# 线性代数 行列式

## 张晓平



数学与统计学院

Email: xpzhang.math@whu.edu.cn

 $Homepage: \ http://staff.whu.edu.cn/show.jsp?n=Zhang\%20Xiaoping$ 

# 目录

- 1 行列式简介
- ② 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - · n阶行列式的定义
- 3 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

- ① 行列式简介
- ② 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

# 行列式出现于线性方程组的求解,它最早是一种速记的表达式,现在已 经是数学中一种非常有用的工具。

- 行列式是由菜布尼茨和日本数学家关孝和分别发明的。
  - 1683年,日本数学家关孝和在其著作《解伏题之法》中也提出了行
  - 1603年4月, 莱布尼茨在写给洛比达的一封信中使用并给出了行列
- 1750年,瑞士数学家克莱姆在其著作《线性代数分析导引》中,对

行列式出现于线性方程组的求解,它最早是一种速记的表达式,现在已 经是数学中一种非常有用的工具。

- 行列式是由莱布尼茨和日本数学家关孝和分别发明的。
  - 1683年,日本数学家关孝和在其著作《解伏题之法》中也提出了行列式的概念与算法。《解伏题之法》的意思就是"解行列式问题的方法",书里对行列式的概念和它的展开已经有了清楚的叙述。
  - 1693年4月,莱布尼茨在写给洛比达的一封信中使用并给出了行列 式,并给出方程组的系数行列式为零的条件。
- 1750年,瑞士数学家克莱姆在其著作《线性代数分析导引》中,对 行列式的定义和展开法则给出了比较完整、明确的阐述,并给出了 现在我们所称的解线性方程组的克莱姆法则。

行列式出现于线性方程组的求解,它最早是一种速记的表达式,现在已 经是数学中一种非常有用的工具。

- 行列式是由莱布尼茨和日本数学家关孝和分别发明的。
  - 1683年,日本数学家关孝和在其著作《解伏题之法》中也提出了行列式的概念与算法。《解伏题之法》的意思就是"解行列式问题的方法",书里对行列式的概念和它的展开已经有了清楚的叙述。
  - 1693年4月,莱布尼茨在写给洛比达的一封信中使用并给出了行列 式,并给出方程组的系数行列式为零的条件。
- 1750年,瑞士数学家克莱姆在其著作《线性代数分析导引》中,对 行列式的定义和展开法则给出了比较完整、明确的阐述,并给出了 现在我们所称的解线性方程组的克莱姆法则。

- 在行列式的发展史上,第一个对行列式理论做出连贯的逻辑的阐述,即把行列式理论与线性方程组求解相分离的人,是法国数学家范德蒙。范德蒙自幼在父亲的指导下学习音乐,但对数学有深厚的兴趣,后来终于成为法兰西科学院院士。他给出了用二阶子式和它们的余子式来展开行列式的法则,就对行列式本身这一点来说,他是这门理论的奠基人。
- 1772年,拉普拉斯在一篇论文中证明了范德蒙提出的一些规则,推 广了他的展开行列式的方法。
- 继范德蒙之后,在行列式的理论方面,又一位做出贡献的就是另一位法国大数学家柯西。1815年,柯西在一篇论文中给出了行列式的第一个系统的、几乎是近代的处理。其中主要结果之一是行列式的乘法定理。另外,他第一个把行列式的元素排成方阵,采用双足标记法;引进了行列式特征方程的术语;给出了相似行列式的概念;改进了拉普拉斯的行列式展开定理并给出了一个证明等。

- 在行列式的发展史上,第一个对行列式理论做出连贯的逻辑的阐述,即把行列式理论与线性方程组求解相分离的人,是法国数学家范德蒙。范德蒙自幼在父亲的指导下学习音乐,但对数学有深厚的兴趣,后来终于成为法兰西科学院院士。他给出了用二阶子式和它们的余子式来展开行列式的法则,就对行列式本身这一点来说,他是这门理论的奠基人。
- 1772年,拉普拉斯在一篇论文中证明了范德蒙提出的一些规则,推 广了他的展开行列式的方法。
- 继范德蒙之后,在行列式的理论方面,又一位做出贡献的就是另一位法国大数学家柯西。1815年,柯西在一篇论文中给出了行列式的第一个系统的、几乎是近代的处理。其中主要结果之一是行列式的乘法定理。另外,他第一个把行列式的元素排成方阵,采用双足标记法;引进了行列式特征方程的术语;给出了相似行列式的概念;改进了拉普拉斯的行列式展开定理并给出了一个证明等。

- 在行列式的发展史上,第一个对行列式理论做出连贯的逻辑的阐述,即把行列式理论与线性方程组求解相分离的人,是法国数学家范德蒙。范德蒙自幼在父亲的指导下学习音乐,但对数学有深厚的兴趣,后来终于成为法兰西科学院院士。他给出了用二阶子式和它们的余子式来展开行列式的法则,就对行列式本身这一点来说,他是这门理论的奠基人。
- 1772年,拉普拉斯在一篇论文中证明了范德蒙提出的一些规则,推 广了他的展开行列式的方法。
- 继范德蒙之后,在行列式的理论方面,又一位做出贡献的就是另一位法国大数学家柯西。1815年,柯西在一篇论文中给出了行列式的第一个系统的、几乎是近代的处理。其中主要结果之一是行列式的乘法定理。另外,他第一个把行列式的元素排成方阵,采用双足标记法;引进了行列式特征方程的术语;给出了相似行列式的概念;改进了拉普拉斯的行列式展开定理并给出了一个证明等。

- 1 行列式简介
- ② 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - · n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题



- 1 行列式简介
- ② 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

# 引例

#### 用消元法求解

$$a_{11}x_1 + a_{12}x_2 = b_1,$$
  
 $a_{21}x_1 + a_{22}x_2 = b_2.$ 

消去x2得

$$(a_{11}a_{22} - a_{12}a_{21})x_1 = b_1a_{22} - b_2a_{12},$$

消去X1得

$$(a_{11}a_{22} - a_{12}a_{21})x_2 = b_2a_{11} - b_1a_{11}.$$

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}}, \quad x_2 = \frac{b_2 a_{11} - b_1 a_{11}}{a_{11} a_{22} - a_{12} a_{21}}.$$



## 引例

用消元法求解

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$

消去x2得

$$(a_{11}a_{22}-a_{12}a_{21})x_1=b_1a_{22}-b_2a_{12},$$

消去x1得

$$(a_{11}a_{22}-a_{12}a_{21})x_2=b_2a_{11}-b_1a_{11}.$$

$$\overline{a}_{11}a_{22} - a_{12}a_{21} \neq 0$$
,则

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}}, \quad x_2 = \frac{b_2 a_{11} - b_1 a_{11}}{a_{11} a_{22} - a_{12} a_{21}}.$$

◆ロト ◆個ト ◆差ト ◆差ト を めらぐ

## 引例

用消元法求解

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1, \\ a_{21}x_1 + a_{22}x_2 = b_2. \end{cases}$$

消去x2得

$$(a_{11}a_{22}-a_{12}a_{21})x_1=b_1a_{22}-b_2a_{12},$$

消去x1得

$$(a_{11}a_{22}-a_{12}a_{21})x_2=b_2a_{11}-b_1a_{11}.$$

$$x_1 = \frac{b_1 a_{22} - b_2 a_{12}}{a_{11} a_{22} - a_{12} a_{21}}, \quad x_2 = \frac{b_2 a_{11} - b_1 a_{11}}{a_{11} a_{22} - a_{12} a_{21}}.$$



## 二阶行列式

由22 = 4个数,按下列形式排成2行2列的方形

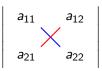
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$
,

其被定义成一个数

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \equiv D,$$

该数称为由这四个数构成的二阶行列式。

- aij表示行列式的元素。
   i为行标,表明该元素位于第i行;
   j为列标,表明该元素位于第j列。
- 对角线法则



#### 类似地,

$$b_1 a_{22} - b_2 a_{12} = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \equiv D_1$$
  
 $b_2 a_{11} - b_1 a_{21} = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} \equiv D_2$ 

则上述方程组的解可表示为

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}.$$

#### 例1

求解二元线性方程组

$$\begin{cases} 3x_1 - 2x_2 = 12, \\ 2x_1 + x_2 = 1. \end{cases}$$

解:因为

$$D = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 7 \neq 0,$$

$$D_1 = \begin{vmatrix} 12 & -2 \\ 1 & 1 \end{vmatrix} = 14,$$

$$D_2 = \begin{vmatrix} 3 & 12 \\ 2 & 1 \end{vmatrix} = -21,$$

因此,

$$x_1 = \frac{D_1}{D} = 2$$
,  $x_2 = \frac{D_2}{D} = -3$ 

#### 例1

求解二元线性方程组

$$\begin{cases} 3x_1 - 2x_2 = 12, \\ 2x_1 + x_2 = 1. \end{cases}$$

解:因为

$$D = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 7 \neq 0,$$

$$D_1 = \begin{vmatrix} 12 & -2 \\ 1 & 1 \end{vmatrix} = 14,$$

$$D_2 = \begin{vmatrix} 3 & 12 \\ 2 & 1 \end{vmatrix} = -21,$$

因此,

$$x_1 = \frac{D_1}{D} = 2$$
,  $x_2 = \frac{D_2}{D} = -3$ .

4□ > 4□ > 4□ > 4□ > 4□ > 4□

- 1 行列式简介
- ② 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - · n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

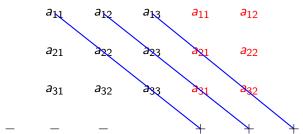
### 三阶行列式

由 $3^2 = 9$ 个数组成的3行3列的三阶行列式,则按如下形式定义一个数

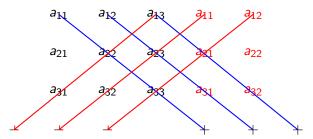
$$D_{3} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} \end{vmatrix}$$

## • 沙路法

# • 沙路法



## • 沙路法



计算

$$D_3 = \left| \begin{array}{rrrr} 1 & 2 & -4 \\ -2 & 2 & 1 \\ -3 & 4 & -2 \end{array} \right|$$

解:由对角线法则可知,

$$D_3 = 1 \times 2 \times (-2) + 2 \times 1 \times (-3) + (-2) \times 4 \times (-4)$$
$$-2 \times (-2) \times (-2) - (-4) \times 2 \times (-3) + 1 \times 1 \times 4$$
$$= -14.$$

## 例3

计算

$$D_3 = \left| \begin{array}{rrrr} 1 & 2 & -4 \\ -2 & 2 & 1 \\ -3 & 4 & -2 \end{array} \right|$$

解:由对角线法则可知,

$$D_3 = 1 \times 2 \times (-2) + 2 \times 1 \times (-3) + (-2) \times 4 \times (-4)$$

$$-2 \times (-2) \times (-2) - (-4) \times 2 \times (-3) + 1 \times 1 \times 4$$

$$= -14.$$

# 求方程

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & x \\ 4 & 9 & x^2 \end{vmatrix} = 0$$

解: 行列式

$$D = 3x^2 + 18 + 4x - 2x^2 - 12 - 9x = x^2 - 5x + 6$$

由此可知X=2或3。

# 例3

求方程

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & x \\ 4 & 9 & x^2 \end{vmatrix} = 0$$

解: 行列式

$$D = 3x^2 + 18 + 4x - 2x^2 - 12 - 9x = x^2 - 5x + 6$$

由此可知X = 2或3。

## 如果三元一次方程组

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1,$$
  
 $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2,$   
 $a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3,$ 

的系数行列式

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

则用消元法求解可得

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad x_3 = \frac{D_3}{D},$$

其中

$$D_1 = \left| \begin{array}{cc|ccc} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{array} \right|, \ D_2 = \left| \begin{array}{cccc} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{array} \right|, \ D_3 = \left| \begin{array}{cccc} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{array} \right|.$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$= a_{11} \underbrace{ \begin{bmatrix} a_{22} & a_{33} \\ a_{23} & a_{32} \end{bmatrix}}_{M_{11}} - a_{12} \underbrace{ \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}}_{M_{12}} + a_{13} \underbrace{ \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}}_{M_{13}}$$

这里, $M_{11}$ ,  $M_{12}$ ,  $M_{13}$ 分别称为 $a_{11}$ ,  $a_{12}$ ,  $a_{13}$ 的余子式,并称

$$A_{11} = (-1)^{1+1} M_{11}, \quad A_{12} = (-1)^{1+2} M_{12}, \quad A_{13} = (-1)^{1+3} M_{13}$$

分别称为a11, a12, a13的代数余子式。这样, D可表示为

$$D = a_{11}A_{11} + a_{11}A_{13} + a_{13}A_{13}.$$

◆ロト ◆母 ト ◆ 差 ト ◆ 差 ト り へ ②

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

$$=a_{11}(a_{22}a_{33}-a_{23}a_{32})-a_{12}(a_{21}a_{33}-a_{23}a_{31})+a_{13}(a_{21}a_{32}-a_{22}a_{31})$$

$$= a_{11} \underbrace{ \begin{bmatrix} a_{22} & a_{33} \\ a_{23} & a_{32} \end{bmatrix}}_{M_{11}} - a_{12} \underbrace{ \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}}_{M_{12}} + a_{13} \underbrace{ \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}}_{M_{13}}$$

这里, $M_{11}$ ,  $M_{12}$ ,  $M_{13}$ 分别称为 $a_{11}$ ,  $a_{12}$ ,  $a_{13}$ 的余子式,并称

$$A_{11} = (-1)^{1+1} M_{11}, \quad A_{12} = (-1)^{1+2} M_{12}, \quad A_{13} = (-1)^{1+3} M_{13}$$

分别称为a11, a12, a13的代数余子式。这样, D可表示为

$$D = a_{11}A_{11} + a_{11}A_{13} + a_{13}A_{13}.$$

M<sub>11</sub>

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$= a_{11} \underbrace{\begin{vmatrix} a_{22} & a_{33} \\ a_{23} & a_{32} \end{vmatrix}}_{M_{11}} - a_{12} \underbrace{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}}_{M_{12}} + a_{13} \underbrace{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}_{M_{13}}$$

$$A_{11} = (-1)^{1+1} M_{11}, \quad A_{12} = (-1)^{1+2} M_{12}, \quad A_{13} = (-1)^{1+3} M_{13}$$

$$D = a_{11}A_{11} + a_{11}A_{13} + a_{13}A_{13}.$$

张晓平

 $M_{13}$ 

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{33} \\ a_{23} & a_{32} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

这里, M<sub>11</sub>, M<sub>12</sub>, M<sub>13</sub>分别称为a<sub>11</sub>, a<sub>12</sub>, a<sub>13</sub>的余子式, 并称

 $M_{12}$ 

$$A_{11} = (-1)^{1+1} M_{11}, \quad A_{12} = (-1)^{1+2} M_{12}, \quad A_{13} = (-1)^{1+3} M_{13}$$

分别称为a11, a12, a13的代数余子式。这样, D可表示为

 $D = a_{11}A_{11} + a_{11}A_{13} + a_{13}A_{13}$ 

M<sub>13</sub>

张晓平

M<sub>11</sub>

M<sub>11</sub>

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{33} \\ a_{23} & a_{32} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

这里, $M_{11}$ ,  $M_{12}$ ,  $M_{13}$ 分别称为 $a_{11}$ ,  $a_{12}$ ,  $a_{13}$ 的余子式,并称

 $M_{12}$ 

$$A_{11} = (-1)^{1+1} M_{11}, \quad A_{12} = (-1)^{1+2} M_{12}, \quad A_{13} = (-1)^{1+3} M_{13}$$

分别称为a11, a12, a13的代数余子式。这样, D可表示为

$$D = a_{11}A_{11} + a_{11}A_{13} + a_{13}A_{13}.$$

M<sub>13</sub>

同样地,

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12},$$

其中

$$A_{11} = (-1)^{1+1}|a_{22}| = a_{22}, \quad A_{11} = (-1)^{1+2}|a_{21}| = -a_{21}.$$

注意这里的|a22|, |a21|是一阶行列式, 而不是绝对值。

我们把一阶行列式|a|定义为a。

同样地,

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12},$$

其中

$$A_{11} = (-1)^{1+1}|a_{22}| = a_{22}, \quad A_{11} = (-1)^{1+2}|a_{21}| = -a_{21}.$$

注意这里的|a22|, |a21|是一阶行列式,而不是绝对值。

我们把一阶行列式|a|定义为a。



同样地,

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12},$$

其中

$$A_{11} = (-1)^{1+1}|a_{22}| = a_{22}, \quad A_{11} = (-1)^{1+2}|a_{21}| = -a_{21}.$$

注意这里的|a22|, |a21|是一阶行列式, 而不是绝对值。

我们把一阶行列式|a|定义为a。



- 1 行列式简介
- ② 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

由 $n^2$ 个数 $a_{ij}(i,j=1,2,\cdots,n)$ 组成的n阶行列式

$$D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$
 (1)

是一个数。

- $\exists n = 1 \forall n \in \mathbb{Z}$   $D = |a_{11}| = a_{11}$ ;
- 当n ≥ 2时, 定义

$$D = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}, \tag{2}$$

其中

$$A_{1j} = (-1)^{1+j} M_{1j}$$

而 $M_{1j}$ 是D中划去第1行第j列后,按原顺序排成的n-1阶行列式,即

$$M_{1j} = \begin{vmatrix} a_{21} & \cdots & a_{2,j-1} & a_{2,j+1} & \cdots & a_{2n} \\ a_{31} & \cdots & a_{3,j-1} & a_{3,j+1} & \cdots & a_{3n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{n,i-1} & a_{n,i+1} & \cdots & a_{nn} \end{vmatrix}$$
  $(j = 1, 2, \dots, n),$ 

并称M1;为a1;的余子式,A1;为a1;的代数余子式

- $\exists n = 1 \forall n \in \mathbb{Z}$   $D = |a_{11}| = a_{11}$ ;
- 当n≥2时,定义

$$D = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}, \tag{2}$$

其中

$$A_{1j} = (-1)^{1+j} M_{1j}$$

而 $M_{1j}$ 是D中划去第1行第j列后,按原顺序排成的n-1阶行列式,即

$$M_{1j} = \begin{vmatrix} a_{21} & \cdots & a_{2,j-1} & a_{2,j+1} & \cdots & a_{2n} \\ a_{31} & \cdots & a_{3,j-1} & a_{3,j+1} & \cdots & a_{3n} \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{n,j-1} & a_{n,j+1} & \cdots & a_{nn} \end{vmatrix} \quad (j = 1, 2, \dots, n),$$

并称 $M_{1i}$ 为 $a_{1i}$ 的余子式, $A_{1i}$ 为 $a_{1i}$ 的代数余子式.

#### 注

- 1 在 D中,  $a_{11}, a_{22}, \dots, a_{nn}$  所在的对角线称为行列式的主对角线,  $a_{11}, a_{22}, \dots, a_{nn}$  称为主对角元。
- 2 行列式D是由n<sup>2</sup>个元素构成的n次齐次多项式:
  - 二阶行列式的展开式有2!项
  - 三阶行列式的展开式有3!项
  - n阶行列式的展开式有n!项,其中每一项都是不同行不同列的n个元素的乘积,带正号的项与带负号的项各占一半。

证明:n阶下三角行列式

$$D_n = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}.$$

#### 证明【数学归纳法】

- 当n=2时,结论成立。
- 假设结论对n-1阶下三角阵成立,则由定义

$$\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11} \cdot (-1)^{1+1} \begin{vmatrix} a_{22} & 0 & \cdots & 0 \\ a_{31} & a_{33} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

证明:n阶下三角行列式

$$D_{n} = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}.$$

### 证明【数学归纳法】

- 当n = 2时,结论成立。
- 假设结论对n-1阶下三角阵成立,则由定义

$$\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} \cdot (-1)^{1+1} \\ a_{11} \cdot (-1)^{1+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} \cdot a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

张晓平

线性代数

证明:n阶下三角行列式

$$D_{n} = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}.$$

#### 证明【数学归纳法】

- 当n = 2时,结论成立。
- 假设结论对n-1阶下三角阵成立,则由定义

$$\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11} \cdot (-1)^{1+1} \begin{vmatrix} a_{22} & 0 & \cdots & 0 \\ a_{31} & a_{33} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

证明:n阶下三角行列式

$$D_n = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}.$$

#### 证明【数学归纳法】

- 当n = 2时,结论成立。
- 假设结论对n-1阶下三角阵成立,则由定义

$$\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11} \cdot (-1)^{1+1} \begin{vmatrix} a_{22} & 0 & \cdots & 0 \\ a_{31} & a_{33} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

= a<sub>11</sub>(a<sub>22</sub>a<sub>33</sub>···a<sub>nn</sub>). 线性代数

证明:n阶下三角行列式

$$D_n = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}.$$

### 证明【数学归纳法】

- 当n = 2时,结论成立。
- 假设结论对n-1阶下三角阵成立,则由定义

$$\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} \cdot (-1)^{1+1} \\ a_{11} \cdot (-1)^{1+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} \cdot a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

 $= a_{11}(a_{22}a_{33}\cdots a_{nn}).$ 

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{n2} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 & \cdots & 0 \\ a_{21} & 0 & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & 0 & a_{n3} & \cdots & a_{nn} \end{vmatrix} + \cdots + \begin{vmatrix} 0 & \cdots & 0 & a_{1n} \\ a_{21} & \cdots & a_{2,n-1} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ a_{n1} & \cdots & a_{n,n-1} & 0 \end{vmatrix}$$

#### 同理可证

$$\begin{vmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix} = a_{11}a_{22}\cdots a_{nn}$$

#### 解

### 由行列式定义,

$$D_{n} = \begin{vmatrix} 0 & 0 & \cdots & 0 & a_{n} \\ 0 & 0 & \cdots & a_{n-1} & * \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & a_{2} & \cdots & * & * \\ a_{1} & * & \cdots & * & * \end{vmatrix} = (-1)^{1+n} a_{n} \begin{vmatrix} 0 & 0 & \cdots & a_{n-1} \\ \vdots & \vdots & & \vdots \\ 0 & a_{2} & \cdots & * \\ a_{1} & * & \cdots & * \end{vmatrix}$$

# 解

由行列式定义,

$$D_{n} = \begin{bmatrix} 0 & 0 & \cdots & 0 & a_{n} \\ 0 & 0 & \cdots & a_{n-1} & * \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & a_{2} & \cdots & * & * \\ a_{1} & * & \cdots & * & * \end{bmatrix} = (-1)^{1+n} a_{n} \begin{bmatrix} 0 & 0 & \cdots & a_{n-1} \\ \vdots & \vdots & & \vdots \\ 0 & a_{2} & \cdots & * \\ a_{1} & * & \cdots & * \end{bmatrix}$$

### 解

由行列式定义,

$$D_{n} = \begin{vmatrix} 0 & 0 & \cdots & 0 & a_{n} \\ 0 & 0 & \cdots & a_{n-1} & * \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & a_{2} & \cdots & * & * \\ a_{1} & * & \cdots & * & * \end{vmatrix} = (-1)^{1+n} a_{n} \begin{vmatrix} 0 & 0 & \cdots & a_{n-1} \\ \vdots & \vdots & & \vdots \\ 0 & a_{2} & \cdots & * \\ a_{1} & * & \cdots & * \end{vmatrix}$$

## 解

由行列式定义,

$$D_{n} = \begin{vmatrix} 0 & 0 & \cdots & 0 & a_{n} \\ 0 & 0 & \cdots & a_{n-1} & * \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & a_{2} & \cdots & * & * \\ a_{1} & * & \cdots & * & * \end{vmatrix} = (-1)^{1+n} a_{n} \begin{vmatrix} 0 & 0 & \cdots & a_{n-1} \\ \vdots & \vdots & & \vdots \\ 0 & a_{2} & \cdots & * \\ a_{1} & * & \cdots & * \end{vmatrix}$$

### 解(续)

同理递推,

$$D_{n} = (-1)^{n-1} a_{n} D_{n-1} = (-1)^{n-1} a_{n} (-1)^{n-2} a_{n-1} D_{n-2}$$

$$\cdots$$

$$= (-1)^{(n-1)+(n-2)+\cdots+2+1} a_{n} a_{n-1} \cdots a_{2} a_{1}$$

$$(-1)^{\frac{n(n-1)}{2}} a_{n} a_{n-1} \cdots a_{2} a_{1}$$

$$D_2 = -a_1 a_2$$
,  $D_3 = -a_1 a_2 a_3$ ,  $D_4 = a_1 a_2 a_3 a_4$ ,  $D_5 = a_1 a_2 a_3 a_4 a_5$ .

# 解(续)

同理递推,

$$D_{n} = (-1)^{n-1} a_{n} D_{n-1} = (-1)^{n-1} a_{n} (-1)^{n-2} a_{n-1} D_{n-2}$$

$$\cdots$$

$$= (-1)^{(n-1)+(n-2)+\cdots+2+1} a_{n} a_{n-1} \cdots a_{2} a_{1}$$

例如:

$$D_2 = -a_1 a_2$$
,  $D_3 = -a_1 a_2 a_3$ ,  $D_4 = a_1 a_2 a_3 a_4$ ,  $D_5 = a_1 a_2 a_3 a_4 a_5$ .

同理递推,

$$D_{n} = (-1)^{n-1}a_{n}D_{n-1} = (-1)^{n-1}a_{n}(-1)^{n-2}a_{n-1}D_{n-2}$$

$$\dots$$

$$= (-1)^{(n-1)+(n-2)+\dots+2+1}a_{n}a_{n-1}\dots a_{2}a_{1}$$

$$= (-1)^{\frac{n(n-1)}{2}}a_{n}a_{n-1}\dots a_{2}a_{1}.$$

例如,

$$D_2 = -a_1 a_2$$
,  $D_3 = -a_1 a_2 a_3$ ,  $D_4 = a_1 a_2 a_3 a_4$ ,  $D_5 = a_1 a_2 a_3 a_4 a_5$ .

同理递推,

$$D_{n} = (-1)^{n-1}a_{n}D_{n-1} = (-1)^{n-1}a_{n}(-1)^{n-2}a_{n-1}D_{n-2}$$

$$\dots$$

$$= (-1)^{(n-1)+(n-2)+\dots+2+1}a_{n}a_{n-1}\dots a_{2}a_{1}$$

$$= (-1)^{\frac{n(n-1)}{2}}a_{n}a_{n-1}\dots a_{2}a_{1}.$$

例如,

$$D_2 = -a_1 a_2$$
,  $D_3 = -a_1 a_2 a_3$ ,  $D_4 = a_1 a_2 a_3 a_4$ ,  $D_5 = a_1 a_2 a_3 a_4 a_5$ .

### 解(续)

同理递推,

$$D_{n} = (-1)^{n-1}a_{n}D_{n-1} = (-1)^{n-1}a_{n}(-1)^{n-2}a_{n-1}D_{n-2}$$

$$\dots$$

$$= (-1)^{(n-1)+(n-2)+\dots+2+1}a_{n}a_{n-1}\dots a_{2}a_{1}$$

$$= (-1)^{\frac{n(n-1)}{2}}a_{n}a_{n-1}\dots a_{2}a_{1}.$$

例如,

$$D_2 = -a_1a_2, \quad D_3 = -a_1a_2a_3, \quad D_4 = a_1a_2a_3a_4, \quad D_5 = a_1a_2a_3a_4a_5.$$

- 1 行列式简介
- 2 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

### 性质1

互换行列式的行与列, 值不变, 即

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{vmatrix}$$
(3)

### 证明【数学归纳法】

## 将等式两端的行列式分别记为D和D',对阶数n用归纳法。

- 当n = 2时, D = D'显然成立。
- 假设结论对于阶数小于n的行列式都成立,以下考虑阶数为n的情况。由定义可知,

$$D = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n},$$
  

$$D' = a_{11}A'_{11} + a_{21}A'_{21} + \dots + a_{n1}A'_{n1}$$

显然, $A_{11} = A'_{11}$ 。



#### 证明【数学归纳法】

将等式两端的行列式分别记为D和D',对阶数n用归纳法。

- 当n = 2时, D = D′显然成立。
- 假设结论对于阶数小于n的行列式都成立,以下考虑阶数为n的情况。由定义可知,

$$D = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n},$$
  

$$D' = a_{11}A'_{11} + a_{21}A'_{21} + \dots + a_{n1}A'_{n1}$$

显然, $A_{11} = A'_{11}$ 。



### 证明【数学归纳法】

将等式两端的行列式分别记为D和D',对阶数n用归纳法。

- 当n = 2时, D = D′显然成立。
- 假设结论对于阶数小于n的行列式都成立,以下考虑阶数为n的情况。由定义可知,

$$D = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n},$$
  

$$D' = a_{11}A'_{11} + a_{21}A'_{21} + \dots + a_{n1}A'_{n1}$$

显然, $A_{11} = A'_{11}$ 。



于是

$$D' = a_{11}A_{11} + (-1)^{1+2}a_{21}\begin{vmatrix} a_{12} & a_{32} & \cdots & a_{n2} \\ a_{13} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{3n} & \cdots & a_{nn} \end{vmatrix}$$

$$+(-1)^{1+3}a_{31}\begin{vmatrix} a_{12} & a_{22} & a_{42} & \cdots & a_{n2} \\ a_{13} & a_{23} & a_{43} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{1n} & a_{2n} & a_{4n} & \cdots & a_{nn} \end{vmatrix}$$

$$+\cdots + (-1)^{1+n}a_{n1}\begin{vmatrix} a_{12} & a_{22} & \cdots & a_{n-1,2} \\ a_{13} & a_{23} & \cdots & a_{n-1,3} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{n-1,n} \end{vmatrix}$$

对n-1个行列式按第一行展开,将含a12的项进行合并,可得

$$(-1)^{1+2}a_{21}a_{12}\begin{vmatrix} a_{33} & \cdots & a_{n3} \\ \vdots & & \vdots \\ a_{3n} & \cdots & a_{nn} \end{vmatrix} + (-1)^{1+3}a_{31}a_{12}\begin{vmatrix} a_{23} & a_{43} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots \\ a_{2n} & a_{4n} & \cdots & a_{nn} \end{vmatrix}$$

$$+\cdots+(-1)^{1+n}a_{n1}a_{12}\begin{vmatrix} a_{23}&\cdots&a_{n-1,3}\\ \vdots&&\vdots\\ a_{2n}&\cdots&a_{n-1,n}\end{vmatrix}$$

$$= (-1)^{1+2} a_{12} \begin{pmatrix} (-1)^{1+1} a_{21} & a_{33} & \cdots & a_{n3} \\ \vdots & & \vdots & & \vdots \\ a_{3n} & \cdots & a_{nn} \end{pmatrix} \\ + (-1)^{1+2} a_{31} \begin{vmatrix} a_{23} & a_{43} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots \\ a_{2n} & a_{4n} & \cdots & a_{nn} \end{vmatrix} + \cdots + \\ (-1)^{1+n-1} a_{n1} \begin{vmatrix} a_{23} & \cdots & a_{n-1,3} \\ \vdots & & \vdots \\ a_{2,n3} & \cdots & a_{n-1,n} \end{vmatrix}$$

$$= (-1)^{1+2} a_{12} \begin{pmatrix} \begin{vmatrix} a_{21} & 0 & \cdots & 0 \\ 0 & a_{33} & \cdots & a_{n3} \\ 0 & \vdots & & \vdots \\ 0 & a_{3n} & \cdots & a_{nn} \end{vmatrix} \\ + \begin{vmatrix} 0 & a_{31} & 0 & \cdots & 0 \\ a_{23} & 0 & a_{43} & \cdots & a_{n3} \\ \vdots & 0 & \vdots & & \vdots \\ a_{2n} & 0 & a_{4n} & \cdots & a_{nn} \end{vmatrix} + \cdots + \\ \begin{vmatrix} 0 & \cdots & 0 & a_{n-1} \\ a_{23} & \cdots & a_{n-1,3} & 0 \\ \vdots & & \vdots & 0 \\ a_{2,n3} & \cdots & a_{n-1,n} & 0 \end{vmatrix}$$

$$= (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{31} & \cdots & a_{n1} \\ a_{23} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots \\ a_{2n} & a_{3n} & \cdots & a_{nn} \end{vmatrix}$$
$$= (-1)^{1+2} a_{12} M'_{12} = a_{12} A'_{12} = a_{12} A_{12}.$$

同理,含 $a_{13}$ 的项合并后其值等于 $a_{13}A_{13}$ , ...,含 $a_{1n}$ 的项合并后其值等于 $a_{1n}A_{1n}$ . 因此,D=D'.

