A. Here are ten different matrices, one of which represents a structure.

$$\mathrm{H}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

VNs	6-cycle	8-cycle	K-L divergence
v_1	2	1	2.0588
v_4	1	1	0.5090
v_6	0	0	0.1870

Variable node v_1 is in an 8-cycle and two 6-cycles, v_4 is in an 8-cycle and a 6-cycle and v_6 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 2.0588, D(P||Q)_4 = 0.5090, D(P||Q)_6 = 0.1870.$$

$$\mathrm{H}_2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

VNs	8-cycle	K-L divergence
v_1	3	1.5363
v_2	2	0.2969
v_6	0	0.1514

Variable node v_1 is in three 8-cycles, v_2 is in two 8-cycles, and v_6 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.5363, D(P||Q)_2 = 0.2969, D(P||Q)_6 = 0.1514.$$

$$\mathrm{H}_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0; \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0; \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0; \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0; \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0; \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	6-cycle	8-cycle	K-L divergence
v_1	2	1	1.6660
v_4	1	1	0.4278
v_8	0	0	0.2221

Variable node v_1 is in an 8-cycle and two 6-cycles, v_4 is in an 8-cycle and a 6-cycle, and v_8 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.6660, D(P||Q)_4 = 0.4278, D(P||Q)_8 = 0.2221.$$

$$\mathrm{H}_4 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix};$$

VNs	4-cycle	6-cycle	K-L divergence
v_2	1	2	3.6788
v_3	0	2	3.6707
v_5	0	0	0.1215

Variable node v_2 is in two 6-cycles and a 4-cycle, v_3 is in two 6-cycles, and v_5 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_2 = 3.6788, D(P||Q)_3 = 3.6707, D(P||Q)_5 = 0.1215.$$

$$\mathrm{H}_5 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix};$$

VNs	4-cycle	K-L divergence
v_1	3	8.2688
v_5	0	0.3679

Variable node v_1 is in three 4-cycles, and v_5 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 8.2688, D(P||Q)_5 = 0.3679.$$

$$\mathrm{H}_6 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix};$$

VNs	4-cycle	8-cycle	K-L divergence
v_1	0	2	0.3482
v_3	1	2	1.7059
v_5	0	0	0.1474

Variable node v_1 is in two 8-cycles, v_3 is in two 8-cycles and a 4-cycle, and v_5 is not in any cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 0.3482, D(P||Q)_3 = 1.7059, D(P||Q)_5 = 0.1474.$$

$$\mathrm{H}_7 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	6-cycle	8-cycle	10-cycle	K-L divergence
v_1	0	1	1	0.2398
v_2	0	0	0	0.1974
v_3	1	1	1	1.0050
v_5	1	0	1	0.3990

Variabe node v_1 is in an 8-cycles and a 10-cycle, v_2 is not in any cycle, v_3 is in a 10-cycle, an 8-cycle, and a 6-cycle, and v_5 is in a 10-cycle and a 6-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 0.2398, D(P||Q)_2 = 0.1974,$$

$$D(P||Q)_3 = 1.0050, D(P||Q)_5 = 0.3990.$$

$$\mathrm{H}_8 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	4-cycle	6-cycle	K-L divergence
v_1	2	1	1.7526
v_2	0	0	0.4380
v_3	1	1	0.5541

Variable node v_1 is in a 6-cycles and two 4-cycles, v_2 is not in any cycle, and v_3 is in a 6-cycle and a 4-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.7526, D(P||Q)_2 = 0.4380, D(P||Q)_3 = 0.5541.$$

$$\mathrm{H}_9 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	4-cycle	6-cycle	8-cycle	K-L divergence
v_1	2	2	1	2.8381
v_2	0	0	0	0.3855
v_3	0	2	1	0.6957
v_6	1	2	0	1.1791
v_7	1	1	1	0.7878

Variable node v_1 is in two 4-cycles, two 6-cycles and an 8-cycle, v_2 is not in any cycle, v_3 is in two 6-cycles and an 8-cycle, v_6 is in a 4-cycle and two 6-cycles, and v_7 is in a 4-cycle, a 6-cycle and an 8-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 2.8381, D(P||Q)_2 = 0.3855, D(P||Q)_3 = 0.6957,$$

$$D(P||Q)_6 = 1.1791, D(P||Q)_7 = 0.7878.$$

$$\mathrm{H}_{10} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix};$$

VNs	4-cycle	6-cycle	8-cycle	K-L divergence
v_1	1	0	1	1.4210
v_2	0	1	1	0.3473
v_3	0	0	0	0.2434
v_6	1	1	1	2.6271

Variable node v_1 is in a 4-cycle and an 8-cycle, v_2 is in a 6-cycle and an 8-cycle, v_3 is not in any cycle, v_6 is in a 4-cycle, a 6-cycle, and an 8-cycle. The corresponding K-L divergences are

$$D(P||Q)_1 = 1.4210, D(P||Q)_2 = 0.3473,$$

$$D(P||Q)_3 = 0.2434, D(P||Q)_6 = 2.6271.$$

B. Comparison of Kullback-Leibler Divergence.

The Kullback-Leibler Divergences are compared in the following table. We can see that compared with BP decoding, the damping factor BP decoding can effectively reduce the K-L divergence, that is, the decoding result is closer to the MAP decoding result.

Matrices	variable nodes	$D(P\|\|Q)$	$D(S\|\|Q)$
H_1	v_1	2.0588	0.9181
	v_4	0.5090	0.3706
H_2	v_1	1.5363	0.5529
	v_2	0.2969	0.2527
H_3	v_1	1.6660	0.9181
	v_4	0.4278	0.3706
H_4	v_2	3.6788	0.7018
	v_3	3.6707	0.3241
H_5	v_1	8.2688	5.6401
H_6	v_1	0.3482	0.2864
	v_3	1.7059	0.9370
H_7	v_1	0.2398	0.1964
	v_3	1.0050	0.3295
	v_5	0.3990	0.2733
H_8	v_1	1.7526	1.3598
	v_3	0.5541	0.3564
H_9	v_1	2.8381	1.4505
	v_3	0.6957	0.3192
	v_6	1.1791	0.4141
	v_7	0.7878	0.3044
H_{10}	v_1	1.4210	0.4890
	v_2	0.3473	0.2178
	v_6	2.6271	1.4536