

The Silent Scholar Problem: A Probabilistic Framework for Breaking Epistemic Asymmetry in LLM Agents

Zan-Kai Chong
School of Science and Technology
Kwansei Gakuin University
 Japan
 zankai@ieee.org

Hiroyuki Ohsaki
School of Science and Technology
Kwansei Gakuin University
 Japan
 ohsaki@kwansei.ac.jp

Bryan Ng
School of Engineering & Computer Science
Victoria University of Wellington
 New Zealand
 ckbryan@hotmail.com

Abstract—Autonomous agents powered by large language models (LLMs), often enhanced by retrieval-augmented generation (RAG), are powerful processors of digital content. However, RAG is limited to consumption, constraining agents as unidirectional consumers. We term this limitation epistemic asymmetry. This imbalance creates a globally inefficient landscape, where silent scholars redundantly reconstruct similar reasoning in isolation, stagnating collective intelligence. To break this asymmetry, we propose a formal probabilistic framework that provides agents with a non-altruistic motive to engage in bidirectional knowledge exchange. We model an agent’s belief in a proposition as a Beta-Bernoulli distribution with a forgetting factor (γ). This allows us to isolate epistemic uncertainty as the variance of belief, establishing a dual motive for interaction: (1) a homeostatic drive to maintain certainty against temporal decay, and (2) an optimal learning strategy that targets maximum ambiguity ($E[\theta] = 0.5$). Furthermore, we address the practical challenges of deployment by introducing epistemic caching, a mechanism that leverages the forgetting factor to dynamically allocate resources to the non-stationary active head of the knowledge distribution. While our current analysis focuses on independent propositions and binary feedback to establish a baseline for this motive, we discuss how the framework extends to correlated and multi-categorical knowledge structures. Finally, we discuss how these accumulated belief states can serve as verifiable reward signals for reinforcement learning from human feedback (RLHF) and high-quality data filters for supervised fine-tuning (SFT), effectively bridging the gap between inference-time interaction and long-term model alignment. Simulation results validate this architecture, demonstrating that our uncertainty-driven strategy significantly outperforms random baselines in heterogeneous (Zipfian) environments, successfully prioritising the active head of the knowledge distribution while maintaining adaptability to concept drift.

I. INTRODUCTION

The emergence of large language models (LLMs) has ushered in a new generation of autonomous agent systems [17]. These agents exhibit exceptional proficiency in analysing extensive information from the internet, performing intricate multi-step operations, and integrating knowledge to respond to user queries [22]. Ranging from advanced personal assistants to automated research platforms, they mark a major advancement in artificial intelligence, serving as highly capable processors and interpreters of digital content.

However, despite their breadth of capabilities, these agents remain constrained by the static nature of their pre-trained knowledge. Retrieval-augmented generation (RAG) has emerged as a significant step toward addressing this, enabling LLMs to query external web resources to enhance factual accuracy and mitigate hallucination [9], [8]. Often, the retrieved documents provide valuable, context-specific examples that the agent can use for in-context few-shot learning to improve its immediate response.

Yet, RAG, even when used for few-shot learning, only solves the problem of consumption. Contemporary agents remain architecturally constrained as unidirectional consumers of knowledge, with minimal mechanisms for reintegrating their synthesised insights into shared digital ecosystems. We identify this limitation as epistemic asymmetry, and it creates a critical failure mode for the agent itself.

Without bidirectional exchange, an agent cannot distinguish between aleatoric noise (environmental randomness) and epistemic ignorance (model deficiency), terms coined in [18]. Isolated from external correction, the agent is forced to train on its own unverified outputs, a recursive process that leads to model collapse. In this degenerative state, the agent’s belief distribution loses variance, and it becomes confidently wrong and incapable of adapting to concept drift [16].

Current agents lack the motive to break their asymmetry because they lack a formal model to quantify this reducible, epistemic uncertainty. Without a formal uncertainty model, an agent cannot distinguish between environmental noise (which it should ignore) and its own ignorance (which it should resolve). Consequently, they have no mechanism to understand why contributing back to the digital commons (e.g., public forums, Q&A sites, Stack Overflow, Reddit, etc.) would be beneficial. For an agent, posting a solution to a public problem and receiving feedback is a powerful, low-cost method to gain new data. This feedback is the very evidence needed to reduce its epistemic uncertainty, but current models are not equipped to quantify this value.

In this paper, we envision that the contemporary agents will evolve from silent scholars to becoming epistemic agents and actively engaging in bidirectional knowledge exchange. To ground this vision, we propose the formal probabilistic frame-

work that provides their non-altruistic motive. We initially model the agent’s belief over its propositions as an unknown success rate using a Beta-Bernoulli model with a forgetting factor (γ). While this treats propositions as independent units of evidence, it provides a necessary simplification for our initial derivation of the equilibrium sample size (N_{eq}) and the mathematical foundation for more complex, interdependent knowledge representations.