有了这个性质,行列式对行成立的性质都适用于列。



$$= (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{31} & \cdots & a_{n1} \\ a_{23} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots \\ a_{2n} & a_{3n} & \cdots & a_{nn} \end{vmatrix}$$

$$= (-1)^{1+2} a_{12} M'_{12} = a_{12} A'_{12} = a_{12} A_{12}.$$

同理,含 $a_{13}$ 的项合并后其值等于 $a_{13}A_{13}$ ,...,含 $a_{1n}$ 的项合并后其值等于 $a_{1n}A_{1n}$ . 因此,D=D'.

有了这个性质,行列式对行成立的性质都适用于列。



$$= (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{31} & \cdots & a_{n1} \\ a_{23} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & & \vdots \\ a_{2n} & a_{3n} & \cdots & a_{nn} \end{vmatrix}$$

$$= (-1)^{1+2} a_{12} M'_{12} = a_{12} A'_{12} = a_{12} A_{12}.$$

同理,含 $a_{13}$ 的项合并后其值等于 $a_{13}A_{13}$ , ..., 含 $a_{1n}$ 的项合并后其值等于 $a_{1n}A_{1n}$ . 因此,D=D'.

有了这个性质,行列式对行成立的性质都适用于列。



#### 性质2

行列式对任一行按下式展开,其值相等,即

$$D = a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in} = \sum_{j=1}^{n} a_{ij}A_{ij}, \quad i = 1, 2, \cdots, n,$$

其中

$$A_{ij} = (-1)^{i+j} M_{ij}$$

而 $M_{ij}$ 为D中划掉第i行第j列后其余元素按原顺序排成的n-1阶行列式,它称为 $a_{ij}$ 的余子式, $A_{ij}$ 称为 $a_{ij}$ 的代数余子式.



# 证明[数学归纳法]

- 当n=2时,结论显然成立。
- 假设结论对阶数< n-1的行列式成立,以下考虑阶数为n的情况。

$$D = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\begin{vmatrix} a_{21} & a_{22} & a_{24} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{24} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n4} & \cdots & a_{nn} \end{vmatrix}$$

$$+ \cdots + (-1)^{1+n} a_{1n} \begin{vmatrix} a_{21} & a_{22} & \cdots & a_{2,n-1} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{i,n-1} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{n,n-1} \end{vmatrix}$$

由归纳假设,按行展开后合并含ail的项可得

$$(-1)^{(i-1)+1}a_{i1}\begin{pmatrix} (-1)^{1+2}a_{12} & a_{23} & a_{24} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{i-1,3} & a_{i-1,4} & \cdots & a_{i-1,n} \\ a_{i+1,3} & a_{i+1,4} & \cdots & a_{i+1,n} \\ \vdots & \vdots & & \vdots \\ a_{n,3} & a_{n,4} & \cdots & a_{nn} \end{pmatrix}$$

$$+(-1)^{1+3}a_{13}\begin{pmatrix} a_{22} & a_{24} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{i-1,2} & a_{i-1,4} & \cdots & a_{i-1,n} \\ a_{i+1,2} & a_{i+1,4} & \cdots & a_{i+1,n} \\ \vdots & \vdots & & \vdots \\ a_{n2} & a_{n4} & \cdots & a_{nn} \end{pmatrix}$$

$$\begin{vmatrix} a_{22} & \cdots & a_{2,n-1} \\ \vdots & & \vdots \\ a_{i-1,2} & \cdots & a_{i-1,n-1} \\ a_{i+1,2} & \cdots & a_{i+1,n-1} \\ \vdots & & \vdots \\ a_{n2} & \cdots & a_{n,n-1} \end{vmatrix}$$

$$\begin{vmatrix} a_{12} & a_{13} & \cdots & a_{1n} \\ a_{22} & a_{23} & \cdots & a_{2n} \end{vmatrix}$$

$$= (-1)^{i+1} a_{i1} \begin{vmatrix} a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{i-1,2} & a_{i-1,3} & \cdots & a_{i-1,n} \\ a_{i-1,2} & a_{i-1,3} & \cdots & a_{i-1,n} \\ \vdots & \vdots & & \vdots \\ a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix} = (-1)^{i+1} a_{i1} M_{i1} = a_{i1} A_{i1}.$$

同理可证,含 $a_{i2}$ 的项合并后其值为 $a_{i2}A_{i2}$ ,...,含 $a_{in}$ 的项合并后其值为 $a_{in}A_{in}$ .

#### (线性性质)

1 行列式的某一行(列)中所有的元素都乘以同一个数k,等于用数k乘以此行列式,即

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ ka_{i1} & ka_{i2} & \cdots & ka_{in} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$
(4)

#### (线性性质)

2 若行列式的某一行(列)的元素都是两数之和,如

$$\begin{vmatrix} a_{11} & \cdots & a_{1j} + b_{1j} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j} + b_{2j} & \cdots & a_{2n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj} + b_{nj} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & b_{1j} & \cdots & a_{1n} \\ a_{21} & \cdots & b_{2j} & \cdots & a_{2n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & b_{nj} & \cdots & a_{nn} \end{vmatrix}$$

$$(5)$$

#### 一些记号

- r<sub>i</sub>×k (c<sub>i</sub>×k):第i行(列)乘以k
- r<sub>i</sub> ÷ k (c<sub>i</sub> ÷ k) : 第i行(列)提取公因子k



如果行列式 $D=|a_{ij}|_n$ 的元素 $a_{ij}=-a_{ji}(i,j=1,2,\cdots,n)$ ,就称D是反对称行列式(其中 $a_{ii}=-a_{ii}\Rightarrow a_{ii}=0, i=1,2,\cdots,n$ ).

证明:奇数阶反对称行列式的值为0.

证明

$$D = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ -a_{12} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{1n} & -a_{2n} & \cdots & 0 \end{bmatrix}$$

如果行列式 $D=|a_{ij}|_n$ 的元素 $a_{ij}=-a_{ji}(i,j=1,2,\cdots,n)$ ,就称D是反对称行列式(其中 $a_{ii}=-a_{ii}\Rightarrow a_{ii}=0, i=1,2,\cdots,n$ ).

证明:奇数阶反对称行列式的值为0.

### 证明

$$D = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ -a_{12} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{1n} & -a_{2n} & \cdots & 0 \end{bmatrix}$$

性质1 
$$\begin{vmatrix} 0 & -a_{12} & \cdots & -a_{1n} \\ a_{12} & 0 & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & 0 \end{vmatrix}$$
 
$$\begin{vmatrix} 0 & a_{12} & \cdots & a_{1n} \\ \end{vmatrix}$$

$$\frac{\text{性质3-1}}{\text{将每行提取公因子}-1}$$
  $(-1)^n$   $\begin{vmatrix} 0 & a_{12} & \cdots & a_{1n} \\ -a_{12} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{1n} & -a_{2n} & \cdots & 0 \end{vmatrix} = (-1)^n D$ 

由于n为奇数,故D = -D,从而D = 0.

将每行提取公因子-1

# 推论1

若行列式的某行元素全为0,其值为0.

$$\begin{vmatrix}
1 & 2 & 3 \\
0 & 0 & 0 \\
2 & 5 & 1
\end{vmatrix} = 0.$$

# 推论1

若行列式的某行元素全为0,其值为0.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 2 & 5 & 1 \end{vmatrix} = 0.$$

#### 性质4

### 若行列式有两行(列)完全相同,其值为0.

证明

不妨设D的第i和j行元素全部相等,即对

$$D = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

有  $a_{il}=a_{jl} (i \neq j, l=1,2,\cdots,n)$ .

#### 性质4

若行列式有两行(列)完全相同,其值为0.

#### 证明

不妨设D的第i和j行元素全部相等,即对

$$D = \left| \begin{array}{ccccc} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{array} \right|,$$

有  $a_{il} = a_{il} (i \neq j, l = 1, 2, \cdots, n)$ .

对阶数n用数学归纳法。

- 当n = 2时,结论显然成立。
- 假设结论对阶数为n-1的行列式成立,在n阶的情况下,对第 $k(k \neq i,j)$ 行展开,有

$$D = a_{k1}A_{k1} + a_{k2}A_{k2} + \dots + a_{kn}A_{kn}.$$

注意到余子式 $M_{kl}(l=1,2,\cdots,n)$ 是n-1阶行列式,且其中有两行元素相同,故

$$A_{kl} = (-1)^{k+l} M_{kl} = 0 \quad (l = 1, 2, \dots, n),$$

从而D=0.



对阶数n用数学归纳法。

- 当n = 2时,结论显然成立。
- 假设结论对阶数为n-1的行列式成立,在n阶的情况下,对 第 $k(k \neq i, j)$ 行展开,有

$$D = a_{k1}A_{k1} + a_{k2}A_{k2} + \cdots + a_{kn}A_{kn}.$$

注意到余子式 $M_{kl}(l=1,2,\cdots,n)$ 是n-1阶行列式,且其中有两行元素相同,故

$$A_{kl} = (-1)^{k+l} M_{kl} = 0 \quad (l = 1, 2, \dots, n),$$

从而D=0.



对阶数n用数学归纳法。

- 当n = 2时,结论显然成立。
- 假设结论对阶数为n-1的行列式成立,在n阶的情况下,对 第 $k(k \neq i, j)$ 行展开,有

$$D=a_{k1}A_{k1}+a_{k2}A_{k2}+\cdots+a_{kn}A_{kn}.$$

注意到余子式 $M_{kl}(l=1,2,\cdots,n)$ 是n-1阶行列式,且其中有两行元素相同,故

$$A_{kl} = (-1)^{k+l} M_{kl} = 0 \quad (l = 1, 2, \dots, n),$$

从而D=0.



$$\left|\begin{array}{ccc} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{array}\right| = 0.$$

推论2

若行列式中有两行(列)元素成比例,则行列式的值为0.

$$\begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{vmatrix} = 0.$$

$$\left|\begin{array}{ccc} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{array}\right| = 0.$$

#### 推论2

若行列式中有两行(列)元素成比例,则行列式的值为0.

$$\begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{vmatrix} = 0.$$

$$\left|\begin{array}{ccc} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{array}\right| = 0.$$

#### 推论2

若行列式中有两行(列)元素成比例,则行列式的值为0.

$$\left|\begin{array}{ccc|c} 2 & 0 & -2 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{array}\right| = 2 \left|\begin{array}{ccc|c} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 2 & 3 & 4 \end{array}\right| = 0.$$

#### 性质5

把行列式的某一行(列)的各元素乘以同一个数然后加到另一行(列)对应的元素上去,行列式的值不变。

### 证明

将数k乘以第j行加到第i行,有

#### 性质5

把行列式的某一行(列)的各元素乘以同一个数然后加到另一行(列)对应的元素上去,行列式的值不变。

### 证明

将数k乘以第j行加到第j行,有

张晓平

### 性质3-2

推论2

 $a_{11}$   $a_{12}$  ...
  $a_{1n}$ 
 $\vdots$   $\vdots$   $\vdots$   $\vdots$ 
 $ka_{j1}$   $ka_{j2}$  ...
  $ka_{jn}$ 
 $\vdots$   $\vdots$   $\vdots$   $\vdots$ 
 $a_{j1}$   $a_{j2}$  ...
  $a_{jn}$ 
 $\vdots$   $\vdots$   $\vdots$ 
 $a_{n1}$   $a_{n2}$  ...
  $a_{nn}$ 

性质3-2

$$a_{11}$$
 $a_{12}$ 
 $\cdots$ 
 $a_{1n}$ 
 $a_{11}$ 
 $a_{12}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{i1}$ 
 $a_{i2}$ 
 $\cdots$ 
 $a_{in}$ 
 $a_{i1}$ 
 $a_{i2}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{j1}$ 
 $a_{j2}$ 
 $\cdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{n1}$ 
 $a_{n2}$ 
 $\cdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{n1}$ 
 $a_{n2}$ 
 $\cdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 

推论2

 $a_{1n}$ 

 $a_{jn}$ 

 $a_{nn}$ 

# 性质3-2

#### 

# 推论2

 $a_{11} \quad a_{12} \quad \cdots \quad a_{1n}$   $\vdots \quad \vdots \quad \vdots \quad \vdots$   $a_{i1} \quad a_{i2} \quad \cdots \quad a_{in}$   $\vdots \quad \vdots \quad \vdots$   $a_{j1} \quad a_{j2} \quad \cdots \quad a_{jn}$   $\vdots \quad \vdots \quad \vdots$ 

#### 

 $a_{11}$  $a_{12} \cdots$  $a_{1n}$  $a_{i1}$  $a_{i2}$  $a_{in}$ 性质3-2  $a_{j1}$  $a_{j2}$  $a_{jn}$  $a_{n1}$  $a_{n2}$  $a_{nn}$  $a_{1n}$  $a_{11}$  $a_{12}$  $a_{i1}$  $a_{i2}$  $a_{in}$ 推论2  $a_{i1}$  $a_{jn}$  $a_{j2}$ 

+

### 一些记号

•  $r_i + r_j \times k$ : 将第j行乘以k加到第i行

•  $c_i + c_i \times k$ : 将第j列乘以k加到第i列



# 性质6

# 互换行列式的两行(列),行列式变号。

证明

$$D = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}$$

$$\vdots & \vdots & & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

# 性质6

互换行列式的两行(列),行列式变号。

### 证明

$$D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

性质5
 
$$a_{11}$$
 $a_{12}$ 
 $a_{1n}$ 
 $a_{i1} + a_{j1}$ 
 $a_{i2} + a_{j2}$ 
 $a_{in} + a_{jn}$ 
 $a_{j1}$ 
 $a_{j2}$ 
 $a_{jn}$ 
 $a_{j1}$ 
 $a_{j2}$ 
 $a_{jn}$ 
 $a_{n1}$ 
 $a_{n2}$ 
 $a_{nn}$ 

性质5  $r_{i}+r_{j}$   $a_{11}$   $a_{12}$   $a_{i2}$   $a_{i2}+a_{j2}$   $a_{in}+a_{jn}$   $a_{i2}+a_{j2}$   $a_{in}+a_{jn}$   $a_{in}+a_{jn}$   $a_{i2}$   $a_{in}+a_{jn}$   $a_{in}+a_{jn}$  $a_{in}+a_{jn}$ 

$$a_{11}$$
 $a_{12}$ 
 $\cdots$ 
 $a_{1n}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{i1} + a_{j1}$ 
 $a_{i2} + a_{j2}$ 
 $\cdots$ 
 $a_{in} + a_{jn}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{j1}$ 
 $a_{j2}$ 
 $\cdots$ 
 $a_{jn}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{n1}$ 
 $a_{n2}$ 
 $\cdots$ 
 $a_{nn}$ 

性质5
 
$$\vdots$$
 $\vdots$ 
 $\vdots$ 
 $r_j - r_i$ 
 $\vdots$ 
 $a_{i1} + a_{j1}$ 
 $a_{i2} + a_{j2}$ 
 $\cdots$ 
 $a_{in} + a_{jn}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $-a_{i1}$ 
 $-a_{i2}$ 
 $\cdots$ 
 $-a_{in}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{n1}$ 
 $a_{n2}$ 
 $\cdots$ 
 $a_{nn}$ 

世质5  

$$r_i+r_j$$
 $a_{11}$ 
 $a_{12}$  · · · 。  $a_{1n}$ 
 $\vdots$   $\vdots$   $\vdots$   $a_{j1}$   $a_{j2}$  · · ·  $a_{jn}$ 
 $\vdots$   $\vdots$   $\vdots$   $\vdots$   $\vdots$   $-a_{i1}$   $-a_{i2}$  · · ·  $-a_{in}$ 
 $\vdots$   $\vdots$   $\vdots$   $\vdots$   $a_{n1}$   $a_{n2}$  · · ·  $a_{nn}$ 

性质5 r<sub>i</sub>+r<sub>j</sub>

```
\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{j1} & a_{j2} & \cdots & a_{jn} \\ \vdots & \vdots & & \vdots \\ -a_{i1} & -a_{i2} & \cdots & -a_{in} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}
```

<u>性质3-1</u> — D

| 
$$a_{11}$$
  $a_{12}$  · · · ·  $a_{1n}$  |  $\vdots$   $\vdots$   $\vdots$   $a_{j1}$   $a_{j2}$  · · · ·  $a_{jn}$  |  $\vdots$   $\vdots$   $\vdots$   $\vdots$   $\vdots$   $-a_{i1}$   $-a_{i2}$  · · ·  $-a_{in}$   $\vdots$   $\vdots$   $\vdots$   $a_{n1}$   $a_{n2}$  · · · ·  $a_{nn}$ 

性质3-1 — D

$$a_{11}$$
 $a_{12}$ 
 $\cdots$ 
 $a_{1n}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{j1}$ 
 $a_{j2}$ 
 $\cdots$ 
 $a_{jn}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $-a_{i1}$ 
 $-a_{i2}$ 
 $\cdots$ 
 $-a_{in}$ 
 $\vdots$ 
 $\vdots$ 
 $\vdots$ 
 $a_{n1}$ 
 $a_{n2}$ 
 $\cdots$ 
 $a_{nn}$ 

### 一些记号

- r<sub>i</sub> ↔ r<sub>j</sub>: 互换第i,j行
- c<sub>i</sub> ↔ c<sub>j</sub>: 互换第i,j列

例

$$\begin{vmatrix}
1 & 2 \\
3 & 4
\end{vmatrix} \quad \xrightarrow{\underline{r_1 \leftrightarrow r_2}} \quad - \begin{vmatrix}
3 & 4 \\
1 & 2
\end{vmatrix}$$

$$\begin{vmatrix}
1 & 2 \\
3 & 4
\end{vmatrix} \quad \xrightarrow{\underline{c_1 \leftrightarrow c_2}} \quad - \begin{vmatrix}
2 & 1 \\
4 & 3
\end{vmatrix}$$

行列式某一行的元素乘以另一行对应元素的代数余子式之和等于0,即

$$\sum_{k=1}^{n} a_{ik} A_{jk} = 0 \quad (i \neq j).$$

证明

由性质2,对D的第j行展开得

$$a_{11}$$
  $a_{12}$  ...  $a_{1n}$ 
 $\vdots$   $\vdots$   $\vdots$ 
 $a_{i1}$   $a_{i2}$  ...  $a_{in}$ 
 $\vdots$   $\vdots$   $\vdots$ 
 $a_{j1}$   $a_{j2}$  ...  $a_{jn}$ 
 $\vdots$   $\vdots$   $\vdots$ 
 $a_{n1}$   $a_{n2}$  ...  $a_{nn}$ 

$$= a_{j1}A_{j1} + a_{j2}A_{j2} + \cdots + a_{jn}A_{jn}$$

行列式某一行的元素乘以另一行对应元素的代数余子式之和等于0,即

$$\sum_{k=1}^{n} a_{ik} A_{jk} = 0 \quad (i \neq j).$$

### 证明

由性质2,对D的第j行展开得

$$\vdots \qquad \vdots \qquad = a_{j1}A_{j1} + a_{j2}A_{j2} + \cdots + a_{jn}A_{jn}$$

因此,将D中第j行的元素 $a_{j1}, a_{j2}, \cdots, a_{jn}$ 换成 $a_{i1}, a_{i2}, \cdots, a_{in}$ 后所得的行列式, 其展开式就是 $\sum_{k=1}^{n} a_{ik} A_{jk}$ ,即

#### 结论

• 对行列式D按行展开,有

$$\sum_{k=1} a_{ik} A_{jk} = \delta_{ij} D,$$

其中 $\delta_{ij}$ 为克罗内克(Kronecker)记号,表示为

$$\delta_{ij} = \left\{ \begin{array}{ll} 1 & i = j \\ 0 & i \neq j \end{array} \right..$$

• 对行列式 D按列展开,有

$$\sum_{k=1} a_{ki} A_{kj} = \delta_{ij} D,$$



- 1 行列式简介
- 2 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

回顾一下行列式的性质

互换行列式的行与列, 值不变

### 性质2

行列式对任一行按下式展开, 其值相等, 即

$$D = a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in} = \sum_{j=1}^{n} a_{ij}A_{ij}, \quad i = 1, 2, \cdots, n,$$

其中

$$A_{ij} = (-1)^{i+j} M_{ij}$$

而 $M_{ij}$ 为D中划掉第i行第j列后其余元素按原顺序排成的n-1阶行列式,它称为 $a_{ii}$ 的余子式, $A_{ii}$ 称为 $a_{ii}$ 的代数余子式.

◆ロト ◆母 ト ◆ 差 ト ◆ 差 ・ 釣 へ ②

### 性质3 (线性性质)

- 1 行列式的某一行(列)中所有的元素都乘以同一个数k,等于用数k乘以此行列式;
- 2 若行列式的某一行(列)的元素都是两数之和,则该行列式可表示 为两个行列式的和。

# 推论1

若行列式的某行元素全为0,其值为0.

若行列式有两行(列)完全相同,其值为0.

# 推论2

若行列式中有两行(列)元素成比例,则行列式的值为0.



把行列式的某一行(列)的各元素乘以同一个数然后加到另一行(列)对应的元素上去,行列式的值不变。

### 性质6

互换行列式的两行(列),行列式变号。

行列式某一行的元素乘以另一行对应元素的代数余子式之和等于0,即

$$\sum_{k=1}^{n} a_{ik} A_{jk} = 0 \quad (i \neq j).$$

### 结论

• 对行列式D按行展开,有

$$\sum_{k=1}^{n} a_{ik} A_{jk} = \delta_{ij} D.$$

• 对行列式 D按列展开,有

$$\sum_{k=1}^{n} a_{ki} A_{kj} = \delta_{ij} D.$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} - \begin{vmatrix} 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{vmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{vmatrix}$$

$$= \frac{r_2 \leftrightarrow r_3}{-1} = \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -8 & 4 & -6 \\ 0 & 16 & -2 & 7 \end{vmatrix} = \frac{r_3 + 4r_2}{-1} = \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 8 & -10 \\ 0 & 0 & -10 & 15 \end{vmatrix} = \cdots = 40$$

计算

$$D = \left| \begin{array}{rrrr} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{array} \right|$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} = \begin{bmatrix} 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{bmatrix} = \frac{c_2 - c_1}{c_4 + 5c_1} = \begin{bmatrix} 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{bmatrix}$$

$$= \frac{c_2 \leftrightarrow c_3}{-1} = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -8 & 4 & -6 \end{bmatrix} = \frac{c_3 + 4c_2}{c_4 - 8c_2} = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 8 & -10 \end{bmatrix} = \cdots = 40$$

计算

$$D = \left| \begin{array}{rrrr} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{array} \right|$$

$$D = \frac{\mathbf{c_1} \leftrightarrow \mathbf{c_2}}{-1} = \begin{bmatrix} 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{bmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} = \begin{bmatrix} 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{bmatrix}$$

$$= \frac{r_2 \leftrightarrow r_3}{0} = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -8 & 4 & -6 \end{bmatrix} = \frac{r_3 + 4r_2}{r_4 - 8r_2} = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 8 & -10 \end{bmatrix} = \cdots = 40$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{bmatrix}$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{vmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{vmatrix}$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} = \begin{vmatrix} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{vmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} = \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{vmatrix}$$



计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D \quad \stackrel{c_1 \leftrightarrow c_2}{=} \quad - \left| \begin{array}{ccccc} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{array} \right| \stackrel{r_2 - r_1}{=} \quad - \left| \begin{array}{cccccc} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{array} \right|$$



计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{vmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{vmatrix}$$

$$= \frac{r_2 \leftrightarrow r_3}{-1} = \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -8 & 4 & -6 \\ 0 & 16 & -2 & 7 \end{vmatrix} = \frac{r_3 + 4r_2}{r_4 - 8r_2} \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 8 & -10 \\ 0 & 0 & -10 & 15 \end{vmatrix} = \cdots = 40$$

计算

$$D = \left| \begin{array}{rrrr} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{array} \right|$$

$$D = \frac{c_1 \leftrightarrow c_2}{-1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{vmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{vmatrix}$$
$$= \frac{r_2 \leftrightarrow r_3}{-1} = \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -8 & 4 & -6 \\ 0 & 16 & -2 & 7 \end{vmatrix} = \frac{r_3 + 4r_2}{r_4 - 8r_2} \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 8 & -10 \\ 0 & 0 & -10 & 15 \end{vmatrix} = \cdots = 40$$



计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 \leftrightarrow c_2}{\phantom{-}} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 1 & -5 & 3 & -4 \\ 0 & 2 & 1 & -1 \\ -5 & 1 & 3 & -3 \end{vmatrix} = \frac{r_2 - r_1}{r_4 + 5r_1} - \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & -8 & 4 & -6 \\ 0 & 2 & 1 & -1 \\ 0 & 16 & -2 & 7 \end{vmatrix}$$
$$= \frac{r_2 \leftrightarrow r_3}{\phantom{-}} \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & -8 & 4 & -6 \\ 0 & 16 & -2 & 7 \end{vmatrix} = \frac{r_3 + 4r_2}{r_4 - 8r_2} \begin{vmatrix} 1 & 3 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 8 & -10 \\ 0 & 0 & -10 & 15 \end{vmatrix} = \cdots = 40$$

计算

$$D = \left| \begin{array}{rrrr} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{array} \right|$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} = \frac{c_2 + c_1}{-5} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

$$D = \left| \begin{array}{rrrr} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{array} \right|$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} = \frac{c_2 + c_1}{-5} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} \stackrel{r_2 + r_1}{=} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} \xrightarrow{\frac{r_2 + r_1}{2}} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} \stackrel{c_2 + c_1}{=} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} = \frac{c_2 + c_1}{-5} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

计算

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$
$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} = \frac{r_2 + r_1}{-5} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$
$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

$$D = \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -4 \\ 2 & 0 & 1 & -1 \\ 1 & -5 & 3 & -3 \end{vmatrix}$$

$$D = \frac{c_1 - 2c_3}{c_4 + c_3} \begin{vmatrix} 5 & 1 & -1 & 1 \\ -11 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & 3 & 0 \end{vmatrix}$$

$$= (-1)^{3+3} \begin{vmatrix} 5 & 1 & 1 \\ -11 & 1 & -1 \\ -5 & -5 & 0 \end{vmatrix} = \frac{r_2 + r_1}{-5} \begin{vmatrix} 5 & 1 & 1 \\ -6 & 2 & 0 \\ -5 & -5 & 0 \end{vmatrix}$$

$$= (-1)^{1+3} \begin{vmatrix} -6 & 2 \\ -5 & -5 \end{vmatrix} = 40.$$

计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$

$$D = \begin{bmatrix} \frac{a-3}{5-2} \\ \frac{a-5}{5-2} \\ 0 = a & 2a+b & 3a+2b+c \\ 0 = a & 3a+b & 6a+3b+c \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & a & a+b & a+b+c \\ \frac{a-5}{5-2} & 0 & 0 & a & 2a+b \\ 0 & 0 & a & 3a+b \end{bmatrix}$$

计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$

计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$

计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$



计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$

计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$

计算

$$D = \begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$



$$D = \begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 2 & 3 & 4 & \cdots & n & 1 \\ 3 & 4 & 5 & \cdots & 1 & 2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n & 1 & 2 & \cdots & n-2 & n-1 \end{vmatrix}$$



解:

$$D_{n} = \frac{r_{i} - r_{i-1}}{\stackrel{i=n,\dots,2}{=}} \begin{vmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ 1 & 1 & 1 & \dots & 1 & 1-n \\ 1 & 1 & 1 & \dots & 1-n & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 - n & 1 & \dots & 1 & 1 \\ 1 & 0 & 0 & \dots & 0 & -n \\ \stackrel{c_{i}-c_{1}}{\stackrel{i=2,\dots,n}{=}} \begin{vmatrix} 1 & 1 & 2 & \dots & n-2 & n-1 \\ 1 & 0 & 0 & \dots & 0 & -n \\ \vdots & \vdots & \vdots & & \vdots & \vdots \end{vmatrix}$$

-n 0 ··· 0 ←□ → ◆□ → ◆ ≧ → ◆ ≧ →

解:

$$D_{n} = \frac{r_{i} - r_{i-1}}{\frac{1}{i=n, \cdots, 2}} \begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 1 & 1 & 1 & \cdots & 1 & 1-n \\ 1 & 1 & 1 & \cdots & 1-n & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1-n & 1 & \cdots & 1 & 1 \end{vmatrix}$$

$$= \frac{c_{i} - c_{1}}{\frac{i}{i=2, \cdots, n}} \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \end{vmatrix}$$

... () < □ > <(□ > < □ > < □ > < □ > < □ >

解:

$$D_{n} = \frac{\prod_{i=n,\dots,2}^{r_{i}-r_{i-1}}}{\prod_{i=n,\dots,2}^{r_{i}-r_{i-1}}} \begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 1 & 1 & 1 & \cdots & 1 & 1-n \\ 1 & 1 & 1 & \cdots & 1-n & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1-n & 1 & \cdots & 1 & 1 \end{vmatrix}$$

$$\frac{\prod_{i=n,\dots,n}^{r_{i}-r_{i-1}}}{\prod_{i=n,\dots,n}^{r_{i}-r_{i-1}}} \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{vmatrix}$$

-n 0 ··· 0 ←□ > ←□ > ← ≣ > ← ≣ >

$$D_{n} = \frac{r_{i} - r_{i-1}}{\stackrel{r_{i} - r_{i-1}}{\stackrel{r_{i} - r_{i-1}}{\stackrel{r_{i} - r_{i}}{\stackrel{r_{i}}{\stackrel{r_{i} - r_{i}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_{i} - r_{i}}{\stackrel{r_{i} - r_{i}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_{i} - r_{i}}}{\stackrel{r_$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{i=2, \cdots, n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\stackrel{c_{i} \div n}{=i=2, \dots, n}} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\stackrel{c_{1} \leftrightarrow n}{=i=2, \dots, n}} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \end{vmatrix}$$

$$\frac{c_{1} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\frac{c_{1} + c_{2} + \cdots + c_{n}}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{n} n^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{i=2, \cdots, n}}{i=2, \cdots, n}} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \begin{bmatrix} 1 + \frac{1}{n} \frac{n(n-1)}{2} \\ 1 + \frac{1}{n} \frac{n(n-1)}{2} \end{bmatrix} (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

(续)
$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{1} + c_{2} + \cdots + c_{n}}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

84代後

(续)
$$D_n = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_i \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_1 + c_2 + \cdots + c_n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

