4190.310 Programming Language
The K-- Language

1 Syntax

Expression e	$\rightarrow$	`unit`	unit
	$\mid$	x `:=` e	assignment
	$\mid$	e `;` e	sequence
	$\mid$	`if` e `then` e `else` e	branch
	$\mid$	`while` e `do` e	while loop
	$\mid$	`read` x	input
	$\mid$	`write` e	output
	$\mid$	`let` x `:=` e `in` e	variable binding
	$\mid$	n	integer
	$\mid$	`true` $\mid$ `false`	boolean
	$\mid$	x	identifier
	$\mid$	e `+` e $\mid$ e `-` e $\mid$ e `*` e $\mid$ e `/` e	arithmetic operation
	$\mid$	e `<` e $\mid$ e `=` e $\mid$ `not` e	conditional operation

1.1 Program

A program is an expression.

1.2 Identifiers

Alpha-numeric identifiers are `[a-zA-Z][a-zA-Z0-9_]`*. Identifiers are case sensitive: `z` and `Z` are different. The reserved words cannot be used as identifiers: `unit true false not if then else let in end while do read write`

1.3 Numbers/Comments

Numbers are integers, optionally prefixed with `-`(for negative integer): `-?[0-9]`$^+$.

A comment is any character sequence within the comment block `(* *)`. The comment block can be nested.

1.4 Precedence/Associativity

In parsing K-- program text, the precedence of the K-- constructs in decreasing order is as follows. Symbols in the same set have identical precedence. Symbols with subscript L (respectively R) are left (respectively right) associative. Symbols without subscript are nonassociative.

$$\begin{array}{l}
\{\texttt{not}\}_R, \\
\{\texttt{*}, \texttt{/}\}_L, \\
\{\texttt{+}, \texttt{-}\}_L, \\
\{\texttt{=}, \texttt{<}\}_L, \\
\{\texttt{write}\}_R, \\
\{\texttt{:=}\}_R, \\
\{\texttt{else}\}, \\
\{\texttt{then}\}, \\
\{\texttt{do}\}, \\
\{\texttt{;}\}_L, \\
\{\texttt{in}\}
\end{array}$$

For example, K-- program

```
x := e1; e2                ⇒  (x := e1) ; e2
while e do e1; e2          ⇒  (while e do e1); e2
if e1 then e2 else e3; e4  ⇒  (if e1 then e2 else e3); e4
```

Rule of thumb: for your test programs, if your programs are hard to read (hence can be parsed not as you expected) then put parentheses around.

2 Domains

$$\begin{array}{llrcl}
n & \in & \mathbb{Z} & & \text{integer} \\
b & \in & \mathbb{B} & & \text{boolean} \\
v & \in & \mathit{Val} & = & \mathbb{Z} + \mathbb{B} + \{\cdot\} \\
\sigma & \in & \mathit{Env} & = & \mathit{Id} \xrightarrow{\text{fin}} \mathit{Addr} \\
M & \in & \mathit{Mem} & = & \mathit{Addr} \xrightarrow{\text{fin}} \mathit{Val} \\
x, y & \in & \mathit{Id} & & \text{identifier} \\
l & \in & \mathit{Addr} & & \text{address}
\end{array}$$

3 Semantics

[True] $$\frac{}{\sigma, M \vdash \texttt{true} \Rightarrow \mathit{true}, M}$$

[False] $$\frac{}{\sigma, M \vdash \texttt{false} \Rightarrow \mathit{false}, M}$$

[Num] $$\frac{}{\sigma, M \vdash n \Rightarrow n, M}$$

[Unit] $$\frac{}{\sigma, M \vdash \texttt{unit} \Rightarrow \cdot, M}$$

[Var] $$\frac{}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

[Add] $$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \ \texttt{+} \ e_2 \Rightarrow n_1 + n_2, M''}$$

[Sub] $$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \ \texttt{-} \ e_2 \Rightarrow n_1 - n_2, M''}$$

[Mul] $$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \times e_2 \Rightarrow \mathtt{n_1} \times \mathtt{n_2}, M''}$$

[Div] $$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \ \texttt{/} \ e_2 \Rightarrow n_1/n_2, M''}$$

[EqualTrue] $$\frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_2, M'' \qquad e_1 = e_2}{\sigma, M \vdash e_1 \ \texttt{=} \ e_2 \Rightarrow \mathit{true}, M''}$$

[EqualFalse] $$\frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_2, M'' \qquad e_1 \neq e_2}{\sigma, M \vdash e_1 \ \texttt{=} \ e_2 \Rightarrow \mathit{false}, M''}$$

[Less] $$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow n_2, M''}{\sigma, M \vdash e_1 \ \texttt{<} \ e_2 \Rightarrow n_1 < n_2, M''}$$

[Not] $$\frac{\sigma, M \vdash e \Rightarrow b, M'}{\sigma, M \vdash \texttt{not} \ e \Rightarrow \mathit{not}\ b, M'}$$

[Assign] $$\frac{\sigma, M \vdash e \Rightarrow v, M'}{\sigma, M \vdash x \ \texttt{:=} \ e \Rightarrow v, M'\{\sigma(x) \mapsto v\}}$$

[Seq] $$\frac{\sigma, M \vdash e_1 \Rightarrow v_1, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v_2, M''}{\sigma, M \vdash e_1 \ \texttt{;} \ e_2 \Rightarrow v_2, M''}$$

[IfTrue] $$\frac{\sigma, M \vdash e \Rightarrow \mathit{true}, M' \qquad \sigma, M' \vdash e_1 \Rightarrow v, M''}{\sigma, M \vdash \texttt{if} \ e \ \texttt{then} \ e_1 \ \texttt{else} \ e_2 \Rightarrow v, M''}$$

[IfFalse] $$\frac{\sigma, M \vdash e \Rightarrow \mathit{false}, M' \qquad \sigma, M' \vdash e_2 \Rightarrow v, M''}{\sigma, M \vdash \texttt{if} \ e \ \texttt{then} \ e_1 \ \texttt{else} \ e_2 \Rightarrow v, M''}$$

[WhileTrue] $$\frac{\begin{array}{c}\sigma, M \vdash e_1 \Rightarrow \mathit{true}, M' \\ \sigma, M' \vdash e_2 \Rightarrow v_1, M_1 \qquad \sigma, M_1 \vdash \texttt{while} \ e_1 \ \texttt{do} \ e_2 \Rightarrow v_2, M_2\end{array}}{\sigma, M \vdash \texttt{while} \ e_1 \ \texttt{do} \ e_2 \Rightarrow v_2, M_2}$$

[WhileFalse] $$\frac{\sigma, M \vdash e_1 \Rightarrow \mathit{false}, M'}{\sigma, M \vdash \texttt{while} \ e_1 \ \texttt{do} \ e_2 \Rightarrow \cdot, M'}$$

[Let] $$\frac{\begin{array}{c}\sigma, M \vdash e_1 \Rightarrow v, M' \\ \sigma\{x \mapsto l\}, M'\{l \mapsto v\} \vdash e_2 \Rightarrow v', M'' \qquad l \notin \mathit{Dom}\ M'\end{array}}{\sigma, M \vdash \texttt{let} \ x \ \texttt{:=} \ e_1 \ \texttt{in} \ e_2 \Rightarrow v', M''}$$

[Read] $$\frac{}{\sigma, M \vdash \texttt{read} \ x \Rightarrow n, M\{\sigma(x) \mapsto n\}}$$

[Write] $$\frac{\sigma, M \vdash e \Rightarrow n, M'}{\sigma, M \vdash \texttt{write} \ e \Rightarrow n, M'}$$