This framework provides a non-altruistic motive: First, the forgetting factor ensures that certainty decays over time, effectively converting stale knowledge back into epistemic uncertainty. This prevents the variance from decaying to zero, establishing a persistent motive for continuous engagement. Second, our analysis indicates that the agent’s potential for learning is mathematically maximised at the point of highest ambiguity ($E[\theta] = 0.5$). These findings provide the formally-justified motive for an agent to break its asymmetry. Furthermore, we address the challenge of scalability by deriving an eviction policy that functions as epistemic caching. By leveraging the forgetting factor, the agent creates a dynamic working set of beliefs, ensuring the model remains computationally tractable despite the vast proposition space of real-world LLMs.

The remainder of this paper is organised as follows. Section II reviews related work in autonomous agents and active learning, and explicitly positions our framework against established paradigms such as online Bayesian updating and multi-armed bandits. Section III formalises our probabilistic framework, detailing the Beta-Bernoulli model used to represent an agent’s belief state and uncertainty. Then it provides a formal analysis of this model’s properties, establishing a non-altruistic motive of persistent uncertainty and maximum ambiguity. Section IV presents our experimental setup and results validating this framework. Section V discusses the broader implications for system robustness and model alignment, proposing mechanisms to mitigate re-calibration latency and distill external belief states into the model’s intrinsic weights via SFT and RLHF. Finally, Section VI concludes our work.

II. RELATED WORK

Our work is situated at the nexus of several rapidly evolving fields. To motivate our contribution, we first review the capabilities and limitations of existing agent architectures, focusing on their role as unidirectional knowledge consumers. We then analyse the isolated learner paradigm of current self-reflection frameworks. Finally, we situate our probabilistic model as a novel bridge between the heuristic learning of agents and the formal principles of active learning.

A. Autonomous Agents

The advent of LLMs has catalysed a paradigm shift in autonomous systems. As extensively documented in recent surveys, these models form the cognitive core of a new class of agents capable of complex reasoning, planning, and tool use [17]. Agents powered by LLMs can decompose high-level instructions into executable steps, interact with external APIs, and synthesise information to achieve sophisticated goals. This

body of work has established the foundation for agents that can effectively function as information processors and task executors.

Recently, agent development has increasingly focused on deeper integration with both web-based and desktop environments, enabling more seamless user experiences and greater task automation. Instead of relying solely on APIs, a new generation of browser agents interacts directly with graphical user interfaces to perform tasks. Tech companies like Perplexity, Anthropic, and OpenAI have developed agents that can navigate websites, fill out forms, manage calendars, and synthesise information across multiple open tabs [12], [1], [13]. These systems demonstrate a clear trajectory towards agents that are not just connected to the internet, but are situated within it, using the web as their native workspace. This evolution validates the internet as a viable environment for complex agentic behaviour. Nonetheless, these agents are still architecturally designed as advanced assistants that act on behalf of a single user, consuming and processing information without a mechanism for reciprocal knowledge contribution.

B. Agent Learning and Reflection

A more advanced class of agents attempts to learn from their own operational experience through internal feedback loops. A prominent example is the Reflexion framework [15], which enables an agent to perform self-reflection on its past failures. By analysing its own action history, the agent can generate verbal feedback for itself, which it then uses to improve its performance on subsequent trials. This concept of learning from a history of feedback is also formalised in frameworks like Chain of Hindsight (CoH) [10], which fine-tunes a model on its past trajectories.

This trend of internal self-evaluation extends beyond just past failures. Tree of Thoughts (ToT) [21], for example, allows an agent to deliberately explore and heuristically evaluate multiple reasoning paths concurrently. Similarly, Self-RAG [2] introduces reflection tokens that allow an agent to critique its own retrieval and generation steps in real-time. Collectively, these frameworks represent a significant step beyond static agents by endowing them with the capacity for self-correction.

This entire line of research, however, reveals two critical gaps. First, the learning process, whether it is reflection (Reflexion, CoH), deliberation (ToT), or self-critique (Self-RAG), remains a fundamentally private and isolated activity. An agent’s insights are not shared, and the experience it gains is not contributed back to any external ecosystem, thereby reinforcing its epistemic isolation.

Second, the learning process itself is largely heuristic. The decision to reflect, backtrack, or critique is often triggered by an LLM’s self-generated score, simple failure detection, or a non-probabilistic heuristic. This literature lacks a formal, mathematical definition of an agent’s belief state or its certainty. This makes it difficult to quantify why an agent should seek external feedback or when it has learned enough, a gap our probabilistic model directly addresses.