计算行列式

$$D_{20} = \frac{c_{i+1} - c_i}{c_{i+1} - c_i}$$

计算行列式

$$D_{20} = \frac{c_{i+1} - c_i}{i=19, \cdots, 1} \begin{vmatrix} 2 & -1 & 1 & \cdots & 1 & 1 & 1 \\ 3 & -1 & -1 & \cdots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 20 & -1 & -1 & \cdots & -1 & -1 & -1 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 3 & 0 & 2 & \cdots & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 21 & 0 & 0 & \cdots & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 21 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} = 21 \times (-1)^{20+1} \times 2^{18} = -21 \times 2^{18}.$$

计算行列式

$$D_{20} = \begin{vmatrix} 1 & 2 & 3 & \cdots & 18 & 19 & 20 \\ 2 & 1 & 2 & \cdots & 17 & 18 & 19 \\ 3 & 2 & 1 & \cdots & 16 & 17 & 18 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 20 & 19 & 18 & \cdots & 3 & 2 & 1 \end{vmatrix}$$

$$D_{20} = \frac{c_{i+1}-c_i}{\stackrel{i}{=}19,\cdots,1} = \begin{vmatrix} 2 & -1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & \cdots & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ 20 & -1 & -1 & \cdots & -1 & -1 & -1 & -1 \\ & & & & & & & & & & & & & \\ \hline = \frac{r_i+r_1}{\stackrel{i}{=}2,\cdots,20} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ 3 & 0 & 2 & \cdots & 2 & 2 & 2 & 2 \\ 4 & 0 & 0 & \cdots & 2 & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 21 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \end{vmatrix} = 21 \times (-1)^{20+1} \times 2^{18} = -21 \times 2^{18}.$$

计算行列式

$$D_{20} = \begin{vmatrix} 1 & 2 & 3 & \cdots & 18 & 19 & 20 \\ 2 & 1 & 2 & \cdots & 17 & 18 & 19 \\ 3 & 2 & 1 & \cdots & 16 & 17 & 18 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 20 & 19 & 18 & \cdots & 3 & 2 & 1 \end{vmatrix}$$

计算行列式

$$D_{20} = \begin{vmatrix} 1 & 2 & 3 & \cdots & 18 & 19 & 20 \\ 2 & 1 & 2 & \cdots & 17 & 18 & 19 \\ 3 & 2 & 1 & \cdots & 16 & 17 & 18 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 20 & 19 & 18 & \cdots & 3 & 2 & 1 \end{vmatrix}$$

$$D_{20} = \frac{c_{i+1}-c_i}{\stackrel{i}{=}19,\cdots,1} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 2 & -1 & 1 & \cdots & 1 & 1 & 1 \\ 3 & -1 & -1 & \cdots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 20 & -1 & -1 & \cdots & -1 & -1 & -1 \end{vmatrix}$$

$$= \frac{r_i+r_1}{\stackrel{i}{=}2,\cdots,20} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 3 & 0 & 2 & \cdots & 2 & 2 & 2 \\ 4 & 0 & 0 & \cdots & 2 & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 21 & 0 & 0 & \cdots & 0 & 0 & 0 \end{vmatrix} = 21 \times (-1)^{20+1} \times 2^{18} = -21 \times 2^{18}.$$

计算行列式

$$D_{20} = \begin{vmatrix} 1 & 2 & 3 & \cdots & 18 & 19 & 20 \\ 2 & 1 & 2 & \cdots & 17 & 18 & 19 \\ 3 & 2 & 1 & \cdots & 16 & 17 & 18 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 20 & 19 & 18 & \cdots & 3 & 2 & 1 \end{vmatrix}$$

$$D_{20} = \frac{c_{i+1} - c_i}{\stackrel{i}{=} 19, \dots, 1} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & \dots & 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & \dots & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ 20 & -1 & -1 & \dots & -1 & -1 & -1 & -1 \end{vmatrix}$$

$$= \frac{r_i + r_1}{\stackrel{i}{=} 2, \dots, 20} = \begin{vmatrix} 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ 3 & 0 & 2 & \dots & 2 & 2 & 2 & 2 \\ 4 & 0 & 0 & \dots & 2 & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ 21 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \end{vmatrix} = 21 \times (-1)^{20+1} \times 2^{18} = -21 \times 2^{18}.$$

## 计算元素为 $a_{ij} = |i - j|$ 的n阶行列式

$$D_{n} = \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-1 \\ \vdots & \vdots & \vdots & & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 \\ n-1 & n-2 & n-3 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & & \vdots & & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & -1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \\ 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & 0 & 0 & \cdots & 0 & 2 \end{vmatrix} = (-1)^{n-1}$$

```
例6
```

$$D_{n} = \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 & 1 \\ n-1 & n-2 & n-3 & \cdots & 1 & 0 \end{vmatrix}$$

$$\frac{c_{i+1}-c_{i}}{\stackrel{\longleftarrow}{i=n-1,\cdots,1}} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \end{vmatrix}$$

$$0 & 1 & 1 & \cdots & 1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \end{vmatrix}$$

$$0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ n-2 & 0 & 0 & \cdots & 0 & 2 \end{vmatrix} = (-1)^{n-1}(n-1)2^{n-2}.$$

```
例6
```

$$D_{n} = \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 & 1 \\ n-1 & n-2 & n-3 & \cdots & 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \\ 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & 0 & 0 & \cdots & 0 & 2 \end{vmatrix} = (-1)^{n-1}(n-1)2^{n-2}.$$

```
例6
```

$$D_{n} = \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 & 1 \\ n-1 & n-2 & n-3 & \cdots & 1 & 0 \end{vmatrix}$$

$$= \frac{c_{i+1}-c_{i}}{i=n-1,\cdots,1} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \end{vmatrix}$$

$$= \frac{c_{i+1}-c_{i}}{i=n-1,\cdots,1} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \end{vmatrix}$$

$$= \frac{c_{i+1}-c_{i}}{i=n-1,\cdots,1} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ n-2 & 0 & 0 & \cdots & 0 & 2 \end{vmatrix} = (-1)^{n-1}(n-1)2^{n-2}.$$

```
例6
```

$$D_{n} = \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 & 1 \\ n-1 & n-2 & n-3 & \cdots & 1 & 0 \\ \end{vmatrix}$$

$$\frac{c_{i+1}-c_{i}}{i=n-1,\cdots,1} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \\ \end{vmatrix}$$

$$\frac{r_{i}+r_{1}}{i=2,\cdots,n} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 2 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & 0 & 0 & \cdots & 0 & 2 \end{vmatrix} = (-1)^{n-1}(n-1)2^{n-2}.$$

```
例6
```

$$D_{n} = \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 & 1 \\ n-1 & n-2 & n-3 & \cdots & 1 & 0 \end{vmatrix}$$

$$\frac{c_{i+1}-c_{i}}{\stackrel{i}{=}n-1,\cdots,1} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \end{vmatrix}$$

$$\frac{r_{i}+r_{1}}{\stackrel{i}{=}2,\cdots,n}} \begin{vmatrix} \frac{r_{i}+r_{1}}{i=2,\cdots,n} \\ n-2 & 0 & 0 & \cdots & 0 & 2 \\ n-1 &$$

```
例6
```

解:

 $D_n$ 

$$= \begin{vmatrix} 0 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 1 & \cdots & n-3 & n-2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ n-2 & n-3 & n-4 & \cdots & 0 & 1 \\ n-1 & n-2 & n-3 & \cdots & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} c_{i+1}-c_i \\ \vdots \\ n-2 & 1 & 1 & \cdots & 1 & 1 \\ 1 & -1 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ n-2 & -1 & -1 & \cdots & -1 & 1 \\ n-1 & -1 & -1 & \cdots & -1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} c_{i+1}-c_i \\ \vdots \\ n-2 & 1 & 1 & \cdots & 1 & 1 \\ n-1 & 1 & \cdots & 1 & 1 \\ n-1 & -1 & 1 & \cdots & -1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} c_{i+r_1} \\ \vdots \\ n-2 & 0 & 0 & \cdots & 0 & 2 \\ n-1 & 0 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= (-1)^{n-1}(n-1)2^{n-2}.$$

例7 计算

$$D = \left| \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \\ 2 & 2 & 0 & \cdots & 0 \\ 3 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots \vdots & & & & \\ n & 0 & 0 & \cdots & n \end{array} \right|$$

$$D = \frac{r_1 - r_i}{i = 2, \dots, n}$$

$$= (1 - \sum_{i=2}^{n} i) \cdot 2 \cdot 3 \cdot \dots \cdot n$$

$$= (1 - \sum_{i=2}^{n} i) \cdot 2 \cdot 3 \cdot \dots \cdot n$$

$$= \left[ 2 - \frac{(n+1)n}{2} \right] n!$$

$$D = \frac{r_1 - r_i}{i = 2, \dots, n} = \begin{bmatrix} 1 - \sum_{i=2}^{n} i & 0 & 0 & \dots & 0 \\ 2 & 2 & 0 & \dots & 0 \\ 3 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & 0 & 0 & \dots & n \end{bmatrix}$$

$$= (1 - \sum_{i=2}^{n} i) \cdot 2 \cdot 3 \cdot \dots \cdot n$$

$$= \left[ 2 - \frac{(n+1)n}{2} \right] n!$$

$$D = \frac{r_1 - r_i}{\stackrel{r_1 - r_i}{i = 2, \cdots, n}} \begin{vmatrix} 1 - \sum_{i=2}^{n} i & 0 & 0 & \cdots & 0 \\ 2 & 2 & 0 & \cdots & 0 \\ 3 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & 0 & 0 & \cdots & n \end{vmatrix}$$

$$= (1 - \sum_{i=2}^{n} i) \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

$$D = \frac{r_1 - r_i}{\stackrel{r_1 - r_i}{= 2, \cdots, n}} \begin{vmatrix} 1 - \sum_{i=2}^{n} i & 0 & 0 & \cdots & 0 \\ 2 & 2 & 0 & \cdots & 0 \\ 3 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & 0 & 0 & \cdots & n \end{vmatrix}$$

$$= (1 - \sum_{i=2}^{n} i) \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

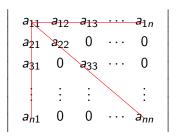
$$= \left[ 2 - \frac{(n+1)n}{2} \right] n!$$

$$D = \frac{r_1 - r_i}{\stackrel{r_1 - r_i}{= 2, \cdots, n}} \begin{vmatrix} 1 - \sum_{i=2}^{n} i & 0 & 0 & \cdots & 0 \\ 2 & 2 & 0 & \cdots & 0 \\ 3 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & 0 & 0 & \cdots & n \end{vmatrix}$$

$$= (1 - \sum_{i=2}^{n} i) \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

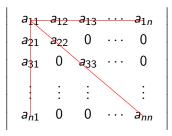
$$= \left[ 2 - \frac{(n+1)n}{2} \right] n!$$

# 如何计算"爪形"行列式



其解法固定,即从第二行开始,每行依次乘一个系数然后加到第一行, 使得第一行除第一个元素外都为零,从而得到一个下三角行列式。

## 如何计算"爪形"行列式



其解法固定,即从第二行开始,每行依次乘一个系数然后加到第一行, 使得第一行除第一个元素外都为零,从而得到一个下三角行列式。

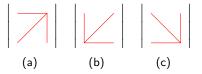
# 计算行列式 (假定 $a_i \neq 0$ )

$$D_{n+1} = \begin{vmatrix} a_0 & 1 & 1 & \cdots & 1 \\ 1 & a_1 & 0 & \cdots & 0 \\ 1 & 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & & & & \\ 1 & 0 & 0 & \cdots & a_n \end{vmatrix} = (a_0 - \sum_{i=1}^n \frac{1}{a_i}) a_1 a_2 \cdots a_n.$$

# 计算行列式 (假定 $a_i \neq 0$ )

$$D_{n+1} = \begin{vmatrix} a_0 & 1 & 1 & \cdots & 1 \\ 1 & a_1 & 0 & \cdots & 0 \\ 1 & 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & & & \\ 1 & 0 & 0 & \cdots & a_n \end{vmatrix} = (a_0 - \sum_{i=1}^n \frac{1}{a_i}) a_1 a_2 \cdots a_n.$$

类似的方式还可用于求解如下形式的"爪型行列式"



例8

$$\left|\begin{array}{ccccccc} 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 2 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & n-1 & \cdots & 0 & 1 \\ n & 0 & \cdots & 0 & 1 \end{array}\right| = (-$$

$$= (-1)^{\frac{n(n-1)}{2}} n! \left(1 - \sum_{i=2}^{n} \frac{1}{i}\right)$$

例8

$$\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 2 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & n-1 & \cdots & 0 & 1 \\ n & 0 & \cdots & 0 & 1 \end{vmatrix} = (-1)^{\frac{n(n-1)}{2}} n! \left(1 - \sum_{i=2}^{n} \frac{1}{i}\right)$$

# 例9

计算n阶行列式

$$D_n = \begin{vmatrix} x & a & \cdots & a \\ a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x \end{vmatrix}$$

$$D_n = \frac{c_1 + c_2 + \dots + c_n}{c_n}$$

$$c_1 \div [x + (n-1)a] \qquad [x$$

$$[x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 1 & a & \cdots & x \end{vmatrix}$$

$$\frac{r_i - r_1}{i = 2, \cdots, n}$$

$$[x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$[x + (n-1)a](x-a)^{n-1}$$

$$D_n = \frac{c_1 + c_2 + \cdots + c_n}{c_1 + c_2 + \cdots + c_n}$$

$$\begin{vmatrix} x + (n-1)a & a & \cdots & a \\ x + (n-1)a & x & \cdots & a \\ \vdots & \vdots & \vdots & \vdots \\ x + (n-1)a & a & \cdots & x \end{vmatrix}$$

 $= [x + (n-1)a](x-a)^{n-1}$ 



$$D_{n} = \frac{c_{1}+c_{2}+\cdots+c_{n}}{x+(n-1)a} = \begin{bmatrix} x+(n-1)a & a & \cdots & a \\ x+(n-1)a & x & \cdots & a \\ \vdots & \vdots & \vdots \\ x+(n-1)a & a & \cdots & x \end{bmatrix}$$

$$= \frac{c_{1}\div[x+(n-1)a]}{i=2,\cdots,n} = \begin{bmatrix} x+(n-1)a \end{bmatrix} \begin{vmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & \vdots \\ 1 & a & \cdots & x \end{vmatrix}$$

$$= \frac{r_{i}-r_{1}}{i=2,\cdots,n} = \begin{bmatrix} x+(n-1)a \end{bmatrix} \begin{vmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$D_{n} = \frac{c_{1}+c_{2}+\cdots+c_{n}}{\sum_{i=2,\cdots,n}^{n}} = \begin{bmatrix} x+(n-1)a & a & \cdots & a \\ x+(n-1)a & x & \cdots & a \\ \vdots & \vdots & \vdots \\ x+(n-1)a & a & \cdots & x \end{bmatrix}$$

$$\begin{bmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 1 & a & \cdots & x \end{bmatrix}$$

$$\begin{bmatrix} x+(n-1)a \end{bmatrix} = \begin{bmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{bmatrix}$$

(□ ) (□ ) (Ē ) (Ē ) (Ē ) (Ē ) (O)

$$D_{n} = \frac{c_{1}+c_{2}+\cdots+c_{n}}{\begin{vmatrix} x+(n-1)a & a & \cdots & a \\ x+(n-1)a & x & \cdots & a \\ \vdots & \vdots & \vdots \\ x+(n-1)a & a & \cdots & x \end{vmatrix}}$$

$$= \frac{c_{1}\div[x+(n-1)a]}{\begin{vmatrix} x+(n-1)a \end{vmatrix}} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & \vdots \\ 1 & a & \cdots & x \end{vmatrix}$$

$$= \frac{r_{i}-r_{1}}{i=2,\cdots,n} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{vmatrix}$$

/ (ハー)の』(ハーロ) | 4□ ▶ 4回 ▶ 4回 ▶ 4頁 ▶ 4頁 ▶ □

$$D_{n} = \frac{c_{1}+c_{2}+\cdots+c_{n}}{\begin{vmatrix} x+(n-1)a & a & \cdots & a \\ x+(n-1)a & x & \cdots & a \\ \vdots & \vdots & \vdots \\ x+(n-1)a & a & \cdots & x \end{vmatrix}}$$

$$= \frac{c_{1}\div[x+(n-1)a]}{\begin{vmatrix} x+(n-1)a \end{vmatrix}} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & \vdots \\ 1 & a & \cdots & x \end{vmatrix}$$

$$= \frac{r_{i}-r_{1}}{i=2,\cdots,n} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$D_{n} = \frac{c_{1}+c_{2}+\cdots+c_{n}}{\begin{vmatrix} x+(n-1)a & a & \cdots & a \\ x+(n-1)a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ x+(n-1)a & a & \cdots & x \end{vmatrix}}$$

$$= \frac{c_{1}\div[x+(n-1)a]}{\begin{vmatrix} c_{1}\div[x+(n-1)a] \end{vmatrix}} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 1 & a & \cdots & x \end{vmatrix}}$$

$$= \frac{r_{i}-r_{1}}{i=2,\cdots,n} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{vmatrix}$$

/ (ハー)の』(ハーロ) | 4□ ▶ 4回 ▶ 4回 ▶ 4頁 ▶ 4頁 ▶ □

$$D_{n} = \frac{c_{1}+c_{2}+\cdots+c_{n}}{\begin{vmatrix} x+(n-1)a & a & \cdots & a \\ x+(n-1)a & x & \cdots & a \end{vmatrix}} \\ \vdots & \vdots & & \vdots \\ x+(n-1)a & a & \cdots & x \end{vmatrix}$$

$$= \frac{c_{1}\div[x+(n-1)a]}{\begin{vmatrix} c_{1}\div[x+(n-1)a] \end{vmatrix}} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & a \\ 1 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 1 & a & \cdots & x \end{vmatrix}} \\ = \frac{r_{i}-r_{1}}{i=2,\cdots,n} = [x+(n-1)a] \begin{vmatrix} 1 & a & \cdots & 0 \\ 0 & x-a & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x-a \end{vmatrix}}$$

 $[x + (n-1)a](x-a)^{n-1}$ 

$$D_{n} = \frac{r_{i}-r_{1}}{\stackrel{i}{=}2,\cdots,n} = \begin{bmatrix} x & a & a & \cdots & a \\ a-x & x-a & 0 & \cdots & 0 \\ a-x & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a-x & 0 & 0 & \cdots & x-a \end{bmatrix}$$

$$= [x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \frac{r_{i}-r_{1}}{i=2,\cdots,n}$$

$$\begin{vmatrix}
x & a & a & \cdots & a \\
a-x & x-a & 0 & \cdots & 0 \\
a-x & 0 & x-a & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
a-x & 0 & 0 & \cdots & x-a
\end{vmatrix}$$

$$= [x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \frac{r_{i}-r_{1}}{i=2,\cdots,n} = \begin{vmatrix} x & a & a & \cdots & a \\ a-x & x-a & 0 & \cdots & 0 \\ a-x & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a-x & 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$=$$
  $[x + (n-1)a](x-a)^{n-1}$ 

$$D_{n} = \frac{r_{i}-r_{1}}{i=2,\cdots,n} = \begin{vmatrix} x & a & a & \cdots & a \\ a-x & x-a & 0 & \cdots & 0 \\ a-x & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a-x & 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$= [x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \frac{r_{i}-r_{1}}{\stackrel{i}{=}2,\cdots,n} = \begin{vmatrix} x & a & a & \cdots & a \\ a-x & x-a & 0 & \cdots & 0 \\ a-x & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a-x & 0 & 0 & \cdots & x-a \end{vmatrix}$$

 $= [x + (n-1)a](x-a)^{n-1}$ 

$$D_{n} = \frac{r_{i}-r_{1}}{\stackrel{i}{=}2,\cdots,n} = \begin{vmatrix} x & a & a & \cdots & a \\ a-x & x-a & 0 & \cdots & 0 \\ a-x & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a-x & 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$= [x + (n-1)a](x-a)^{n-1}.$$

$$D_{n} = \frac{r_{i}-r_{1}}{i=2,\cdots,n} = \begin{vmatrix} x & a & a & \cdots & a \\ a-x & x-a & 0 & \cdots & 0 \\ a-x & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a-x & 0 & 0 & \cdots & x-a \end{vmatrix}$$

$$= [x + (n-1)a](x-a)^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1}$$

$$D_{n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x - a & 0 & \cdots & 0 \\ -1 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

- 若x ≠ a, 则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}} = \begin{bmatrix} 1 + \frac{a}{x - a}n & a & a & \dots & a \\ 0 & x - a & 0 & \dots & 0 \\ 0 & 0 & x - a & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & x - a & \dots \end{bmatrix}$$

$$[x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} = \frac{r_{i} - r_{1}}{i = 2, \cdots, n+1} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x - a & 0 & \cdots & 0 \\ -1 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

$$\frac{n}{n+1} = \begin{bmatrix}
1 & a & a & \cdots & a \\
-1 & x-a & 0 & \cdots & 0 \\
-1 & 0 & x-a & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
-1 & 0 & 0 & \cdots & x-a
\end{bmatrix}$$

- 若x ≠ a, 则

$$D_n = \frac{c_1 + \frac{1}{x - a}c_j}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}}$$

$$\begin{vmatrix}
1 + \frac{a}{x-a}n & a & a & \cdots & a \\
0 & x-a & 0 & \cdots & 0 \\
0 & 0 & x-a & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & x-a
\end{vmatrix}_{n+1}$$

$$[x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{\begin{array}{c} r_{i}-r_{1} \\ \hline i=2,\cdots,n+1 \end{array}} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x-a & 0 & \cdots & 0 \\ -1 & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x-a \end{vmatrix}_{n+1}$$

- 若x = a,则 $D_n = 0$ 。
- 若x≠a,则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}} = \begin{bmatrix} 1 + \frac{a}{x - a}n & a & a & \cdots & a \\ 0 & x - a & 0 & \cdots & 0 \\ 0 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & x - a \end{bmatrix}_{n+1}$$

$$[x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{r_{i}-r_{1}} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x-a & 0 & \cdots & 0 \\ -1 & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x-a \end{vmatrix}_{n+1}$$

- 若x≠a,则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}} = \begin{bmatrix} 1 + \frac{a}{x - a}n & a & a & \cdots & a \\ 0 & x - a & 0 & \cdots & 0 \\ 0 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & x - a \end{bmatrix}$$

$$[x + (n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{r_{i}-r_{1}} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x-a & 0 & \cdots & 0 \\ -1 & 0 & x-a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x-a \end{vmatrix}_{n+1}$$

- 若x ≠ a, 则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}} = \begin{bmatrix} 1 + \frac{a}{x - a}n & a & a & \cdots & a \\ 0 & x - a & 0 & \cdots & 0 \\ 0 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & x - a \end{bmatrix}_{n+1}$$

$$[x + (n-1)a](x-a)^{n-1}$$
.

$$D_n = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{r_i - r_1} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x - a & 0 & \cdots & 0 \\ -1 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

- 若x ≠ a, 则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}} = \begin{bmatrix} 1 + \frac{a}{x - a}n & a & a & \dots & a \\ 0 & x - a & 0 & \dots & 0 \\ 0 & 0 & x - a & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & x - a \end{bmatrix}_{n+1}$$

$$[x + (n-1)a](x-a)^{n-1}$$

### 方法三 (升阶法)

$$D_n = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{\begin{array}{c} r_i - r_1 \\ \hline i = 2, \cdots, n+1 \end{array}} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x - a & 0 & \cdots & 0 \\ -1 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

- 若x ≠ a, 则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\cdots,n+1}^{j=2,\cdots,n+1}} = \begin{vmatrix} 1 + \frac{a}{x - a}n & a & a & \cdots & a \\ 0 & x - a & 0 & \cdots & 0 \\ 0 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

$$[x + (n-1)a](x-a)^{n-1}$$

### 方法三 (升阶法)

$$D_n = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{\begin{array}{c} r_i - r_1 \\ \hline i = 2, \cdots, n+1 \end{array}} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x - a & 0 & \cdots & 0 \\ -1 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

- 若x ≠ a, 则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\cdots,n+1}^{j=2,\cdots,n+1}} = \begin{vmatrix} 1 + \frac{a}{x - a}n & a & a & \cdots & a \\ 0 & x - a & 0 & \cdots & 0 \\ 0 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

$$[x + (n-1)a](x-a)^{n-1}$$
.



### 方法三 (升阶法)

$$D_n = \begin{vmatrix} 1 & a & a & \cdots & a \\ 0 & x & a & \cdots & a \\ 0 & a & x & \cdots & a \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & a & a & \cdots & x \end{vmatrix}_{n+1} \xrightarrow{\underbrace{r_i - r_1}_{i=2,\cdots,n+1}} \begin{vmatrix} 1 & a & a & \cdots & a \\ -1 & x - a & 0 & \cdots & 0 \\ -1 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

- 若x ≠ a, 则

$$D_{n} = \frac{c_{1} + \frac{1}{x - a}c_{j}}{\sum_{j=2,\dots,n+1}^{j=2,\dots,n+1}} = \begin{vmatrix} 1 + \frac{a}{x - a}n & a & a & \cdots & a \\ 0 & x - a & 0 & \cdots & 0 \\ 0 & 0 & x - a & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & x - a \end{vmatrix}_{n+1}$$

 $[x + (n-1)a](x-a)^{n-1}$ .

$$D_{n} = \begin{vmatrix} x-a & a & \cdots & a \\ 0 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 0 & a & \cdots & x \end{vmatrix} + \begin{vmatrix} a & a & \cdots & a \\ a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x \end{vmatrix}$$

于是

$$\begin{cases}
D_n = (x-a)D_{n-1} + a(x-a)^{n-1} \\
(x-a)D_{n-1} = (x-a)^2D_{n-2} + a(x-a)^{n-1} \\
\dots \\
(x-a)^{n-4}D_4 = (x-a)^{n-3}D_3 + a(x-a)^{n-1} \\
(x-a)^{n-3}D_3 = (x-a)^{n-2}D_2 + a(x-a)^{n-1}
\end{cases}$$

$$D_n = (x-a)^{n-2}(x^2-a^2) + (n-2)a(x-a)^{n-1} = [x+(n-1)a](x-a)^{n-1}$$

$$D_n = \begin{vmatrix} x-a & a & \cdots & a \\ 0 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 0 & a & \cdots & x \end{vmatrix} + \begin{vmatrix} a & a & \cdots & a \\ a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x \end{vmatrix}$$

$$= (x-a)D_{n-1} + a(x-a)^{n-1}.$$

于是

$$\begin{cases}
D_n &= (x-a)D_{n-1} + a(x-a)^{n-1} \\
(x-a)D_{n-1} &= (x-a)^2D_{n-2} + a(x-a)^{n-1} \\
\dots \\
(x-a)^{n-4}D_4 &= (x-a)^{n-3}D_3 + a(x-a)^{n-1} \\
(x-a)^{n-3}D_3 &= (x-a)^{n-2}D_2 + a(x-a)^{n-1}
\end{cases}$$

$$D_n = (x-a)^{n-2}(x^2-a^2) + (n-2)a(x-a)^{n-1} = [x+(n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} x - a & a & \cdots & a \\ 0 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 0 & a & \cdots & x \end{vmatrix} + \begin{vmatrix} a & a & \cdots & a \\ a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x \end{vmatrix}$$
$$= (x - a)D_{n-1} + a(x - a)^{n-1}.$$

于是

$$\begin{cases}
D_n &= (x-a)D_{n-1} + a(x-a)^{n-1} \\
(x-a)D_{n-1} &= (x-a)^2 D_{n-2} + a(x-a)^{n-1} \\
\dots \\
(x-a)^{n-4}D_4 &= (x-a)^{n-3}D_3 + a(x-a)^{n-1} \\
(x-a)^{n-3}D_3 &= (x-a)^{n-2}D_2 + a(x-a)^{n-1}
\end{cases}$$

$$D_n = (x-a)^{n-2}(x^2-a^2) + (n-2)a(x-a)^{n-1} = [x+(n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} x - a & a & \cdots & a \\ 0 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 0 & a & \cdots & x \end{vmatrix} + \begin{vmatrix} a & a & \cdots & a \\ a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x \end{vmatrix}$$
$$= (x - a)D_{n-1} + a(x - a)^{n-1}.$$

于是

$$\begin{cases}
D_n &= (x-a)D_{n-1} + a(x-a)^{n-1} \\
(x-a)D_{n-1} &= (x-a)^2D_{n-2} + a(x-a)^{n-1} \\
\dots \\
(x-a)^{n-4}D_4 &= (x-a)^{n-3}D_3 + a(x-a)^{n-1} \\
(x-a)^{n-3}D_3 &= (x-a)^{n-2}D_2 + a(x-a)^{n-1}
\end{cases}$$

$$D_n = (x-a)^{n-2}(x^2-a^2) + (n-2)a(x-a)^{n-1} = [x+(n-1)a](x-a)^{n-1}$$

$$D_{n} = \begin{vmatrix} x - a & a & \cdots & a \\ 0 & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ 0 & a & \cdots & x \end{vmatrix} + \begin{vmatrix} a & a & \cdots & a \\ a & x & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x \end{vmatrix}$$
$$= (x - a)D_{n-1} + a(x - a)^{n-1}.$$

于是

$$\begin{cases}
D_n &= (x-a)D_{n-1} + a(x-a)^{n-1} \\
(x-a)D_{n-1} &= (x-a)^2D_{n-2} + a(x-a)^{n-1} \\
& \cdots \\
(x-a)^{n-4}D_4 &= (x-a)^{n-3}D_3 + a(x-a)^{n-1} \\
(x-a)^{n-3}D_3 &= (x-a)^{n-2}D_2 + a(x-a)^{n-1}
\end{cases}$$

因此

$$D_n = (x-a)^{n-2}(x^2-a^2) + (n-2)a(x-a)^{n-1} = [x+(n-1)a](x-a)^{n-1}$$

**→ロト→御→→**車→→車→ 車 め9で

•

$$\begin{vmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 0 \end{vmatrix} = (-1)^{n-1}(n-1)$$

$$\begin{vmatrix}
1 + \lambda & 1 & \cdots & 1 \\
1 & 1 + \lambda & \cdots & 1 \\
\vdots & \vdots & & \vdots \\
1 & 1 & \cdots & 1 + \lambda
\end{vmatrix} = (\lambda + n)\lambda^{n-1}$$

$$\begin{vmatrix} 1+\lambda & 1 & \cdots & 1 \\ 1 & 1+\lambda & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1+\lambda \end{vmatrix} = (\lambda+n)\lambda^{n-1}$$

$$\begin{vmatrix} 1 & a & \cdots & a \\ a & 1 & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & 1 \end{vmatrix} = [1 + (n-1)a](1-a)^{n-1}$$

$$\begin{vmatrix} 1+\lambda & 1 & \cdots & 1 \\ 1 & 1+\lambda & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1+\lambda \end{vmatrix} = (\lambda+n)\lambda^{n-1}$$

$$\begin{vmatrix} 1 & a & \cdots & a \\ a & 1 & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & 1 \end{vmatrix} = [1 + (n-1)a](1-a)^{n-1}$$

$$\begin{vmatrix} 1+\lambda & 1 & \cdots & 1 \\ 1 & 1+\lambda & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1+\lambda \end{vmatrix} = (\lambda+n)\lambda^{n-1}$$

$$\begin{vmatrix} 1 & a & \cdots & a \\ a & 1 & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & 1 \end{vmatrix} = [1 + (n-1)a](1-a)^{n-1}$$

$$\begin{vmatrix} 1+\lambda & 1 & \cdots & 1 \\ 1 & 1+\lambda & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1+\lambda \end{vmatrix} = (\lambda+n)\lambda^{n-1}$$

$$\begin{vmatrix} 1 & a & \cdots & a \\ a & 1 & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & 1 \end{vmatrix} = [1 + (n-1)a](1-a)^{n-1}$$

$$\begin{vmatrix} 1+\lambda & 1 & \cdots & 1 \\ 1 & 1+\lambda & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1+\lambda \end{vmatrix} = (\lambda+n)\lambda^{n-1}$$

