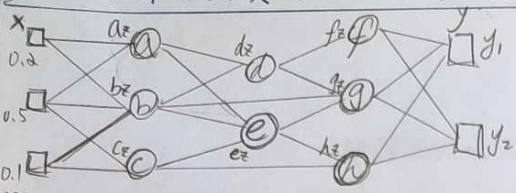
人工智慧導論與實作 HW1 資訊系 F74096124 楊憲閔



(a) $a_z = 0.2\times0.2 + 0.5\times0.2 = 0.14$ $a = \varphi(a_z) = \varphi(0.14) = 0.8$

$b_z = 0.2\times0.2 + 0.5\times0.2 + 0.1\times0.2 = 0.16$ $b = \varphi(b_z) = \varphi(0.16) = 0.83$

$c_z = 0.5\times0.2 + 0.1\times0.2 = 0.12$ $c = \varphi(c_z) = \varphi(0.12) = 0.777$

$d_z = 0.8\times0.5 + 0.83\times0.5 = 0.815$ $d = \varphi(d_z) = \varphi(0.815) \fallingdotseq 1$

$e_z = 0.8\times-0.5 + 0.83\times0.5 + 0.777\times-0.5 = -0.37$ $e = \varphi(e_z) = \varphi(-0.37) = 0.02$

$f_z = 1\times0.1 = 0.1$ $f = \varphi(f_z) = \varphi(0.1) = 0.73$

$g_z = 1\times-0.9 + 0.83\times-0.5 + 0.02\times0.9 = -1.297$ $g = \varphi(g_z) = \varphi(-1.297) \fallingdotseq 0$

$h_z = 0.02\times-0.1 + 0.777\times0.5 = 0.383$ $h = \varphi(h_z) = \varphi(0.383) \fallingdotseq 0.98$

∴ each hidden layer: $a=0.8$ $b=0.83$ $c=0.777$ $d=1$ $e=0.02$ $f=0.73$ $g=0$ $h=0.98$

$y_1 = 0.73\times0.6 + 0\times-0.3 + 0.98\times0.1 = 0.536$ $y_2 = 0.73\times0.3 + 0\times-0.9 + 0.98\times-0.1 = 0.121$

∴ $y_{pred} = [0.536, 0.121]^T$

(b) 更新 $W_{3,1}$ 之值：① $d\to f\to y_1$

② $d\to f\to y_2$

$$d\to f\to y_1: \frac{\partial E}{\partial W_{3,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial W_{3,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial W_{3,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial W_{3,1}} = (y_1-y_{real})(W_{4,1})(\varphi'(f_z))(d)$$

$$= (0.536-1)(0.6)(1.97)(1) = -0.548448$$

$$d\to f\to y_2: \frac{\partial E}{\partial W_{3,1}} = (y_2-y_{real})(W_{4,2})(\varphi(f_z))(d) = (0.121)(0.3)(1.97)(1) = 0.071511$$

∴ $L = -0.548448 + 0.071511 \fallingdotseq -0.477 \Rightarrow W_{3,1,new} = W_{3,1,old} - 0.5\times-0.477 = 0.339$

更新 $W_{3,2}$ 之值：① $d\to g\to y_1$

② $d\to g\to y_2$

$$d\to g\to y_1: \frac{\partial E}{\partial W_{3,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial W_{3,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial W_{3,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial W_{3,2}} = (y_1-y_{real})(W_{4,3})(\varphi'(g_z))(d)$$

$$= (0.536-1)(-0.3)(0)(1) = 0$$

$$d\to g\to y_2: \frac{\partial E}{\partial W_{3,2}} = (y_2-y_{real})(W_{4,4})(\varphi'(g_z))(d) = 0$$

∴ $L = 0 \Rightarrow W_{3,2,new} = W_{3,2,old} - 0.5\times0 \Rightarrow W_{3,2,new} = -0.9$

