

# Pattern Recognition Homework1

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1.

$$g_1(\mathbf{x}) = p(\mathbf{x}|\omega_1)P(\omega_1)$$

$$g_2(\mathbf{x}) = p(\mathbf{x}|\omega_2)P(\omega_2)$$

$$P(\omega_1) = 0.6$$

$$P(\omega_2) = 0.4$$

$$p(\mathbf{x}|\omega_1) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right] = \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{(x+1)^2}{8}\right]$$

$$p(\mathbf{x}|\omega_2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right] = \frac{1}{\sqrt{3}\sqrt{2\pi}} \exp\left[-\frac{(x-2)^2}{6}\right]$$

$$\text{Let } g_1(\mathbf{x}) = g_2(\mathbf{x})$$

$$\Rightarrow \frac{1}{2\sqrt{2\pi}} \exp\left[-\frac{(x+1)^2}{8}\right] * 0.6 = \frac{1}{\sqrt{3}\sqrt{2\pi}} \exp\left[-\frac{(x-2)^2}{6}\right] * 0.4$$

$$\Rightarrow \frac{3\sqrt{3}}{4} = \exp\left[\frac{(x+1)^2}{8} - \frac{(x-2)^2}{6}\right]$$

$$\Rightarrow \frac{3\sqrt{3}}{4} = \exp\left[\frac{6(x+1)^2 - 8(x-2)^2}{48}\right]$$

$$\Rightarrow \ln \frac{3\sqrt{3}}{4} = \frac{6(x+1)^2 - 8(x-2)^2}{48}$$

$$\Rightarrow \ln \frac{3\sqrt{3}}{4} = \frac{-x^2 + 22x - 26}{24}$$

$$\Rightarrow x \doteq 0.91432 \text{ or } 21.08568$$

$$\text{Ans : } R_1 = (\infty, 0.91432] \cup [21.08568, \infty), \quad R_2 = [0.91432, 21.08568]$$

2.

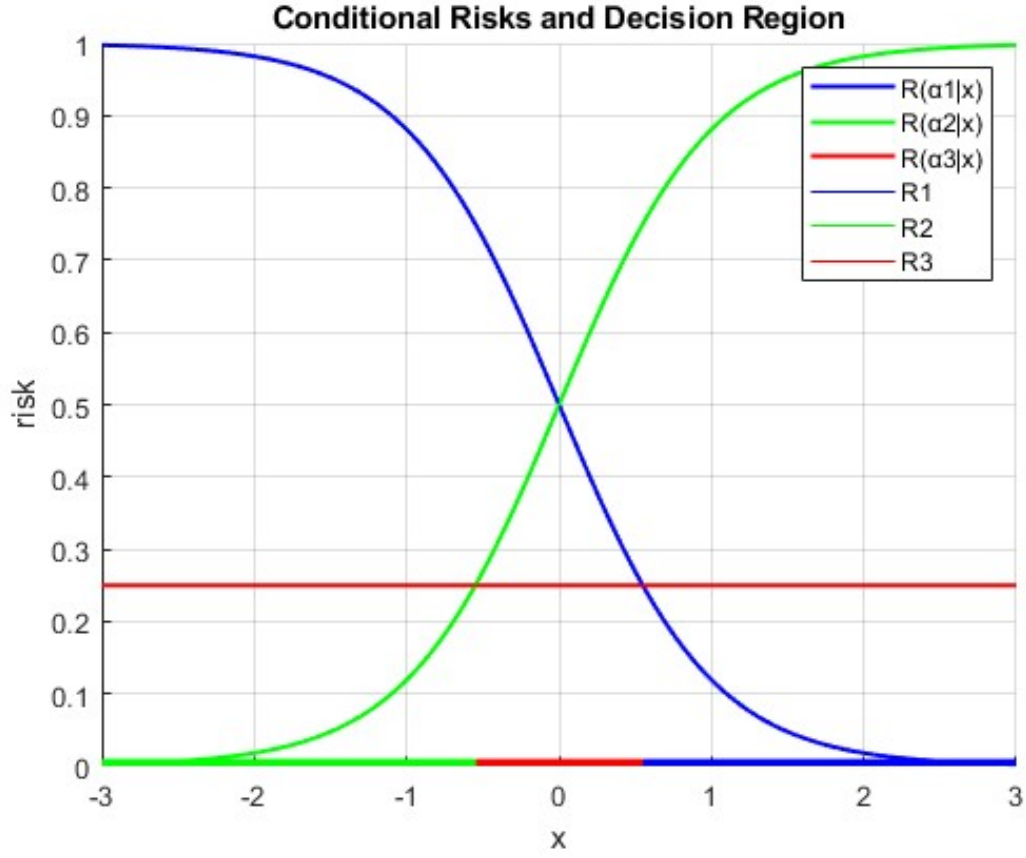


Figure 1: Conditional Risks and Decision Region

3.

