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MATHEMATICAL QUESTIONS

Question 1

In an LTI circuit, the zero-state response $y(t) = \delta(t)$ has been obtained for the input $x(t) = (e^{-t} - \sin t)u(t)$.

(a) Determine the impulse response of the circuit.

there are different ways to solve this part,but because no limitations has been mentioned at the first, i use my differential equation knowledge to solve it using laplace method.as we know $\delta(t) = ((e^{-t} - \sin t)u(t)) * h(t)$ which h(t) is the impulse response of the circuit.now take laplace from booth sides to find h(t):

$$1 = (\frac{1}{s+1} - \frac{1}{s^2+1})H(s) \Rightarrow H(s) = \frac{s^3+s^2+s+1}{s^2-s} = s + 2 - \frac{1}{s} + \frac{4}{s-1}$$

then by taking laplace-inverse we have:

$$h(t) = (4e^t - 1)u(t) + 2\delta(t) + \delta^{'}(t)$$

note that it was possible to find the same answer by derivating and applying linear algebra operations such add or other to input and output signal too.

(b) Does there exist any initial condition for which the zero-input response grows unbounded as time approaches infinity?

from previuse part we know what h(t) is, and actually we know that it is a combination of homoginios equations answer and some delta terms, as you can see we have root 1 there $((4e^t - 1)u(t))$ so it is absoulotely possible to determine initial conditions for which this term still apears at zero-input response and causes it grows unbounded...

Question 2

Find the element or elements that must be in the closed container of the figure bellow to satisfy the following conditions.
a. Average power to the circuit $= 3000W$**.**
b. Circuit has a lagging power factor.

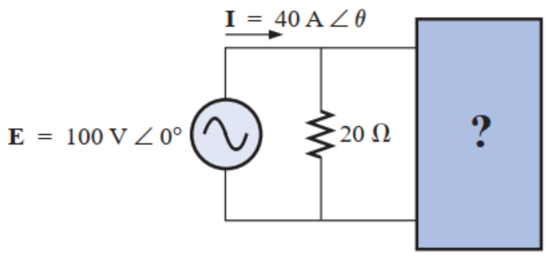

Figure 1: A circuit in sinusoidal steady state.

Circuit has a lagging power factor.it means that the voltage source should see an inductive impedance and the current lags behind the voltage.this makes us that $\theta < 0$.

$$P_{av} = I_{rms}V_{rms}cos(-\theta) = 4000cos(-\theta) = 3000 \Rightarrow cos(-\theta) = 0.75$$
$$\theta = -41.04$$

note that the current of resistor is $5\angle 0$.now write a kcl for top node to find current of black-box:

$$I_{bb} = I - I_R \Rightarrow I_{bb} = 40\angle - 41.04 - 5\angle 0 = 36.39\angle - 46.61$$

and the voltage of black-box is the same with voltage source.so:

$$Z_{bb} = \frac{100\angle 0}{36.39\angle - 46.61} = 2.75\angle 46.61 = 1.89 + 2j$$

it clears all things. at the black-box we should put a resistor R=1.89 and an inductor for which Lw = 2 in series. while the w of voltage source dont define, the value of L cant be determined explicitly!

Question 3

Consider the LTI circuit shown in Fig. 2.

Question 3 continued on next page...

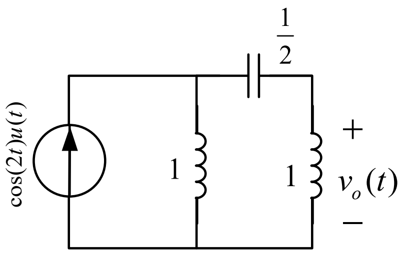

Figure 2: An LC circuit.

(a) *Find the differential equation of $v_o(t)$ using D operator.*

just writing 2 kcls for top nodes(x is the voltage of top-left node,and L1 is the middle inductor)...

$$-I + i_{L1}(0^-) + \int_{0^-}^{t} x dt + 0.5\frac{d(x - v_o)}{dt} = 0$$

by derivative and simplifying:

$$(1 + \frac{D^2}{2})x = DI + 0.5D^2 v_o$$

actually with the same steps for other node we have that:

$$i_{L2}(0^-) + \int_{0^-}^{t} v_o dt + 0.5\frac{d(v_o - x)}{dt} = 0 \Rightarrow (1 + \frac{D^2}{2})v_o = 0.5D^2 x$$

now lets substitute x from seconde equation on the first one to get vo:

$$x = \frac{2 + D^2}{D^2} v_o \Rightarrow (4 + 4D^2 + D^4)v_o = 2D^3 I + D^4 v_o$$

and return it to the normal form:

$$\frac{d^2 v_o}{dt^2} + v_o = 0.5\frac{d^3 I}{dt^3}$$

(b) *Solve the differential equation. Assume that the initial conditions are zero.*

