

A. Here are ten different matrices, one of which represents a structure.

$$H_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

VNs	6-cycle	8-cycle	K-L divergence
v_1	2	1	1.6279
v_4	1	1	0.4426
v_6	0	0	0.1819

Variable node v_1 is in an 8-cycle and two 6-cycles, v_4 is in an 8-cycle and a 6-cycle and v_6 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.6279, D(P||Q)_4 = 0.4426, D(P||Q)_6 = 0.1819.$$

$$H_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

VNs	8-cycle	K-L divergence
v_1	3	0.8867
v_2	2	0.2281
v_6	0	0.1772

Variable node v_1 is in three 8-cycles, v_2 is in two 8-cycles, and v_6 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 0.8867, D(P||Q)_2 = 0.2281, D(P||Q)_6 = 0.1772.$$

$$H_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	6-cycle	8-cycle	K-L divergence
v_1	2	1	1.2537
v_4	1	1	0.4023
v_8	0	0	0.1881

Variable node v_1 is in an 8-cycle and two 6-cycles, v_4 is in an 8-cycle and a 6-cycle, and v_8 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.2537, D(P||Q)_4 = 0.4023, D(P||Q)_8 = 0.1881.$$

$$H_4 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

VNs	4-cycle	6-cycle	K-L divergence
v_2	1	2	2.7613
v_5	0	0	0.1511

Variable node v_2 is in two 6-cycles and a 4-cycle, and v_5 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_2 = 2.7631, D(P||Q)_5 = 0.1511.$$

$$H_5 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix};$$

VNs	4-cycle	K-L divergence
v_1	3	7.3640
v_5	0	0.3631

Variable node v_1 is in three 4-cycles, and v_5 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 7.3640, D(P||Q)_5 = 0.3631.$$

$$H_6 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix};$$

VNs	4-cycle	8-cycle	K-L divergence
v_1	0	2	0.3213
v_3	1	2	1.4089
v_5	0	0	0.1626

Variable node v_1 is in two 8-cycles, v_3 is in two 8-cycles and a 4-cycle, and v_5 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 0.3213, D(P||Q)_3 = 1.4089, D(P||Q)_5 = 0.1626.$$

$$H_7 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	6-cycle	8-cycle	10-cycle	K-L divergence
v_1	0	1	1	0.2608
v_2	0	0	0	0.1965
v_3	1	1	1	0.7236
v_5	1	0	1	0.3719

Variable node v_1 is in an 8-cycle and a 10-cycle, v_2 is not in any cycle, v_3 is in a 10-cycle, an 8-cycle, and a 6-cycle, and v_5 is in a 10-cycle and a 6-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 0.2608, D(P||Q)_2 = 0.1965,$$

$$D(P||Q)_3 = 0.7236, D(P||Q)_5 = 0.3719.$$

$$H_8 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	4-cycle	6-cycle	K-L divergence
v_1	2	1	1.5401
v_2	0	0	0.3275
v_3	1	1	0.4985

Variable node v_1 is in a 6-cycle and two 4-cycles, v_2 is not in any cycle, and v_3 is in a 6-cycle and a 4-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.5401, D(P||Q)_2 = 0.3275, D(P||Q)_3 = 0.4985.$$

$$H_9 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	4-cycle	6-cycle	8-cycle	K-L divergence
v_1	2	2	1	2.4027
v_2	0	0	0	0.2785
v_3	0	2	1	0.5986
v_6	1	2	0	0.9270
v_7	1	1	1	0.6296

Variable node v_1 is in two 4-cycles, two 6-cycles and an 8-cycle, v_2 is not in any cycle, v_3 is in two 6-cycles and an 8-cycle, v_6 is in a 4-cycle and two 6-cycles, and v_7 is in a 4-cycle, a 6-cycle and an 8-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 2.4027, D(P||Q)_2 = 0.2785, D(P||Q)_3 = 0.5986, \\ D(P||Q)_6 = 0.9270, D(P||Q)_7 = 0.6296.$$

$$H_{10} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	4-cycle	6-cycle	8-cycle	K-L divergence
v_1	1	0	1	1.1697
v_2	0	1	1	0.2715
v_3	0	0	0	0.2010
v_6	1	1	1	2.2601

Variable node v_1 is in a 4-cycle and an 8-cycle, v_2 is in a 6-cycle and an 8-cycle, v_3 is not in any cycle, v_6 is in a 4-cycle, a 6-cycle, and an 8-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.1697, D(P||Q)_2 = 0.2715, \\ D(P||Q)_3 = 0.2010, D(P||Q)_6 = 2.2601.$$