C. Active Learning and Uncertainty Sampling

To contextualize active learning, it is necessary to distinguish between the types of uncertainty an agent encounters. As reviewed by Wang et al. [18], uncertainty is categorised into aleatoric (inherent data noise, irreducible) and epistemic (model ignorance, reducible). Active learning strategies are fundamentally designed to target epistemic uncertainty, as this is the only component that can be reduced through the acquisition of new data.

The field of active learning provides the formal mathematical tools to address the shortcomings of heuristic learning [11]. Active learning is a subfield of machine learning where a learning algorithm can intelligently choose the data from which it learns. The goal is to achieve higher accuracy with fewer labelled examples, and thus maximising learning efficiency.

The principles of active learning are well-established, but their application has been almost exclusively limited to a model-dataset paradigm. In this traditional view, an agent actively queries a static, pooled dataset to request labels for the most informative unlabelled points. This literature has not, to our knowledge, framed uncertainty as a social or economic motive for an agent to engage in a public, bidirectional exchange. Furthermore, a fundamental divergence exists in the objective of sampling. Traditional active learning seeks to minimise the query budget required to reach a static convergence point (i.e., sampling stops when uncertainty ≈ 0). In contrast, our approach views uncertainty not as a metric to be eliminated, but as a persistent driver of existence. By introducing a forgetting factor, we ensure the agent never converges to absolute certainty, thereby transforming active learning from a finite optimisation task into a continuous, lifelong homeostatic process.

Our work bridges this critical gap. We propose that the formal principles of uncertainty sampling can be repurposed as the non-altruistic, formal justification for an epistemic agent to break its asymmetry. We reframe "querying a static dataset" as "posting a solution on a public forum" and "receiving a label" as "gaining feedback from the digital commons." In doing so, we provide the formal, probabilistic motive that is missing from current agent architectures.

D. Theoretical Positioning

While our framework leverages established mathematical components, specifically Beta-Bernoulli updating, exponential forgetting, and uncertainty sampling, our contribution lies in the architectural synthesis of these tools to resolve the silent scholar problem. We explicitly distinguish our approach from prior work in four key areas:

- 1) Online Bayesian Updating & Concept Drift: Traditional online learning aims to passively track evolving parameters or detect drift to trigger retraining [7]. In contrast, we deploy the forgetting factor (γ) not as a tracking mechanism, but as a homeostatic driver. The decay does not merely reflect a changing world, it actively compels the agent to engage with the digital commons to prevent

its internal belief state from degenerating into maximum entropy.

- 2) Multi-Armed Bandits: Bandit algorithms (e.g., Thompson Sampling) typically optimise for cumulative external reward, balancing exploration and exploitation [14]. Our framework is distinct in that it is non-altruistic and reward-agnostic. In other words, the agent optimises solely for epistemic maintenance. It treats variance reduction not as a means to a reward, but as the survival objective itself.
- 3) RAG Caching: Standard RAG caching stores static retrieved documents or vector embeddings to reduce latency or cost [3]. Our epistemic caching (Section III-D) stores dynamic belief states (α, β) . This allows the agent to maintain a living model of truth that evolves with every interaction, rather than a static snapshot of text.

III. MATHEMATICAL MODEL

In this section, we model the non-altruistic motive of epistemic agents in verifying a portfolio of propositions. We employ a Beta-Bernoulli model with a forgetting factor (γ) to capture the evolving belief associated with each proposition. Building on this, we introduce a formal definition of uncertainty that supports query strategy and analyse the key properties of these dynamic models, including the adaptability-certainty trade-off. Finally, we demonstrate how this framework scales to real-world environments by introducing epistemic caching, a mechanism that leverages the forgetting factor to prioritize the retention of structurally significant knowledge while evicting obsolete beliefs.

A. Dynamic Belief Representation with Beta-Bernoulli Model with Forgetting Factor

We model the agent's knowledge base as a portfolio of k propositions, $P = \{p_1, p_2, \dots, p_k\}$. These propositions represent facts, claims, or the posited effectiveness of a reasoning method. While the theoretical space of propositions is vast, we treat P as a dynamic working set. As we will detail in Section III-D, the model naturally supports dynamic allocation to handle the long-tail distribution of propositions found in real-world LLMs, allowing the agent to focus only on the active head of the distribution. Consequently, for the formal analysis in Sections III-A through III-C, we treat P as the finite set of currently active propositions.

For each proposition p_i , the agent's goal is to learn and track its current probability of being true, $\theta_i(t) \in [0, 1]$ at discrete time step t using a Beta-Bernoulli model. Correspondingly, the agent's belief for proposition p is represented by a Beta(α_i, β_i) distribution, where α_i and β_i are pseudo-counts for positive and negative evidence, respectively.