lets find impulse response and then convolve it with I.suppose $h(t) = (c_1 sin(t) + c_2 cos(t))u(t) + k_1\delta(t) + k_2\delta^{'}(t)$ based on part a and homoginiouse answers.so $h^{'}(t) = (-c_2 sin(t) + c_1 cos(t))u(t) + c_2\delta(t) + k_1\delta^{'}(t) + k_2\delta^{''}(t)$ and $h^{''}(t) = (-c_1 sin(t) - c_2 cos(t))u(t) + c_1\delta(t) + c_2\delta^{'}(t) + k_1\delta^{''}(t) + k_2\delta^{'''}(t)$
lets substitute h(t) on the equation and find variables:

$$(k_1 + c_1)\delta(t) + (k_2 + c_2)\delta^{'}(t) + k_1\delta^{''}(t) + k_2\delta^{'''}(t) = 0.5\delta^{'''}(t)$$

so $k_1 = 0, k_2 = 0.5, c_1 = 0, c_2 = 0.5$ and $h(t) = -0.5cos(t)u(t) + 0.5\delta^{'}(t)$.this was the first step.but the seconde step will be the convolotion of h(t) and I=cos(2t)u(t).we have some formolas to evaluate convolotion of cos(at)u(t) and cos(bt)u(t) where a and be equal or not which help us to calculate theme easily,and note that $\delta'(t)$ works as a derivator:

$$v_o(t) = h(t) * cos(2t)u(t) = -0.5(\frac{2}{3}sin(2t) + \frac{1}{3}sin(t))u(t) - sin(2t)u(t) + 0.5\delta(t)$$
$$= (-\frac{4}{3}sin(2t) - \frac{1}{6}sin(t))u(t) + 0.5\delta(t)$$

(c) Calculate $v_o(t)$ in steady state.

clearly just the sin terms will stay alive!:

$$-\frac{4}{3}sin(2t) - \frac{1}{6}sin(t)$$

see how the phasor method makes mistake to find this part and it is because of there is sinosoidal terms at the homoginiuse equation answers!!

(d) Calculate $v_o(t)$ in steady state if the frequency of the current source is 1 rad/s.

here we have a = b (return to the part b!) so based on formola:

$$v_o(t) = h(t) * cos(2t)u(t) = -0.5(\frac{1}{2}sin(t) + \frac{t}{2}cos(t))u(t) - 0.5sin(t)u(t) + 0.5\delta(t)$$
$$= (-\frac{3}{4}sin(t) - \frac{t}{4}cos(t))u(t) + 0.5\delta(t)$$

Question 4

Let $R_4 = 1$, $L_1 = \frac{3}{2}$, and $L_3 = \frac{1}{2}$ in the filter circuit $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$ of Fig. 5. Calculate C_2 such that the filter has its maximum amplitude frequency and 3-dB cutoff frequency at $\omega_m = 0$ and $\omega_c = 1$, respectively.

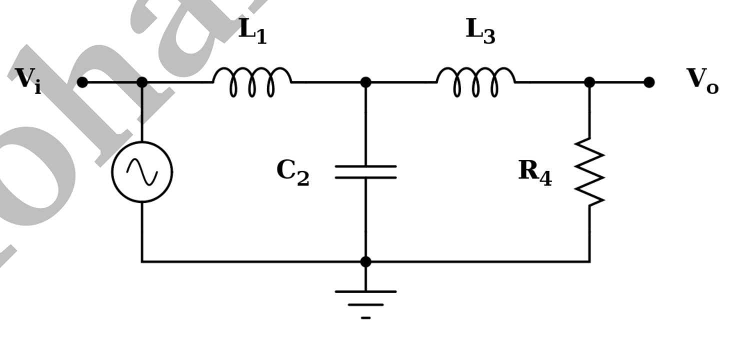

Figure 3: A filter circuit.

this is my favorite filter (T-filter) as we go to solve it:
clearly it is a lowpass filter.whene w = 0, we can see the inductors are short circuit and

Question 4 continued on next page. . .

capacitor is opencircuit so all voltage lies on R4 and H(jw)=1.hence at cut off frequency we need to have for w = 1 -> |H(jw)| = $\frac{1}{\sqrt{2}}$.H(jw) can be found by a simple phasore voltage division(let vx be the voltage of midd node,z1 be the impedance seen from midd node and ground to the right,and zt be the total impedance of the circuit):

$$v_o = \frac{1}{1+0.5wj}v_x = \frac{1}{1+0.5wj}\frac{w-2j}{4w-1.5w^3c+(3w^2c-2)j}v_i$$

some parts of calculating equivalent impedances are hidden because of simplifiying the solution path.