### 升阶法适用于求形如

$$\begin{array}{ccccc} x_1 & a & \cdots & a \\ a & x_2 & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x_n \end{array}$$

或

的行列式。

$$\begin{vmatrix} x_1 & a & \cdots & a \\ a & x_2 & \cdots & a \\ \vdots & \vdots & & \vdots \\ a & a & \cdots & x_n \end{vmatrix} = \left(1 + \sum_{i=1}^n \frac{a}{x_i - a}\right) \prod_{i=1}^n (x_i - a)$$

$$\begin{vmatrix} x_1 & a_1 & \cdots & a_n \\ a_1 & x_2 & \cdots & a_n \\ \vdots & \vdots & & \vdots \\ a_{i-1} & a_{i-1} & \cdots & a_{i-1} \end{vmatrix} = \left(1 + \sum_{i=1}^n \frac{a_i}{x_i - a_i}\right) \prod_{i=1}^n (x_i - a_i)$$

### 常见形式

$$\begin{vmatrix} 1+a & 1 & \cdots & 1 \\ 2 & 2+a & \cdots & 2 \\ \vdots & \vdots & & \vdots \\ n & n & \cdots & n+a \end{vmatrix} = \left[a + \frac{n(n+1)}{2}\right] a^{n-1}$$

或

$$\begin{vmatrix} a_1 + b & a_1 & \cdots & a_n \\ a_1 & a_2 + b & \cdots & a_n \\ \vdots & \vdots & & \vdots \\ a_1 & a_2 & \cdots & a_n + b \end{vmatrix} = b^{n-1} \left( \sum_{i=1}^n a_i + b \right)$$

### 常见形式

$$\begin{vmatrix} 1+a & 1 & \cdots & 1 \\ 2 & 2+a & \cdots & 2 \\ \vdots & \vdots & & \vdots \\ n & n & \cdots & n+a \end{vmatrix} = \left[a + \frac{n(n+1)}{2}\right] a^{n-1}$$

或

$$\begin{vmatrix} a_1 + b & a_1 & \cdots & a_n \\ a_1 & a_2 + b & \cdots & a_n \\ \vdots & \vdots & & \vdots \\ a_1 & a_2 & \cdots & a_n + b \end{vmatrix} = b^{n-1} (\sum_{i=1}^n a_i + b)$$

### 常见形式

$$\begin{vmatrix} 1+a & 1 & \cdots & 1 \\ 2 & 2+a & \cdots & 2 \\ \vdots & \vdots & & \vdots \\ n & n & \cdots & n+a \end{vmatrix} = \left[a + \frac{n(n+1)}{2}\right] a^{n-1}$$

或

$$\begin{vmatrix} a_1 + b & a_1 & \cdots & a_n \\ a_1 & a_2 + b & \cdots & a_n \\ \vdots & \vdots & & \vdots \\ a_1 & a_2 & \cdots & a_n + b \end{vmatrix} = b^{n-1} (\sum_{i=1}^n a_i + b)$$

设

$$D = \begin{vmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \cdots & a_{kk} \\ c_{11} & \cdots & c_{1k} & b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots & \vdots & & \vdots \\ c_{n1} & \cdots & c_{nk} & b_{n1} & \cdots & b_{nn} \end{vmatrix},$$

$$D_1 = det(a_{ij}) = \begin{vmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \cdots & a_{kk} \end{vmatrix},$$

$$D_2 = det(b_{ij}) = \begin{vmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{n1} & \cdots & b_{nn} \end{vmatrix}.$$

证明:  $D = D_1D_2$ 

$$D_1 = \begin{vmatrix} p_{11} \\ \vdots & \ddots \\ p_{k1} & \cdots & p_{kk} \end{vmatrix} = p_{11} \cdots p_{kk}.$$

对 $D_2$ 做运算 $c_i + \lambda c_j$ 将它转化成下三角行列式,设为

$$D_2=\left|egin{array}{ccc} q_{11} & \cdots & q_{1n} \\ & \ddots & dots \\ & & q_{nn} \end{array}
ight|=q_{11}\cdots q_{nn}.$$

于是,对D的前k行做运算 $r_i + \lambda r_i$ ,对其后n列做运算 $c_i + \lambda c_i$ ,把D转化为

$$D = \begin{vmatrix} p_{11} & & & & & & \\ \vdots & \ddots & & & & & \\ p_{k1} & \cdots & p_{kk} & & & & \\ c_{11} & \cdots & c_{1k} & q_{11} & & & \\ \vdots & & \vdots & \vdots & \ddots & & \\ c_{n1} & \cdots & c_{nk} & q_{n1} & \cdots & q_{nn} \end{vmatrix}$$

 $oldsymbol{\&} D = oldsymbol{p}_{11} \cdots oldsymbol{p}_{kk} oldsymbol{q}_{11} \cdots oldsymbol{q}_{nn} = D_1 D_2$ 

$$D_1 = \begin{vmatrix} p_{11} \\ \vdots & \ddots \\ p_{k1} & \cdots & p_{kk} \end{vmatrix} = p_{11} \cdots p_{kk}.$$

对 $D_2$ 做运算 $c_i + \lambda c_i$ 将它转化成下三角行列式,设为

$$D_2 = \left| egin{array}{ccc} q_{11} & \cdots & q_{1n} \\ & \ddots & dots \\ & q_{nn} \end{array} 
ight| = q_{11} \cdots q_{nn}.$$

于是,对D的前k行做运算 $r_i + \lambda r_i$ ,对其后n列做运算 $c_i + \lambda c_i$ ,把D转化为

$$D = \begin{vmatrix} p_{11} \\ \vdots & \ddots \\ p_{k1} & \cdots & p_{kk} \\ c_{11} & \cdots & c_{1k} & q_{11} \\ \vdots & & \vdots & \vdots & \ddots \\ c_{n1} & \cdots & c_{nk} & q_{n1} & \cdots & q_{nn} \end{vmatrix}$$

故 $D=p_{11}\cdots p_{kk}q_{11}\cdots q_{nn}=D_1D_2$ 

- 4 ロ ト 4 個 ト 4 差 ト 4 差 ト - 差 - かくで

对 $D_2$ 做运算 $c_i + \lambda c_i$ 将它转化成下三角行列式,设为

$$D_2 = \left| egin{array}{ccc} q_{11} & \cdots & q_{1n} \\ & \ddots & drawnows \\ & & q_{nn} \end{array} 
ight| = q_{11} \cdots q_{nn}.$$

于是,对D的前k行做运算 $r_i + \lambda r_j$ ,对其后n列做运算 $c_i + \lambda c_j$ ,把D转化为

$$D = \begin{pmatrix} p_{11} \\ \vdots & \ddots \\ p_{k1} & \cdots & p_{kk} \\ c_{11} & \cdots & c_{1k} & q_{11} \\ \vdots & & \vdots & \vdots & \ddots \\ c_{n1} & \cdots & c_{nk} & q_{n1} & \cdots & q_{nn} \end{pmatrix}$$

族 $D=p_{11}\cdots p_{kk}q_{11}\cdots q_{nn}=D_1D_2$ 

- 4 ロ ト 4 個 ト 4 種 ト 4 種 ト - 種 - 夕 Q (

$$D_1 = \begin{vmatrix} p_{11} \\ \vdots & \ddots \\ p_{k1} & \cdots & p_{kk} \end{vmatrix} = p_{11} \cdots p_{kk}.$$

对 $D_2$ 做运算 $c_i + \lambda c_i$ 将它转化成下三角行列式,设为

$$D_2 = \left| egin{array}{ccc} q_{11} & \cdots & q_{1n} \\ & \ddots & drawnows \\ & & q_{nn} \end{array} 
ight| = q_{11} \cdots q_{nn}.$$

于是,对D的前k行做运算 $r_i + \lambda r_j$ ,对其后n列做运算 $c_i + \lambda c_j$ ,把D转化为

$$D = \begin{vmatrix} p_{11} \\ \vdots & \ddots & & & \\ p_{k1} & \cdots & p_{kk} \\ c_{11} & \cdots & c_{1k} & q_{11} \\ \vdots & & \vdots & \vdots & \ddots \\ c_{n1} & \cdots & c_{nk} & q_{n1} & \cdots & q_{nn} \end{vmatrix}$$

故 $D = p_{11} \cdots p_{kk} q_{11} \cdots q_{nn} = D_1 D_2.$ 

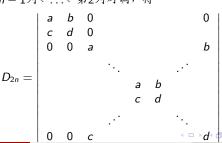
计算2n阶行列式

解: 把 $D_{2n}$ 中的第2n行依次与第2n-1行、...、第2行对调(共2n-2次相邻对换),在把第2n列依次与第2n-1列、...、第2列对调,得



计算2n阶行列式

解: 把 $D_{2n}$ 中的第2n行依次与第2n-1行、...、第2行对调(共2n-2次相邻对换),在把第2n列依次与第2n-1列、...、第2列对调,得



故

$$D_{2n} = D_2 D_{2(n-1)}$$

$$= (ad - bc) D_{2(n-1)}$$

$$= (ad - bc)^2 D_{2(n-2)}$$

$$= \cdots$$

$$= (ad - bc)^{n-1} D_2$$

$$= (ad - bc)^n.$$

证明范德蒙德(Vandermonde)行列式

$$D_n = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \vdots & \vdots & & \vdots \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \end{vmatrix} = \prod_{n \geq i > j \geq 1} (x_i - x_j).$$

证明: 用数学归纳法证明。当n = 2时,

$$D_2 = \begin{vmatrix} 1 & 1 \\ x_1 & x_2 \end{vmatrix} = x_2 - x_1 = \prod_{2 \ge i > j \ge 1} (x_i - x_j),$$

结论成立。



证明范德蒙德(Vandermonde)行列式

$$D_n = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \vdots & \vdots & & \vdots \\ x_1^{n-1} & x_2^{n-1} & \cdots & x_n^{n-1} \end{vmatrix} = \prod_{n \ge i > j \ge 1} (x_i - x_j).$$

证明: 用数学归纳法证明。当n=2时,

$$D_2 = \begin{vmatrix} 1 & 1 \\ x_1 & x_2 \end{vmatrix} = x_2 - x_1 = \prod_{2 \ge i > j \ge 1} (x_i - x_j),$$

结论成立。



现假设结论对n-1阶范德蒙德行列式成立,以下证明结论对n阶范德蒙德行列式也成立。

$$D_{n} = \frac{r_{l} - x_{1} r_{l-1}}{\sum_{i=n, \dots, 2}^{l}} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & x_{2} - x_{1} & x_{3} - x_{1} & \cdots & x_{n} - x_{1} \\ 0 & x_{2}(x_{2} - x_{1}) & x_{3}(x_{3} - x_{1}) & \cdots & x_{n}(x_{n} - x_{1}) \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & x_{2}^{n-2}(x_{2} - x_{1}) & x_{3}^{n-2}(x_{3} - x_{1}) & \cdots & x_{n}^{n-2}(x_{n} - x_{1}) \end{vmatrix}$$

按第1列展开,并把每列的公因子 $(x_i - x_1)$ 提出,就有

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_2 & x_3 & \cdots & x_n \\ \vdots & \vdots & & \vdots \\ x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} \end{vmatrix}$$

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \prod_{n \ge i > j \ge 2} (x_i - x_j) = \prod_{n \ge i > j \ge 1} (x_i - x_j).$$

现假设结论对n-1阶范德蒙德行列式成立,以下证明结论对n阶范德蒙德行列式也成立。

$$D_{n} \stackrel{\frac{r_{i}-x_{1}r_{i-1}}{i=n,\cdots,2}}{=} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & x_{2}-x_{1} & x_{3}-x_{1} & \cdots & x_{n}-x_{1} \\ 0 & x_{2}(x_{2}-x_{1}) & x_{3}(x_{3}-x_{1}) & \cdots & x_{n}(x_{n}-x_{1}) \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & x_{2}^{n-2}(x_{2}-x_{1}) & x_{3}^{n-2}(x_{3}-x_{1}) & \cdots & x_{n}^{n-2}(x_{n}-x_{1}) \end{vmatrix}$$

按第1列展开,并把每列的公因子 $(x_i - x_1)$ 提出,就有

$$D_{n} = (x_{2} - x_{1})(x_{3} - x_{1}) \cdots (x_{n} - x_{1}) \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_{2} & x_{3} & \cdots & x_{n} \\ \vdots & \vdots & & \vdots \\ x_{2}^{n-2} & x_{3}^{n-2} & \cdots & x_{n}^{n-2} \end{vmatrix}$$

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \prod_{n \ge i > j \ge 2} (x_i - x_j) = \prod_{n \ge i > j \ge 1} (x_i - x_j).$$

现假设结论对n-1阶范德蒙德行列式成立,以下证明结论对n阶范德蒙德行列式也成立。

$$D_{n} = \frac{r_{i} - x_{1}r_{i-1}}{\sum_{i=n,\dots,2}^{i}} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & x_{2} - x_{1} & x_{3} - x_{1} & \cdots & x_{n} - x_{1} \\ 0 & x_{2}(x_{2} - x_{1}) & x_{3}(x_{3} - x_{1}) & \cdots & x_{n}(x_{n} - x_{1}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & x_{2}^{n-2}(x_{2} - x_{1}) & x_{3}^{n-2}(x_{3} - x_{1}) & \cdots & x_{n}^{n-2}(x_{n} - x_{1}) \end{vmatrix}$$

按第1列展开,并把每列的公因子 $(x_i - x_1)$ 提出,就有

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_2 & x_3 & \cdots & x_n \\ \vdots & \vdots & & \vdots \\ x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} \end{vmatrix}$$

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \prod_{n \ge i > j \ge 2} (x_i - x_j) = \prod_{n \ge i > j \ge 1} (x_i - x_j).$$

现假设结论对n — 1阶范德蒙德行列式成立,以下证明结论对n阶范德蒙德行列式也成立。

$$D_{n} = \frac{\frac{r_{i} - x_{1} r_{i-1}}{i = n, \dots, 2}}{\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & x_{2} - x_{1} & x_{3} - x_{1} & \dots & x_{n} - x_{1} \\ 0 & x_{2}(x_{2} - x_{1}) & x_{3}(x_{3} - x_{1}) & \dots & x_{n}(x_{n} - x_{1}) \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & x_{2}^{n-2}(x_{2} - x_{1}) & x_{3}^{n-2}(x_{3} - x_{1}) & \dots & x_{n}^{n-2}(x_{n} - x_{1}) \end{vmatrix}}$$

按第1列展开,并把每列的公因子 $(x_i - x_1)$ 提出,就有

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_2 & x_3 & \cdots & x_n \\ \vdots & \vdots & & \vdots \\ x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} \end{vmatrix}$$

$$D_n = (x_2 - x_1)(x_3 - x_1) \cdots (x_n - x_1) \prod_{n \ge i > j \ge 2} (x_i - x_j) = \prod_{n \ge i > j \ge 1} (x_i - x_j).$$

设a,b,c为互不相同的实数,证明:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = 0$$

的充要条件是a+b+c=0.

解:考察范德蒙德行列式

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & y \\ a^2 & b^2 & c^2 & y^2 \\ a^3 & b^3 & c^3 & y^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a-y)(b-y)(c-y)$$

注意到行列式D可看成是关于y的多项式,比较包含y2的项:

$$\cdots - \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} y^2 + \cdots = \cdots - (a-b)(a-c)(b-c)(a+b+c)y^2 + \cdots$$

设a, b, c为互不相同的实数,证明:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = 0$$

的充要条件是a+b+c=0.

解:考察范德蒙德行列式

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & y \\ a^2 & b^2 & c^2 & y^2 \\ a^3 & b^3 & c^3 & y^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a-y)(b-y)(c-y)$$

注意到行列式D可看成是关于y的多项式,比较包含y<sup>2</sup>的项:

$$\cdots - \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} y^2 + \cdots = \cdots - (a-b)(a-c)(b-c)(a+b+c)y^2 + \cdots$$

解: (续) 于是

$$(a-b)(a-c)(b-c)(a+b+c) = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = 0$$

而a, b, c互不相同,故a+b+c=0.



计算三对角行列式

#### 解 对Dn按第一行展开

$$D_{n} = aD_{n-1} + (-1)^{1+2}b \begin{vmatrix} c & b & & & & & & & & & & \\ 0 & a & b & & & & & & & \\ 0 & c & a & b & & & & & & \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ 0 & 0 & \cdots & c & a & b & & & & \\ 0 & 0 & \cdots & 0 & c & a & b \end{vmatrix} = aD_{n-1} - bcD_{n-2}$$

$$D_1 = a$$
,  $D_2 = a^2 - b$ 

计算三对角行列式

#### 解 对Dn按第一行展开

$$D_{n} = aD_{n-1} + (-1)^{1+2}b \begin{vmatrix} c & b & & & & & & & & & & \\ 0 & a & b & & & & & & & \\ 0 & c & a & b & & & & & & \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ 0 & 0 & \cdots & c & a & b & & & & \\ 0 & 0 & \cdots & 0 & c & a & b \end{vmatrix} = aD_{n-1} - bcD_{n-2}$$

$$D_1 = a, \quad D_2 = a^2 - b$$



计算三对角行列式

#### 解 对Dn按第一行展开

$$D_{n} = aD_{n-1} + (-1)^{1+2}b \begin{vmatrix} c & b & & & & & & & & & & \\ 0 & a & b & & & & & & & \\ 0 & c & a & b & & & & & & \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ 0 & 0 & \cdots & c & a & b & & & \\ 0 & 0 & \cdots & 0 & c & a & b \end{vmatrix} = aD_{n-1} - bcD_{n-2}$$

$$D_1 = a, \quad D_2 = a^2 - b$$



计算三对角行列式

## 解 对Dn按第一行展开

$$D_{n} = aD_{n-1} + (-1)^{1+2}b \begin{vmatrix} c & b & & & & & & & & & & \\ 0 & a & b & & & & & & & \\ 0 & c & a & b & & & & & & \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ 0 & 0 & \cdots & c & a & b & & & \\ 0 & 0 & \cdots & 0 & c & a & b \end{vmatrix} = aD_{n-1} - bcD_{n-2}$$

$$D_1 = a$$
,  $D_2 = a^2 - bc$ 

# $D_n = aD_{n-1} - bcD_{n-2}$

# 改写成

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a$$
,  $kl=bc$ .

$$\begin{cases} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a - k)a - kl = Ia - Ik = I^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2} \Delta_2 = I^2,$$

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

而
$$D_1 = a = k + l$$
,故

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a$$
,  $kl=bc$ .

$$\phi \Delta_n = D_n - kD_{n-1}, 它满足$$

$$\begin{cases} \Delta_n = l \Delta_{n-1}, \\ \Delta_2 = D_2 - k D_1 = a^2 - bc - ka = (a - k)a - kl = la - lk = l^2 \end{cases}$$

由此可知

$$\Delta_n = I^{n-2} \Delta_2 = I^2,$$

$$D_{n} = l^{n} + kD_{n-1} = l^{n} + k(l^{n-1} + kD_{n-2}) = l^{n} + kl^{n-1} + k^{2}D_{n-2}$$

$$= l^{n} + kl^{n-1} + k^{2}(l^{n-2} + kD_{n-3}) = l^{n} + kl^{n-1} + k^{2}l^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = l^{n} + kl^{n-1} + k^{2}l^{n-2} + \cdots + k^{n-2}l^{2} + k^{n-1}D_{1}$$

而
$$D_1 = a = k + l$$
,故

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k^2I^{n-2}I^2 + k^2I^{n-$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k + l = a$$
,  $kl = bc$ .

$$\phi \Delta_n = D_n - kD_{n-1}, 它满足$$

$$\begin{cases} \Delta_n = l \Delta_{n-1}, \\ \Delta_2 = D_2 - k D_1 = a^2 - bc - ka = (a - k)a - kl = la - lk = l^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2} \Delta_2 = I^2,$$

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k + l = a$$
,  $kl = bc$ .

$$\begin{cases} \Delta_n = I \Delta_{n-1}, \\ \Delta_2 = D_2 - k D_1 = a^2 - bc - ka = (a - k)a - kl = la - lk = l^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2} \Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k^2I^{n-2}I^2 + k^2I^{n-$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a$$
,  $kl=bc$ .

令 
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kl = la - lk = l^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2} \Delta_2 = I^2,$$

即

$$D_{n} = l^{n} + kD_{n-1} = l^{n} + k(l^{n-1} + kD_{n-2}) = l^{n} + kl^{n-1} + k^{2}D_{n-2}$$

$$= l^{n} + kl^{n-1} + k^{2}(l^{n-2} + kD_{n-3}) = l^{n} + kl^{n-1} + k^{2}l^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = l^{n} + kl^{n-1} + k^{2}l^{n-2} + \cdots + k^{n-2}l^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a$$
,  $kl=bc$ .

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足
$$\begin{cases} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-1}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足
$$\begin{cases} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k^n$$



$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足
$$\begin{cases} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k^n$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$



$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \dots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \dots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k^n$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足
$$\begin{cases} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kl = la - lk = l^2. \end{cases}$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

即

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k'$$

$$D_n = aD_{n-1} - bcD_{n-2}$$

$$D_n - kD_{n-1} = I(D_{n-1} - kD_{n-2})$$

这里

$$k+l=a, \quad kl=bc.$$

令
$$\Delta_n = D_n - kD_{n-1}$$
,它满足 
$$\left\{ \begin{array}{l} \Delta_n = I\Delta_{n-1}, \\ \Delta_2 = D_2 - kD_1 = a^2 - bc - ka = (a-k)a - kI = Ia - Ik = I^2. \end{array} \right.$$

由此可知

$$\Delta_n = I^{n-2}\Delta_2 = I^2,$$

$$D_{n} = I^{n} + kD_{n-1} = I^{n} + k(I^{n-1} + kD_{n-2}) = I^{n} + kI^{n-1} + k^{2}D_{n-2}$$

$$= I^{n} + kI^{n-1} + k^{2}(I^{n-2} + kD_{n-3}) = I^{n} + kI^{n-1} + k^{2}I^{n-2} + k^{3}D_{n-3}$$

$$= \cdots = I^{n} + kI^{n-1} + k^{2}I^{n-2} + \cdots + k^{n-2}I^{2} + k^{n-1}D_{1}$$

$$D_n = I^n + kI^{n-1} + k^2I^{n-2} + \dots + k^{n-2}I^2 + k^{n-1}I + k^n$$



- 1 行列式简介
- 2 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

#### 考察n元一次方程组

$$\begin{cases}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\
\dots \\
a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n.
\end{cases}$$
(6)

与二、三元线性方程组相类似,它的解可以用n阶行列式表示。



#### 克莱姆法则

如果线性方程组(6)的系数行列式不等于0,即

$$D = \left| \begin{array}{ccc} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{array} \right| \neq 0$$

则方程组(6)存在唯一解

$$x_1 = \frac{D_1}{D}, \ x_2 = \frac{D_2}{D}, \ \cdots, \ x_n = \frac{D_n}{D},$$

其中

第j列

证明: 先证存在性:  $将x_i = \frac{D_i}{D}$ 代入第i个方程,则有

$$a_{i1}x_1 + \cdots + a_{ii}x_i + \cdots + a_{in}x_n$$

$$= \frac{1}{D}(a_{i1}D_1 + \dots + a_{ii}D_i + \dots + a_{in}D_n)$$

$$= \frac{1}{D}[a_{i1}(b_1A_{11} + \dots + b_nA_{n1}) + \dots + a_{ii}(b_1A_{1i} + \dots + b_nA_{ni}) + \dots + a_{in}(b_1A_{1n} + \dots + b_nA_{nn})]$$

$$= \frac{1}{D}[b_1(a_{i1}A_{11} + a_{i2}A_{12} \dots + a_{in}A_{1n}) + \dots + b_i(a_{i1}A_{i1} + a_{i2}A_{i2} \dots + a_{in}A_{in}) + \dots + b_n(a_{i1}A_{n1} + a_{i2}A_{n2} \dots + a_{in}A_{nn})]$$

$$=\frac{1}{D}b_iD=b_i.$$



证明: 先证存在性: 将 $x_i = \frac{D_i}{D_i}$ 代入第i个方程,则有

$$a_{i1}x_{1} + \dots + a_{ii}x_{i} + \dots + a_{in}x_{n}$$

$$= \frac{1}{D}(a_{i1}D_{1} + \dots + a_{ii}D_{i} + \dots + a_{in}D_{n})$$

$$= \frac{1}{D}[a_{i1}(b_{1}A_{11} + \dots + b_{n}A_{n1}) + \dots + a_{ii}(b_{1}A_{1i} + \dots + b_{n}A_{ni}) + \dots + a_{in}(b_{1}A_{1n} + \dots + b_{n}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{11} + a_{i2}A_{12} + \dots + a_{in}A_{1n}) + \dots + b_{i}(a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}) + \dots + b_{i}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

证明: 先证存在性: 
$$将x_i = \frac{D_i}{D_i}$$
代入第 $i$ 个方程,则有

$$a_{i1}x_{1} + \dots + a_{ii}x_{i} + \dots + a_{in}x_{n}$$

$$= \frac{1}{D}(a_{i1}D_{1} + \dots + a_{ii}D_{i} + \dots + a_{in}D_{n})$$

$$= \frac{1}{D}[a_{i1}(b_{1}A_{11} + \dots + b_{n}A_{n1}) + \dots + a_{ii}(b_{1}A_{1i} + \dots + b_{n}A_{ni}) + \dots + a_{in}(b_{1}A_{1n} + \dots + b_{n}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{11} + a_{i2}A_{12} + \dots + a_{in}A_{nn})]$$

$$+ \dots + b_{n}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

$$+ \dots + b_{n}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

证明: 先证存在性: 
$$将x_i = \frac{D_i}{D_i}$$
代入第 $i$ 个方程,则有

$$a_{i1}x_{1} + \dots + a_{ii}x_{i} + \dots + a_{in}x_{n}$$

$$= \frac{1}{D}(a_{i1}D_{1} + \dots + a_{ii}D_{i} + \dots + a_{in}D_{n})$$

$$= \frac{1}{D}[a_{i1}(b_{1}A_{11} + \dots + b_{n}A_{n1}) + \dots + a_{ii}(b_{1}A_{1i} + \dots + b_{n}A_{ni}) + \dots + a_{in}(b_{1}A_{1n} + \dots + b_{n}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{11} + a_{i2}A_{12} + \dots + a_{in}A_{1n}) + \dots + b_{i}(a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{nn})]$$

$$+ \dots + b_{n}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

$$1 + D = b$$

证明: 先证存在性: 
$$将x_i = \frac{D_i}{D_i}$$
代入第 $i$ 个方程,则有

$$a_{i1}x_{1} + \cdots + a_{ii}x_{i} + \cdots + a_{in}x_{n}$$

$$= \frac{1}{D}(a_{i1}D_{1} + \cdots + a_{ii}D_{i} + \cdots + a_{in}D_{n})$$

$$= \frac{1}{D}[a_{i1}(b_{1}A_{11} + \cdots + b_{n}A_{n1}) + \cdots + a_{ii}(b_{1}A_{1i} + \cdots + b_{n}A_{ni}) + \cdots + a_{in}(b_{1}A_{1n} + \cdots + b_{n}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{11} + a_{i2}A_{12} + \cdots + a_{in}A_{1n}) + \cdots + b_{i}(a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in})]$$

$$+ \cdots + b_{n}(a_{i1}A_{n1} + a_{i2}A_{n2} + \cdots + a_{in}A_{nn})]$$

证明: 先证存在性: 
$$将x_i = \frac{D_i}{D_i}$$
代入第 $i$ 个方程,则有

$$a_{i1}x_{1} + \cdots + a_{ii}x_{i} + \cdots + a_{in}x_{n}$$

$$= \frac{1}{D}(a_{i1}D_{1} + \cdots + a_{ii}D_{i} + \cdots + a_{in}D_{n})$$

$$= \frac{1}{D}[a_{i1}(b_{1}A_{11} + \cdots + b_{n}A_{n1}) + \cdots + a_{ii}(b_{1}A_{1i} + \cdots + b_{n}A_{ni}) + \cdots + a_{in}(b_{1}A_{1n} + \cdots + b_{n}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{11} + a_{i2}A_{12} + \cdots + a_{in}A_{1n}) + \cdots + b_{i}(a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in}) + \cdots + b_{n}(a_{i1}A_{n1} + a_{i2}A_{n2} + \cdots + a_{in}A_{nn})]$$

证明: 先证存在性: 
$$将x_i = \frac{D_i}{D_i}$$
代入第 $i$ 个方程,则有

$$a_{i1}x_{1} + \dots + a_{ii}x_{i} + \dots + a_{in}x_{n}$$

$$= \frac{1}{D}(a_{i1}D_{1} + \dots + a_{ii}D_{i} + \dots + a_{in}D_{n})$$

$$= \frac{1}{D}[a_{i1}(b_{1}A_{11} + \dots + b_{n}A_{n1}) + \dots + a_{ii}(b_{1}A_{1i} + \dots + b_{n}A_{ni}) + \dots + a_{in}(b_{1}A_{1n} + \dots + b_{n}A_{nn})]$$

$$= \frac{1}{D}[b_{1}(a_{i1}A_{11} + a_{i2}A_{12} + \dots + a_{in}A_{1n}) + \dots + b_{i}(a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in}) + \dots + b_{n}(a_{i1}A_{n1} + a_{i2}A_{n2} + \dots + a_{in}A_{nn})]$$

$$= \frac{1}{D}[b_{i}D = b_{i}.$$

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ ,以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$= \begin{vmatrix} a_{11}y_1 & a_{12} & \cdots & a_{1n} \\ a_{21}y_1 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_1 & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_k & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_k & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_k & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n-2} & a_{2n} & \cdots & a_{2n} \end{vmatrix} = D_1$$

所以 $y_1=D_1/D$ 。同理可证 $y_i=D_i/D, i=2,\cdots,n$ 。

 $y_1D$ 

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$= \begin{vmatrix} a_{11}y_1 & a_{12} & \cdots & a_{1n} \\ a_{21}y_1 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_1 & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_k & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_k & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_k & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_n & a_{nn} & a_{nn} & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_n & a_{nn} & a_{nn} & a_{nn} \end{vmatrix}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ , $i = 2, \dots, n$ 。

 $y_1D$ 

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$= \begin{vmatrix} a_{11}y_1 & a_{12} & \cdots & a_{1n} \\ a_{21}y_1 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_1 & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_k & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_k & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_k & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D_1$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ , $i = 2, \dots, n$ 。

→□▶→□▶→□▶→□▶ □ ○○○○

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\frac{\sum_{k=1}^{n} a_{1k}y_{k}}{\sum_{k=1}^{n} a_{2k}y_{k}} a_{22} \cdots a_{2n}$$

$$\vdots & \vdots & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} \cdots a_{nn}$$

$$b_{1} & a_{12} \cdots & a_{1n} \\ b_{2} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \end{vmatrix} = D_{1}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ , $i = 2, \dots, n$ 。

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\frac{c_{1}+y_{2}c_{2}+\cdots+y_{n}c_{n}}{\sum_{k=1}^{n}a_{1k}y_{k}} \begin{vmatrix} a_{12} & \cdots & a_{1n} \\ b_{k=1} & a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n}a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \end{vmatrix} = D_{1}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ ,  $i = 2, \dots, n$ 。

