8.9828 & 11.8517 \end{bmatrix}$

ω_3 covariance vector $= \begin{bmatrix} 7.4743 & 6.7005 \\ 6.7005 & 7.7044 \end{bmatrix}$

(b) ω_1 error rate = 70%

ω_2 error rate = 30%

ω_3 error rate = 10%

(c) ω_1 mean vector = $\begin{bmatrix} -0.4400 \\ -1.7490 \\ -0.7660 \end{bmatrix}$

ω_2 mean vector = $\begin{bmatrix} -0.5430 \\ -0.7620 \\ -0.5420 \end{bmatrix}$

ω_3 mean vector = $\begin{bmatrix} 3.8830 \\ 1.3760 \\ 1.5800 \end{bmatrix}$

ω_1 covariance vector = $\begin{bmatrix} 12.9425 & 6.9258 & 3.7101 \\ 6.9258 & 13.1608 & 3.5162 \\ 3.7101 & 3.5162 & 17.7521 \end{bmatrix}$

ω_2 covariance vector = $\begin{bmatrix} 33.1464 & 8.9828 & -14.7301 \\ 8.9828 & 11.8517 & 0.3681 \\ -14.7301 & 0.3681 & 16.5791 \end{bmatrix}$

ω_3 covariance vector = $\begin{bmatrix} 7.4743 & 6.7005 & 11.8346 \\ 6.7005 & 7.7044 & 10.4477 \\ 11.8346 & 10.4477 & 42.5586 \end{bmatrix}$

ω_1 error rate = 50%

ω_2 error rate = 30%

ω_3 error rate = 0%

4.

(a)

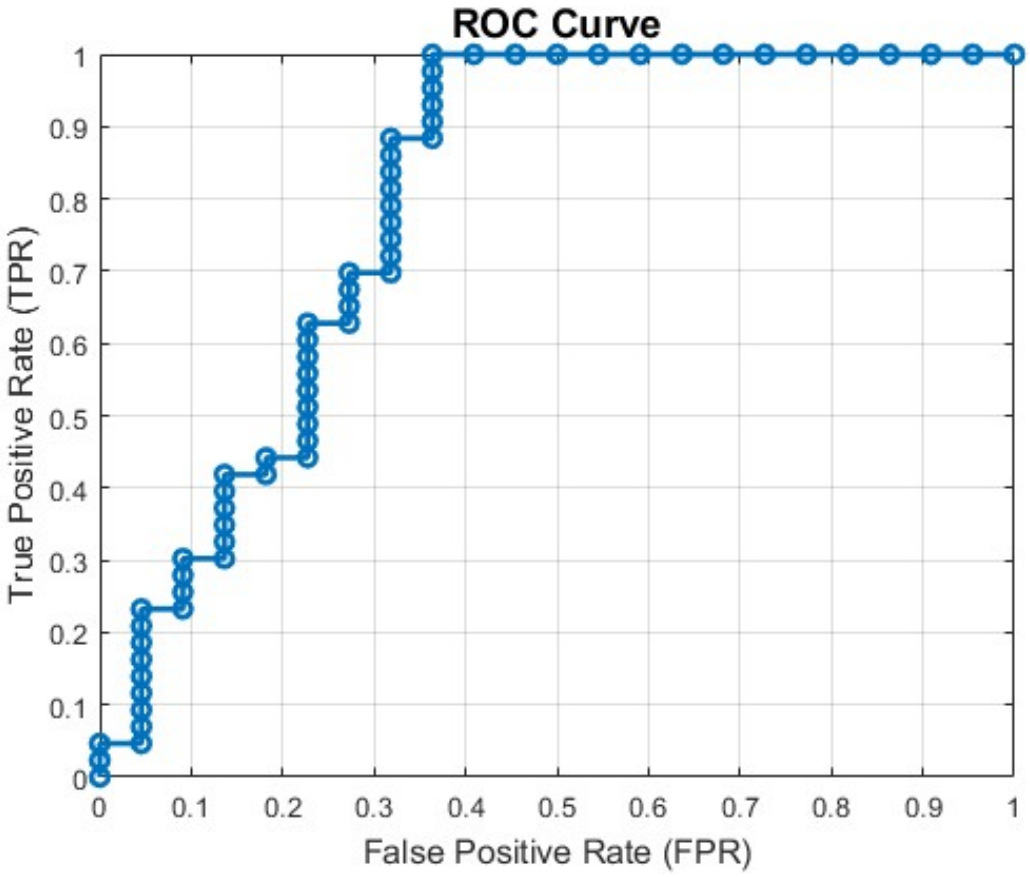

Figure 2: ROC curve

(b) AUC = 0.80