$$\frac{v_o}{v_i} = \frac{2w-4j}{10w-6w^3c+(4w^2+6w^2c-1.5w^4c-4)j}$$

now find w=1 -> H(j):

$$H(j) = \frac{2-4j}{10-6c=4.5cj} \Rightarrow |H(j)| = \frac{2\sqrt{5}}{\sqrt{100-120c+56.25c^2}}$$

and $|H(j)| = \frac{|H(0)|}{\sqrt{2}}$:

$$\frac{20}{100-120c+56.25c^2} = 0.5 \Rightarrow 56.25c^2 - 120c + 60 = 0$$

and c = $\frac{120+900}{112.5} = 9.0666666...$ (c>0)

Question 5

The circuit of Fig. 8 is in sinusoidal steady state. For which positive frequency, the shown current and voltage phasors are in-phase? For this frequency, calculate the steady state $v_{ss}(t)$ corresponding to the phasor V if $i_{ss}(t) = 4\cos(\omega t)$.

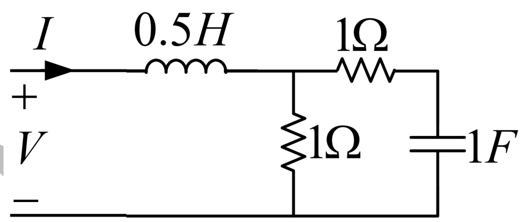

Figure 4: An LTI circuit in sinusoidal steady state.

current and voltage phasors should be in-phase and it occurs if and only if the total impedance of circuit be a real resistor and contains no complex part.it means the circuit is in resonance mode!:

$$z = \frac{1-\frac{1}{w}j}{2-\frac{1}{w}j} + 0.5wj = \frac{1+2w^2}{1+4w^2} + (0.5w - \frac{w}{1+4w^2})j$$

$$Im(z) = 0 \Rightarrow 0.5w - \frac{w}{1+4w^2} = 0 \Rightarrow w(0.5 - \frac{1}{1+4w^2}) = 0$$

$$\begin{cases} w = 0 \\ or \\ 1+4w^2 = 0 \Rightarrow w = 0.5 \quad (w > 0) \end{cases}$$

so w=0.5 and the input impedance is w->0.5 $z = \frac{1+2w^2}{1+4w^2}$, z=0.75 and $V = IZ = IR = 3\cos(0.5t)$

Question 6

Consider the circuit shown in Fig. 5.

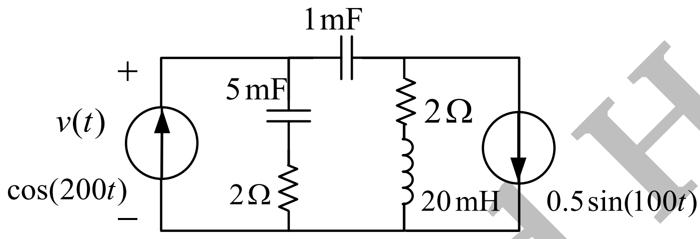

Figure 5: An LTI circuit.

(a) Find the steady state expression of $v(t)$.

the frequency of current sources are not equal and we have to use supperposition principle to find v(t) [supperposition just works for time domain, not phasor domain]:
firstly turn off the right current source:

$$V_1 = \frac{(2-j)(2-j)}{4-2j} I_1 \quad I_1 = 1\angle 0 \quad (cos(200t))$$

now turn the left current source off to find its impact on v:

$$V_2 = \frac{(2-12j)(2+2j)}{4-10j} I_2 \quad I_2 = 0.5\angle -\frac{\pi}{2} = -0.5j \quad (0.5sin(100t))$$

observe that after returning to time domain,$V_1(t) = 1.11cos(200t - 26.56)$ and $V_2(t) = -1.59sin(100t + 212.66)$ so $V(t) = V_1(t) + V_2(t) = 1.11cos(200t - 26.56) - 1.59sin(100t + 212.66)$

(b) Find the apparent delivered power of the left current source.

as you know we have supperposition for apparent power even if there be different frequencies! so lets turn right source off.from part a know that $V_1(t) = 1.11cos(200t - 26.56)$ which in phasore domain $V_1 = 0.99 - 0.49j$ and at this state $Z_t = \frac{(2-j)(2-j)}{4-2j}$.lets evaluate apparent power in 2 ways:

$$P = 0.5VI^* = 0.5 \times (0.99 - 0.49j) \times 1 = 0.495 - 0.248j$$

or:

$$P = 0.5Z_t|I|^2 = 0.5 \times \frac{(2-j)(2-j)}{4-2j} = 0.5 - 0.25j$$

as you can see the powers are roughly equal!

(c) *Find the average delivered power of the left current source.*

based on definition the real part of apparent power is the average delivered power so it is $0.5w$.