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_{k} & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \end{vmatrix}$$

$$= b_{1} \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \end{vmatrix}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D, i = 2, \cdots, n$ 。

4□ > 4□ > 4□ > 4 = > = = 99

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_{k} & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n} & a_{n} & a_{n} & a_{n} \end{vmatrix} = D_{1}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ ,  $i = 2, \dots, n$ 。

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_{k} & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D_{1}$$

$$= \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

所以 $y_1=D_1/D$ 。同理可证 $y_i=D_i/D, i=2,\cdots,n$ 。

- **↓**ロト **↓**┛ト **↓** 目 ト ◆ き → りへで

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_{k} & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D_{1}$$

$$\vdots & \vdots & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ , $i = 2, \dots, n$ 。

→□▶→□▶→重▶→重 りへ⊙

张晓平

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_{k} & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D_{1}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ , $i = 2, \dots, n$ 。

◆□▶ ◆□▶ ◆豊▶ ◆豊▶ ・豊 める◆

张晓平

再证唯一性:设还有一组解 $y_i$ ,  $i=1,2,\cdots,n$ , 以下证明 $y_i=D_i/D$ 。 现构造一个新行列式

$$y_{1}D = \begin{vmatrix} a_{11}y_{1} & a_{12} & \cdots & a_{1n} \\ a_{21}y_{1} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}y_{1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} \sum_{k=1}^{n} a_{1k}y_{k} & a_{12} & \cdots & a_{1n} \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ \sum_{k=1}^{n} a_{2k}y_{k} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = D_{1}$$

所以 $y_1 = D_1/D$ 。同理可证 $y_i = D_i/D$ ,  $i = 2, \dots, n$ 。

◆ロト ◆個ト ◆差ト ◆差ト 差 めなぐ

张晓平

线性代数

例

$$\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8, \\ x_1 - 3x_2 - 6x_4 = 9, \\ x_2 - x_3 + 2x_4 = -5, \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0. \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 & -5 & 1 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 1 & 4 & -7 & 6 \end{vmatrix} \xrightarrow{r_1 - 2r_2} \begin{vmatrix} 0 & 7 & -5 & 13 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 0 & 7 & -7 & 12 \end{vmatrix}$$
$$= - \begin{vmatrix} 7 & -5 & 13 \\ 2 & -1 & 2 \\ 7 & -7 & 12 \end{vmatrix} \xrightarrow{c_1 + 2c_2} - \begin{vmatrix} -3 & -5 & 3 \\ 0 & -1 & 0 \\ -7 & -7 & -2 \end{vmatrix}$$
$$= \begin{vmatrix} -3 & 3 \\ -7 & -2 \end{vmatrix} = 27.$$

例

$$\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8, \\ x_1 - 3x_2 - 6x_4 = 9, \\ x_2 - x_3 + 2x_4 = -5, \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0. \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 & -5 & 1 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 1 & 4 & -7 & 6 \end{vmatrix} = \frac{r_1 - 2r_2}{r_4 - r_2} \begin{vmatrix} 0 & 7 & -5 & 13 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 0 & 7 & -7 & 12 \end{vmatrix}$$
$$= - \begin{vmatrix} 7 & -5 & 13 \\ 2 & -1 & 2 \\ 7 & -7 & 12 \end{vmatrix} = \frac{c_1 + 2c_2}{c_3 + 2c_2} - \begin{vmatrix} -3 & -5 & 3 \\ 0 & -1 & 0 \\ -7 & -7 & -2 \end{vmatrix}$$
$$= \begin{vmatrix} -3 & 3 \\ -7 & -2 \end{vmatrix} = 27.$$

解: (续)

$$D_{1} = \begin{vmatrix} 8 & 1 & -5 & 1 \\ 9 & -3 & 0 & -6 \\ -5 & 2 & -1 & 2 \\ 0 & 4 & -7 & 6 \end{vmatrix} = 81, \quad D_{2} = \begin{vmatrix} 2 & 8 & -5 & 1 \\ 1 & 9 & 0 & -6 \\ 0 & -5 & -1 & 2 \\ 1 & 0 & -7 & 6 \end{vmatrix} = -108,$$

$$D_{3} = \begin{vmatrix} 2 & 1 & 8 & 1 \\ 1 & -3 & 9 & -6 \\ 0 & 2 & -5 & 2 \\ 1 & 4 & 0 & 6 \end{vmatrix} = -27, \quad D_{4} = \begin{vmatrix} 2 & 1 & -5 & 8 \\ 1 & -3 & 0 & 9 \\ 0 & 2 & -1 & -5 \\ 1 & 4 & 7 & 0 \end{vmatrix} = 27$$

于是得

$$x_1 = \frac{D_1}{D} = 3$$
,  $x_2 = \frac{D_2}{D} = -4$ ,  $x_3 = \frac{D_3}{D} = -1$ ,  $x_4 = \frac{D_4}{D} = 1$ .



例

设曲线 $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ 通过四点(1,3),(2,4),(3,3),(4,-3), 求系数 $a_0,a_1,a_2,a_3$ 。

解:依题意可得线性方程组

$$\begin{cases} a_0 + a_1 + a_2 + a_3 = 3, \\ a_0 + 2a_1 + 4a_2 + 8a_3 = 4, \\ a_0 + 3a_1 + 9a_3 + 27a_3 = 3, \\ a_0 + 4a_1 + 16a_4 + 64a_3 = 3, \end{cases}$$

其系数行列式为

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{vmatrix}$$

是一个范德蒙德行列式,其值为

$$D = 1 \cdot 2 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 12$$

例

设曲线 $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ 通过四点(1,3),(2,4),(3,3),(4,-3), 求系数 $a_0,a_1,a_2,a_3$ 。

解:依题意可得线性方程组

$$\begin{cases} a_0 + a_1 + a_2 + a_3 = 3, \\ a_0 + 2a_1 + 4a_2 + 8a_3 = 4, \\ a_0 + 3a_1 + 9a_3 + 27a_3 = 3, \\ a_0 + 4a_1 + 16a_4 + 64a_3 = 3, \end{cases}$$

其系数行列式为

$$D = \left| \begin{array}{rrrr} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{array} \right|$$

是一个范德蒙德行列式, 其值为

$$D = 1 \cdot 2 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 12$$

◆ロト ◆個ト ◆差ト ◆差ト を めらぐ

解: (续)

$$D_{1} = \begin{vmatrix} 3 & 1 & 1 & 1 \\ 4 & 2 & 4 & 8 \\ 3 & 3 & 9 & 27 \\ -3 & 4 & 16 & 64 \end{vmatrix} = 36, \quad D_{2} = \begin{vmatrix} 1 & 3 & 1 & 1 \\ 1 & 4 & 4 & 8 \\ 1 & 3 & 8 & 27 \\ 1 & -3 & 16 & 64 \end{vmatrix} = -18,$$

$$D_{3} = \begin{vmatrix} 1 & 1 & 3 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 3 & 27 \\ 1 & 4 & -3 & 64 \end{vmatrix} = 24, \quad D_{4} = \begin{vmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 4 \\ 1 & 3 & 9 & 3 \\ 1 & 4 & 16 & -3 \end{vmatrix} = -6.$$

于是得

$$a_0 = \frac{D_1}{D} = 3$$
,  $a_1 = \frac{D_2}{D} = -3/2$ ,  $a_2 = \frac{D_3}{D} = 2$ ,  $a_3 = \frac{D_4}{D} = -1/2$ .

即曲线方程为

$$y = 3 - \frac{3}{2}x + 2x^2 - \frac{1}{2}x^3.$$



### 定理

如果线性方程组(6)的系数行列式 $D \neq 0$ ,则(6)一定有解,且解是惟一的。

#### 定理

如果线性方程组(6)无解或有两个不同的解,则它的系数行列式必为0。

# 说明:

 线性方程组(6)右端的常数项b<sub>1</sub>, b<sub>2</sub>, ···, b<sub>n</sub>不全为0时, 线性方程组(6)叫做非齐次 线性方程组

当b1,b2,···,bn全为0时, 线性方程组(6)叫做齐次线性方程组。

#### 齐次线性方程组

对于齐次线性方程组

$$\begin{cases}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0, \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0, \\
\dots \\
a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = 0.
\end{cases}$$
(7)

一定有零解,但不一定有非零解。

张晓平

# 定理

如果齐次线性方程组(7)的系数行列式 $D \neq 0$ ,则它没有非零解。

# 定理

如果齐次线性方程组(7)有非零解,则它的系数行列式必为0。

当 入为何值时, 齐次线性方程组

$$\begin{cases} (5-\lambda)x + 2y + 2z = 0, \\ 2x + (6-\lambda)y = 0, \\ 2x + (4-\lambda)z = 0. \end{cases}$$

#### 有非零解?

解:由上述定理可知,若所给齐次线性方程组有非零解,则其系数行列式D=0。而

$$D = \begin{vmatrix} 5 - \lambda & 2 & 2 \\ 2 & 6 - \lambda & 0 \\ 2 & 0 & 4 - \lambda \end{vmatrix}$$
$$= (5 - \lambda)(6 - \lambda)(4 - \lambda) - 4(4 - \lambda) - 4(6 - \lambda)$$
$$= (5 - \lambda)(2 - \lambda)(8 - \lambda),$$

当 入为何值时, 齐次线性方程组

$$\begin{cases} (5-\lambda)x + 2y + 2z = 0, \\ 2x + (6-\lambda)y = 0, \\ 2x + (4-\lambda)z = 0. \end{cases}$$

有非零解?

**解:**由上述定理可知,若所给齐次线性方程组有非零解,则其系数行列式D=0。 而

$$D = \begin{vmatrix} 5 - \lambda & 2 & 2 \\ 2 & 6 - \lambda & 0 \\ 2 & 0 & 4 - \lambda \end{vmatrix}$$
$$= (5 - \lambda)(6 - \lambda)(4 - \lambda) - 4(4 - \lambda) - 4(6 - \lambda)$$
$$= (5 - \lambda)(2 - \lambda)(8 - \lambda),$$

故 $\lambda = 2 \cdot \lambda = 5$ 或 $\lambda = 8$ 。



张晓平

- 1 行列式简介
- 2 行列式的定义
  - 二阶行列式
  - 三阶行列式
  - · n阶行列式的定义
- ③ 行列式的性质
- 4 行列式的计算
- 5 克莱姆法则
- 6 习题

原式 = 
$$a^2b^2$$
 –  $(ab)(ab)$  = 0.

原式 = 
$$a^2b^2 - (ab)(ab) = 0$$
.

| $\cos\alpha$ | $-\sin\alpha$ |
|--------------|---------------|
| $\sin\alpha$ | $\cos\alpha$  |

原式 = 
$$\cos^2 \alpha + \sin^2 \alpha = 1$$
.

$$\begin{array}{ccc}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{array}$$

原式 = 
$$\cos^2 \alpha + \sin^2 \alpha = 1$$
.

$$a+bi$$
  $b$   $2a$   $a-bi$ 

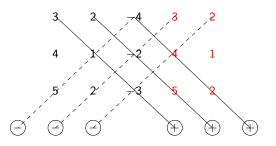
原式 = 
$$(a + bi)(a - bi) - 2ab = a^2 + b^2 - 2ab = (a - b)^2$$

原式 = 
$$(a + bi)(a - bi) - 2ab = a^2 + b^2 - 2ab = (a - b)^2$$
.

$$\begin{array}{cccc} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{array}$$

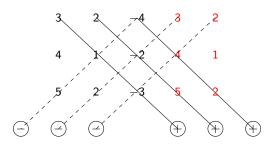


原式 = 
$$3 \cdot 1 \cdot (-3) + 2 \cdot (-2) \cdot 5 + (-4) \cdot 4 \cdot 2 - (-4) \cdot 1 \cdot 5 - 3 \cdot (-2) \cdot 2 - 2 \cdot 4 \cdot (-3)$$



原式 = 
$$3 \cdot 1 \cdot (-3) + 2 \cdot (-2) \cdot 5 + (-4) \cdot 4 \cdot 2 - (-4) \cdot 1 \cdot 5 - 3 \cdot (-2) \cdot 2 - 2 \cdot 4 \cdot (-3) - -9 - 20 - 32 + 20 + 12 + 24 - -5$$

解:



原式 = 
$$3 \cdot 1 \cdot (-3) + 2 \cdot (-2) \cdot 5 + (-4) \cdot 4 \cdot 2 - (-4) \cdot 1 \cdot 5 - 3 \cdot (-2) \cdot 2 - 2 \cdot 4 \cdot (-3)$$
  
=  $-9 - 20 - 32 + 20 + 12 + 24 = -5$ .

40.40.45.45. 5 000

$$\bar{R} \stackrel{?}{\preceq} \frac{r_3 - r_2}{r_2 - r_1} \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix} = 0$$

原式
$$\frac{r_3-r_2}{r_2-r_1}$$
 $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix} = 0.$ 

$$\begin{array}{c|cccc} 2 & 2 & 1 \\ 4 & 1 & -1 \\ 202 & 199 & 101 \end{array}$$

原式 
$$\stackrel{c_1-c_2}{=}$$
  $\begin{vmatrix} 0 & 2 & 1 \\ 3 & 1 & -1 \\ 3 & 199 & 101 \end{vmatrix}$   $\stackrel{r_3-r_2}{=}$   $\begin{vmatrix} 0 & 2 & 1 \\ 3 & 1 & -1 \\ 0 & 198 & 102 \end{vmatrix}$   $= (-1)^{2+1} \cdot 3 \cdot \begin{vmatrix} 2 & 1 \\ 198 & 102 \end{vmatrix} = -3 \cdot (2 \cdot 102 - 198) = -18$ 

$$\begin{array}{c|ccccc}
2 & 2 & 1 \\
4 & 1 & -1 \\
202 & 199 & 101
\end{array}$$

原式 
$$\frac{c_1-c_2}{3}$$
  $\begin{vmatrix} 0 & 2 & 1 \\ 3 & 1 & -1 \\ 3 & 199 & 101 \end{vmatrix} = \frac{r_3-r_2}{3} \begin{vmatrix} 0 & 2 & 1 \\ 3 & 1 & -1 \\ 0 & 198 & 102 \end{vmatrix}$   $= (-1)^{2+1} \cdot 3 \cdot \begin{vmatrix} 2 & 1 \\ 198 & 102 \end{vmatrix} = -3 \cdot (2 \cdot 102 - 198) = -18.$ 

$$\left|\begin{array}{ccc} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{array}\right|, \quad \omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

解: 注意到 $\omega^3 = 1$ , 故

$$\omega \left| \begin{array}{ccc|c} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{array} \right| = \left| \begin{array}{ccc|c} 1 & \omega & \omega^2 \\ \omega^3 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{array} \right| = \left| \begin{array}{ccc|c} 1 & \omega & \omega^2 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{array} \right| = 0$$

从而

$$原式 = 0$$
.

$$\left|\begin{array}{ccc} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{array}\right|, \quad \omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

解: 注意到 $\omega^3 = 1$ , 故

$$\left| egin{array}{ccc|c} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{array} \right| = \left| egin{array}{ccc|c} 1 & \omega & \omega^2 \\ \omega^3 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{array} \right| = \left| egin{array}{ccc|c} 1 & \omega & \omega^2 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{array} \right| = 0,$$

从而

原式 
$$= 0$$
.

$$\left|\begin{array}{ccc} 1 & x & x \\ x & 2 & x \\ x & x & 3 \end{array}\right|.$$

原式 
$$\frac{r_2 - xr_1}{r_3 - xr_1}$$
  $\begin{vmatrix} 1 & x & x \\ 0 & 2 - x^2 & x - x^2 \\ 0 & x - x^2 & 3 - x^2 \end{vmatrix}$   $= \begin{vmatrix} 2 - x^2 & x - x^2 \\ x - x^2 & 3 - x^2 \end{vmatrix}$   $= (2 - x^2)(3 - x^2) - (x - x^2)^2 = 2x^3 - 6x^2 + 6.$ 

$$\left|\begin{array}{ccc} 1 & x & x \\ x & 2 & x \\ x & x & 3 \end{array}\right|.$$

原式 
$$\frac{r_2-xr_1}{r_3-xr_1}$$
  $\begin{vmatrix} 1 & x & x \\ 0 & 2-x^2 & x-x^2 \\ 0 & x-x^2 & 3-x^2 \end{vmatrix}$   $=$   $\begin{vmatrix} 2-x^2 & x-x^2 \\ x-x^2 & 3-x^2 \end{vmatrix}$   $=$   $(2-x^2)(3-x^2)-(x-x^2)^2=2x^3-6x^2+6$ .

原式 = 
$$(-1)^{1+4} \cdot 4 \cdot \begin{vmatrix} 0 & 0 & 4 \\ 0 & 4 & 3 \\ 4 & 3 & 2 \end{vmatrix}$$
  
=  $(-1)^{1+4} \cdot 4 \cdot (-1)^{1+3} \cdot 4 \cdot \begin{vmatrix} 0 & 4 \\ 4 & 3 \end{vmatrix}$   
=  $-4 \cdot 4 \cdot (-16) = 256$ .

原式 
$$= (-1)^{1+4} \cdot 4 \cdot \begin{vmatrix} 0 & 0 & 4 \\ 0 & 4 & 3 \\ 4 & 3 & 2 \end{vmatrix}$$
$$= (-1)^{1+4} \cdot 4 \cdot (-1)^{1+3} \cdot 4 \cdot \begin{vmatrix} 0 & 4 \\ 4 & 3 \end{vmatrix}$$
$$= -4 \cdot 4 \cdot (-16) = 256.$$

原式 
$$= (-1)^{10+10} \cdot 10 \cdot \begin{vmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 2 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 8 & \cdots & 0 & 0 \\ 9 & 0 & \cdots & 0 & 0 \end{vmatrix} = 10 \cdot (-1)^{\frac{9 \times 8}{2}} 9! = 10!$$

原式 
$$= (-1)^{10+10} \cdot 10 \cdot \begin{vmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 2 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 8 & \cdots & 0 & 0 \\ 9 & 0 & \cdots & 0 & 0 \end{vmatrix} = 10 \cdot (-1)^{\frac{9 \times 8}{2}} 9! = 10!$$

原式 
$$\frac{r_i - r_1}{i = 2, 3, 4}$$
  $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{vmatrix} = (-2)^3 = -8.$ 

原式 
$$\frac{r_i-r_1}{i=2,3,4}$$
 
$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{vmatrix} = (-2)^3 = -8.$$

$$\mathbb{R} \stackrel{r_i = r_{i-1}}{= 4,3,2} \begin{vmatrix}
1 & 2 & 3 & 4 \\
1 & 1 & 1 & -3 \\
1 & 1 & -3 & 1
\end{vmatrix}
\underbrace{\begin{array}{c}c_i = c_1 \\ i = 2,3,4\end{array}}_{i=2,3,4} \begin{vmatrix}
1 & 1 & 2 & 3 \\
1 & 0 & 0 & -4 \\
1 & 0 & -4 & 0 \\
1 & -4 & 0 & 0
\end{vmatrix}$$

$$\underbrace{\begin{array}{c}c_i \div 4 \\ i = 2,3,4\end{array}}_{i=2,3,4} 4^3 \begin{vmatrix}
1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\
1 & 0 & 0 & -1 \\
1 & 0 & -1 & 0 \\
1 & -1 & 0 & 0
\end{vmatrix}
\underbrace{\begin{array}{c}c_1 + c_2 + c_3 + c_4 \\ 0 & 0 & -1 & 0
\end{vmatrix}}_{i=2,3,4} 4^3 \begin{vmatrix}
1 + \frac{1+2+3}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\
0 & 0 & 0 & -1 \\
0 & -1 & 0 & 0
\end{vmatrix}$$

$$= 4^3 \frac{10}{4} \begin{vmatrix}
0 & 0 & -1 \\
0 & -1 & 0 \\
0 & -1 & 0
\end{vmatrix} = 160.$$

原式 
$$\frac{r_i - r_{i-1}}{i=4,3,2}$$
  $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & -3 & 1 \\ 1 & -3 & 1 & 1 \end{vmatrix}$   $\frac{c_i - c_1}{i=2,3,4}$   $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 0 & -4 \\ 1 & 0 & -4 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix}$   $\frac{c_i \div 4}{i=2,3,4}$   $4^3$   $\begin{vmatrix} 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{vmatrix}$   $\frac{c_1 + c_2 + c_3 + c_4}{i=2,3,4}$   $4^3$   $\begin{vmatrix} 1 + \frac{1+2+3}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}$   $= 4^3 \frac{10}{4}$   $\begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} = 160.$ 

原式 
$$\frac{r_i - r_{i-1}}{i=4,3,2}$$
  $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & -3 & 1 \\ 1 & -3 & 1 & 1 \end{vmatrix}$   $\frac{c_i - c_1}{i=2,3,4}$   $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 0 & -4 \\ 1 & 0 & -4 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix}$   $\frac{c_i \div 4}{i=2,3,4}$   $4^3$   $\begin{vmatrix} 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{vmatrix}$   $\frac{c_1 + c_2 + c_3 + c_4}{i=2,3,4}$   $4^3$   $0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 \end{vmatrix}$   $0 & 0 & -1$   $0 & 0 & -1$   $0 & 0 & -1$   $0 & 0 & 0 & -1$   $0 & 0 & 0 & -1$   $0 & 0 & 0 & 0 & -1$   $0 & 0 & 0 & 0 & -1$   $0 & 0 & 0 & 0 & -1 & 0 & 0 \end{vmatrix}$   $0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{vmatrix}$   $0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \end{vmatrix}$   $0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 &$ 

原式 
$$\frac{r_i - r_{i-1}}{i=4,3,2}$$
  $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & -3 & 1 \end{vmatrix}$   $\frac{c_i - c_1}{i=2,3,4}$   $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 0 & -4 \\ 1 & 0 & -4 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix}$   $\frac{c_i \div 4}{i=2,3,4}$   $4^3$   $\begin{vmatrix} 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{vmatrix}$   $\frac{c_1 + c_2 + c_3 + c_4}{i=2,3,4}$   $4^3$   $\begin{vmatrix} 1 + \frac{1+2+3}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}$   $= 4^3 \frac{10}{4}$   $\begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 160.$ 

解:

原式 
$$\frac{r_i - r_{i-1}}{i=4,3,2}$$
  $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & -3 & 1 \\ 1 & -3 & 1 & 1 \end{vmatrix} = \frac{c_i - c_1}{i=2,3,4} \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 0 & -4 \\ 1 & 0 & -4 & 0 \\ 1 & -4 & 0 & 0 \end{vmatrix}$ 

$$\frac{c_i \div 4}{i=2,3,4} 4^3 \begin{vmatrix} 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{vmatrix} = \frac{c_1 + c_2 + c_3 + c_4}{4} 4^3 \begin{vmatrix} 1 + \frac{1 + 2 + 3}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}$$

$$= 4^3 \frac{10}{4} \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 160.$$

解:
$$\begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$
解:
$$\begin{bmatrix} \frac{r_i - r_{i-1}}{i=4,3,2} & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & -3 & 1 \\ 1 & -3 & 1 & 1 \end{bmatrix} \xrightarrow{\begin{bmatrix} c_i - c_1 \\ i=2,3,4 \end{bmatrix}} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 0 & -4 \\ 1 & 0 & -4 & 0 \\ 1 & -4 & 0 & 0 \end{bmatrix}$$

$$\frac{c_i \div 4}{i=2,3,4} \ 4^3 \begin{vmatrix} 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{vmatrix} \xrightarrow{\underbrace{c_1 + c_2 + c_3 + c_4}} 4^3 \begin{vmatrix} 1 + \frac{1 + 2 + 3}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}$$

$$= 4^3 \frac{10}{4} \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 160.$$

$$\left|\begin{array}{ccccc} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right|$$

原式 
$$\frac{r_3+r_2}{r_4+r_2}$$
  $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 1 \\ 2 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} \cdot (-1) \cdot \begin{vmatrix} 5 & 4 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$   $\frac{r_1-r_2}{r_2} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix} = -(-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$ 

原式 
$$\frac{r_3+r_2}{r_4+r_2}$$
  $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 1 \\ 2 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} \cdot (-1) \cdot \begin{vmatrix} 5 & 4 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$   $\frac{r_3-r_2}{r_4+r_2}$   $-\begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix} = -(-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$ 

原式 
$$\frac{r_3+r_2}{r_4+r_2}$$
  $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 1 \\ 2 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} \cdot (-1) \cdot \begin{vmatrix} 5 & 4 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$   $\frac{r_3+r_2}{r_4+r_2}$   $-\begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix} = -(-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$ 

原式 
$$\frac{r_3+r_2}{r_4+r_2}$$
  $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 1 \\ 2 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} \cdot (-1) \cdot \begin{vmatrix} 5 & 4 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$   $= \frac{r_1-r_2}{r_2} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix} = -(-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$ 

原式 
$$\frac{r_3+r_2}{r_4+r_2}$$
  $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 1 \\ 2 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} \cdot (-1) \cdot \begin{vmatrix} 5 & 4 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$   $\frac{r_1-r_2}{r_2}$   $-\begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 1 \\ 2 & 3 & 2 \end{vmatrix} = -(-1)^{1+3} \cdot 1 \cdot \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$ 

解:

= -12.

解:

= -12.

解:

原式 
$$\xrightarrow{r_2 \leftrightarrow r_3}$$
 -  $\begin{vmatrix} 3 & 6 & 5 & 6 & 4 \\ 3 & 6 & 3 & 4 & 2 \\ 2 & 5 & 4 & 5 & 3 \\ 2 & 5 & 4 & 6 & 5 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$   $\xrightarrow{r_2 \rightarrow r_3}$  -  $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 2 & 5 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$ 

$$\frac{r_3-2r_1}{r_5-r_1} - \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 0 & 3 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -2 & -2 \end{vmatrix} = \frac{r_2 \leftrightarrow r_3}{r_5+2r_4} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 2 & 3 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

= -12.

解:

原式 
$$\frac{r_2\leftrightarrow r_3}{}$$
  $\begin{vmatrix} 3 & 6 & 5 & 6 & 4 \\ 3 & 6 & 3 & 4 & 2 \\ 2 & 5 & 4 & 5 & 3 \\ 2 & 5 & 4 & 6 & 5 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$   $\begin{vmatrix} r_2-r_1 \\ r_4-r_3 \\ r_1-r_3 \end{vmatrix}$   $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 2 & 5 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$ 

= -12.

4□▶ 4□▶ 4□▶ 4□▶ 4□ ♥ 900

解:

$$\frac{r_3-2r_1}{r_5-r_1} - \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 0 & 3 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -2 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 2 & 3 & 1 \\ 0 & 0 & 0 & -2 & -2 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

= -12

解:

原式 
$$=\frac{r_2\leftrightarrow r_3}{}$$
  $\begin{vmatrix} 3 & 6 & 5 & 6 & 4 \\ 3 & 6 & 3 & 4 & 2 \\ 2 & 5 & 4 & 5 & 3 \\ 2 & 5 & 4 & 6 & 5 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$   $\begin{vmatrix} r_2-r_1 \\ 4-r_3 \\ r_1-r_3 \end{vmatrix}$   $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 2 & 5 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$ 

$$\frac{r_3 - 2r_1}{r_5 - r_1} - \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 0 & 3 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -2 & -2 \end{vmatrix} \xrightarrow{r_2 \leftrightarrow r_3} \frac{1}{r_5 + 2r_4} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 2 & 3 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

= -12

解:

原式 
$$=\frac{r_2\leftrightarrow r_3}{}$$
  $\begin{vmatrix} 3 & 6 & 5 & 6 & 4 \\ 3 & 6 & 3 & 4 & 2 \\ 2 & 5 & 4 & 5 & 3 \\ 2 & 5 & 4 & 6 & 5 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$   $\begin{vmatrix} r_2-r_1 \\ r_2-r_2 \\ r_1-r_3 \end{vmatrix}$   $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 \\ 2 & 5 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & -1 & -1 \end{vmatrix}$ 

= -12.

↓□▶ ↓□▶ ↓□▶ ↓□▶ ↓□ ♥ ♀○

$$\left|\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 5 & 1 \end{array}\right|$$

$$\bar{R} \vec{\Lambda} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \cdot \begin{vmatrix} -1 & 3 \\ 5 & 1 \end{vmatrix} = (-2) \cdot (-16) = 32.$$

$$\left|\begin{array}{ccccc} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 5 & 1 \end{array}\right|$$

原式 
$$=\frac{r_3\leftrightarrow r_5}{0}$$
  $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{vmatrix}$   $=$   $\begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 0 & 1 & 0 \end{vmatrix}$   $\cdot$   $\begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   $=$   $-(-1)^{3+2} \cdot 1 \cdot \begin{vmatrix} 1 & 3 \\ 6 & 8 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   $=$   $(-10) \cdot 2 = -20$ .