更新 $W_{3,3}$ 之值：① $e\to g\to y_1$

② $e\to g\to y_2$

$$e\to g\to y_1: \frac{\partial E}{\partial W_{3,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial W_{3,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial W_{3,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial W_{3,3}} = (y_1-y_{real})(W_{4,3})(\varphi'(g_z))(e)$$

$$= (0.536-1)(-0.3)(0)(0.02) = 0$$

$$e\to g\to y_2: \frac{\partial E}{\partial W_{3,3}} = (y_2-y_{real})(W_{4,4})(\varphi'(g_z))(e) = 0$$

∴ $L = 0 \Rightarrow W_{3,3,new} = W_{3,3,old} - 0.5\times0 \Rightarrow W_{3,3,new} = 0.9$

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更新 $W_{3,4}$ = ① $e \to h \to y_1$

② $e \to h \to y_2$

$e \to h \to y_1$: $\frac{\partial E}{\partial W_{3,4}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial W_{3,4}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial h}\frac{\partial h}{\partial W_{3,4}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial h}\frac{\partial h}{\partial z_e}\frac{\partial z_e}{\partial W_{3,4}} = (y_1 - y_{real})(W_{4,5})(\varphi'(h_z))(e)$

$= (0.536-1)(0.1)(0.2)(0.02) = -0.0001856$

$e \to h \to y_2$: $\frac{\partial E}{\partial W_{3,4}} = (y_2 - y_{real})(W_{4,6})(\varphi'(h_z))(e) = (0.121)(-0.1)(0.2)(0.02) = -0.0000484$

$\therefore L = -0.000234 \Rightarrow W_{3,4,new} = W_{3,4,old} - 0.5 \times -0.000234 = -0.099883$

$\therefore$ W_3 更新完為 :

	1	2	3	4
W_3	0.339	-0.9	0.9	-0.099883

(c) 更新 $W_{2,1}$: ① $a \to d \to f \to y_1$

② $a \to d \to f \to y_2$

③ $a \to d \to g \to y_1$

④ $a \to d \to g \to y_2$

$a \to d \to f \to y_1$: $\frac{\partial E}{\partial W_{2,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial W_{2,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial W_{2,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial W_{2,1}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial d}\frac{\partial d}{\partial W_{2,1}}$

$= \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial d}\frac{\partial d}{\partial d_z}\frac{\partial d_z}{\partial W_{2,1}} = (y_1 - y_{real})(W_{4,1})(\varphi'(f_z))(W_{3,1})(\varphi'(d_z))(a) = 0$

$a \to d \to f \to y_2$: $\frac{\partial E}{\partial W_{2,1}} = (y_2 - y_{real})(W_{4,2})(\varphi'(f_z))(W_{3,1})(\varphi'(d_z))(a) = 0$

$a \to d \to g \to y_1$: $\frac{\partial E}{\partial W_{2,1}} = (y_1 - y_{real})(W_{4,3})(\varphi'(g_z))(W_{3,2})(\varphi'(d_z))(a) = 0$

$a \to d \to g \to y_2$: $\frac{\partial E}{\partial W_{2,1}} = (y_2 - y_{real})(W_{4,4})(\varphi'(g_z))(W_{3,2})(\varphi'(d_z))(a) = 0$

$W_{2,1,new} = W_{2,1,old} - 0.5 \times 0 \Rightarrow W_{2,1,new} = 0.5$

更新 $W_{2,2}$ = ① $a \to e \to g \to y_1$

② $a \to e \to g \to y_2$

③ $a \to e \to h \to y_1$

④ $a \to e \to h \to y_2$

$a \to e \to g \to y_1$: $\frac{\partial E}{\partial W_{2,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial W_{2,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial W_{2,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial z_g}\frac{\partial z_g}{\partial W_{2,2}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial z_g}\frac{\partial z_g}{\partial e}\frac{\partial e}{\partial W_{2,2}}$

$= \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial z_g}\frac{\partial z_g}{\partial e}\frac{\partial e}{\partial z_e}\frac{\partial z_e}{\partial W_{2,2}} = (y_1 - y_{real})(W_{4,3})(\varphi'(g_z))(W_{3,2})(\varphi'(e_z))(a)$