(d) *Find the reactive delivered power of the left current source.*

based on definition the imaginary part of apparent power is the reactive delivered power so it is $-0.25w$.

Question 7

A first-order series RL circuit is driven by a sinusoidal voltage source of $v_s(t) = V_m \cos(\omega t + \phi)u(t)$. Calculate ϕ that removes the transient part in the zero-state response of the circuit current $i(t)$.

in an RL series circuit we can write the equation of currrent:

$$\frac{di}{dt} + \frac{R}{L}i = \frac{v_s}{L}$$

the homouginious answer is $c_1 e^{-\frac{L}{R}t}$.the steady state response which is actually the particular answer of the main equation can be calculated using phasor on the equation as bellow:

$$(wj + \frac{R}{L})I = \frac{V}{L} \Rightarrow I = \frac{V}{R + Lwj}$$

and $V = V_m e^{j\phi}$ so $i_P = \frac{V_m}{\sqrt{R^2+L^2w^2}} cos(wt + \phi - tan^-(\frac{LW}{R}))$ and the complete answer is:

$$i = c_1 e^{-\frac{L}{R}t} + \frac{V_m}{\sqrt{R^2 + L^2w^2}} cos(wt + \phi - tan^-(\frac{Lw}{R}))$$

we want the transient part be zero.so it is nessecery to choose $c_1 = 0$.now we have continiuity of inductors currnt which means at $t = 0^+$ the current should be 0:

$$i(0^+) = 0 \Rightarrow \frac{V_m}{\sqrt{R^2 + L^2w^2}} cos(\phi - tan^-(\frac{Lw}{R})) = 0$$

$$\phi - tan^-(\frac{Lw}{R}) = k\pi + \frac{\pi}{2} \Rightarrow \phi = k\pi + \frac{\pi}{2} + tan^-(\frac{Lw}{R})$$

which k is an integer.

Question 8

For the circuit shown in Fig. 6,

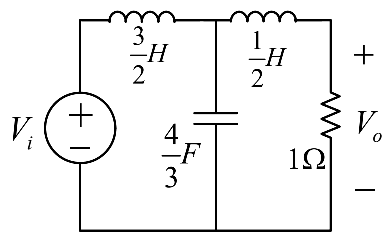

Figure 6: An LTI network.

(a) *Calculate the network function* $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$.

this is a simple voltage divition problem.lets V_x be the voltage of capacitor.so:

$$V_x = \frac{\frac{(1+0.5jw)\frac{3}{4jw}}{1+0.5jw}}{\frac{(1+0.5jw)\frac{3}{4jw}}{1+0.5jw} + 1.5jw}$$

and clearly $V_o = \frac{1}{1+0.5jw}V_x$ and by simplifying(there may be some false while calculating,please dont be carefully with this and see the main concept!):

$$\frac{V_o}{V_i} = \frac{6+3jw}{6+15jw+18(jw)^2+12(jw)^3+3(jw)^4}$$

(b) *Calculate the amplitude response and plot it approximately.*

we can rewrite H again to recognize imaginary and real part of H to find the amplitiude response:

$$H(jw) = \frac{6+3jw}{6-18w^2+3w^4+(15w-12w^3)j}$$

$$|H| = \frac{\sqrt{36+9w^2}}{\sqrt{(6-18w^2+3w^4)^2+(15w-12w^3)^2}}$$

this is a very logical response.for w=0 , capacitor is open circuit but inductors are short circuit and this behavior causes all voltage drops on the resistor and the same amplitude...at the infinit w, inductors are open circuit and the voltage of resistor will go to zero...

Question 8 continued on next page...

(c) Calculate the 3-dB stop frequencies.

