

Practice – Backward Propagation Detailed Explanation

First of all, the symbols are defined as below for explanation:

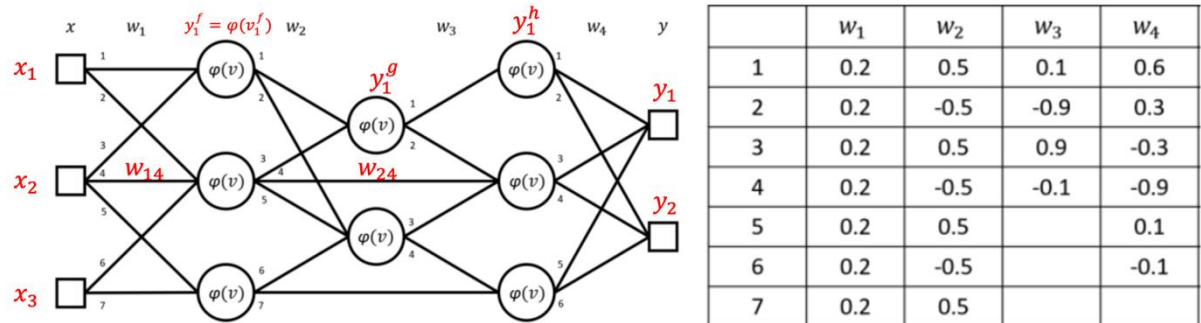


Figure 1.

v	-0.53	-0.37	-0.38	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.37	0.38	0.39	0.40	0.53
$\varphi(v)$	0	0.02	0.02	0.73	0.75	0.77	0.79	0.8	0.82	0.83	0.98	0.98	0.98	0.98	1
$\varphi'(v)$	0	0.2	0.2	1.97	1.88	1.77	1.66	1.6	1.48	1.41	0.2	0.2	0.2	0.2	0

Figure 2.

(a)

$$v_1^f = w_{11}x_1 + w_{13}x_2 = 0.2 * 0.2 + 0.2 * 0.5 = 0.14$$

$$y_1^f = \varphi(v_1^f) = \varphi(0.14) = 0.8$$

$$v_2^f = w_{12}x_1 + w_{14}x_2 + w_{16}x_3 = 0.2 * 0.2 + 0.2 * 0.5 + 0.2 * 0.1 = 0.16$$

$$y_2^f = \varphi(v_2^f) = \varphi(0.16) = 0.83$$

$$v_3^f = w_{15}x_2 + w_{17}x_3 = 0.2 * 0.5 + 0.2 * 0.1 = 0.12$$

$$y_3^f = \varphi(v_3^f) = \varphi(0.12) = 0.77$$

$$v_1^g = w_{21}y_1^f + w_{23}y_2^f = 0.5 * 0.8 + 0.5 * 0.83 = 0.815$$

$$y_1^g = \varphi(v_1^g) = \varphi(0.815) = 1$$

$$v_2^g = w_{22}y_1^f + w_{25}y_2^f + w_{26}y_3^f = (-0.5) * 0.8 + 0.5 * 0.83 + (-0.5) * 0.77 = -0.37$$

$$y_2^g = \varphi(v_2^g) = \varphi(-0.37) = 0.02$$

$$v_1^h = w_{31}y_1^g = 0.1 * 1 = 0.1$$

$$y_1^h = \varphi(v_1^h) = \varphi(0.1) = 0.73$$

$$v_2^h = w_{32}y_1^g + w_{24}y_2^f + w_{33}y_2^g = (-0.9) * 1 + (-0.5) * 0.83 + 0.9 * 0.02 = -1.297$$

$$y_2^h = \varphi(v_2^h) = \varphi(-1.297) = 0$$

$$v_3^h = w_{34}y_2^g + w_{27}y_3^f = (-0.1) * 0.02 + 0.5 * 0.77 = 0.383$$

$$y_3^h = \varphi(v_3^h) = \varphi(0.383) = 0.98$$

$$y_1 = w_{41}y_1^h + w_{43}y_2^h + w_{45}y_3^h = 0.6 * 0.73 + (-0.3) * 0 + 0.1 * 0.98 = 0.536$$

$$y_2 = w_{42}y_1^h + w_{44}y_2^h + w_{46}y_3^h = 0.3 * 0.73 + (-0.9) * 0 + (-0.1) * 0.98 = 0.121$$