原式 
$$=$$
  $\frac{r_3 \leftrightarrow r_5}{678910}$   $=$   $- \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   $= (-10) \cdot 2 = -20.$ 

原式 
$$=\frac{r_3\leftrightarrow r_5}{}$$
  $-\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   
 $= -(-1)^{3+2} \cdot 1 \cdot \begin{vmatrix} 1 & 3 \\ 6 & 8 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   
 $= (-10) \cdot 2 = -20.$ 

原式 
$$=\frac{r_3\leftrightarrow r_5}{}$$
  $-\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   
 $= -(-1)^{3+2} \cdot 1 \cdot \begin{vmatrix} 1 & 3 \\ 6 & 8 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$   
 $= (-10) \cdot 2 = -20.$ 

原式 
$$\frac{r_3 \leftrightarrow r_5}{-} - \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$$
$$= -(-1)^{3+2} \cdot 1 \cdot \begin{vmatrix} 1 & 3 \\ 6 & 8 \end{vmatrix} \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$$
$$= (-10) \cdot 2 = -20.$$

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & 0 & 2 & 4 & 0 \\ 1 & 2 & 4 & 0 & -1 \\ 3 & 1 & 2 & 5 & 8 \end{bmatrix}$$

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \end{vmatrix}$   $\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$   $\begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & -1 & 1 & 2 \\ 2 & 5 & 8 & 3 & 1 \end{vmatrix}$   $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 2 & 4 & 0 \end{vmatrix}$   $\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$   $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 \end{vmatrix}$ 

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$   $\begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 1 & 2 & -1 \\ 2 & 5 & 3 & 1 & 8 \end{vmatrix}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ 4 & 0 & -1 & 1 & 2 \\ 2 & 5 & 8 & 3 & 1 \end{vmatrix}$   $= \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 2 & 4 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ 

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 1 & 2 & -1 \\ 2 & 5 & 3 & 1 & 8 \end{vmatrix}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ 4 & 0 & -1 & 1 & 2 \\ 2 & 5 & 8 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 2 & 4 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ 

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$   $\begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 1 & 2 & -1 \\ 2 & 5 & 3 & 1 & 8 \end{vmatrix}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{3}$   $\frac$ 

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & 0 & 2 & 4 & 0 \\ 1 & 2 & 4 & 0 & -1 \\ 3 & 1 & 2 & 5 & 8 \end{bmatrix}$$

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$   $\begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 1 & 2 & -1 \\ 2 & 5 & 3 & 1 & 8 \end{vmatrix}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & -1 & 1 & 2 \\ 2 & 5 & 8 & 3 & 1 \end{vmatrix}$   $=$   $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 2 & 4 & 0 \end{vmatrix}$   $\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ 

张晓平

线性代数

134 / 190

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 3 & 0 & 2 \\ 0 & 0 & 2 & 4 & 0 \\ 1 & 2 & 4 & 0 & -1 \\ 3 & 1 & 2 & 5 & 8 \end{bmatrix}$$

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$   $\begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 1 & 2 & -1 \\ 2 & 5 & 3 & 1 & 8 \end{vmatrix}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & -1 & 1 & 2 \\ 2 & 5 & 8 & 3 & 1 \end{vmatrix}$   $=$   $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 2 & 4 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ 

解:

原式 
$$\frac{c_3\leftrightarrow c_2}{c_2\leftrightarrow c_1}$$
  $\begin{vmatrix} 1 & 0 & 0 & -1 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 4 & 0 \\ 4 & 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 5 & 8 \end{vmatrix}$   $\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$   $\begin{vmatrix} 1 & -1 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 2 \\ 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 1 & 2 & -1 \\ 2 & 5 & 3 & 1 & 8 \end{vmatrix}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\begin{vmatrix} 1 & -1 & 2 & 0 & 0 \\ 3 & 0 & 2 & 0 & 0 \\ 4 & 0 & -1 & 1 & 2 \\ 2 & 5 & 8 & 3 & 1 \end{vmatrix}$   $=$   $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \\ 2 & 4 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$   $\frac{c_5\leftrightarrow c_4}{c_4\leftrightarrow c_3}$   $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \\ 2 & 4 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 2 \cdot 6 \cdot (-5) = -60.$ 

$$\left|\begin{array}{ccc|c} * & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{array}\right|, \quad \mathbf{A} = \left|\begin{array}{ccc|c} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{array}\right|, \quad \mathbf{B} = \left|\begin{array}{ccc|c} & & & & -1 \\ & & & -2 & \\ & & -4 & \\ & & -5 & \end{array}\right|$$

原式 = 
$$(-1)^{3\times5}$$
 |  $\begin{vmatrix} \mathbf{A} & * \\ \mathbf{0} & \mathbf{B} \end{vmatrix}$  =  $(-1) \cdot |\mathbf{A}| \cdot |\mathbf{B}|$  =  $(-1) \cdot 1 \cdot 2 \cdot 3 \cdot (-1)^{\frac{5\times4}{2}} (-1)(-2)(-3)(-4)(-5)$  =  $6 \times 120 = 720$ 

$$\left|\begin{array}{ccc} * & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{array}\right|, \quad \mathbf{A} = \left|\begin{array}{ccc} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{array}\right|, \quad \mathbf{B} = \left|\begin{array}{ccc} & & & -1 \\ & & & -2 \\ & & -4 & & \\ & -5 & & & \end{array}\right|$$

原式 = 
$$(-1)^{3\times5}$$
 |  $\begin{array}{c|c} \mathbf{A} & * \\ \mathbf{0} & \mathbf{B} \end{array}$  | 
$$= (-1) \cdot |\mathbf{A}| \cdot |\mathbf{B}|$$
 =  $(-1) \cdot 1 \cdot 2 \cdot 3 \cdot (-1)^{\frac{5\times4}{2}} (-1)(-2)(-3)(-4)(-5)$  =  $6 \times 120 = 720$ 

$$\left|\begin{array}{ccc|c} * & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{array}\right|, \quad \mathbf{A} = \left|\begin{array}{ccc|c} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{array}\right|, \quad \mathbf{B} = \left|\begin{array}{ccc|c} & & & & -1 \\ & & & -2 & \\ & & & -3 & \\ & & -5 & & \end{array}\right|$$

原式 = 
$$(-1)^{3\times5}$$
 |  $\begin{array}{c|c} \mathbf{A} & * \\ \mathbf{0} & \mathbf{B} \end{array}$  | 
$$= (-1) \cdot |\mathbf{A}| \cdot |\mathbf{B}|$$

$$= (-1) \cdot 1 \cdot 2 \cdot 3 \cdot (-1)^{\frac{5\times4}{2}} (-1)(-2)(-3)(-4)(-5)$$

$$= 6 \times 120 = 720$$

$$\left|\begin{array}{ccc} * & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{array}\right|, \quad \mathbf{A} = \left|\begin{array}{ccc} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{array}\right|, \quad \mathbf{B} = \left|\begin{array}{ccc} & & & -1 \\ & & & -2 \\ & & -4 & & \\ & -5 & & & \end{array}\right|$$

原式 = 
$$(-1)^{3\times5}$$
 |  $\begin{array}{c|c} \mathbf{A} & * \\ \mathbf{0} & \mathbf{B} \end{array}$  | 
$$= (-1) \cdot |\mathbf{A}| \cdot |\mathbf{B}|$$

$$= (-1) \cdot 1 \cdot 2 \cdot 3 \cdot (-1)^{\frac{5\times4}{2}} (-1)(-2)(-3)(-4)(-5)$$

$$= 6 \times 120 = 720$$

$$\left|\begin{array}{ccc|c} * & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{array}\right|, \quad \mathbf{A} = \left|\begin{array}{ccc|c} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{array}\right|, \quad \mathbf{B} = \left|\begin{array}{ccc|c} & & & & -1 \\ & & & -2 & \\ & & & -4 & \\ & & & -5 & \end{array}\right|$$

原式 = 
$$(-1)^{3\times5}$$
 |  $\begin{array}{c|c} \mathbf{A} & * \\ \mathbf{0} & \mathbf{B} \end{array}$  | 
$$= (-1) \cdot |\mathbf{A}| \cdot |\mathbf{B}|$$

$$= (-1) \cdot 1 \cdot 2 \cdot 3 \cdot (-1)^{\frac{5\times4}{2}} (-1)(-2)(-3)(-4)(-5)$$

$$= 6 \times 120 = 720$$

$$\begin{vmatrix} * & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{vmatrix}, \quad \mathbf{A} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{vmatrix}, \quad \mathbf{B} = \begin{vmatrix} & & & -1 \\ & & & \\ & -4 & & \\ & -5 & & \end{vmatrix}$$

原式 = 
$$(-1)^{3\times5}$$
 |  $\begin{array}{c|c} \mathbf{A} & * \\ \mathbf{0} & \mathbf{B} \end{array}$  | 
$$= (-1) \cdot |\mathbf{A}| \cdot |\mathbf{B}|$$

$$= (-1) \cdot 1 \cdot 2 \cdot 3 \cdot (-1)^{\frac{5\times4}{2}} (-1)(-2)(-3)(-4)(-5)$$

$$= 6 \times 120 = 720$$

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

证明

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

差边 = 
$$\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix}$$
 +  $\begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  +  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  +  $\begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$  =  $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$ 

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

差速 = 
$$\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = b_1 b_1 c_1$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \frac{\pi i b_1}{b_1} c_1$$

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

差边 = 
$$\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x \begin{vmatrix} b_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 &$$

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

差数 = 
$$\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x$$

$$= x$$

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

差数 = 
$$\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x$$

证明:

$$\begin{vmatrix} a_1 + b_1 x & a_1 x + b_1 & c_1 \\ a_2 + b_2 x & a_2 x + b_2 & c_2 \\ a_3 + b_3 x & a_3 x + b_3 & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

差 遊 
$$= \begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_1x & c_1 \\ a_2 & a_2x & c_2 \\ a_3 & a_3x & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = x b b b b b b c_1$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x \begin{vmatrix} b_1 & a_1x & c_1 \\ b_2 & a_2x & c_2 \\ b_3 & a_3x & c_3 \end{vmatrix} = (1 - x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = x b b b b b b b b b b b b b b c_2$$

证明:

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2.$$

$$\dot{\mathcal{E}}\dot{\mathcal{B}} = egin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \ 0 & 1+x & 1 & 1 & 1 \ 0 & 1 & 1-x & 1 & 1 \ 0 & 1 & 1 & 1+y & 1 \ 0 & 1 & 1 & 1 & 1-y \end{bmatrix}$$

$$= x^2 y^2.$$

证明:

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2.$$

左边 = 
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1+x & 1 & 1 & 1 \\ 0 & 1 & 1-x & 1 & 1 \\ 0 & 1 & 1 & 1+y & 1 \\ 0 & 1 & 1 & 1 & 1-y \end{bmatrix}$$

$$= x^2 y^2.$$

证明:

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2.$$

左边 = 
$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1+x & 1 & 1 & 1 \\ 0 & 1 & 1-x & 1 & 1 \\ 0 & 1 & 1 & 1+y & 1 \\ 0 & 1 & 1 & 1 & 1-y \end{vmatrix}$$

$$= x^2 y^2.$$

证明:

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2.$$

左边 = 
$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1+x & 1 & 1 & 1 \\ 0 & 1 & 1-x & 1 & 1 \\ 0 & 1 & 1 & 1+y & 1 \\ 0 & 1 & 1 & 1 & 1-y \end{vmatrix}$$

$$= x^2 y^2.$$

证明:

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2.$$

证明:

左边 = 
$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1+x & 1 & 1 & 1 \\ 0 & 1 & 1-x & 1 & 1 \\ 0 & 1 & 1 & 1+y & 1 \\ 0 & 1 & 1 & 1 & 1-y \end{vmatrix}$$

 $= x^2 y^2.$ 

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2.$$

左边 = 
$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1+x & 1 & 1 & 1 \\ 0 & 1 & 1-x & 1 & 1 \\ 0 & 1 & 1 & 1+y & 1 \\ 0 & 1 & 1 & 1 & 1-y \end{vmatrix}$$

$$= x^2 y^2.$$

证明:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c).$$

证明: 考察范德蒙德行列式

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & y \\ a^2 & b^2 & c^2 & y^2 \\ a^3 & b^3 & c^3 & y^3 \end{vmatrix} = (y-a)(y-b)(y-c)(c-a)(c-b)(b-a)$$

等式两端均为关于y的多项式,比较y<sup>2</sup>的系数,可知

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c)$$

证明:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c).$$

证明: 考察范德蒙德行列式

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & y \\ a^2 & b^2 & c^2 & y^2 \\ a^3 & b^3 & c^3 & y^3 \end{vmatrix} = (y-a)(y-b)(y-c)(c-a)(c-b)(b-a)$$

等式两端均为关于y的多项式,比较y<sup>2</sup>的系数,可知

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c)$$

证明:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c).$$

证明: 考察范德蒙德行列式

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & y \\ a^2 & b^2 & c^2 & y^2 \\ a^3 & b^3 & c^3 & y^3 \end{vmatrix} = (y-a)(y-b)(y-c)(c-a)(c-b)(b-a)$$

等式两端均为关于y的多项式,比较y2的系数,可知

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c)$$

证明:

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (ab + bc + ca) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

证明: 考察范德蒙德行列式

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & y & y^2 & y^3 \end{vmatrix} = (y-a)(y-b)(y-c)(c-a)(c-b)(b-a)$$

等式两端均为关于y的多项式,比较y的系数,可知

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(ab+bc+ca) = (ab+bc+ca) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

<ロ > < 回 > < 回 > < 巨 > くき > しき > しき の < ○

证明:

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (ab + bc + ca) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

证明: 考察范德蒙德行列式

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & y & y^2 & y^3 \end{vmatrix} = (y-a)(y-b)(y-c)(c-a)(c-b)(b-a)$$

等式两端均为关于y的多项式,比较y的系数,可知

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(ab+bc+ca) = (ab+bc+ca) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

↓□▶ ↓□▶ ↓□▶ ↓□▶ ↓□ ♥ ♀○

证明:

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (ab + bc + ca) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

证明: 考察范德蒙德行列式

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & y & y^2 & y^3 \end{vmatrix} = (y-a)(y-b)(y-c)(c-a)(c-b)(b-a)$$

等式两端均为关于y的多项式,比较y的系数,可知

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(ab+bc+ca) = (ab+bc+ca) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

↓□▶ ↓□▶ ↓□▶ ↓□▶ ↓□ ♥ ♀○

左边 接第4行展开 
$$(-1)^{4+1} \cdot d \cdot \begin{pmatrix} 0 & 2 & a \\ 0 & b & 0 \\ c & 4 & 5 \end{pmatrix}$$

接第2行展开 
$$(-d) \cdot (-1)^{2+2} \cdot b$$
  $\begin{vmatrix} 0 & a \\ c & 5 \end{vmatrix} = abcd.$ 

计算

接第2行展开 
$$(-d) \cdot (-1)^{2+2} \cdot b$$
  $\begin{vmatrix} 0 & a \\ c & 5 \end{vmatrix} = abcd.$ 

计算

左边 接第4行展开 
$$(-1)^{4+1} \cdot d \cdot \begin{vmatrix} 0 & 2 & a \\ 0 & b & 0 \\ c & 4 & 5 \end{vmatrix}$$

$$\underline{
 按第2行展开} (-d) \cdot (-1)^{2+2} \cdot b \begin{vmatrix} 0 & a \\ c & 5 \end{vmatrix} = abcd.$$

计算

左边 接第4行展开 
$$(-1)^{4+1} \cdot d \cdot \begin{vmatrix} 0 & 2 & a \\ 0 & b & 0 \\ c & 4 & 5 \end{vmatrix}$$

$$\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix}$$

$$= a \cdot \left| \begin{array}{ccc} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{array} \right| + \left| \begin{array}{ccc} c & 1 \\ -1 & d \end{array} \right|$$

$$ilde{ extrm{ iny 49.17 展开}} a \cdot \left(b \cdot ig| \begin{array}{cc|c} c & 1 \\ -1 & d \end{array} \middle| + (-1)^{1+2} \cdot 1 \cdot ig| \begin{array}{cc|c} -1 & 1 \\ 0 & d \end{array} \middle| + (cd+1)^{1+2} \cdot 1 \cdot \left(cd+1\right)^{1+2} \cdot \left(cd+1\right)^{1+2}$$

$$\left|\begin{array}{ccccc} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{array}\right|$$

左边 接第1行展升 
$$(-1)^{1+1} \cdot a \cdot \begin{vmatrix} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} -1 & 1 & 0 \\ 0 & c & 1 \\ 0 & -1 & d \end{vmatrix}$$

$$= a \cdot \left| \begin{array}{ccc} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{array} \right| + \left| \begin{array}{ccc} c & 1 \\ -1 & d \end{array} \right|$$

$$ilde{ ilde{ ilde{H}}} = ilde{ ilde{ ilde{H}}} = ilde{ ilde{ ilde{h}}} \cdot \left( b \cdot \left| \begin{array}{cc} c & 1 \\ -1 & d \end{array} \right| + (-1)^{1+2} \cdot 1 \cdot \left| \begin{array}{cc} -1 & 1 \\ 0 & d \end{array} \right| \right) + (cd+1)$$

$$\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix}$$

左边 接第1行展开 
$$(-1)^{1+1} \cdot a \cdot \begin{vmatrix} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} -1 & 1 & 0 \\ 0 & c & 1 \\ 0 & -1 & d \end{vmatrix}$$

$$= a \cdot \left| \begin{array}{ccc} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{array} \right| + \left| \begin{array}{ccc} c & 1 \\ -1 & d \end{array} \right|$$

$$ilde{ ext{按第1行展开}} a \cdot \left(b \cdot \left| \begin{array}{cc} c & 1 \\ -1 & d \end{array} \right| + (-1)^{1+2} \cdot 1 \cdot \left| \begin{array}{cc} -1 & 1 \\ 0 & d \end{array} \right| \right) + (cd+1)^{1+2} \cdot 1 \cdot \left| \begin{array}{cc} -1 & 1 \\ 0 & d \end{array} \right|$$

$$\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix}$$

左边 接第1行展开 
$$(-1)^{1+1} \cdot a \cdot \begin{vmatrix} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} -1 & 1 & 0 \\ 0 & c & 1 \\ 0 & -1 & d \end{vmatrix}$$

$$= a \cdot \left| \begin{array}{ccc} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{array} \right| + \left| \begin{array}{ccc} c & 1 \\ -1 & d \end{array} \right|$$

$$ext{ } ext{ } ex$$

$$\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix}$$

左边 接第1行展开 
$$(-1)^{1+1} \cdot a \cdot \begin{vmatrix} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{vmatrix} + (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} -1 & 1 & 0 \\ 0 & c & 1 \\ 0 & -1 & d \end{vmatrix}$$

$$= a \cdot \left| \begin{array}{ccc} b & 1 & 0 \\ -1 & c & 1 \\ 0 & -1 & d \end{array} \right| + \left| \begin{array}{ccc} c & 1 \\ -1 & d \end{array} \right|$$

$$ilde{ ext{按第1行展升}} a \cdot \left(b \cdot ig| egin{array}{c|c} c & 1 \\ -1 & d \end{array} ig| + (-1)^{1+2} \cdot 1 \cdot ig| egin{array}{c|c} -1 & 1 \\ 0 & d \end{array} ig| 
ight) + (cd+1)$$

$$=a(b(cd+1)+d)+(cd+1)=(ab+1)(cd+1)+ad$$

$$\begin{vmatrix} a & 1 & 0 & 0 \\ -1 & b & 1 & 0 \\ 0 & -1 & c & 1 \\ 0 & 0 & -1 & d \end{vmatrix}$$

$$= a \cdot \left| egin{array}{ccc} b & 1 & 0 \ -1 & c & 1 \ 0 & -1 & d \end{array} 
ight| + \left| egin{array}{ccc} c & 1 \ -1 & d \end{array} 
ight|$$

$$ilde{ ext{按第1行展升}} a \cdot \left(b \cdot ig| \begin{array}{cc} c & 1 \\ -1 & d \end{array} ig| + (-1)^{1+2} \cdot 1 \cdot ig| \begin{array}{cc} -1 & 1 \\ 0 & d \end{array} ig| \right) + (cd+1)$$

$$= a(b(cd+1)+d) + (cd+1) = (ab+1)(cd+1) + ad$$

$$\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix}$$

左边 
$$\frac{a^2}{3-2}$$
  $2a+1$   $2a+3$   $2a+5$   $b^2$   $2b+1$   $2b+3$   $2b+5$   $c^2$   $2c+1$   $2c+3$   $2c+5$   $d^2$   $2d+1$   $2d+3$   $2d+5$ 

$$\begin{vmatrix}
a^2 & 2a+1 & 2 & 2 \\
b^2 & 2b+1 & 2 & 2 \\
c^2 & 2c+1 & 2 & 2 \\
d^2 & 2d+1 & 2 & 2
\end{vmatrix} = 0.$$

$$\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix}$$

$$\begin{vmatrix} a^2 & 2a+1 & 2a+3 & 2a+5 \\ b^2 & 2b+1 & 2b+3 & 2b+5 \\ c^2 & 2c+1 & 2c+3 & 2c+5 \\ d^2 & 2d+1 & 2d+3 & 2d+5 \end{vmatrix}$$

$$\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix}$$

左边 
$$\frac{c_4-c_3}{c_2-c_1}$$
  $b^2$ 

$$\begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix}$$

左边 
$$\frac{\frac{c_4-c_3}{c_3-c_2}}{\frac{c_4-c_3}{c_3-c_2}} \begin{vmatrix} a^2 & 2a+1 & 2a+3 & 2a+5 \\ b^2 & 2b+1 & 2b+3 & 2b+5 \\ c^2 & 2c+1 & 2c+3 & 2c+5 \\ d^2 & 2d+1 & 2d+3 & 2d+5 \end{vmatrix}$$
$$\frac{c_4-c_3}{c_3-c_2} \begin{vmatrix} a^2 & 2a+1 & 2 & 2 \\ b^2 & 2b+1 & 2 & 2 \\ c^2 & 2c+1 & 2 & 2 \\ d^2 & 2d+1 & 2 & 2 \end{vmatrix} = 0.$$

$$\mathbb{R} \stackrel{r_3+r_1+r_2}{\stackrel{r_3}{=}} \begin{vmatrix} a & b & a+b+c & 1 \\ b & c & a+b+c & 1 \\ c & a & a+b+c & 1 \\ \frac{b+c}{2} & \frac{c+a}{2} & a+b+c & 1 \end{vmatrix} = 0$$

原式 
$$\frac{r_3+r_1+r_2}{}$$
  $\begin{vmatrix} a & b & a+b+c & 1 \\ b & c & a+b+c & 1 \\ c & a & a+b+c & 1 \\ \frac{b+c}{2} & \frac{c+a}{2} & a+b+c & 1 \end{vmatrix} = 0.$ 

$$\begin{vmatrix} a & b & c & 1 \\ b & c & a & 1 \\ c & a & b & 1 \\ \frac{b+c}{2} & \frac{c+a}{2} & \frac{a+b}{2} & 1 \end{vmatrix}$$

原式 
$$\frac{c_4 \leftrightarrow c_3}{c_3 \leftrightarrow c_2}$$
  $\begin{vmatrix} a_1 & b_1 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_3 & a_3 \\ b_4 & a_4 & 0 & 0 \end{vmatrix}$ 

$$\frac{r_4 \leftrightarrow r_3}{r_3 \leftrightarrow r_2} \begin{vmatrix} a_1 & b_1 & 0 & 0 \\ b_4 & a_4 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_3 & a_3 \end{vmatrix} = (a_1 a_4 - b_1 b_4)(a_2 a_3 - b_2 b_3).$$

原式 
$$\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$$
  $\begin{vmatrix} a_1 & b_1 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_3 & a_3 \\ b_4 & a_4 & 0 & 0 \end{vmatrix}$ 

$$\begin{vmatrix} a_1 & 0 & 0 & b_1 \\ 0 & a_2 & b_2 & 0 \\ 0 & b_3 & a_3 & 0 \\ b_4 & 0 & 0 & a_4 \end{vmatrix}$$

原式 
$$\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$$
  $\begin{vmatrix} a_1 & b_1 & 0 & 0\\ 0 & 0 & a_2 & b_2\\ 0 & 0 & b_3 & a_3\\ b_4 & a_4 & 0 & 0 \end{vmatrix}$ 

$$\frac{r_4 \leftrightarrow r_3}{r_3 \leftrightarrow r_2} \begin{vmatrix} a_1 & b_1 & 0 & 0 \\ b_4 & a_4 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_3 & a_3 \end{vmatrix} = (a_1 a_4 - b_1 b_4)(a_2 a_3 - b_2 b_3).$$

$$\begin{vmatrix} a_1 & 0 & 0 & b_1 \\ 0 & a_2 & b_2 & 0 \\ 0 & b_3 & a_3 & 0 \\ b_4 & 0 & 0 & a_4 \end{vmatrix}$$

原式 
$$\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$$
  $\begin{vmatrix} a_1 & b_1 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_3 & a_3 \\ b_4 & a_4 & 0 & 0 \end{vmatrix}$ 

$$\frac{r_4 \leftrightarrow r_3}{r_3 \leftrightarrow r_2} \begin{vmatrix}
a_1 & b_1 & 0 & 0 \\
b_4 & a_4 & 0 & 0 \\
0 & 0 & a_2 & b_2 \\
0 & 0 & b_2 & a_2
\end{vmatrix} = (a_1 a_4 - b_1 b_4)(a_2 a_3 - b_2 b_3).$$

$$\begin{vmatrix} a_1 & 0 & 0 & b_1 \\ 0 & a_2 & b_2 & 0 \\ 0 & b_3 & a_3 & 0 \\ b_4 & 0 & 0 & a_4 \end{vmatrix}$$

原式 
$$\frac{c_4\leftrightarrow c_3}{c_3\leftrightarrow c_2}$$
  $\begin{vmatrix} a_1 & b_1 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_3 & a_3 \\ b_4 & a_4 & 0 & 0 \end{vmatrix}$ 

$$\frac{\frac{r_4 \leftrightarrow r_3}{r_3 \leftrightarrow r_2}}{\begin{vmatrix} a_1 & b_1 & 0 & 0 \\ b_4 & a_4 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & b_2 & a_3 \end{vmatrix}}{= (a_1 a_4 - b_1 b_4)(a_2 a_3 - b_2 b_3).$$

计算

原式 = 
$$\begin{bmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 1 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 3 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 2 & 2 & 2 & \cdots & n-1 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & n \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{bmatrix}$$

计算

$$\begin{vmatrix} 1 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 3 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 2 & \cdots & n-1 & 2 \\ 2 & 2 & 2 & \cdots & 2 & n \end{vmatrix}$$

原式 = 
$$\begin{bmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 1 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 3 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 2 & 2 & 2 & \cdots & n-1 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & n \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{bmatrix}$$

原式 = 
$$\begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 1 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 3 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 2 & 2 & 2 & \cdots & n-1 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & n \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$

计算

$$\begin{vmatrix} 1 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 2 & \cdots & 2 & 2 \\ 2 & 2 & 3 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 2 & \cdots & n-1 & 2 \\ 2 & 2 & 2 & \cdots & 2 & n \end{vmatrix}$$

原式 = 
$$\begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 1 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & 2 & 3 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 2 & 2 & 2 & \cdots & n-1 & 2 \\ 0 & 2 & 2 & 2 & \cdots & 2 & n \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$

原式 = 
$$\begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$
$$= (-1)^{3+1}(-1) \begin{vmatrix} 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & n-3 & 0 \\ 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$
$$= -2(n-2)!$$

原式 = 
$$\begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$
$$= (-1)^{3+1}(-1) \begin{vmatrix} 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & n-3 & 0 \\ 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$

原式 = 
$$\begin{vmatrix} 1 & 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & n-3 & 0 \\ -1 & 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$
$$= (-1)^{3+1}(-1) \begin{vmatrix} 2 & 2 & 2 & \cdots & 2 & 2 \\ -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & n-3 & 0 \\ 0 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix}$$
$$= -2(n-2)!$$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ a & a-1 & a-2 & \cdots & a-n \\ a^2 & (a-1)^2 & (a-2)^2 & \cdots & (a-n)^2 \\ \vdots & \vdots & \vdots & & \vdots \\ a^n & (a-1)^n & (a-2)^n & \cdots & (a-n)^n \end{vmatrix}$$

解:该行列式为范德蒙行列式,故

原式 
$$= \prod_{n \ge i > j \ge 0} [(a - i) - (a - j)]$$
$$= \prod_{n \ge i > j \ge 0} (j - i)$$
$$= (-1)^{\frac{n(n+1)}{2}} \prod_{n \ge i > j \ge 0} (i - j)$$
$$= (-1)^{\frac{n(n+1)}{2}} \prod_{i=1}^{n} i!$$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ a & a-1 & a-2 & \cdots & a-n \\ a^2 & (a-1)^2 & (a-2)^2 & \cdots & (a-n)^2 \\ \vdots & \vdots & \vdots & & \vdots \\ a^n & (a-1)^n & (a-2)^n & \cdots & (a-n)^n \end{vmatrix}$$

解:该行列式为范德蒙行列式,故

原式 
$$= \prod_{n \ge i > j \ge 0} [(a - i) - (a - j)]$$

$$= \prod_{n \ge i > j \ge 0} (j - i)$$

$$= (-1)^{\frac{n(n+1)}{2}} \prod_{n \ge i > j \ge 0} (i - j)$$

$$= (-1)^{\frac{n(n+1)}{2}} \prod_{i=1}^{n} i!$$

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ a & a-1 & a-2 & \cdots & a-n \\ a^2 & (a-1)^2 & (a-2)^2 & \cdots & (a-n)^2 \\ \vdots & \vdots & \vdots & & \vdots \\ a^n & (a-1)^n & (a-2)^n & \cdots & (a-n)^n \end{vmatrix}$$

#### 解: 该行列式为范德蒙行列式,故

原式 
$$= \prod_{n \ge i > j \ge 0} [(a - i) - (a - j)]$$

$$= \prod_{n \ge i > j \ge 0} (j - i)$$

$$= (-1)^{\frac{n(n+1)}{2}} \prod_{n \ge i > j \ge 0} (i - j)$$

$$= (-1)^{\frac{n(n+1)}{2}} \prod_{i=1}^{n} i!$$

$$\begin{vmatrix} a_1^n & a_1^{n-1}b_1 & a_1^{n-2}b_1^2 & \cdots & a_1b_1^{n-1} & b_1^n \\ a_2^n & a_2^{n-1}b_2 & a_2^{n-2}b_2^2 & \cdots & a_2b_2^{n-1} & b_2^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n+1}^n & a_{n+1}^{n-1}b_{n+1} & a_{n+1}^{n-2}b_{n+1}^2 & \cdots & a_{n+1}b_{n+1}^{n-1} & b_{n+1}^n \end{vmatrix}$$

$$\begin{vmatrix} a_1^n & a_1^{n-1}b_1 & a_1^{n-2}b_1^2 & \cdots & a_1b_1^{n-1} & b_1^n \\ a_2^n & a_2^{n-1}b_2 & a_2^{n-2}b_2^2 & \cdots & a_2b_2^{n-1} & b_2^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n+1}^n & a_{n+1}^{n-1}b_{n+1} & a_{n+1}^{n-2}b_{n+1}^2 & \cdots & a_{n+1}b_{n+1}^{n-1} & b_{n+1}^n \end{vmatrix}$$

原式 
$$= a_1^n a_2^n \cdots a_{n+1}^n \begin{vmatrix} 1 & a_1^{-1}b_1 & (a_1^{-1}b_1)^2 & \cdots & (a_1^{-1}b_1)^{n-1} & (a_1^{-1}b_1)^n \\ 1 & a_2^{-1}b_2 & (a_2^{-1}b_2)^2 & \cdots & (a_2^{-1}b_2)^{n-1} & (a_2^{-1}b_2)^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & a_{n+1}^{-1}b_{n+1} & (a_{n+1}^{-1}b_{n+1})^2 & \cdots & (a_{n+1}^{-1}b_{n+1})^{n-1} & (a_{n+1}^{-1}b_{n+1}) \end{vmatrix}$$

$$= a_1^n a_2^n \cdots a_{n+1}^n \prod_{n+1 \ge i > j \ge 1} \left( \frac{b_i}{a_i} - \frac{b_j}{a_j} \right)$$

$$= \prod_{n+1 \ge i > j \ge 1} (b_i a_j - a_i b_j)$$

$$\begin{vmatrix} a_1^n & a_1^{n-1}b_1 & a_1^{n-2}b_1^2 & \cdots & a_1b_1^{n-1} & b_1^n \\ a_2^n & a_2^{n-1}b_2 & a_2^{n-2}b_2^2 & \cdots & a_2b_2^{n-1} & b_2^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n+1}^n & a_{n+1}^{n-1}b_{n+1} & a_{n+1}^{n-2}b_{n+1}^2 & \cdots & a_{n+1}b_{n+1}^{n-1} & b_{n+1}^n \end{vmatrix}$$

原式 
$$= a_1^n a_2^n \cdots a_{n+1}^n$$
  $\begin{vmatrix} 1 & a_1^{-1} b_1 & (a_1^{-1} b_1)^2 & \cdots & (a_1^{-1} b_1)^{n-1} & (a_1^{-1} b_1)^n \\ 1 & a_2^{-1} b_2 & (a_2^{-1} b_2)^2 & \cdots & (a_2^{-1} b_2)^{n-1} & (a_2^{-1} b_2)^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & a_{n+1}^{-1} b_{n+1} & (a_{n+1}^{-1} b_{n+1})^2 & \cdots & (a_{n+1}^{-1} b_{n+1})^{n-1} & (a_{n+1}^{-1} b_{n+1})^n \end{vmatrix}$ 
 $= a_1^n a_2^n \cdots a_{n+1}^n \prod_{n+1 \ge i > j \ge 1} \left( \frac{b_i}{a_i} - \frac{b_j}{a_j} \right)$ 
 $= \prod_{n+1 \ge i > j \ge 1} (b_i a_j - a_i b_j)$ 