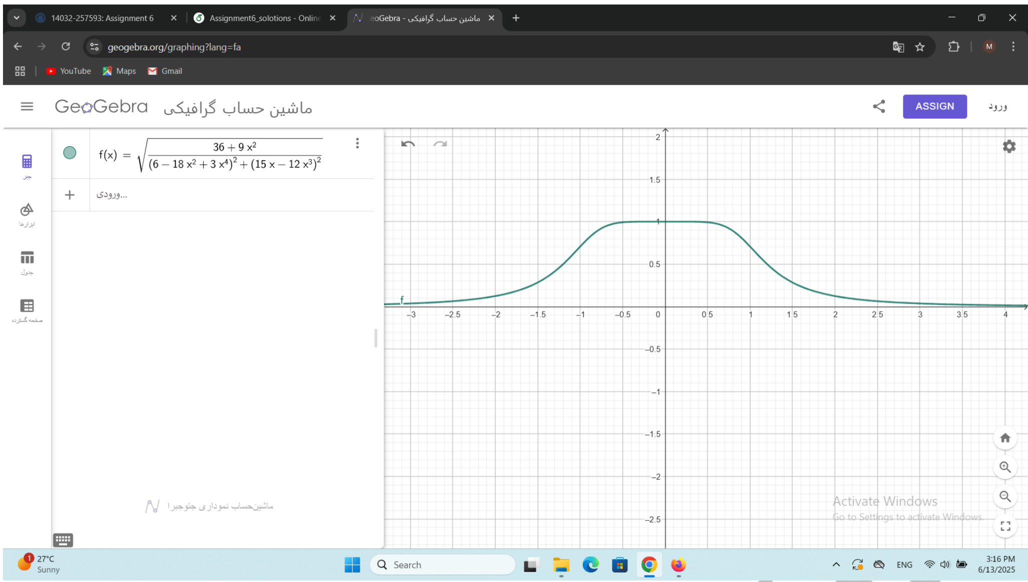

Figure 7: AmplitudeResponse1

as you can see on diagram above,the maximum of this function is 1 and we are going to find frequencies which it catchs $\frac{1}{\sqrt{2}}$.lets do it using calculating apps because solving this equation is hard. the stop frequencies are w=+1 and w=-1 which the equivalent frequencies are about f=+0.159 and f=-0.159 Hz;

Question 9

The network function corresponding to a desired response of an LTI circuit is $H(j\omega) = \frac{Y(j\omega)}{W(j\omega)} = \frac{1-j\omega^3}{2-\omega^2+3j\omega}$. Find the impulse response and the zero-state response of the circuit to the input $e^{-t}u(t)$.

from the given network function we can gain the main differential equation of input and output.if we substitute every power of jw with one derivative we can see that:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = w + \frac{d^3w}{dt^3}$$

now lets find impulse response.the roots of charachteristic equation are -1 and -2.so lets get answer to form $(c_1e^{-t} + c_2e^{-2t})u(t) + k_1\delta(t) + k_2\delta'(t)$ and by substitution on the equation:

$$k_2\delta''' + (k_1 + 3k_2)\delta'' + (c_1 + c_2 + 3k_1 + 2k_2)\delta' + (2c_1 + c_2 + 2k_1)\delta = \delta + \delta'''$$

and :

$$k_2 = 1$$

Question 9 continued on next page. . .

$$k_1 + 3k_2 = 0$$

$$c_1 + c_2 + 3k_1 + 2k_2 = 0$$

$$2c_1 + c_2 + 2k_1 = 1$$

by solving system above we have:

$$k_2 = 1, k_1 = -3, c_1 = 0, c_2 = 7$$

so the response to the impulse input will ganna be $7e^{-2t}u(t) - 3\delta(t) + \delta^{'}(t)$.note that this answer could be calculated using laplace transform too by substitution any jw with S and then applying laplace inverse to catch impulse response.However, after finding impulse response of the circuit,lets use convolution to find its response to the input $e^{-t}u(t)$.

$$y = e^{-t}u(t) * (7e^{-2t}u(t) - 3\delta(t) + \delta^{'}(t))$$

$$= e^{-t}u(t) * 7e^{-2t}u(t) - 3e^{-t}u(t) * \delta(t) + e^{-t}u(t) * \delta^{'}(t)$$

using properties of convolution and this point that $e^{-at}u(t)*e^{-bt}u(t) = \frac{1}{b-a}(e^{-at}-e^{-bt})u(t)$ we have:

$$y = (3e^{-t} - 7e^{-2t})u(t) + \delta(t)$$

Question 10

Let $H_1(j\omega) = \frac{I}{I_{s1}}\Big|_{\substack{V_{s2}=0\\ Vs3=0}} = \frac{2+j\omega+3(j\omega)^3}{1+j\omega}$, $H_2(j\omega) = \frac{I}{V_{s2}}\Big|_{\substack{I_{s1}=0\\ Vs3=0}} = \frac{3-2j\omega}{1+j\omega}$, and $H_3(j\omega) = \frac{I}{V_{s3}}\Big|_{\substack{I_{s1}=0\\ Vs2=0}} = \frac{4+j\omega}{1+j\omega+(j\omega)^2}$ in the circuit shown in Fig. 8.