$= 0$

$a \to e \to g \to y_2$: $\frac{\partial E}{\partial W_{2,2}} = (y_2 - y_{real})(W_{4,4})(\varphi'(g_z))(W_{3,2})(\varphi'(e_z))(a) = 0$

$a \to e \to h \to y_1$: $\frac{\partial E}{\partial W_{2,2}} = (y_1 - y_{real})(W_{4,5})(\varphi'(h_z))(W_{3,4})(\varphi'(e_z))(a) = (0.536-1)(0.1)(0.2)(-0.1)(0.2)(0.8)$

$= 0.00014848$

$a \to e \to h \to y_2$: $\frac{\partial E}{\partial W_{2,2}} = (y_2 - y_{real})(W_{4,6})(\varphi'(h_z))(W_{3,4})(\varphi'(e_z))(a) = (0.121-0)(-0.1)(0.2)(-0.1)(0.2)(0.8)$

$= 0.0000387$

$W_{2,2,new} = W_{2,2,old} - 0.5 \times (0.00014848 + 0.0000387) = -0.5000936$

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更新 $W_{2,3}$ 之值：① $b \to d \to f \to y_1$

② $b \to d \to f \to y_2$

③ $b \to d \to g \to y_1$

④ $b \to d \to g \to y_2$

$b \to d \to f \to y_1$：$\frac{\partial E}{\partial w_{b,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial w_{b,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial w_{b,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial w_{b,3}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial d}\frac{\partial d}{\partial w_{b,3}}$

$= \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial f}\frac{\partial f}{\partial f_z}\frac{\partial f_z}{\partial d}\frac{\partial d}{\partial d_z}\frac{\partial d_z}{\partial w_{b,3}} = (y_1 - y_{real})(w_{4,1})(\varphi'(f_z))(w_{3,1})(\varphi'(d_z))(b) = 0$

$b \to d \to f \to y_2$：$\frac{\partial E}{\partial w_{b,3}} = (y_2 - y_{real})(w_{4,2})(\varphi'(f_z))(w_{3,1})(\varphi'(d_z))(b) = 0$

$b \to d \to g \to y_1$：$\frac{\partial E}{\partial w_{b,3}} = (y_1 - y_{real})(w_{4,3})(\varphi'(f_z))(w_{3,2})(\varphi'(d_z))(b) = 0$

$b \to d \to g \to y_2$：$\frac{\partial E}{\partial w_{b,3}} = (y_2 - y_{real})(w_{4,4})(\varphi'(f_z))(w_{3,2})(\varphi'(d_z))(b) = 0$

$W_{3,3,new} = W_{3,3,old} - 0.5 \times 0 \Rightarrow W_{3,3,new} = 0.5$

更新 $W_{3,4}$ 之值：① $b \to g \to y_1$

② $b \to g \to y_2$

$b \to g \to y_1$：$\frac{\partial E}{\partial w_{b,4}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial w_{b,4}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial w_{b,4}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial w_{b,4}} = (y_1 - y_{real})(w_{4,3})(\varphi'(g_z))(b) = 0$

$b \to g \to y_2$：$\frac{\partial E}{\partial w_{b,4}} = (y_2 - y_{real})(w_{4,4})(\varphi'(g_z))(b) = 0$

$W_{3,4,new} = W_{3,4,old} - 0.5 \times 0 \Rightarrow W_{3,4,new} = -0.5$

更新 $W_{b,5}$ 之值：① $b \to e \to g \to y_1$

② $b \to e \to g \to y_2$

③ $b \to e \to h \to y_1$

④ $b \to e \to h \to y_2$

$b \to e \to g \to y_1$：$\frac{\partial E}{\partial w_{b,5}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial w_{b,5}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial w_{b,5}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial w_{b,5}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial e}\frac{\partial e}{\partial w_{b,5}}$

$= \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial e}\frac{\partial e}{\partial e_z}\frac{\partial e_z}{\partial w_{b,5}} = (y_1 - y_{real})(w_{4,3})(\varphi'(g_z))(w_{3,2})(\varphi'(e_z))(b) = 0$