(b)

$$\begin{aligned}\frac{\partial E}{\partial w_{31}} &= \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_1^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_1^h} \right) \frac{\partial y_1^h}{\partial v_1^h} \frac{\partial v_1^h}{\partial w_{31}} \\ &= [(y_1 - d_1) * w_{41} + (y_2 - d_2) * w_{42}] * \varphi'(v_1^h) * y_1^g \\ &= [(0.536 - 1) * 0.6 + (0.121 - 0) * 0.3] * 1.97 * 1 = -0.477 \\ w_{31}(n+1) &= w_{31}(n) - \eta \frac{\partial E}{\partial w_{31}} = 0.1 - 0.5 * (-0.477) = 0.339\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial w_{32}} &= \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_2^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial w_{32}} \\ &= [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * y_1^g \\ &= [(0.536 - 1) * (-0.3) + (0.121 - 0) * (-0.9)] * 0 * 1 = 0 \\ w_{32}(n+1) &= w_{32}(n) - \eta \frac{\partial E}{\partial w_{32}} = (-0.9) - 0.5 * 0 = -0.9\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial w_{33}} &= \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_2^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial w_{33}} \\ &= [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * y_2^g \\ &= [(0.536 - 1) * (-0.3) + (0.121 - 0) * (-0.9)] * 0 * 0.02 = 0 \\ w_{33}(n+1) &= w_{33}(n) - \eta \frac{\partial E}{\partial w_{33}} = 0.9 - 0.5 * 0 = 0.9\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial w_{34}} &= \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_3^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_3^h} \right) \frac{\partial y_3^h}{\partial v_3^h} \frac{\partial v_3^h}{\partial w_{34}} \\ &= [(y_1 - d_1) * w_{45} + (y_2 - d_2) * w_{46}] * \varphi'(v_3^h) * y_2^g \\ &= [(0.536 - 1) * 0.1 + (0.121 - 0) * (-0.1)] * 0.2 * 0.02 = -0.000234 \\ w_{34}(n+1) &= w_{34}(n) - \eta \frac{\partial E}{\partial w_{34}} = (-0.1) - 0.5 * (-0.000234) = -0.099883\end{aligned}$$

(c)

$$\begin{aligned}
\frac{\partial E}{\partial w_{21}} &= \left[\left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_1^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_1^h} \right) \frac{\partial y_1^h}{\partial v_1^h} \frac{\partial v_1^h}{\partial y_1^g} + \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_2^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial y_1^g} \right] \frac{\partial y_1^g}{\partial v_1^g} \frac{\partial v_1^g}{\partial w_{21}} \\
&= \{ [(y_1 - d_1) * w_{41} + (y_2 - d_2) * w_{42}] * \varphi'(v_1^h) * w_{31} \\
&\quad + [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{32} \} * \varphi'(v_1^g) * y_1^f \\
&= \{ [(y_1 - d_1) * w_{41} + (y_2 - d_2) * w_{42}] * \varphi'(v_1^h) * w_{31} \\
&\quad + [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{32} \} * 0 * y_1^f = 0
\end{aligned}$$

$$w_{21}(n+1) = w_{21}(n) - \eta \frac{\partial E}{\partial w_{21}} = 0.5 - 0.5 * 0 = 0.5$$

$$\begin{aligned}
\frac{\partial E}{\partial w_{22}} &= \left[\left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_1^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial y_2^g} + \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_3^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_3^h} \right) \frac{\partial y_3^h}{\partial v_3^h} \frac{\partial v_3^h}{\partial y_2^g} \right] \frac{\partial y_2^g}{\partial v_2^g} \frac{\partial v_2^g}{\partial w_{22}} \\
&= \{ [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{33} \\
&\quad + [(y_1 - d_1) * w_{45} + (y_2 - d_2) * w_{46}] * \varphi'(v_3^h) * w_{34} \} * \varphi'(v_2^g) * y_1^f \\
&= \{ [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * 0 * w_{33} \\
&\quad + [(0.536 - 1) * 0.1 + (0.121 - 0) * (-0.1)] * 0.2 * (-0.1) \} * 0.2 * 0.8 \\
&= 0.0001872
\end{aligned}$$

$$w_{22}(n+1) = w_{22}(n) - \eta \frac{\partial E}{\partial w_{22}} = (-0.5) - 0.5 * 0.0001872 = -0.5000936$$