$$(a) \omega_1 \text{ mean vector} = \begin{bmatrix} -0.4400 \\ -1.7490 \end{bmatrix}$$

$$\omega_2 \text{ mean vector} = \begin{bmatrix} -0.5430 \\ -0.7620 \end{bmatrix}$$

$$\omega_3 \text{ mean vector} = \begin{bmatrix} 3.8830 \\ 1.3760 \end{bmatrix}$$

$$\omega_1 \text{ covariance vector} = \begin{bmatrix} 12.9425 & 6.9258 \\ 6.9258 & 13.1608 \end{bmatrix}$$

$$\omega_2 \text{ covariance vector} = \begin{bmatrix} 33.1464 & 8.9828 \\ 8.9828 & 11.8517 \end{bmatrix}$$

$$\omega_3 \text{ covariance vector} = \begin{bmatrix} 7.4743 & 6.7005 \\ 6.7005 & 7.7044 \end{bmatrix}$$

(b)  $\omega_1$  error rate = 70%

$\omega_2$  error rate = 30%

$\omega_3$  error rate = 10%

(c)  $\omega_1$  mean vector =  $\begin{bmatrix} -0.4400 \\ -1.7490 \\ -0.7660 \end{bmatrix}$

$\omega_2$  mean vector =  $\begin{bmatrix} -0.5430 \\ -0.7620 \\ -0.5420 \end{bmatrix}$

$\omega_3$  mean vector =  $\begin{bmatrix} 3.8830 \\ 1.3760 \\ 1.5800 \end{bmatrix}$

$\omega_1$  covariance vector =  $\begin{bmatrix} 12.9425 & 6.9258 & 3.7101 \\ 6.9258 & 13.1608 & 3.5162 \\ 3.7101 & 3.5162 & 17.7521 \end{bmatrix}$

$\omega_2$  covariance vector =  $\begin{bmatrix} 33.1464 & 8.9828 & -14.7301 \\ 8.9828 & 11.8517 & 0.3681 \\ -14.7301 & 0.3681 & 16.5791 \end{bmatrix}$

$\omega_3$  covariance vector =  $\begin{bmatrix} 7.4743 & 6.7005 & 11.8346 \\ 6.7005 & 7.7044 & 10.4477 \\ 11.8346 & 10.4477 & 42.5586 \end{bmatrix}$

$\omega_1$  error rate = 50%

$\omega_2$  error rate = 30%

$\omega_3$  error rate = 0%

## 4.

(a)

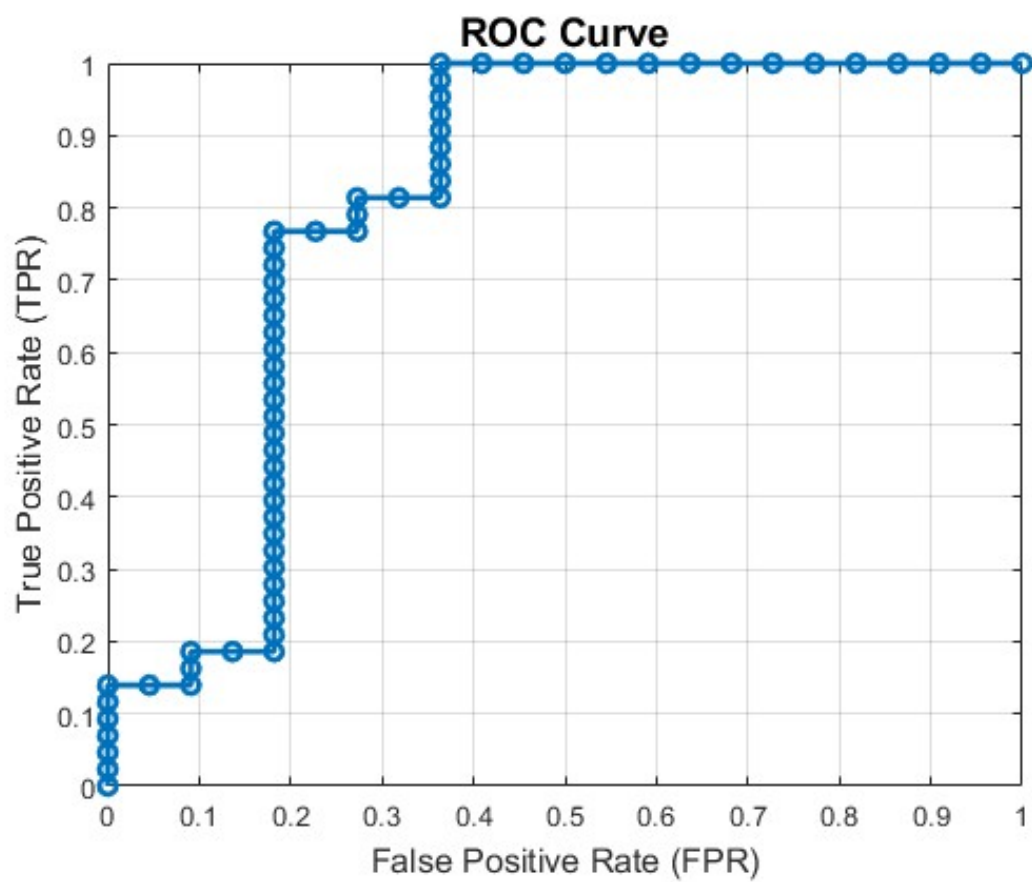


Figure 2: ROC curve

(b)  $AUC = 0.81$