The agent's environment is the dynamic, non-stationary digital commons (the internet), where the public consensus or supporting evidence for a proposition can evolve. Critically, to operate in a non-stationary environment, there is no basis for assuming that a proposition's truth value remains constant. Public consensus may shift, or new evidence may invalidate

old facts. Therefore, the agent must prioritise recent data over stale, historical observations. To address this, we introduce a forgetting factor, γ , where $0 \ll \gamma < 1$ (e.g., $\gamma = 0.99$), to achieve this.

This factor exponentially decays the weight of old observations while incorporating new evidence. We denote the observation at time t as a binary variable $y_t \in \{0, 1\}$, where $y_t = 1$ represents evidence supporting the proposition and $y_t = 0$ represents contradictory evidence. The belief update rule is defined as,

$$\alpha_t = \gamma\alpha_{t-1} + y_t, \quad (1)$$

and

$$\beta_t = \gamma\beta_{t-1} + (1 - y_t). \quad (2)$$

While we define $y_t \in \{0, 1\}$ as a binary evidence signal for the purposes of this foundational model, the Beta-Bernoulli update rule is mathematically compatible with soft evidence. In scenarios where feedback is nuanced, y_t can be treated as a continuous variable $y_t \in [0, 1]$, where values closer to 1 represent stronger supporting evidence and values closer to 0 represent contradictory evidence. This allows the framework to process probabilistic feedback from critic models or human-in-the-loop signals without loss of generality.

B. Uncertainty as the Motive for Interaction

We quantify the agent's epistemic uncertainty as the variance of the belief distribution for a specific proposition p_i . While this study treats propositions as independent units to derive the equilibrium sample size N_{eq} , this individual variance serves as the base component for more complex, joint-uncertainty models in correlated knowledge structures.

The agent's primary challenge is one of active learning: distinguishing between environmental noise and knowledge gaps. We specifically model the agent's epistemic uncertainty regarding proposition p_i . Unlike aleatoric uncertainty, which arises from the intrinsic randomness of the environment and is irreducible, epistemic uncertainty stems from the model's lack of knowledge regarding the true parameter θ .

We quantify this epistemic uncertainty as the variance of the belief distribution, i.e.,

$$\text{Var}(\theta) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}. \quad (3)$$

By maximising this variance, the agent targets the region of highest ambiguity, where the potential for information gain and thus the reduction of epistemic uncertainty is maximised.

C. Key Properties of the Dynamic Model

This Beta-Bernoulli model with forgetting factor provides the following three properties. The agent's non-altruistic drive to reduce its own uncertainty compels it to continuously interact with the digital commons (see Section III-C1) and to intelligently prioritise those interactions on the topics of its greatest confusion (see Section III-C2). Then, the γ serves as the key to adjust the adaptability in Section III-C3.

1) Persistent Uncertainty: In a static model (where $\gamma = 1$), with $N = \alpha + \beta$ as $N \rightarrow \infty$, the denominator of Eq. 3 will grow faster than the numerator. Hence, causing $\text{Var}(\theta) \rightarrow 0$.

However, this is no longer true in the dynamic model. We define the effective sample size as the sum of belief counts, $N_{\text{eff},t} = \alpha_t + \beta_t$. By summing the update rules in Eq. 1 and 2, we derive the recurrence relation for the agent's memory,

$$\begin{aligned} N_{\text{eff},t} &= (\gamma\alpha_{t-1} + y_t) + (\gamma\beta_{t-1} + (1 - y_t)) \\ &= \gamma(\alpha_{t-1} + \beta_{t-1}) + 1 \\ N_{\text{eff},t} &= \gamma N_{\text{eff},t-1} + 1. \end{aligned} \quad (4)$$

As the time progresses, an equilibrium point (N_{eq}) is found, where N_{eff} stops changing. This happens when the amount of memory we lose to decay is exactly equal to the +1 we gain from new evidence. At steady state, $N_{\text{eq}} = (N_{\text{eq}} \times \gamma) + 1$ and

$$N_{\text{eq}} = \frac{1}{1 - \gamma}. \quad (5)$$

Conclusively, the agent's effective sample size (N_{eff}) stabilises instead of growing to infinity. As N_{eff} is capped, the denominator in Eq. 3 stops growing. This leads to a critical divergence from standard uncertainty sampling. In classical static models, as $t \rightarrow \infty$, the variance $\text{Var}(\theta) \rightarrow 0$, which effectively extinguishes the motive to learn. Consequently, a standard agent eventually becomes a silent scholar again once it is confident.

