Quant Source

This folder contains a series of quant code

1. Vanilla option

This folder includes the vanilla option price calculated by Black-Scholes model. Here we will derive the model as follows. Suppose we have a Browian motion with the follow form, where S is the underlying asset value, and W is the Wiener process.

$$\frac{dS}{S} = \mu dt + \sigma dW$$

According to Ito's lemma, we can calculate the analytic form of stochastic process S. Let me rephrase Ito's lemma as follows.

Give a brownian motion, where W_t is a Wiener process,

$$dX_t = \mu_t dt + \sigma_t dW_t$$

Let $f(x, t)$ be a function of random variable X_t and t, then we have

$$df(X_t, t) = \frac{\partial f}{\partial t} dt + \mu_t \frac{\partial f}{\partial X_t} dt + \frac{\sigma_t^2}{2} \frac{\partial^2 f}{\partial X_t^2} dt + \sigma_t \frac{\partial f}{\partial X_t} dW_t$$

What we want to know the the evolution of target price V as function of S and t, so we have the final formula like follows.

$$dV = \frac{\partial V}{\partial t} dt + \mu S \frac{\partial V}{\partial S} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt + \sigma S \frac{\partial V}{\partial S} dW_t$$

Then we use delta-hedge portfolio, which can be stated as follows,

$$P = -V + S \frac{\partial V}{\partial S}$$

$$dP = -dV + \frac{\partial V}{\partial S} dS = -\frac{\partial V}{\partial t} dt - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt$$

Then we suppose the portfolio is risk free, which indicates,

$$rPdt = dP$$

So in concusion, we have the following equation

$$r(-V + S\frac{\partial V}{\partial S}) = -\frac{\partial V}{\partial t} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}$$

By rearranging the equation, we obtain the following parabolic PDE,

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0$$

So we can solve the PDE, and finally obtain, where cdf is the cumulative density function of normal distribution, and cdf function has relationship with erf function.

$$V(S,t) = S \cdot cdf(d_1) - Ke^{-rT} \cdot cdf(d_2)$$

$$cdf\,(x) = \frac{1}{2}\left(erf(\frac{x}{\sqrt{2}}) + 1\right)$$

where

$$d_1 = \frac{ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

We have the code in the class vanilla_option.

2. Monte Carlo Simulation of Vanilla option

Starting with the above formula, we will start the deriation of Monte Carlo simulation of vanilla option. With the following stochastic process,

$$dS = rSdt + \sigma SdW$$

According to Ito's formula, we have the following SDE,

$$d\ln S = \frac{1}{S}dS - \frac{1}{2}\sigma^2 dt$$

$$d\ln S = \left(r - \frac{1}{2}\sigma^2\right)dt + \sigma dW$$

Therefore we can simulate the trajectory of S w.r.t the change of of the time.

$$S = S_0 e^{\left(r-\frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}N(0,1)}$$

if we need to calculate the call/put payoffs for the S, we just need to average them using MC method, like follows (assume risk neural).

$$e^{-rT}\mathbf{E}\left(f\left(S_0 e^{\left(r-\frac{1}{2}\sigma^2\right)T+\sigma\sqrt{T}N(0,1)}\right)\right)$$

3. Methods for the Greeks

Here we will introduce some greeks (compared to the alpha and beta greeks). The detailed greeks can be listed as follows.

Name	Formula	Calls	Puts
Delta	$\frac{\partial C}{\partial S}$	$cdf(d_1)$	$cdf(d_1)-1$
Gamma	$\frac{\partial^2 C}{\partial S^2}$	$\frac{N(d_1)}{S\sigma\sqrt{T-t}}$	$\frac{N(d_1)}{S\sigma\sqrt{T-t}}$
Vega	$\frac{\partial C}{\partial \sigma}$	$N(d_1)S\sqrt{T-t}$	$N(d_1)S\sqrt{T-t}$
Theta	$\frac{\partial C}{\partial t}$	$-\frac{S\sigma N(d_1)}{2\sqrt{T-t}}-rKe^{-r(T-t)}N(d_2)$	$-\frac{S\sigma N(d_1)}{2\sqrt{T-t}}+rKe^{-r(T-t)}N(-d_2)$
Rho	$\frac{\partial C}{\partial r}$	$K(T-t)e^{-r(T-t)}N(d_2)$	$-K(T-t)e^{-r(T-t)}N(-d_2)$

We will try to implement all these greeks.