$b \to e \to g \to y_2$：$\frac{\partial E}{\partial w_{b,5}} = (y_2 - y_{real})(w_{4,4})(\varphi'(g_z))(w_{3,2})(\varphi'(e_z))(b) = 0$

$b \to e \to h \to y_1$：$\frac{\partial E}{\partial w_{b,5}} = (y_1 - y_{real})(w_{4,5})(\varphi'(h_z))(w_{3,4})(\varphi'(e_z))(b) = (0.536 - 1)(0.1)(0.2)(-0.1)(0.2)(0.83)$

$= 0.000154048$

$b \to e \to h \to y_2$：$\frac{\partial E}{\partial w_{b,5}} = (y_2 - y_{real})(w_{4,6})(\varphi'(h_z))(w_{3,4})(\varphi'(e_z))(b) = (0.1271 - 0)(-0.1)(0.2)(-0.1)(0.2)(0.83)$

$= 0.000040172$

$W_{b,5,new} = W_{2,5,old} - 0.5 \times 0.000194220 = 0.499902897$

更新 $W_{2,b}$ 之值：① $c \to e \to g \to y_1$

② $c \to e \to g \to y_2$

③ $c \to e \to h \to y_1$

④ $c \to e \to h \to y_2$

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$c \to e \to g \to y_1$：$\frac{\partial E}{\partial w_{5,6}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial w_{5,6}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial w_{5,6}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial w_{5,6}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial e}\frac{\partial e}{\partial w_{5,6}}$

$= \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial g}\frac{\partial g}{\partial g_z}\frac{\partial g_z}{\partial e}\frac{\partial e_z}{\partial e_z}\frac{\partial e_z}{\partial w_{5,6}} = (y_1 - y_{real})(w_{4,3})(\varphi'(g_z))(w_{3,2})(\varphi'(e_z))(c) = 0$

$c \to e \to g \to y_2$：$\frac{\partial E}{\partial w_{5,6}} = (y_2 - y_{real})(w_{4,4})(\varphi'(g_z))(w_{3,2})(\varphi'(e_z))(c) = 0$

$c \to e \to h \to y_1$：$\frac{\partial E}{\partial w_{5,6}} = (y_1 - y_{real})(w_{4,5})(\varphi'(h_z))(w_{3,4})(\varphi'(e_z))(c) = (0.536-1)(0.1)(0.2)(-0.1)(0.2)(0.77)$

$= 0.000014912$

$c \to e \to h \to y_2$：$\frac{\partial E}{\partial w_{5,6}} = (y_2 - y_{real})(w_{4,6})(\varphi'(h_z))(w_{3,4})(\varphi'(e_z))(c) = (0.1291-0)(-0.1)(0.2)(-0.1)(0.2)(0.77)$

$= 0.000037268$

$w_{5,6,new} = w_{5,6,old} - 0.5 \times 0.00018018 = -0.5000900\dot{9}$

更新$w_{5,7}$之值：① $c \to h \to y_1$

② $c \to h \to y_2$

$c \to h \to y_1$：$\frac{\partial E}{\partial w_{5,7}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial w_{5,7}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial h}\frac{\partial h}{\partial w_{5,7}} = \frac{\partial E}{\partial y_1}\frac{\partial y_1}{\partial h}\frac{\partial h}{\partial h_z}\frac{\partial h_z}{\partial w_{5,7}} = (y_1 - y_{real})(w_{4,5})(\varphi'(h_z))(c) =$

$= (0.536-1)(0.1)(0.2)(0.77) = -0.0071456$

$c \to h \to y_2$：$\frac{\partial E}{\partial w_{5,7}} = (y_2 - y_{real})(w_{4,6})(\varphi'(h_z))(c) = -0.0018634$

$w_{5,7,new} = w_{5,7,old} - 0.5 \times -0.009009 = 0.5045045$

∴w_5更新完為：

	1	2	3	4	5	6	7
w_5	0.5	-0.5000936	0.5	-0.5	0.4999089	-0.5000900$\dot{9}$	0.5045045