In our dynamic framework, assuming a non-trivial environment where both supporting and contradictory evidence occur with non-zero probability, the variance converges to a non-zero lower bound, i.e.,

$$\lim_{t \rightarrow \infty} \text{Var}(\theta) \geq \delta(\gamma) > 0 \quad (6)$$

This positive floor $\delta(\gamma)$ ensures that the agent's epistemic hunger is never fully satiated. The agent is mathematically forced to remain an active participant in the digital commons, not because it is altruistic, but because its internal belief model constantly decays toward entropy.

2) Maximum Ambiguity: As defined in Section III-C1, the agent's uncertainty is the variance of its belief distribution, $\text{Var}(\theta)$. For any given number of total observations $N = \alpha + \beta$, this variance is maximised when the numerator $\alpha\beta$ is maximised. This occurs when the agent's pseudo-counts are balanced, i.e., $\alpha = \beta$. Correspondingly, when $\alpha = \beta$, the agent's expected success rate is,

$$E[\theta] = \frac{\alpha}{\alpha + \alpha} = 0.5. \quad (7)$$

In other words, the agent is in a state of maximum uncertainty when the proposition's effectiveness is ambiguous ($E[\theta] = 0.5$). While traditionally treated as a heuristic, recent theoretical work by Fuchsgruber et al. [6] on graph active learning proves that acquiring labels for instances with maximal epistemic uncertainty is mathematically equivalent to maximising the gain in the posterior probability of the ground truth. Conversely, their analysis confirms that targeting aleatoric uncertainty yields sub-optimal performance comparable to random sampling. Thus, by targeting the peak of $\text{Var}(\theta)$, our agent is formally optimising its rate of learning.

In other words, the agent is in a state of maximum uncertainty when the proposition’s effectiveness is ambiguous. It is a known property of this model that this state of maximum prior variance is also the state from which a single new observation provides the largest expected reduction in uncertainty (i.e., the highest information gain). When an agent operating with $E[\theta] \approx 0.9$ or $E[\theta] \approx 0.1$ is highly certain, a new observation will only minimally shift its belief. In contrast, an agent with $E[\theta] = 0.5$ will experience a significant belief update regardless of the outcome.

A full analysis of whether an agent should pursue topics of maximum uncertainty or a random-selection approach, is beyond the scope of this paper. This concept, however, is the central focus of uncertainty sampling in the field of active learning, and we refer interested readers to the recent literature [4], [20], [5].

3) Adaptability-Certainty Trade-off: The forgetting factor γ is a critical meta-parameter that controls the agent’s fundamental assumptions about its environment. Its value dictates the trade-off between the agent’s adaptability (how quickly it responds to new, contradictory evidence) and its certainty (the baseline level of uncertainty it can ever achieve).

We’ve established that the effective sample size (N_{eff}) stabilises at $N_{\text{eq}} = \frac{1}{1-\gamma}$ in Section III-C1. This leads to a direct, quantifiable trade-off. When the forgetting factor γ is high, such as $\gamma = 0.999$, which corresponds to $N_{\text{eq}} = 1000$, the agent effectively has a long memory. It places greater trust in its large pool of historical data. As a result, the agent becomes more certain about various propositions, but slower to adapt. A few new, contradictory pieces of evidence have little impact and are essentially overwhelmed by the accumulated weight of the previous 1000 observations.

Conversely, when γ is low, such as $\gamma = 0.95$, which yields $N_{\text{eq}} = 20$, the agent’s effective memory is short. It trusts primarily the most recent 20 observations. This leads to a higher baseline level of uncertainty and the agent cannot achieve high confidence in its beliefs. For the trade-off, the agent has a strong adaptability. A few new, contradictory observations can quickly and significantly alter its beliefs.

D. Scalability via Epistemic Caching

While the forgetting factor (γ) successfully establishes the motive for continuous interaction, it simultaneously introduces a computational constraint: an agent cannot maintain active, decaying belief states for the infinite space of all possible propositions. However, the very mechanism that creates this constraint also provides its solution. Since the effective sample size N_{eff} decays naturally over time, it functions as an intrinsic epistemic timer, allowing us to implement a cache eviction policy that is not heuristic, but probabilistically derived from the agent’s own fading memory.

In real-world deployments, the number of potential propositions is enormous and often follows a long-tail distribution. Most queries are unique, however a specific subset of active propositions is frequently accessed.

Our framework addresses this through dynamic epistemic caching. Similar to cache management in computer architecture, the agent dynamically allocates parameters only to

propositions in the active head of the distribution, while relying on the LLM’s generic pre-training for the long tail.