$$\begin{aligned}
\frac{\partial E}{\partial w_{23}} &= \left[\left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_1^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_1^h} \right) \frac{\partial y_1^h}{\partial v_1^h} \frac{\partial v_1^h}{\partial y_1^g} + \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_2^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial y_1^g} \right] \frac{\partial y_1^g}{\partial v_1^g} \frac{\partial v_1^g}{\partial w_{23}} \\
&= \{ [(y_1 - d_1) * w_{41} + (y_2 - d_2) * w_{42}] * \varphi'(v_1^h) * w_{31} \\
&\quad + [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{32} \} * \varphi'(v_1^g) * y_2^f \\
&= \{ [(y_1 - d_1) * w_{41} + (y_2 - d_2) * w_{42}] * \varphi'(v_1^h) * w_{31} \\
&\quad + [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{32} \} * 0 * y_2^f = 0
\end{aligned}$$

$$w_{23}(n+1) = w_{23}(n) - \eta \frac{\partial E}{\partial w_{23}} = 0.5 - 0.5 * 0 = 0.5$$

$$\begin{aligned}
\frac{\partial E}{\partial w_{24}} &= \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_2^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial w_{24}} \\
&= [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * y_2^f \\
&= [(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * 0 * y_2^f = 0
\end{aligned}$$

$$w_{24}(n+1) = w_{24}(n) - \eta \frac{\partial E}{\partial w_{24}} = (-0.5) - 0.5 * 0 = -0.5$$

$$\begin{aligned}
\frac{\partial E}{\partial w_{25}} &= \left[\left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_1^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial y_2^g} + \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_3^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_3^h} \right) \frac{\partial y_3^h}{\partial v_3^h} \frac{\partial v_3^h}{\partial y_2^g} \right] \frac{\partial y_2^g}{\partial v_2^g} \frac{\partial v_2^g}{\partial w_{25}} \\
&= \{[(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{33} \\
&\quad + [(y_1 - d_1) * w_{45} + (y_2 - d_2) * w_{46}] * \varphi'(v_3^h) * w_{34}\} * \varphi'(v_2^g) * y_2^f \\
&= \{[(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * 0 * w_{33} \\
&\quad + [(0.536 - 1) * 0.1 + (0.121 - 0) * (-0.1)] * 0.2 * (-0.1)\} * 0.2 * 0.83 \\
&= 0.00019422
\end{aligned}$$

$$w_{25}(n+1) = w_{25}(n) - \eta \frac{\partial E}{\partial w_{25}} = 0.5 - 0.5 * 0.00019422 = 0.49990289$$

$$\begin{aligned}
\frac{\partial E}{\partial w_{26}} &= \left[\left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_1^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_2^h} \right) \frac{\partial y_2^h}{\partial v_2^h} \frac{\partial v_2^h}{\partial y_2^g} + \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_3^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_3^h} \right) \frac{\partial y_3^h}{\partial v_3^h} \frac{\partial v_3^h}{\partial y_2^g} \right] \frac{\partial y_2^g}{\partial v_2^g} \frac{\partial v_2^g}{\partial w_{26}} \\
&= \{[(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * \varphi'(v_2^h) * w_{33} \\
&\quad + [(y_1 - d_1) * w_{45} + (y_2 - d_2) * w_{46}] * \varphi'(v_3^h) * w_{34}\} * \varphi'(v_2^g) * y_3^f \\
&= \{[(y_1 - d_1) * w_{43} + (y_2 - d_2) * w_{44}] * 0 * w_{33} \\
&\quad + [(0.536 - 1) * 0.1 + (0.121 - 0) * (-0.1)] * 0.2 * (-0.1)\} * 0.2 * 0.77 \\
&= 0.00018018
\end{aligned}$$

$$w_{26}(n+1) = w_{26}(n) - \eta \frac{\partial E}{\partial w_{26}} = (-0.5) - 0.5 * 0.00018018 = -0.50009009$$

$$\begin{aligned}
\frac{\partial E}{\partial w_{27}} &= \left(\frac{\partial E}{\partial y_1} \frac{\partial y_1}{\partial y_3^h} + \frac{\partial E}{\partial y_2} \frac{\partial y_2}{\partial y_3^h} \right) \frac{\partial y_3^h}{\partial v_3^h} \frac{\partial v_3^h}{\partial w_{27}} \\
&= [(y_1 - d_1) * w_{45} + (y_2 - d_2) * w_{46}] * \varphi'(v_3^h) * y_3^f \\
&= [(0.536 - 1) * 0.1 + (0.121 - 0) * (-0.1)] * 0.2 * 0.77 = -0.009009
\end{aligned}$$

$$w_{27}(n+1) = w_{27}(n) - \eta \frac{\partial E}{\partial w_{27}} = 0.5 - 0.5 * (-0.009009) = 0.5045045$$