Appendix of "Pareto Landscape: Visualising the Landscape of Multi-Objective Optimisation Problems"

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A Two properties of ZDT1

The objective functions of ZDT1 [1] with 2 decision variables are defined as follows:

$$f_1(\mathbf{x}) = x_1, \quad f_2(\mathbf{x}) = (1 + 9x_2)(1 - \sqrt{x_1/(1 + 9x_2)}) \qquad \text{for } 0 \leq x_1, x_2 \leq 1$$

Proposition 1 Given $0 \leq x_1, x_2 \leq 1$, assuming a solution $x \in X$ with a fixed x_2 and then a sufficiently small $\epsilon > 0$ (e.g., $\epsilon = 0.001$) added to x_1, the new solution x' is always non-dominated to x.

Proof. Since $f_1 = x_1$, adding a small constant $\epsilon > 0$ always makes f_1' worse than f_1 (i.e., $f_1' > f_1$). As for f_2, we have $f_2'(x') = (1 + 9x_2)(1 - \sqrt{x_1'/(1 + 9x_2)})$ where $x_1' = x_1 + \epsilon$. We can derive $h = f_2' - f_2 < 0$ that f_2' is always better than f_2. With $f_1'(x') > f_1(x)$ and $f_2'(x') < f_2(x)$, x' is non-dominated to x.

Proposition 2 Given $0 \leq x_1, x_2 \leq 1$, assuming a solution $x \in X$ with a fixed x_1 and then a sufficiently small $\epsilon > 0$ added to x_2, the new solution x' is always dominated by x.

Proof. With a fixed x_1, f_1 will stay unchanged. As for f_2, we have $f_2'(x') = (1 + 9x_2')(1 - \sqrt{x_1/(1 + 9x_2')})$ where $x_2' = x_2 + \epsilon$. We can derive $h = f_2' - f_2 > 0$ that f_2' is always worse than f_2. With $f_1'(x') = f_1(x)$ and $f_2'(x') > f_2(x)$, x' is always dominated by x.

References

1. Zitzler, E., Deb, K., Thiele, L.: Comparison of multiobjective evolutionary algorithms: Empirical results. Evolutionary Computation **8**(2), 173–195 (Jun 2000)

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