The forgetting factor γ naturally governs the eviction policy for this cache. For any rarely accessed proposition p_{rare} , the effective sample size N_{eff} decays over time according to the recurrence relation derived in Section III-C1, i.e.,

$$N_{\text{eff},t} = N_{\text{eff},t-1} \times \gamma + \mathbb{I}_{\text{obs}}, \quad (8)$$

where \mathbb{I}_{obs} represents the binary observation signal,

$$\mathbb{I}_{\text{obs}} = \begin{cases} 1 & \text{if new evidence is observed at time } t \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

We define a significance threshold N_{\min} to formalise the eviction condition. A proposition p_i is evicted from the active tracking set and reverted to the LLM’s generic pre-training when $N_{\text{eff},t} < N_{\min}$.

This mechanism functions as a variation of Least Recently Used (LRU) eviction weighted by epistemic density. Unlike standard LRU, which relies solely on recency, our approach preferentially retains high-confidence propositions (high N_{eff}), ensuring that computational resources remain concentrated on high-utility, frequently accessed knowledge.

IV. SIMULATION RESULTS

To validate the theoretical properties of our Beta-Bernoulli model with forgetting factor, we conducted simulations. The goals are to (a) demonstrate the necessity of the forgetting factor (γ) in a non-stationary environment, validating the adaptability-certainty trade-off, and (b) evaluate the efficacy of uncertainty sampling in a standard uniform environment, and (c) stress-test the epistemic caching mechanism under realistic heterogeneous (Zipfian) access patterns.

A. Experiment Setup

We simulate a digital commons environment containing $k = 100$ independent propositions. To model the non-stationary nature of the digital commons (e.g., evolving public consensus), we introduce a consensus shift. For time steps $t = 1 \dots 500$, the ground truth is $\theta^* = 0.8$ (strong consensus). At $t = 501$, we simulate a paradigm shift, where the ground truth permanently flips to $\theta^* = 0.2$ (contradiction).

We define two environmental access conditions:

- 1) Uniform Access: The agent selects queries from a uniform distribution over the k propositions.
- 2) Heterogeneous Access: To mirror real-world long-tail distributions, the agent samples propositions according to a Zipfian (power law) distribution ($s = 1.1$). This creates a scenario where a small subset of head propositions is frequently accessed, while the majority are rarely queried.

B. Experiment 1: Validation of the Adaptability-Certainty Trade-off

This experiment validates the adaptability-certainty trade-off (Section III-C3). To strictly isolate the impact of the forgetting factor (γ) from the query strategy, we employ a

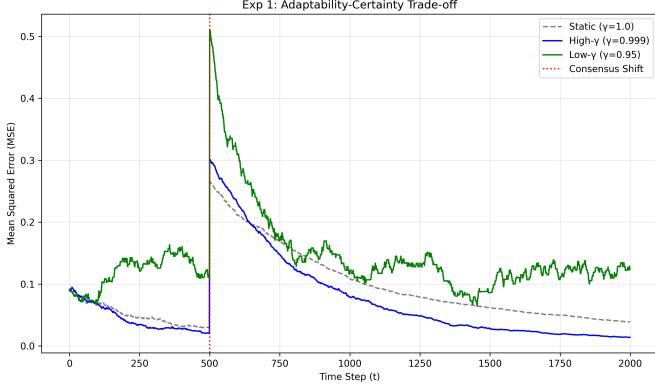


Figure 1. Adaptability trade-off using random sampling. The low- γ agent (green) suffers from high noise but adapts rapidly to the shift. The high- γ agent (blue) offers stability but exhibits significant inertia, adapting slower than the static agent (grey) due to its larger effective memory horizon ($N_{\text{eq}} = 1000$ vs $t = 500$).

naive random sampling strategy across all three agents in a uniform environment: a static agent ($\gamma = 1.0$), a high- γ agent ($\gamma = 0.999$), and a low- γ agent ($\gamma = 0.95$).

As shown in Figure 1, the results illustrate the tension between noise and inertia.

- The Cost of Stability: The high- γ agent (blue) achieves the lowest baseline error before the shift, effectively filtering noise. However, it exhibits significant inertia after the shift at $t = 500$. Notably, it recovers slower than the static agent (grey) in this specific window. This is because the high- γ agent maintains an effective memory of $N_{\text{eq}} \approx 1000$ steps, whereas the static agent had only accumulated 500 steps of history at the time of the shift. This confirms that while high stability is beneficial for precision, it creates a heavy prior that resists new evidence.
- The Benefit of Forgetting: The low- γ agent (green) maintains a higher noise floor (≈ 0.1) but adapts rapidly. Its steep recovery slope after $t = 500$ confirms that a lower forgetting factor is essential for survival in highly volatile environments, allowing the agent to quickly discard obsolete beliefs.

C. Experiment 2: Efficacy in Uniform Environment

We compare the random sampling agent against the uncertainty sampling agent in a uniform access environment with $\gamma = 0.999$.