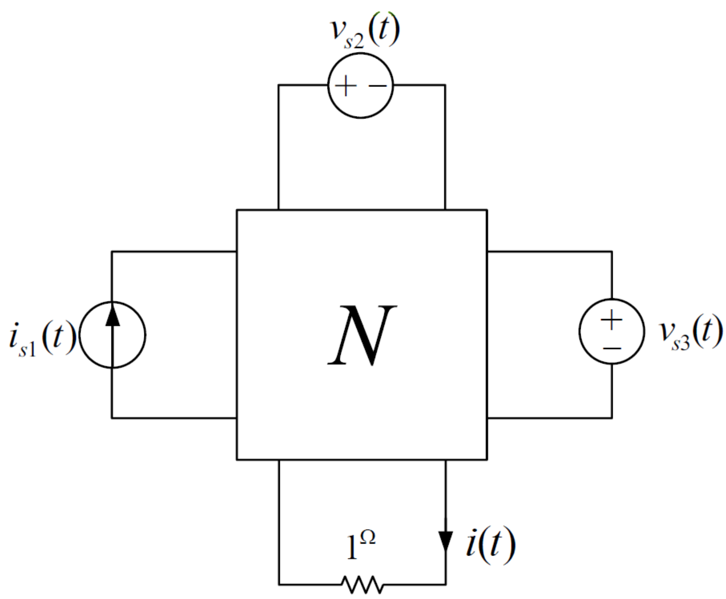

Figure 8: A multi-input circuit in sinusoidal steady state.

Question 10 continued on next page...

note that we have supperposition principle for average power consumed even if the frequency of resources is not equal.

(a) Calculate the average power consumed by the $1\ \Omega$ *resistor if* $i_{s1}(t) = \cos(t)$, $v_{s2}(t) = 2\cos(t)$, *and* $v_{s3}(t) = 3\cos(t)$.

note that here the frequencies are equal so we cant use supperposition for apparent(average+reactive) powers.we should use supperposition on phasore domain to find I_t and then use $P_t = 0.5R|I_t|^2$ to find average power recieved by resistor theorically:
affect of s1:

$$I_1 = \frac{2-2j}{1+j} I_{s1} \Rightarrow I_1 = 2 - 2j$$

affect of s2:

$$I_2 = \frac{3-2j}{1+j} V_{s2} \Rightarrow I_2 = 1 - 5j$$

affect of s3:

$$I_3 = \frac{4+j}{j} V_{s3} \Rightarrow I_3 = 3 - 12j$$

so $I_t = I_1 + I_2 + I_3 = 6 - 19j$ and total adsorbed power of resistor in this case is:

$$P_t = 0.5R|I_t|^2 = 0.5(6^2 + 19^2) = 198.5w$$

(b) Calculate the average power consumed by the $1\ \Omega$ *resistor if* $i_{s1}(t) = \cos(t)$, $v_{s2}(t) = 2\cos(2t)$, *and* $v_{s3}(t) = 3\cos(3t)$.

here frequencies are different and we can use supperposition for apparent(average+reactive) powers.
affect of s1:

$$I = \frac{2-2j}{1+j} I_{s1} \Rightarrow |I| = 2|I_{s1}| = 2 \Rightarrow P = 0.5R|I|^2 = 2w$$

affect of s2:

$$I = \frac{3-4j}{1+2j} V_{s2} \Rightarrow |I| = \sqrt{5}|V_{s2}| = 2\sqrt{5} \Rightarrow P = 0.5R|I|^2 = 10w$$

affect of s3:

$$I = \frac{4+3j}{-8+3j} V_{s3} \Rightarrow |I| = \frac{5}{\sqrt{73}}|V_{s3}| = \frac{15}{\sqrt{73}} \Rightarrow P = 0.5R|I|^2 = 1.54w$$

and total adsorbed power of resistor in this case is sum of those 3 values:

$$P_t = 2 + 10 + 1.54 = 13.54w$$

SOFTWARE QUESTIONS

Question 11

Use AC sweep simulation of PSpice to investigate the filtering response of the circuit shown in Fig. 9. Obtain the amplitude and phase response curves and determine the 3-dB frequency of the filter graphically.

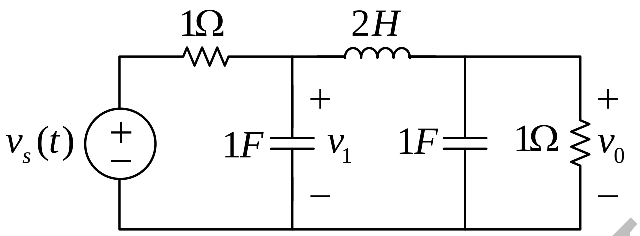

Figure 9: A filtering circuit.

this is a pi-filter.Pi filters are commonly used as low-pass filters, meaning they allow low-frequency signals (like DC) to pass through while attenuating high-frequency signals (like AC ripple). the cutt of 3-db frequency at this case is about 412mHz:

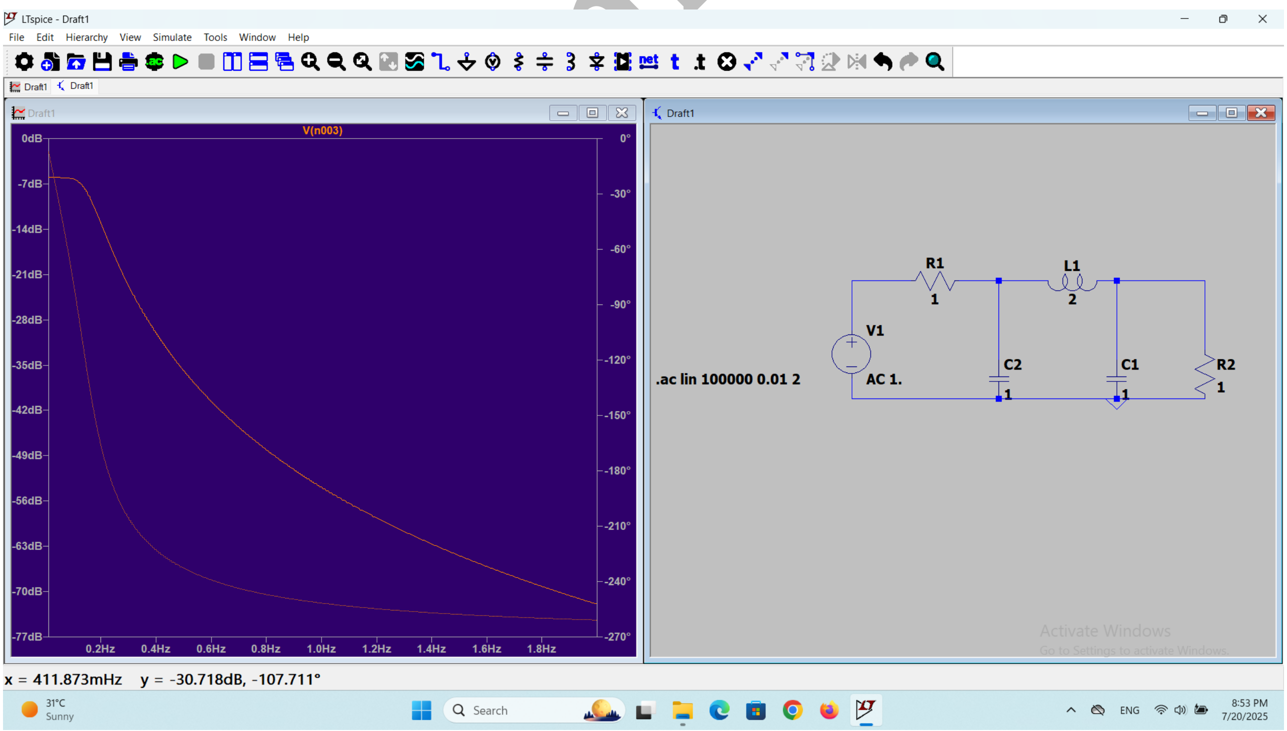

Figure 10: frequency phase Response Of PI-filter

BONUS QUESTIONS

Question 12

The circuit shown in Fig. 11 is called Sallen active bandpass filter, where the triangle abstracts an op-amp amplification circuit with the gain K.

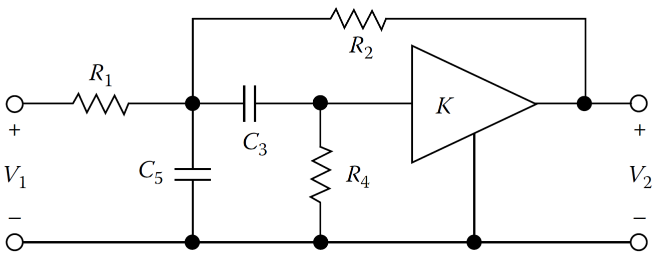

Figure 11: Sallen active bandpass filter.

(a) *Calculate the network function $H(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)}$ of the active filter circuit.*

lets jw=s and the voltage of top-left node be X:

$$\frac{V_2}{kR_4} + \frac{\frac{V_2}{k} - X}{\frac{1}{c_3 s}} = 0$$

$$\frac{X - V_1}{R_1} + \frac{X}{\frac{1}{c_5 s}} + \frac{X - \frac{V_2}{k}}{\frac{1}{c_3 s}} + \frac{X - V_2}{R_2} = 0$$

by solving this system and eliminating X phasore, the network function will be found:

$$H(s) = \frac{V_2}{V_1} = \frac{R_4 R_2 C_3 ks}{R_1 R_2 R_4 C_3 C_5 s^2 + (R_4 R_2 C_3 + R_1 R_2 C_3 + R_1 R_2 C_5)s + R_1 + R_2}$$

now just replace s -> jw to get the network function based on jw...