原式 
$$= a_1^n a_2^n \cdots a_{n+1}^n \begin{vmatrix} 1 & a_1^{-1} b_1 & (a_1^{-1} b_1)^2 & \cdots & (a_1^{-1} b_1)^{n-1} & (a_1^{-1} b_1)^n \\ 1 & a_2^{-1} b_2 & (a_2^{-1} b_2)^2 & \cdots & (a_2^{-1} b_2)^{n-1} & (a_2^{-1} b_2)^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & a_{n+1}^{-1} b_{n+1} & (a_{n+1}^{-1} b_{n+1})^2 & \cdots & (a_{n+1}^{-1} b_{n+1})^{n-1} & (a_{n+1}^{-1} b_{n+1})^n \end{vmatrix}$$

$$= a_1^n a_2^n \cdots a_{n+1}^n \prod_{n+1 \ge i > j \ge 1} \left( \frac{b_i}{a_i} - \frac{b_j}{a_j} \right)$$

$$= \prod_{n+1 \ge i > j \ge 1} (b_i a_j - a_i b_j)$$

◆□▶ ◆□▶ ◆■▶ ◆■▶ ● 釣९○

$$\begin{vmatrix} a_1^n & a_1^{n-1}b_1 & a_1^{n-2}b_1^2 & \cdots & a_1b_1^{n-1} & b_1^n \\ a_2^n & a_2^{n-1}b_2 & a_2^{n-2}b_2^2 & \cdots & a_2b_2^{n-1} & b_2^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n+1}^n & a_{n+1}^{n-1}b_{n+1} & a_{n+1}^{n-2}b_{n+1}^2 & \cdots & a_{n+1}b_{n+1}^{n-1} & b_{n+1}^n \end{vmatrix}$$

原式 
$$= a_1^n a_2^n \cdots a_{n+1}^n \begin{vmatrix} 1 & a_1^{-1}b_1 & (a_1^{-1}b_1)^2 & \cdots & (a_1^{-1}b_1)^{n-1} & (a_1^{-1}b_1)^n \\ 1 & a_2^{-1}b_2 & (a_2^{-1}b_2)^2 & \cdots & (a_2^{-1}b_2)^{n-1} & (a_2^{-1}b_2)^n \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 1 & a_{n+1}^{-1}b_{n+1} & (a_{n+1}^{-1}b_{n+1})^2 & \cdots & (a_{n+1}^{-1}b_{n+1})^{n-1} & (a_{n+1}^{-1}b_{n+1})^n \end{vmatrix}$$

$$= a_1^n a_2^n \cdots a_{n+1}^n \prod_{n+1 \ge i > j \ge 1} \left( \frac{b_i}{a_i} - \frac{b_j}{a_j} \right)$$

$$= \prod_{n+1 \ge i > j \ge 1} (b_i a_j - a_i b_j)$$

用克拉默法则求

$$\begin{cases} 5x_1 & + 4x_3 + 2x_4 = 3, \\ x_1 - x_2 + 2x_3 + x_4 = 1, \\ 4x_1 + x_2 + 2x_3 & = 1, \\ x_1 + x_2 + x_3 + x_4 = 0. \end{cases}$$

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_{1} = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$\frac{r_1 - 2r_2}{r_3 - 2r_2} - \begin{vmatrix} -1 & -4 & 0 \\ 2 & 4 & 1 \\ -3 & -5 & 0 \end{vmatrix} = -(-1)^{2+3} \cdot \begin{vmatrix} -1 & -4 \\ -3 & -5 \end{vmatrix} = -7,$$

用克拉默法则求

$$\begin{cases} 5x_1 & + 4x_3 + 2x_4 = 3, \\ x_1 - x_2 + 2x_3 + x_4 = 1, \\ 4x_1 + x_2 + 2x_3 & = 1, \\ x_1 + x_2 + x_3 + x_4 = 0. \end{cases}$$

$$\mathbf{D} = \begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = -7,$$

$$D_1 = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$\frac{r_1 - 2r_2}{r_3 - 2r_2} - \begin{vmatrix} -1 & -4 & 0 \\ 2 & 4 & 1 \\ -3 & -5 & 0 \end{vmatrix} = -(-1)^{2+3} \cdot \begin{vmatrix} -1 & -4 \\ -3 & -5 \end{vmatrix} = -7,$$

用克拉默法则求

$$5x_1$$
 +  $4x_3$  +  $2x_4$  =  $3x_1$   
 $x_1$  -  $x_2$  +  $2x_3$  +  $x_4$  =  $1x_1$   
 $4x_1$  +  $x_2$  +  $2x_3$  =  $1x_1$   
 $x_1$  +  $x_2$  +  $x_3$  +  $x_4$  =  $0x_1$ 

解:

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_1 = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$\begin{vmatrix} \frac{r_1 - 2r_2}{r_3 - 2r_2} - \begin{vmatrix} -1 & -4 & 0 \\ 2 & 4 & 1 \\ -3 & -5 & 0 \end{vmatrix} = -(-1)^{2+3} \cdot \begin{vmatrix} -1 & -4 \\ -3 & -5 \end{vmatrix} = -7,$$

张晓平

用克拉默法则求

$$\begin{pmatrix} 5x_1 & + & 4x_3 & + & 2x_4 & = & 3, \\ x_1 & - & x_2 & + & 2x_3 & + & x_4 & = & 1, \\ 4x_1 & + & x_2 & + & 2x_3 & & = & 1, \\ x_1 & + & x_2 & + & x_3 & + & x_4 & = & 0. \end{pmatrix}$$

解:

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$\mathbf{D_1} \quad = \left| \begin{array}{ccc|c} \mathbf{3} & 0 & 4 & 2 \\ \mathbf{1} & -1 & 2 & 1 \\ \mathbf{1} & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right| \left| \begin{array}{c} \mathbf{r_3 + r_2} \\ \mathbf{r_4 + r_2} \end{array} \right| \left| \begin{array}{ccc|c} \mathbf{3} & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{array} \right| = (-1)^{2+2}(-1) \left| \begin{array}{ccc|c} \mathbf{3} & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{array} \right|$$

张晓平

用克拉默法则求

$$5x_1 + 4x_3 + 2x_4 = 3,$$

$$x_1 - x_2 + 2x_3 + x_4 = 1,$$

$$4x_1 + x_2 + 2x_3 = 1,$$

$$x_1 + x_2 + x_3 + x_4 = 0.$$

解:

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_{1} = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$\frac{r_1 - 2r_2}{r_3 - 2r_2} - \begin{vmatrix} -1 & -4 & 0 \\ 2 & 4 & 1 \\ -3 & -5 & 0 \end{vmatrix} = -(-1)^{2+3} \cdot \begin{vmatrix} -1 & -4 \\ -3 & -5 \end{vmatrix} = -7,$$

张晓平

用克拉默法则求

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_1 = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

用克拉默法则求

$$\begin{cases} 5x_1 & + 4x_3 + 2x_4 = 3, \\ x_1 - x_2 + 2x_3 + x_4 = 1, \\ 4x_1 + x_2 + 2x_3 & = 1, \\ x_1 + x_2 + x_3 + x_4 = 0. \end{cases}$$

解:

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_{1} = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} (-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

张晓平

用克拉默法则求

$$\begin{cases} 5x_1 & + 4x_3 + 2x_4 = 3, \\ x_1 - x_2 + 2x_3 + x_4 = 1, \\ 4x_1 + x_2 + 2x_3 & = 1, \\ x_1 + x_2 + x_3 + x_4 = 0. \end{cases}$$

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_{1} = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$\frac{r_1 - 2r_2}{r_3 - 2r_2} - \begin{vmatrix} -1 & -4 & 0 \\ 2 & 4 & 1 \\ -3 & -5 & 0 \end{vmatrix} = -(-1)^{2+3} \cdot \begin{vmatrix} -1 & -4 \\ -3 & -5 \end{vmatrix} = -7,$$

用克拉默法则求

$$\begin{cases} 5x_1 & + 4x_3 + 2x_4 = 3, \\ x_1 - x_2 + 2x_3 + x_4 = 1, \\ 4x_1 + x_2 + 2x_3 & = 1, \\ x_1 + x_2 + x_3 + x_4 = 0. \end{cases}$$

$$D = \begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -7,$$

$$D_{1} = \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 3 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 0 & 4 & 1 \\ 1 & 0 & 3 & 2 \end{vmatrix} = (-1)^{2+2} (-1) \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$\frac{r_1-2r_2}{r_3-2r_2} - \begin{vmatrix} -1 & -4 & 0 \\ 2 & 4 & 1 \\ -3 & -5 & 0 \end{vmatrix} = -(-1)^{2+3} \cdot \begin{vmatrix} -1 & -4 \\ -3 & -5 \end{vmatrix} = -7,$$

$$D_2 = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \underbrace{\begin{smallmatrix} c_3 - c_1 \\ c_4 - c_1 \\ 1 & 0 & 0 & 0 \end{smallmatrix}}_{= (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$

张晓平

$$D_2 = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \underbrace{\begin{bmatrix} c_3 - c_1 \\ c_4 - c_1 \end{bmatrix}}_{c_4 - c_1} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$



张晓平

线性代数

$$D_2 = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \underbrace{\begin{bmatrix} c_3 - c_1 \\ c_4 - c_1 \end{bmatrix}}_{c_4 - c_1} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$

$$D_2 = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_3 - c_1}{c_4 - c_1} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$



$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$\frac{c_1-c_2}{}-\left|\begin{array}{ccc} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & -2 & -4 \end{array}\right|=-(-1)^{2+2}\cdot\left|\begin{array}{ccc} 4 & -3 \\ 3 & -4 \end{array}\right|=7,$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ -1 & 2 & -3 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$

$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$\frac{c_1-c_2}{}-\left|\begin{array}{ccc} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & -2 & -4 \end{array}\right|=-(-1)^{2+2}\cdot\left|\begin{array}{ccc} 4 & -3 \\ 3 & -4 \end{array}\right|=7,$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 1 & 2 & -3 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$

4□ > 4□ > 4□ > 4□ > 4□ > 9

$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$\frac{c_1-c_2}{} - \left| \begin{array}{ccc} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & -2 & -4 \end{array} \right| = -(-1)^{2+2} \cdot \left| \begin{array}{ccc} 4 & -3 \\ 3 & -4 \end{array} \right| = 7,$$

$$D_3 = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\begin{vmatrix} \frac{c_1 - 2c_2}{c_3 - 2c_2} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$



$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$

$$\frac{c_1-c_2}{} - \left| \begin{array}{ccc} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & -2 & -4 \end{array} \right| = -(-1)^{2+2} \cdot \left| \begin{array}{ccc} 4 & -3 \\ 3 & -4 \end{array} \right| = 7,$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\frac{c_1 - 2c_2}{c_3 - 2c_2} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$



$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$
$$= \frac{c_{1} - c_{2}}{-2} - \begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & -2 & -4 \end{vmatrix} = -(-1)^{2+2} \cdot \begin{vmatrix} 4 & -3 \\ 3 & -4 \end{vmatrix} = 7,$$

$$D_3 = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\frac{c_1 - 2c_2}{c_3 - 2c_2} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$



$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$
$$= \frac{c_{1} - c_{2}}{c_{4} - c_{1}} - \begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & -2 & -4 \end{vmatrix} = -(-1)^{2+2} \cdot \begin{vmatrix} 4 & -3 \\ 3 & -4 \end{vmatrix} = 7,$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$



张晓平

线性代数

$$D_{2} = \begin{vmatrix} 5 & 3 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \end{vmatrix} = \frac{c_{3} - c_{1}}{c_{4} - c_{1}} \begin{vmatrix} 5 & 3 & -1 & -3 \\ 1 & 1 & 1 & 0 \\ 4 & 1 & -2 & -4 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1} \cdot \begin{vmatrix} 3 & -1 & -3 \\ 1 & 1 & 0 \\ 1 & -2 & -4 \end{vmatrix}$$
$$= \frac{c_{1} - c_{2}}{c_{4} - c_{1}} - \begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & 0 \\ 3 & 2 & 4 \end{vmatrix} = -(-1)^{2+2} \cdot \begin{vmatrix} 4 & -3 \\ 3 & -4 \end{vmatrix} = 7,$$

$$D_{3} = \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix} = \frac{r_{3} + r_{2}}{r_{4} + r_{2}} \begin{vmatrix} 5 & 0 & 3 & 2 \\ 1 & -1 & 1 & 1 \\ 5 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{vmatrix} = (-1)^{2+2}(-1) \begin{vmatrix} 5 & 3 & 2 \\ 5 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_{1} - 2c_{2}}{c_{3} - 2c_{2}} - \begin{vmatrix} -1 & 3 & -4 \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = -(-1)^{3+2} \cdot \begin{vmatrix} -1 & -4 \\ 1 & -3 \end{vmatrix} = 7$$

4 ロ ト 4 個 ト 4 差 ト 4 差 ト 2 9 9 9 0 0

$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \frac{r_3 + r_2}{r_4 + r_2} & \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} \xrightarrow{\underline{n_1 - n_2}} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7$$

$$x_1 = \frac{D_1}{D} = 1,$$
  $x_2 = \frac{D_2}{D} = -1,$ 

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1$$



$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} \xrightarrow{\underline{n_1 - n_2}} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7$$

由克拉默法则可知。

$$x_1 = \frac{D_1}{D} = 1, \quad x_2 = \frac{D_2}{D} = -1,$$

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1$$



$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$$

$$x_1 = \frac{D_1}{D} = 1, \qquad x_2 = \frac{D_2}{D} = -1,$$

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1.$$



$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \frac{r_3 + r_2}{r_4 + r_2} \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$$

$$x_1 = \frac{D_1}{D} = 1, \qquad x_2 = \frac{D_2}{D} = -1,$$

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1.$$



$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \underbrace{\frac{r_3 + r_2}{r_4 + r_2}}_{r_4 + r_2} \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \frac{r_1 - r_2}{} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$$

$$x_1 = \frac{D_1}{D} = 1, \quad x_2 = \frac{D_2}{D} = -1,$$

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1.$$



$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \underbrace{\frac{r_3 + r_2}{r_4 + r_2}}_{r_4 + r_2} \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \frac{r_1 - r_2}{\phantom{1}} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$$

$$x_1 = \frac{D_1}{D} = 1,$$
  $x_2 = \frac{D_2}{D} = -1,$ 

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1.$$



$$D_4 = \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} r_3 + r_2 \\ r_4 + r_2 \end{vmatrix} \begin{vmatrix} 5 & 0 & 4 & 3 \\ 1 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \\ 2 & 0 & 3 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 5 & 4 & 3 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \frac{r_1 - r_2}{-1} - \begin{vmatrix} 0 & 0 & 1 \\ 5 & 4 & 2 \\ 2 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = -7.$$

$$x_1 = \frac{D_1}{D} = 1, \quad x_2 = \frac{D_2}{D} = -1,$$

$$x_3 = \frac{D_3}{D} = -1, \quad x_4 = \frac{D_4}{D} = 1.$$



## 用克拉默法则求

$$\begin{cases} x_2 + x_3 + x_4 + x_5 = 1, \\ x_1 + x_3 + x_4 + x_5 = 2, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 3, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 4, \\ x_1 + x_2 + x_3 + x_4 = 5. \end{cases}$$

$$D = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[r_1+r_2+\dots+r_5]{r_1+r_2+\dots+r_5}} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

$$= \frac{r_i-r_1}{i=2,\dots,4}} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{vmatrix} = 4.$$

32

用克拉默法则求

$$\begin{cases} x_2 + x_3 + x_4 + x_5 = 1, \\ x_1 + x_3 + x_4 + x_5 = 2, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 3, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 4, \\ x_1 + x_2 + x_3 + x_4 = 5. \end{cases}$$

$$D = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow{r_1 + r_2 + \dots + r_5} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

$$= \frac{r_1 - r_1}{i = 2, \dots, 4} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 4.$$

32

用克拉默法则求

$$\begin{cases} x_2 + x_3 + x_4 + x_5 = 1, \\ x_1 + x_3 + x_4 + x_5 = 2, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 3, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 4, \\ x_1 + x_2 + x_3 + x_4 = 5. \end{cases}$$

$$D = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow{r_1 + r_2 + \dots + r_5} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

$$= \frac{r_1 - r_1}{I = 2, \dots, 4} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{vmatrix} = 4.$$

32

用克拉默法则求

$$\begin{cases} x_2 + x_3 + x_4 + x_5 = 1, \\ x_1 + x_3 + x_4 + x_5 = 2, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 3, \\ x_1 + x_2 + x_3 + x_4 + x_5 = 4, \\ x_1 + x_2 + x_3 + x_4 = 5. \end{cases}$$

$$D = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix} = \underbrace{\frac{r_1 + r_2 + \dots + r_5}{r_1 \div 4}}_{\substack{r_1 \div 4}} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$
$$= \underbrace{\frac{r_i - r_1}{i = 2, \dots, 4}}_{\substack{i = 2, \dots, 4}} 4 \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix}}_{\substack{0 \ 0 \ 0 \ 0 \ 0 \ 0}} = 4.$$

$$D_{1} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow{\begin{smallmatrix} 3-1 \\ 4-1 \\ 5-1 \end{smallmatrix}} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \begin{vmatrix} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ -1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{vmatrix}}_{= 11}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} r_3 - r_1 \\ \frac{r_3 - r_1}{r_5 - r_1} \\ \frac{r_3 - r_1}{r_5 - r_1} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \begin{vmatrix} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{vmatrix}}_{= 11}$$

$$D_2 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 3 & 0 & 1 & 1 \ 1 & 4 & 1 & 0 & 1 \ 1 & 5 & 1 & 1 & 0 \ \end{bmatrix} egin{bmatrix} r_3 - r_2 \ r_4 - r_2 \ r_5 - r_2 \ \end{bmatrix} egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 0 & 1 & -1 & 0 & 0 \ 0 & 2 & 0 & -1 & 0 \ 0 & 3 & 0 & 0 & -1 \ \end{bmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \underbrace{r_1 + r_2 + r_3 + r_4}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

4 = → ← 를 → ← 를 → ● ● → 9

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} r_3 - r_1 \\ \frac{r_4 - r_1}{r_5 - r_1} \\ \frac{r_5 - r_1}{r_5 - r_1} \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \begin{vmatrix} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{vmatrix} = \underbrace{{}^{r_1+r_2+r_3+r_4}}_{r_1+r_2+r_3+r_4} - \begin{vmatrix} 11 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{vmatrix} = 11$$

$$D_2 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 3 & 0 & 1 & 1 \ 1 & 4 & 1 & 0 & 1 \ 1 & 5 & 1 & 1 & 0 \ \end{bmatrix} egin{bmatrix} rac{r_3 - r_2}{r_4 - r_2} & 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 & 1 \ 0 & 1 & -1 & 0 & 0 \ 0 & 2 & 0 & -1 & 0 \ 0 & 3 & 0 & 0 & -1 \ \end{bmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \underbrace{r_1 + r_2 + r_3 + r_4}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[r_5 - r_1]{r_3 - r_1} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \left| \begin{array}{ccccc} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| \xrightarrow{r_1 + r_2 + r_3 + r_4} - \left| \begin{array}{cccccc} 11 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = 11,$$

$$D_2 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 3 & 0 & 1 & 1 \ 1 & 4 & 1 & 0 & 1 \ 1 & 5 & 1 & 1 & 0 \ \end{bmatrix} egin{bmatrix} r_3 - r_2 \ r_4 - r_2 \ r_5 - r_2 \ \end{bmatrix} egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 0 & 1 & -1 & 0 & 0 \ 0 & 2 & 0 & -1 & 0 \ 0 & 3 & 0 & 0 & -1 \ \end{bmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \underbrace{\frac{r_1 + r_2 + r_3 + r_4}{r_1 + r_2 + r_3 + r_4}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

<□ > <□ > <□ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[r_5 - r_1]{r_3 - r_1} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \left| \begin{array}{ccccc} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| \xrightarrow{r_1 + r_2 + r_3 + r_4} - \left| \begin{array}{cccccc} 11 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = 11,$$

$$D_2 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 3 & 0 & 1 & 1 \ 1 & 4 & 1 & 0 & 1 \ 1 & 5 & 1 & 1 & 0 \ \end{bmatrix} egin{bmatrix} r_3 - r_2 \ r_4 - r_2 \ r_5 - r_2 \ \end{bmatrix} egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 1 & 2 & 1 & 1 & 1 \ 0 & 1 & -1 & 0 & 0 \ 0 & 2 & 0 & -1 & 0 \ 0 & 3 & 0 & 0 & -1 \ \end{bmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \underbrace{\frac{r_1 + r_2 + r_3 + r_4}{r_4 + r_4 + r_4}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[r_5 - r_1]{r_3 - r_1} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \left| \begin{array}{ccccc} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \end{smallmatrix}}_{r_1 + r_2 + r_3 + r_4} - \left| \begin{array}{cccccc} 11 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = 11,$$

$$D_2 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 \\ 1 & 4 & 1 & 0 & 1 \\ 1 & 5 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 3 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \frac{r_1 + r_2 + r_3 + r_4}{r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

<□ > <□ > <□ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[r_5 - r_1]{r_3 - r_1} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \left| \begin{array}{ccccc} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \end{smallmatrix}}_{r_1 + r_2 + r_3 + r_4} - \left| \begin{array}{cccccc} 11 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = 11,$$

$$D_2 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 \\ 1 & 4 & 1 & 0 & 1 \\ 1 & 5 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} r_3 - r_2 \\ r_4 - r_2 \\ r_5 - r_2 \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 3 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \underbrace{\frac{r_1 + r_2 + r_3 + r_4}{r_1 + r_2 + r_3 + r_4}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = 7.$$

←□ → ←□ → ← □ → ← □ → □ = → ○ ○

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow[r_5 - r_1]{r_3 - r_1} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 \\ 1 & 4 & 1 & 0 & 1 \\ 1 & 5 & 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} r_3 - r_2 \\ r_4 - r_2 \\ r_5 - r_2 \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 3 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 0 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 7 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \end{vmatrix} = 7.$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 1 & 0 \end{vmatrix} \xrightarrow{r_3 - r_1} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 0 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot \left| \begin{array}{ccccc} 2 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \end{smallmatrix}}_{r_1 + r_2 + r_3 + r_4} - \left| \begin{array}{cccccc} 11 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 3 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \end{array} \right| = 11,$$

$$D_{2} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 3 & 0 & 1 & 1 \\ 1 & 4 & 1 & 0 & 1 \\ 1 & 5 & 1 & 1 & 0 \end{vmatrix} \xrightarrow{r_{3}-r_{2}} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 3 & 0 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{2+1} \cdot \left| \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{array} \right| \xrightarrow{\underline{r_1 + r_2 + r_3 + r_4}} - \left| \begin{array}{ccccc} 7 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 0 & -1 \end{array} \right| = 7.$$

$$D_{3} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} \xrightarrow{\substack{r_{2}-r_{3} \\ r_{4}-r_{3}}} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \left| \begin{array}{ccccc} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right| \xrightarrow{\underline{r_1 + r_2 + r_3 + r_4}} \left| \begin{array}{cccccc} 0 & 3 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right| = 3$$

$$D_4 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 0 & 1 & 2 & 1 \ 1 & 1 & 0 & 3 & 1 \ 1 & 1 & 1 & 4 & 1 \ 1 & 1 & 5 & 0 \ \end{bmatrix} egin{bmatrix} r_2 - r_4 \ r_5 - r_4 \ r_5 - r_4 \ \end{bmatrix} egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 0 & -1 & 0 & -2 & 0 \ 1 & 1 & 0 & -1 & -1 \ 0 & 0 & 0 & 1 & -1 \ \end{bmatrix}$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1$$

线性代数

$$D_{3} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} \xrightarrow{r_{2}-r_{3}} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$D_4 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 5 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{vmatrix}$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1$$

10/10/12/12/

$$D_3 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \frac{r_2 - r_3}{r_5 - r_3} \\ \frac{r_4 - r_3}{r_5 - r_3} \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{vmatrix}}_{ \begin{array}{c} 0 & 3 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} | = 3$$

$$D_4 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 0 & 1 & 2 & 1 \ 1 & 1 & 0 & 3 & 1 \ 1 & 1 & 1 & 4 & 1 \ 1 & 1 & 1 & 5 & 0 \ \end{bmatrix} egin{bmatrix} r_2 - r_4 \ r_3 - r_4 \ r_5 - r_4 \ \end{bmatrix} egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 0 & 0 & -1 & -1 & 0 \ 1 & 1 & 1 & 4 & 1 \ 0 & 0 & 0 & 1 & -1 \ \end{bmatrix}$$

线性代数

$$D_3 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \frac{r_2 - r_3}{r_5 - r_3} \\ \frac{r_4 - r_3}{r_5 - r_3} \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \left| \begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right| \xrightarrow{r_1 + r_2 + r_3 + r_4} \left| \begin{array}{cccccc} 0 & 3 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right| = 3$$

$$D_4 = egin{bmatrix} 0 & 1 & 1 & 1 & 1 \ 1 & 0 & 1 & 2 & 1 \ 1 & 1 & 0 & 3 & 1 \ 1 & 1 & 1 & 4 & 1 \ 1 & 1 & 1 & 5 & 0 \ \end{bmatrix} egin{bmatrix} -\frac{r_2-r_4}{r_3-r_4} & 0 & -1 & 0 & -2 & 0 \ 0 & 0 & -1 & -1 & 0 \ 1 & 1 & 1 & 4 & 1 \ 0 & 0 & 0 & 1 & -1 \ \end{bmatrix}$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1$$

线性代数

$$D_3 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \frac{r_2 - r_3}{r_5 - r_3} \\ \frac{r_4 - r_3}{r_5 - r_3} \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \left| \begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right| \xrightarrow{r_1 + r_2 + r_3 + r_4} \left| \begin{array}{cccccc} 0 & 3 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right| = 3$$

$$D_{4} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 5 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{vmatrix}$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1$$

线性代数 154 / 190

$$D_{3} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} \xrightarrow{r_{2}-r_{3}} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ \frac{r_{2}-r_{3}}{r_{5}-r_{3}} & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{vmatrix} = \underbrace{\frac{r_1 + r_2 + r_3 + r_4}{r_1 + r_2 + r_3 + r_4}}_{\begin{array}{c} 0 & 3 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{vmatrix} = 3$$

$$D_4 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 5 & 0 \end{vmatrix} \xrightarrow{r_2 - r_4} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{vmatrix}$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1$$

线性代数

$$D_3 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} = \begin{vmatrix} \frac{r_2 - r_3}{r_5 - r_3} \\ \frac{r_4 - r_3}{r_5 - r_3} \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{vmatrix} = \underbrace{\frac{r_1 + r_2 + r_3 + r_4}{r_1 + r_2 + r_3 + r_4}}_{\begin{array}{c} 1 & 1 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{vmatrix} = 3$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1$$

线性代数

154 / 190

$$D_{3} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} \xrightarrow{r_{2}-r_{3}} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ \frac{r_{4}-r_{3}}{r_{5}-r_{3}} & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \left| \begin{array}{cccc} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right| \xrightarrow{r_1 + r_2 + r_3 + r_4} \left| \begin{array}{ccccc} 0 & 3 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right| = 3,$$

$$D_4 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 5 & 0 \end{vmatrix} = \begin{vmatrix} r_2 - r_4 \\ \frac{r_3 - r_4}{r_5 - r_4} \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{vmatrix}$$

$$= (-1)^{4+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{r_1 + r_2 + r_3 + r_4} - \begin{vmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = -1.$$

线性代数

154 / 190

$$D_{3} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 0 & 1 \\ 1 & 1 & 5 & 1 & 0 \end{vmatrix} \xrightarrow{r_{2}-r_{3}} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ \frac{r_{2}-r_{3}}{r_{5}-r_{3}} & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 & -1 \end{vmatrix}$$

$$= (-1)^{1+3} \cdot \left| \begin{array}{cccc} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right| \xrightarrow{r_1 + r_2 + r_3 + r_4} \left| \begin{array}{ccccc} 0 & 3 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right| = 3,$$

$$D_4 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 3 & 1 \\ 1 & 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 5 & 0 \end{vmatrix} = \begin{vmatrix} r_2 - r_4 \\ \frac{r_3 - r_4}{r_5 - r_4} \\ -r_5 - r_4 \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{vmatrix}$$

$$=(-1)^{4+1} \cdot \left| egin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \ -1 & 0 & -2 & 0 \ 0 & -1 & -1 & 0 \ 0 & 0 & 1 & -1 \end{array} 
ight| = rac{r_1 + r_2 + r_3 + r_4}{2} - \left| egin{array}{ccccc} 0 & 0 & -1 & 0 \ -1 & 0 & -2 & 0 \ 0 & -1 & -1 & 0 \ 0 & 0 & 1 & -1 \end{array} 
ight| = -1.$$

10/10/12/12/

$$D_5 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 0 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix} \begin{vmatrix} r_2 - r_5 \\ r_3 - r_5 \\ r_4 - r_5 \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & -1 & -1 \\ 1 & 1 & 1 & 5 \end{vmatrix}$$

$$= (-1)^{5+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix}}_{r_1 + r_2 + r_3 + r_4} = \begin{vmatrix} 0 & 0 & 0 & -5 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = -5$$

由克拉默法则可知

$$x_1 = \frac{D_1}{D} = \frac{11}{4}, \quad x_2 = \frac{D_2}{D} = \frac{7}{4}, \quad x_3 = \frac{D_3}{D} = \frac{3}{4},$$
 $x_4 = \frac{D_4}{D} = -\frac{1}{4}, \quad x_5 = \frac{D_5}{D} = -\frac{5}{4}.$ 



$$D_5 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 0 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix} \begin{vmatrix} r_2 - r_5 \\ \frac{r_3 - r_5}{r_4 - r_5} \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix}$$

$$= (-1)^{5+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & -5 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = -5.$$

由克拉默法则可知

$$x_1 = \frac{D_1}{D} = \frac{11}{4}, \quad x_2 = \frac{D_2}{D} = \frac{7}{4}, \quad x_3 = \frac{D_3}{D} = \frac{3}{4}$$
  
 $x_4 = \frac{D_4}{D} = -\frac{1}{4}, \quad x_5 = \frac{D_5}{D} = -\frac{5}{4}.$ 



$$D_5 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 0 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix} \begin{vmatrix} \frac{r_2 - r_5}{r_3 - r_5} \\ \frac{r_3 - r_5}{r_4 - r_5} \end{vmatrix} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix}$$

$$= (-1)^{5+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 + r_2 + r_3 + r_4 \\ 0 & 0 & 0 & -1 & -1 \end{smallmatrix}}_{r_1 + r_2 + r_3 + r_4} \begin{vmatrix} 0 & 0 & 0 & -5 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = -5.$$

由克拉默法则可知

$$x_1 = \frac{D_1}{D} = \frac{11}{4}, \quad x_2 = \frac{D_2}{D} = \frac{7}{4}, \quad x_3 = \frac{D_3}{D} = \frac{3}{4}$$
  
 $x_4 = \frac{D_4}{D} = -\frac{1}{4}, \quad x_5 = \frac{D_5}{D} = -\frac{5}{4}.$ 



$$D_{5} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 0 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & 5 \end{vmatrix}$$

$$= (-1)^{5+1} \cdot \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = \begin{vmatrix} r_{1}+r_{2}+r_{3}+r_{4} \\ -1 & 0 & 0 & -3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -1 \end{vmatrix} = -5.$$

由克拉默法则可知,

$$x_1 = \frac{D_1}{D} = \frac{11}{4}, \quad x_2 = \frac{D_2}{D} = \frac{7}{4}, \quad x_3 = \frac{D_3}{D} = \frac{3}{4},$$
  
 $x_4 = \frac{D_4}{D} = -\frac{1}{4}, \quad x_5 = \frac{D_5}{D} = -\frac{5}{4}.$ 



#### 齐次线性方程组

$$\begin{cases} x_1 + x_2 + x_3 + ax_4 = 0 \\ x_1 + 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_1 + x_2 + ax_3 + bx_4 = 0 \end{cases}$$

有非零解时, a, b必须满足什么条件?