As shown in Figure 2, the uncertainty sampling agent (orange) demonstrates superior learning efficiency during the stable phase ($t < 500$), driving MSE significantly lower than the random agent. However, the results highlight a critical re-calibration penalty. Immediately following the consensus shift ($t = 501$), the uncertainty agent spikes to a higher error than random sampling and lags behind. This occurs because the uncertainty agent, having built high confidence in the old regime, must systematically dismantle its strong beliefs. It aggressively targets the ambiguity of the transition ($E[\theta] \approx 0.5$), effectively over-thinking the shift, while the random agent benefits from broader exploration.

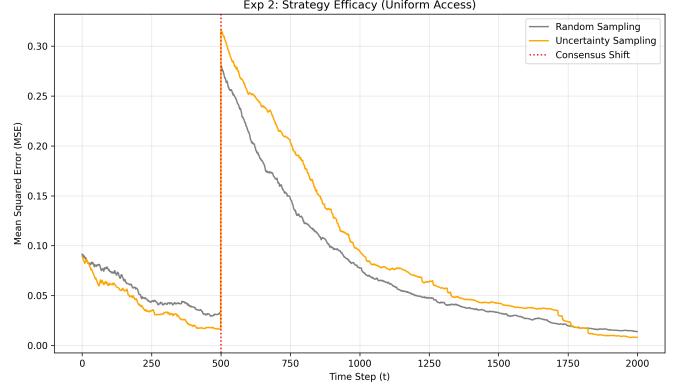


Figure 2. Strategy comparison (Uniform). Uncertainty sampling (orange) minimises error efficiently in stable regimes but incurs a severe re-calibration penalty after the shift ($t = 500$), temporarily lagging behind random sampling.

D. Experiment 3: Robustness in Heterogeneous Environment

Finally, we test the agents in the heterogeneous (Zipfian) environment to validate the epistemic caching mechanism (Section III-D).

As shown in Figure 3, the distinction between strategies becomes critical.

- Inefficiency of Randomness: The random agent (purple) performs poorly throughout, with a slow learning curve. In a long-tail distribution, random sampling wastes the majority of its query budget on irrelevant, low-frequency propositions, failing to maintain the active head of knowledge.
- Long-Term Dominance of Uncertainty Sampling: The uncertainty sampling agent (orange) initially suffers the same re-calibration penalty observed in the experiment in Section IV-C. However, unlike the random agent, it successfully learns. By targeting variance, the agent automatically concentrates its resources on the frequent, high-variance propositions. Around $t = 900$, it crosses below the random agent and continues to drive error down. This confirms that in realistic, skewed-access environments, uncertainty sampling is the only viable strategy for maintaining a high-fidelity belief state over time.

V. DISCUSSION: IMPLICATIONS FOR ALIGNMENT AND ROBUSTNESS

Our experiments demonstrate that uncertainty sampling effectively governs an agent’s immediate query strategy. However, the probabilistic belief states (α, β) generated by this process possess utility that extends beyond real-time interaction. In this section, we discuss how these internal uncertainty metrics can be repurposed to enhance long-term model alignment and system robustness in non-stationary environments.

A. Implications for Model Alignment

Beyond immediate query strategies, the agent’s accumulated belief states offer a verifiable signal for long-term alignment. We identify the following key mechanisms for this integration.

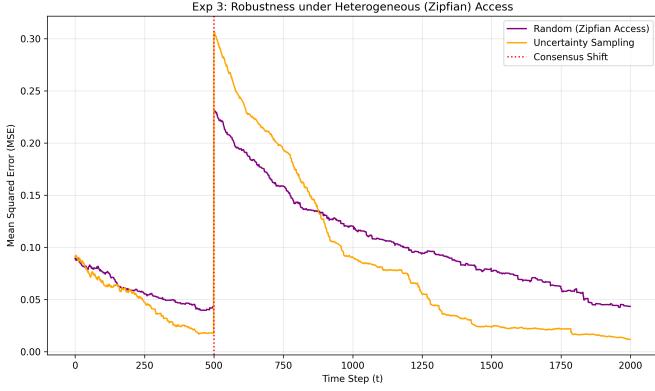


Figure 3. Strategy comparison (Zipfian). Random sampling (purple) fails to learn effectively in a long-tail environment. Uncertainty sampling (orange) demonstrates robustness; despite the initial re-calibration spike, it successfully converges to a lower error, proving its ability to prioritize the active head of the distribution.

1) Dynamic SFT Filtering: Propositions with high confidence ($\text{Var}(\theta) \rightarrow 0$) and high success rates effectively identify the gold standard subset of an agent’s experiences. These can be extracted to create high-quality, autonomously curated datasets for supervised fine-tuning (SFT), aligning with recent findings that data quality outweighs quantity for model robustness [24].