(b) *Calculate and plot the amplitude $|H(j\omega)|$ and phase $\angle H(j\omega)$ of the network function.*

$$|H(jw)| = \frac{R_4 R_2 C_3 kw}{\sqrt{(R_4 R_2 C_3 + R_1 R_2 C_3 + R_1 R_2 C_5)^2 w^2 + (R_1 + R_2 - R_1 R_2 R_4 C_3 C_5 w^2)^2}}$$

and if k > 0 and $R_1 + R_2 - R_1 R_2 R_4 C_3 C_5 w^2 > 0$:

$$\angle H(jw) = \frac{\pi}{2} - tan^{-1}(\frac{(R_4 R_2 C_3 + R_1 R_2 C_3 + R_1 R_2 C_5)w}{R_1 + R_2 - R_1 R_2 R_4 C_3 C_5 w^2})$$

Question 12 continued on next page. . .

these conditions are hold because of arctan's domain and sign of $\frac{\pi}{2}$.here you can see plots for a sample value of elements:

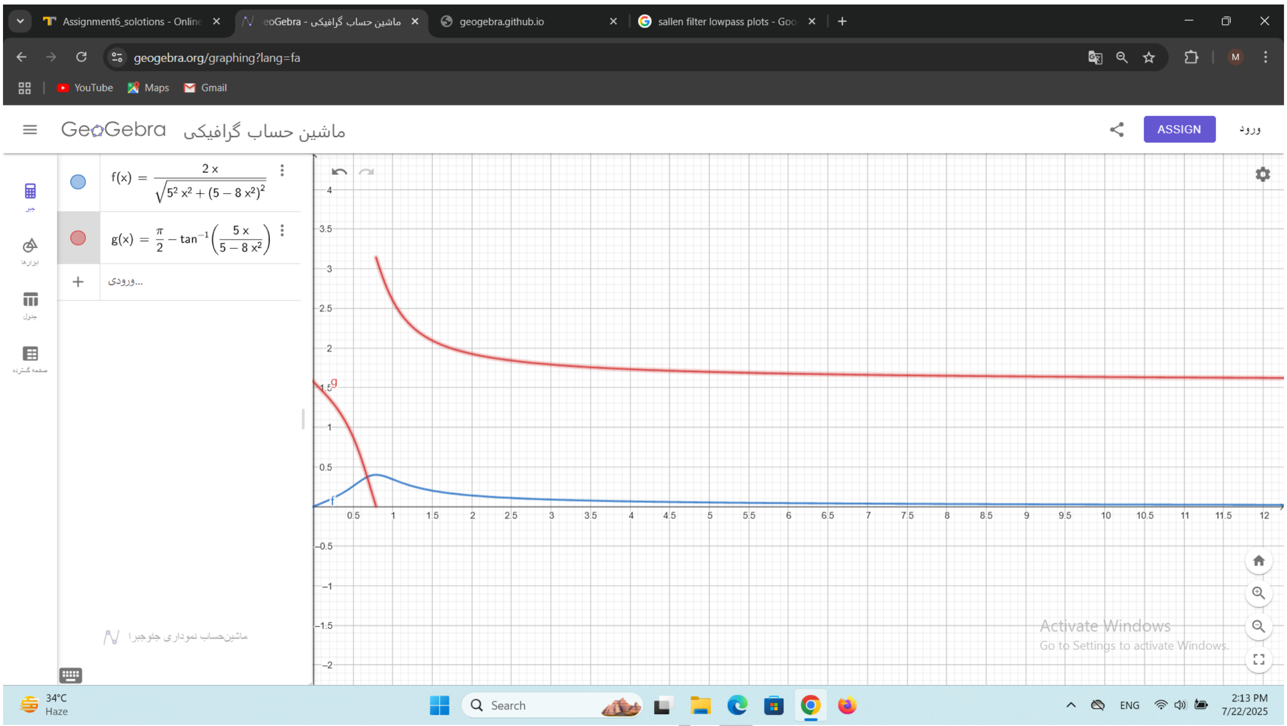

Figure 12: Sallen Active band-Pass Filter (Sample Plots)

(c) Describe the filtering response of the network function and calculate the half-power stop frequencies of the filter.

as you can see, it is clearly a band-pass filter which blocks low and high frequencies and just passes some intermediate frequencies.at this sample example the center-frequency is w=0.79,and stop-frequencies are w=0.54 , w=1.18.but as a mathmatical formola we shoul find the frequency which |H(jw)| is maximum or $\frac{d|H(jw)|}{dt} = 0$:

$$\frac{d|H(jw)|}{dt} = 0 \Rightarrow w = \sqrt{\frac{R_1 + R_2}{R_4 R_2 R_1 C_3 C_5}}$$

now we know that at this w,it is maximum.after finding the max value,we can solve for $|H(jw)| = \frac{H_{max}}{\sqrt{2}}$ to find stop-frequencies as well.it is difficult to solve with parameters but numerically as we did with a sample elements it can be done easily...

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