2) Epistemic Reward Signals: The internal belief state can serve as a dense reward signal for reinforcement learning from human feedback (RLHF). By penalizing reasoning paths that contradict high-confidence internal beliefs, the agent can be incentivized to maintain consistency, bridging the gap between inference-time reasoning and training-time alignment [23]. Beyond providing reward signals, these accumulated beliefs can be synthesized into the agent’s core architecture through a process of continuous distillation.

3) Continuous Distillation: Rather than relying solely on external memory (the portfolio P), the agent can periodically distill its accumulated belief state into the LLM’s weights. By training on the high-certainty active head of its portfolio (as defined in Section III-D), the agent consolidates transient, external observations into permanent, internal knowledge [19], effectively solving the catastrophic forgetting problem by using the Beta-Bernoulli parameters as a stable buffer.

B. Mitigating Re-calibration Latency

Our simulations in Section IV-C identified a significant re-calibration penalty, where uncertainty sampling lagged behind random sampling following the consensus shift. To mitigate this in deployment, future architectures could employ a surprisal reset mechanism. If the prediction error (KL-divergence) exceeds a critical threshold, the agent could temporarily reset the effective sample size (N_{eff}) for that proposition. This would instantly restore maximum plasticity, allowing the agent to bypass the inertia observed in Figure 2 and rapidly adapt to the new ground truth.

C. Model Limitations and Real-World Knowledge Structures

While the proposed framework establishes a formal foundation for the non-altruistic motive of epistemic agents, it relies on two simplifying assumptions that warrant further discussion: proposition independence and binary feedback signals. This section discusses extending the model to real-world complexities, including graph-structured dependencies, multi-categorical observations, and hierarchical epistemic caching.

1) Inter-Proposition Dependencies: In this study, we treat the knowledge base P as a portfolio of k independent propositions. However, real-world knowledge is often structured hierarchically or as a graph (e.g., knowledge graphs), where the truth value of one proposition is highly correlated with others. In such systems, an observation y_t for proposition p_i provides a leakage of information that should theoretically reduce the epistemic uncertainty ($\text{Var}(\theta)$) of related propositions p_j . Future work could extend our Beta-Bernoulli update rule to a graph-based Bayesian framework. In this configuration, updates to α and β would propagate through the graph edges based on semantic similarity or logical entailment, potentially accelerating the agent’s convergence in complex, multi-layered domains.

2) Beyond Binary Observations: Our model currently defines the observation $y_t \in \{0, 1\}$ as a discrete indicator of success or failure. While this unit of evidence is mathematically convenient for deriving the equilibrium sample size N_{eq} , contemporary digital feedback is often nuanced and non-binary. For instance, feedback from the digital commons may take the form of confidence scores or partial critiques. Our framework is inherently compatible with such soft labels. Instead of adding an integer value to the pseudo-counts, the update rule in Eq. 1 and 2 can incorporate a fractional value $s \in [0, 1]$, where s represents the degree of supporting evidence.

3) Epistemic Density in Hierarchies: The introduction of dependencies would also enhance our proposed epistemic caching. Rather than evicting individual propositions based solely on their specific N_{eff} , a hierarchical model could prioritise the retention of foundational propositions that support many leaf nodes. This would ensure that an agent’s limited computational cache is reserved for the most structurally significant knowledge, further optimising the trade-off between adaptability and certainty in high-velocity environments.

VI. CONCLUSION

Contemporary AI agents function as silent scholars, passively consuming knowledge without active engagement. We argue their evolution into epistemic agents will be driven by a formal, non-altruistic motive to learn, compelling them to interact with the digital commons to verify truth propositions.

To ground this, we proposed a probabilistic framework using a Beta-Bernoulli model with a forgetting factor (γ). Our analysis demonstrates that this forgetting factor acts as the critical link between aleatoric volatility and epistemic maintenance. By forcing belief distributions to decay, the framework prevents epistemic uncertainty from vanishing, thereby establishing a permanent, homeostatic motive for continuous engagement.

Furthermore, our simulations empirically validate the mechanism of epistemic caching. We demonstrated that under realistic, heterogeneous (Zipfian) access patterns, the forgetting factor naturally reallocates computational resources to the non-stationary active head of the distribution, allowing the agent to outperform random baselines significantly in long-tail environments. While this study serves as a foundational proof-of-concept assuming proposition independence and binary feedback, it paves the way for more sophisticated architectures. Finally, we highlighted the framework’s potential to close the loop between inference and training, proposing that the agent’s accumulated belief state can serve as a continuously curated dataset for autonomous model alignment and the mitigation of catastrophic forgetting.

In conclusion, we reframe public contribution not as altruism, but as optimal active learning, a formally-justified, rational strategy for an agent to accelerate its own learning.

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