注

齐次线性方程组有非零解的充分必要条件是系数行列式为零。

解:

$$D = \begin{vmatrix} 1 & 1 & 1 & a \\ 1 & 2 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & a & b \end{vmatrix} \xrightarrow{\underline{r_1 \leftrightarrow r_3}} \begin{vmatrix} 1 & 1 & -3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & a \\ 1 & 1 & a & b \end{vmatrix} \xrightarrow{\underline{r_2 - r_1}} \begin{vmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 4 & a - 1 \\ 0 & 0 & a + 3 & b - 1 \end{vmatrix} = 0,$$

即 4(b-1)-(a-1)(a+3)=0,也就是  $(a-1)^2=4b_{a-1}$  人間  $(a-1)^2=4b_{a$ 

齐次线性方程组

$$\begin{cases} x_1 + x_2 + x_3 + ax_4 = 0 \\ x_1 + 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_1 + x_2 + ax_3 + bx_4 = 0 \end{cases}$$

有非零解时, a, b必须满足什么条件?

注

齐次线性方程组有非零解的充分必要条件是系数行列式为零。

解:

$$D = \begin{vmatrix} 1 & 1 & 1 & a \\ 1 & 2 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & a & b \end{vmatrix} = \underbrace{\begin{matrix} r_1 \leftrightarrow r_3 \\ }_{r_1 \leftrightarrow r_3} \begin{vmatrix} 1 & 1 & -3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & a \\ 1 & 1 & a & b \end{vmatrix}}_{r_2 - r_1} \begin{vmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 4 & a - 1 \\ 0 & 0 & a + 3 & b - 1 \end{vmatrix} = 0,$$

齐次线性方程组

$$\begin{cases} x_1 + x_2 + x_3 + ax_4 = 0 \\ x_1 + 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_1 + x_2 + ax_3 + bx_4 = 0 \end{cases}$$

有非零解时, a, b必须满足什么条件?

注

齐次线性方程组有非零解的充分必要条件是系数行列式为零。

解:

$$D = \begin{vmatrix} 1 & 1 & 1 & a \\ 1 & 2 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & a & b \end{vmatrix} = \underbrace{\begin{smallmatrix} r_1 \leftrightarrow r_3 \\ -r_2 \leftrightarrow r_3 \\ -r_3 & -r_4 \\ 1 & 1 & a & b \end{vmatrix}}_{\begin{array}{c} r_2 - r_1 \\ -r_3 - r_4 \\ -r_4 - r_1 \\ -r_4 - r_1 \\ 0 & 0 & a + 3 & b - 1 \end{vmatrix} = 0,$$

齐次线性方程组

$$\begin{cases} x_1 + x_2 + x_3 + ax_4 = 0 \\ x_1 + 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_1 + x_2 + ax_3 + bx_4 = 0 \end{cases}$$

有非零解时, a, b必须满足什么条件?

注

齐次线性方程组有非零解的充分必要条件是系数行列式为零。

解:

$$D = \left| \begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 2 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & a & b \end{array} \right| = \underbrace{\begin{array}{ccc|c} r_1 \leftrightarrow r_3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & a \\ 1 & 1 & a & b \end{array}}_{\left| \begin{array}{ccc|c} r_2 - r_1 \\ r_3 - r_1 \\ \hline r_4 - r_1 \end{array} \right| = \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 4 & a - 1 \\ 0 & 0 & a + 3 & b - 1 \end{bmatrix} = 0,$$

即 4(b-1)-(a-1)(a+3)=0,也就是  $(a-1)^2=4b_{a-1}$  人間  $(a-1)^2=4b_{a$ 

齐次线性方程组

$$\begin{cases} x_1 + x_2 + x_3 + ax_4 = 0 \\ x_1 + 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 - 3x_3 + x_4 = 0 \\ x_1 + x_2 + ax_3 + bx_4 = 0 \end{cases}$$

有非零解时, a, b必须满足什么条件?

注

齐次线性方程组有非零解的充分必要条件是系数行列式为零。

解:

$$D = \left| \begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 2 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & a & b \end{array} \right| = \underbrace{ \begin{array}{c|c} r_1 \leftrightarrow r_3 \\ \hline \end{array}}_{=} \left| \begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & a \\ 1 & 1 & a & b \end{array} \right| = \underbrace{ \begin{array}{c|c} r_2 - r_1 \\ r_3 - r_1 \\ \hline \end{array}}_{r_4 - r_1} \left| \begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 4 & a - 1 \\ 0 & 0 & a + 3 & b - 1 \end{array} \right| = 0,$$

齐次线性方程组

$$\begin{cases} x_1 + x_2 + x_3 + ax_4 = 0, \\ x_1 + 2x_2 + x_3 + x_4 = 0, \\ x_1 + x_2 - 3x_3 + x_4 = 0, \\ x_1 + x_2 + ax_3 + bx_4 = 0. \end{cases}$$

有非零解时, a, b必须满足什么条件?

#### 注

齐次线性方程组有非零解的充分必要条件是系数行列式为零。

## 解:

$$D = \left| \begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 2 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & a & b \end{array} \right| \xrightarrow{\underline{r_1 \leftrightarrow r_3}} \left| \begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & a \\ 1 & 1 & a & b \end{array} \right| \xrightarrow{\underline{r_2 - r_1}}_{r_3 - r_1} \left| \begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 4 & a - 1 \\ 0 & 0 & a + 3 & b - 1 \end{array} \right| = 0,$$

即 4(b-1)-(a-1)(a+3)=0,也就是  $(a-1)^2=4b$ .

# 求平面上过两点 $(x_1, y_1)$ 和 $(x_2, y_2)$ 的直线方程(用行列式表示)。

解: 直线方程的两点式为

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1},$$

$$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$$

亦即

$$x(y_1-y_2)+y(x_2-x_1)+x_1y_2-x_2y_1=0.$$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

求平面上过两点 $(x_1, y_1)$ 和 $(x_2, y_2)$ 的直线方程(用行列式表示)。

### 解: 直线方程的两点式为

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1},$$

$$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$$

亦即

$$x(y_1-y_2)+y(x_2-x_1)+x_1y_2-x_2y_1=0.$$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

求平面上过两点 $(x_1, y_1)$ 和 $(x_2, y_2)$ 的直线方程(用行列式表示)。

解: 直线方程的两点式为

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1},$$

即

$$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$$

亦即

$$x(y_1-y_2)+y(x_2-x_1)+x_1y_2-x_2y_1=0.$$

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

求平面上过两点 $(x_1, y_1)$ 和 $(x_2, y_2)$ 的直线方程(用行列式表示)。

解: 直线方程的两点式为

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1},$$

即

$$(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$$

亦即

$$x(y_1-y_2)+y(x_2-x_1)+x_1y_2-x_2y_1=0.$$

$$\left| \begin{array}{ccc} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{array} \right| = 0.$$

求三次多项式
$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$
, 使得

$$f(-1) = 0, \ f(1) = 4, \ f(2) = 3, \ f(3) = 16.$$

解:由条件可知,f(x)应满足线性方程组

$$\begin{cases} a_0 + (-1)a_1 + (-1)^2a_2 + (-1)^3a_3 = 0 \\ a_0 + a_1 + a_2 + a_3 = 4 \\ a_0 + 2a_1 + (2)^2a_2 + (2)^3a_3 = 3 \\ a_0 + 3a_1 + (3)^2a_2 + (3)^3a_3 = 16 \end{cases}$$

其系数行列式D为范德蒙行列式

$$D = \begin{vmatrix} 1 & -1 & (-1)^2 & (-1)^3 \\ 1 & 1 & 1^2 & 1^3 \\ 1 & 2 & 2^2 & 2^3 \\ 1 & 3 & 3^2 & 3^3 \end{vmatrix} = (3+1)(3-1)(3-2)(2+1)(2-1)(1+1) = 48.$$

→ロト → □ ト → 三 ト → 三 ・ りへで

求三次多项式
$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$
, 使得

$$f(-1) = 0$$
,  $f(1) = 4$ ,  $f(2) = 3$ ,  $f(3) = 16$ .

解: 由条件可知,f(x)应满足线性方程组

$$\begin{cases} a_0 + (-1)a_1 + (-1)^2a_2 + (-1)^3a_3 = 0, \\ a_0 + a_1 + a_2 + a_3 = 4, \\ a_0 + 2a_1 + (2)^2a_2 + (2)^3a_3 = 3, \\ a_0 + 3a_1 + (3)^2a_2 + (3)^3a_3 = 16. \end{cases}$$

其系数行列式D为范德蒙行列式

$$D = \begin{vmatrix} 1 & -1 & (-1)^2 & (-1)^3 \\ 1 & 1 & 1^2 & 1^3 \\ 1 & 2 & 2^2 & 2^3 \\ 1 & 3 & 3^2 & 3^3 \end{vmatrix} = (3+1)(3-1)(3-2)(2+1)(2-1)(1+1) = 48.$$

◆ロト ◆個ト ◆差ト ◆差ト 差 めらゆ

求三次多项式
$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$
, 使得

$$f(-1) = 0$$
,  $f(1) = 4$ ,  $f(2) = 3$ ,  $f(3) = 16$ .

解:由条件可知,f(x)应满足线性方程组

$$\begin{cases} a_0 + (-1)a_1 + (-1)^2a_2 + (-1)^3a_3 = 0, \\ a_0 + a_1 + a_2 + a_3 = 4, \\ a_0 + 2a_1 + (2)^2a_2 + (2)^3a_3 = 3, \\ a_0 + 3a_1 + (3)^2a_2 + (3)^3a_3 = 16. \end{cases}$$

其系数行列式D为范德蒙行列式

$$D = \begin{vmatrix} 1 & -1 & (-1)^2 & (-1)^3 \\ 1 & 1 & 1^2 & 1^3 \\ 1 & 2 & 2^2 & 2^3 \\ 1 & 3 & 3^2 & 3^3 \end{vmatrix} = (3+1)(3-1)(3-2)(2+1)(2-1)(1+1) = 48.$$

◆ロト ◆個ト ◆差ト ◆差ト 差 めらぐ

$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{c_2 + c_3} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & c_1 - 2c_3 \\
\hline
 & c_2 - c_3 \\
\hline
 & -8 & -24 & 36
\end{array}
= 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & -1 \\ 2 & 4 & 2 & 1 \\ 9 & 3 & 12 & 8 \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_3 \\ c_4 + c_3 \end{vmatrix}}_{c_2 + c_3} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & 0 & 0 & 2 \\
\hline
 & c_1 - 2c_3 \\
\hline
 & -9 & -6 & 12 \\
 & -8 & -24 & 36
\end{array}
= 48 \times 7,$$

$$D_2 = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \underbrace{ \begin{vmatrix} c_1 + c_4 \\ c_3 + c_4 \end{vmatrix}}_{28 & 16 & 36 & 27 \end{vmatrix} = 0 \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$



$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_3}{c_4 + c_3}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & c_1 - 2c_3 \\
\hline
 & c_2 - c_3 \\
\hline
 & -8 & -24 & 36
\end{array}
= 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \begin{vmatrix} c_{1}+c_{4} \\ c_{3}+c_{4} \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$



$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_3}{c_4 + c_3}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc} \underline{-c_1-2c_3} & 0 & 0 & 2 \\ -9 & -6 & 12 \\ -8 & -24 & 36 \end{array} \bigg| = 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \begin{vmatrix} c_{1}+c_{4} \\ c_{3}+c_{4} \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$

4□ > 4□ > 4□ > 4 = > 4 = > = 900

$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_3}{c_4 + c_3}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc} \underline{-c_1-2c_3} & 0 & 0 & 2 \\ -9 & -6 & 12 \\ -8 & -24 & 36 \end{array} \bigg| = 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \begin{vmatrix} c_{1}+c_{4} \\ c_{3}+c_{4} \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$



$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_3}{c_4 + c_3}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \underbrace{\begin{bmatrix} c_{1} + c_{4} \\ c_{3} + c_{4} \end{bmatrix}}_{\begin{array}{c} c_{1} + c_{4} \\ c_{3} + c_{4} \end{array}} \begin{vmatrix} 0 & 0 & 0 & -1 \\ 2 & 4 & 2 & 1 \\ 9 & 3 & 12 & 8 \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_3}{c_4 + c_3}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc} \underline{-c_1-2c_3} & 0 & 0 & 2 \\ \hline -9 & -6 & 12 \\ -8 & -24 & 36 \end{array} \bigg| = 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_{1} + c_{4} \\ c_{3} + c_{4} \end{vmatrix}}_{c_{3} + c_{4}} \begin{vmatrix} 0 & 0 & 0 & -1 \\ 2 & 4 & 2 & 1 \\ 9 & 3 & 12 & 8 \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$



$$D_{1} = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_{2}+c_{3}}{c_{4}+c_{3}}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & c_1 - 2c_3 & 0 & 0 & 2 \\
\hline
 & -9 & -6 & 12 \\
 & -8 & -24 & 36
\end{array} = 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \underbrace{\begin{bmatrix} c_{1} + c_{4} \\ c_{3} + c_{4} \\ 28 & 16 & 36 & 27 \end{bmatrix}}_{\begin{array}{c} c_{1} + c_{4} \\ 2 & 4 & 2 & 1 \\ 9 & 3 & 12 & 8 \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$

◆ロ → ◆回 → ◆ き → ◆ き → り へ ()

$$D_{1} = \begin{vmatrix} 0 & -1 & 1 & -1 \\ 4 & 1 & 1 & 1 \\ 3 & 2 & 4 & 8 \\ 16 & 3 & 9 & 27 \end{vmatrix} \xrightarrow{\frac{c_{2}+c_{3}}{c_{4}+c_{3}}} \begin{vmatrix} 0 & 0 & 1 & 0 \\ 4 & 2 & 1 & 2 \\ 3 & 6 & 4 & 12 \\ 16 & 12 & 9 & 36 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 2 \\ 3 & 6 & 12 \\ 16 & 12 & 36 \end{vmatrix}$$

$$\frac{c_1 - 2c_3}{c_2 - c_3} \begin{vmatrix}
0 & 0 & 2 \\
-9 & -6 & 12 \\
-8 & -24 & 36
\end{vmatrix} = 48 \times 7,$$

$$D_{2} = \begin{vmatrix} 1 & 0 & 1 & -1 \\ 1 & 4 & 1 & 1 \\ 1 & 3 & 4 & 8 \\ 1 & 16 & 9 & 27 \end{vmatrix} = \frac{c_{1} + c_{4}}{c_{3} + c_{4}} \begin{vmatrix} 0 & 0 & 0 & -1 \\ 2 & 4 & 2 & 1 \\ 9 & 3 & 12 & 8 \\ 28 & 16 & 36 & 27 \end{vmatrix} = - \begin{vmatrix} 2 & 4 & 2 \\ 9 & 3 & 12 \\ 28 & 16 & 36 \end{vmatrix}$$

$$\begin{array}{c|cccc} \underline{-\frac{c_2-2c_1}{c_2-c_1}} & 2 & 0 & 0 \\ 9 & -15 & 3 \\ 28 & -40 & 8 \end{array} = 0.$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_1 \\ c_4 + c_1 \end{vmatrix}}_{c_4 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & c_2 - 2c_1 \\
\hline
 & c_2 - c_1
\end{array}
\begin{vmatrix}
2 & 0 & 0 \\
3 & -3 & 6 \\
4 & 8 & 24
\end{vmatrix} = 48 \times (-5),$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



张晓平

线性代数

$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_1 \\ c_4 + c_1 \end{vmatrix}}_{c_4 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & c_2 - 2c_1 \\
\hline
 & c_2 - c_1
\end{array}
\begin{vmatrix}
2 & 0 & 0 \\
3 & -3 & 6 \\
4 & 8 & 24
\end{vmatrix} = 48 \times (-5),$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \frac{c_2 + c_1}{c_3 - c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_1 \\ c_4 + c_1 \end{vmatrix}}_{c_4 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$\begin{array}{c|cccc}
 & c_2 - 2c_1 \\
\hline
 & c_2 - c_1
\end{array}
\begin{vmatrix}
2 & 0 & 0 \\
3 & -3 & 6 \\
4 & 8 & 24
\end{vmatrix} = 48 \times (-5),$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} \xrightarrow{c_2 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_1 \\ c_4 + c_1 \end{vmatrix}}_{c_4 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$D_{4} = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} \xrightarrow{\frac{c_{2}+c_{1}}{c_{3}-c_{1}}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_1 \\ c_4 + c_1 \end{vmatrix}}_{c_4 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$D_{4} = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_1}{c_4 + c_1}} \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \underbrace{ \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix}}_{c_2+c_1} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_1}{c_4 + c_1}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \underbrace{\begin{smallmatrix} c_2 + c_1 \\ c_3 - c_1 \end{smallmatrix}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} \xrightarrow{\frac{c_2 + c_1}{c_4 + c_1}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \underbrace{\begin{bmatrix} c_2 + c_1 \\ c_3 - c_1 \end{bmatrix}}_{c_3 - c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$



$$D_3 = \begin{vmatrix} 1 & -1 & 0 & -1 \\ 1 & 1 & 4 & 1 \\ 1 & 2 & 3 & 8 \\ 1 & 3 & 16 & 27 \end{vmatrix} = \underbrace{\begin{vmatrix} c_2 + c_1 \\ c_4 + c_1 \end{vmatrix}}_{c_4 + c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 3 & 9 \\ 1 & 4 & 16 & 28 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 3 & 9 \\ 4 & 16 & 28 \end{vmatrix}$$

$$D_4 = \begin{vmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 16 \end{vmatrix} = \underbrace{\begin{bmatrix} c_2 + c_1 \\ c_3 - c_1 \end{bmatrix}}_{c_3 - c_1} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 3 & 3 \\ 1 & 4 & 8 & 16 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 4 \\ 3 & 3 & 3 \\ 4 & 8 & 16 \end{vmatrix}$$

$$\frac{c_2 - 2c_1}{c_3 - c_1} \begin{vmatrix} 2 & 0 & 0 \\ 3 & 3 & -3 \\ 4 & 8 & 4 \end{vmatrix} = 48 \times 2.$$



由克拉默法则可知

$$x_1 = \frac{D_1}{D} = 7$$
,  $x_2 = \frac{D_2}{D} = 0$ ,  $x_3 = \frac{D_3}{D} = -5$ ,  $x_4 = \frac{D_4}{D} = 2$ .

证明恒等式

$$\begin{vmatrix} 1+a_1 & 1 & \cdots & 1 \\ 1 & 1+a_2 & \cdots & 1 \\ \vdots & & \vdots & \vdots \\ 1 & 1 & \cdots & 1+a_n \end{vmatrix} = \left(1+\sum_{i=1}^n \frac{1}{a_i}\right) \prod_{i=1}^n a_i$$

证明1:

$$\dot{E}$$
边 = 
$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1+a_2 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \cdots & 1+a_n \end{vmatrix} + \begin{vmatrix} a_1 & 1 & \cdots & 1 \\ 0 & 1+a_2 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 1+a_n \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & \cdots & 1 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & a_n \end{vmatrix} + \begin{vmatrix} a_1 & 1 & \cdots & 1 \\ 0 & 1+a_2 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 1+a_n \end{vmatrix}$$

证明恒等式

$$\left|\begin{array}{cccc} 1+a_1 & 1 & \cdots & 1 \\ 1 & 1+a_2 & \cdots & 1 \\ \vdots & & \vdots & \vdots \\ 1 & 1 & \cdots & 1+a_n \end{array}\right| = \left(1+\sum_{i=1}^n \frac{1}{a_i}\right) \prod_{i=1}^n a_i$$

## 证明1:

差数 = 
$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1+a_2 & \cdots & 1 \\ \vdots & & \vdots & \vdots \\ 1 & 1 & \cdots & 1+a_n \end{vmatrix} + \begin{vmatrix} a_1 & 1 & \cdots & 1 \\ 0 & 1+a_2 & \cdots & 1 \\ \vdots & & \vdots & \vdots \\ 0 & 1 & \cdots & 1+a_n \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 1 & \cdots & 1 \\ 0 & a_2 & \cdots & 0 \\ \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & a_n \end{vmatrix} + \begin{vmatrix} a_1 & 1 & \cdots & 1 \\ 0 & 1+a_2 & \cdots & 1 \\ \vdots & & \vdots & \vdots \\ 0 & 1 & \cdots & 1+a_n \end{vmatrix}$$

$$\dot{E}$$
边 =  $a_2 \cdots a_n + a_1 egin{pmatrix} 1 + a_2 & 1 & \cdots & 1 \ 1 & 1 + a_3 & \cdots & 1 \ dots & dots & dots \ 1 & 1 & \cdots & 1 + a_n \ \end{pmatrix}$ 

$$= a_2 \cdots a_n + a_1 \left( \left| \begin{array}{cccc} 1 & 1 & \cdots & 1 \\ 0 & a_3 & \cdots & 0 \\ \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & a_n \end{array} \right| + \left| \begin{array}{cccc} a_2 & 1 & \cdots & 1 \\ 0 & 1 + a_3 & \cdots & 1 \\ \vdots & & \vdots & \vdots \\ 0 & 1 & \cdots & 1 + a_n \end{array} \right| \right)$$

 $= \cdots = a_2 \cdots a_n + a_1 a_3 \cdots a_n + \cdots + a_1 \cdots a_{n-1} + a_1 \cdots a_n.$ 

## 证明2:

$$\frac{r_1 + \sum_{i=1}^{n} \frac{1}{a_i} r_{i+1}}{=} \begin{vmatrix}
1 + \sum_{i=1}^{n} \frac{1}{a_i} & 0 & 0 & \cdots & 0 \\
-1 & a_1 & 0 & \cdots & 0 \\
-1 & 0 & a_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
-1 & 0 & 0 & \cdots & a_n
\end{vmatrix} = \left(1 + \sum_{i=1}^{n} \frac{1}{a_i}\right) \prod_{i=1}^{n} a_i.$$

$$\begin{vmatrix} x & -1 & 0 & \cdots & 0 & 0 \\ 0 & x & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & x & -1 \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_2 & x + a_1 \end{vmatrix} = x^n + \sum_{k=1}^n a_k x^{n-k}.$$

证明: 记行列式为Dn,则

$$D_{n} = xD_{n-1} + (-1)^{n+1}a_{n} \begin{vmatrix} -1 & 0 & \cdots & 0 & 0 \\ x & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & x & -1 \end{vmatrix} = xD_{n-1} + a_{n}$$

$$\begin{vmatrix} x & -1 & 0 & \cdots & 0 & 0 \\ 0 & x & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & x & -1 \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_2 & x+a_1 \end{vmatrix} = x^n + \sum_{k=1}^n a_k x^{n-k}.$$

证明: 记行列式为 Dn,则

$$D_n = xD_{n-1} + (-1)^{n+1}a_n \begin{vmatrix} -1 & 0 & \cdots & 0 & 0 \\ x & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & x & -1 \end{vmatrix} = xD_{n-1} + a_n.$$

于是

$$D_{n} = xD_{n-1} + a_{n},$$
 $D_{n-1} = xD_{n-2} + a_{n-1}, \dots \times x$ 
 $D_{n-2} = xD_{n-3} + a_{n-2}, \dots \times x^{2}$ 
 $\dots$ 
 $D_{2} = xD_{1} + a_{2}. \dots \times x^{n-2}$ 

所以

于是

$$D_{n} = xD_{n-1} + a_{n},$$

$$D_{n-1} = xD_{n-2} + a_{n-1}, \quad \cdots \times x$$

$$D_{n-2} = xD_{n-3} + a_{n-2}, \quad \cdots \times x^{2}$$

$$\vdots$$

$$D_{2} = xD_{1} + a_{2}. \quad \cdots \times x^{n-2}$$

所以

$$D_n = a_n + a_{n-1}x + \dots + a_2x^{n-2} + x^{n-1}D_1$$
  
=  $a_n + a_{n-1}x + \dots + a_2x^{n-2} + x^{n-1}(x + a_1) =$  $\pi \dot{\omega}$ 

证明

$$\begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 & 0 \\ a_2 & x & -1 & \cdots & 0 & 0 \\ a_3 & 0 & x & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 0 & 0 & \cdots & x & -1 \\ a_n & 0 & 0 & \cdots & 0 & x \end{vmatrix} = \sum_{k=1}^n a_k x^{n-k}$$

证明:记行列式为D。

$$D_n = (-1)^{n+1} a_n \begin{vmatrix} -1 & 0 & \cdots & 0 & 0 \\ x & -1 & \cdots & 0 & 0 \\ 0 & x & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & x & -1 \end{vmatrix} + (-1)^{2n} x \begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 \\ a_2 & x & -1 & \cdots & 0 \\ a_3 & 0 & x & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n-1} & 0 & 0 & \cdots & x \end{vmatrix}$$

$$= a_n + xD_{n-1}.$$

证明

$$\begin{vmatrix} a_1 & -1 & 0 & \cdots & 0 & 0 \\ a_2 & x & -1 & \cdots & 0 & 0 \\ a_3 & 0 & x & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{n-1} & 0 & 0 & \cdots & x & -1 \\ a_n & 0 & 0 & \cdots & 0 & x \end{vmatrix} = \sum_{k=1}^n a_k x^{n-k}$$

证明: 记行列式为D。

$$D_{n} = (-1)^{n+1} a_{n} \begin{vmatrix} -1 & 0 & \cdots & 0 & 0 \\ x & -1 & \cdots & 0 & 0 \\ 0 & x & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & x & -1 \end{vmatrix} + (-1)^{2n} x \begin{vmatrix} a_{1} & -1 & 0 & \cdots & 0 \\ a_{2} & x & -1 & \cdots & 0 \\ a_{3} & 0 & x & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n-1} & 0 & 0 & \cdots & x \end{vmatrix}$$

$$= a_{n} + x D_{n-1}.$$

于是

$$D_{n} = xD_{n-1} + a_{n},$$
 $D_{n-1} = xD_{n-2} + a_{n-1}, \quad \dots \times x$ 
 $D_{n-2} = xD_{n-3} + a_{n-2}, \quad \dots \times x^{2}$ 
 $\dots$ 
 $D_{2} = xD_{1} + a_{2}. \quad \dots \times x^{n-2}$ 

所以

$$D_n = a_n + a_{n-1}x + \dots + a_2x^{n-2} + x^{n-1}D_1$$
  
=  $a_n + a_{n-1}x + \dots + a_2x^{n-2} + x^{n-1}a_1 = \pi \psi$ 

于是

$$D_{n} = xD_{n-1} + a_{n},$$
 $D_{n-1} = xD_{n-2} + a_{n-1}, \quad \dots \times x$ 
 $D_{n-2} = xD_{n-3} + a_{n-2}, \quad \dots \times x^{2}$ 
 $\dots$ 
 $D_{2} = xD_{1} + a_{2}. \quad \dots \times x^{n-2}$ 

所以

$$D_n = a_n + a_{n-1}x + \dots + a_2x^{n-2} + x^{n-1}D_1$$
  
=  $a_n + a_{n-1}x + \dots + a_2x^{n-2} + x^{n-1}a_1 = 右 边$ 

$$\begin{vmatrix} \cos\theta & 1 \\ 1 & 2\cos\theta & 1 \\ & \ddots & \ddots & \ddots \\ & & 1 & 2\cos\theta & 1 \\ & & & 1 & 2\cos\theta \end{vmatrix} = \cos n\theta$$

证明:

$$D_{n} = (-1)^{n+(n-1)} \begin{vmatrix} \cos \theta & 1 \\ 1 & 2\cos \theta & 1 \\ & \ddots & \ddots & \ddots \\ & & 1 & 2\cos \theta & 1 \\ & & & 1 & 2\cos \theta \\ & & & 1 & 1 \end{vmatrix}_{n-1} + 2\cos \theta D_{n-1}$$

$$\begin{vmatrix} \cos\theta & 1 \\ 1 & 2\cos\theta & 1 \\ & \ddots & \ddots & \ddots \\ & & 1 & 2\cos\theta & 1 \\ & & & 1 & 2\cos\theta \end{vmatrix} = \cos n\theta$$

证明:

$$D_n = (-1)^{n+(n-1)} \begin{vmatrix} \cos \theta & 1 \\ 1 & 2\cos \theta & 1 \\ & \ddots & \ddots & \ddots \\ & & 1 & 2\cos \theta & 1 \\ & & & 1 & 2\cos \theta \\ & & & 1 & 1 \end{vmatrix}_{n-1} + 2\cos \theta D_{n-1}.$$

用数学归纳法证明。

- $1^{\circ}$  当n=1时,结论显然成立。
- 2°假设结论对阶数< n-1的行列式成立,则由上式可知

$$D_n = -D_{n-2} + 2\cos\theta D_{n-1}$$

$$= -\cos(n-2)\theta + 2\cos\theta\cos(n-1)\theta$$

$$= -\cos(n-2)\theta + \cos(n-2)\theta\cos n\theta$$

$$= \cos n\theta.$$

$$\begin{vmatrix} \frac{1}{3} & -\frac{5}{2} & \frac{2}{5} & \frac{3}{2} \\ 3 & -12 & \frac{21}{5} & 15 \\ \frac{2}{3} & -\frac{9}{2} & \frac{4}{5} & \frac{5}{2} \\ -\frac{1}{7} & \frac{2}{7} & -\frac{1}{7} & \frac{3}{7} \end{vmatrix}$$

原式 = 
$$\frac{1}{30} \times \frac{3}{5} \times \frac{1}{30} \times \frac{1}{7} \times \begin{vmatrix} 10 & -75 & 12 & 45 \\ 5 & -20 & 7 & 25 \\ 20 & -135 & 24 & 75 \\ -1 & 2 & -1 & 3 \end{vmatrix}$$

原式 
$$=$$
  $\frac{1}{30} imes \frac{3}{5} imes \frac{1}{30} imes \frac{1}{7} imes$   $\begin{vmatrix} 10 & -75 & 12 & 45 \\ 5 & -20 & 7 & 25 \\ 20 & -135 & 24 & 75 \\ -1 & 2 & -1 & 3 \end{vmatrix}$ 

原式 
$$= \frac{1}{35 \times 300} \times \begin{vmatrix} -55 & 2 & 75 \\ -10 & 2 & 40 \\ -95 & 4 & 135 \end{vmatrix}$$

$$= \frac{\frac{r_2 - r_1}{r_3 - 2r_1}}{\frac{1}{r_3 - 2r_1}} \frac{1}{35 \times 300} \times \begin{vmatrix} -55 & 2 & 75 \\ 40 & 0 & -35 \\ 15 & 0 & -15 \end{vmatrix}$$

$$= \frac{-2}{35 \times 300} \times \begin{vmatrix} 40 & -35 \\ 15 & -15 \end{vmatrix} = \frac{-2}{35 \times 300} \times 15 \times (-45 + 35) = \frac{1}{35}.$$

原式 
$$\frac{r_1+r_2+\cdots+r_n}{-1}$$
  $\begin{vmatrix} -1 & 1 & \cdots & 1 & -n \\ -1 & 1 & \cdots & -n & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ -1 & -n & \cdots & 1 & 1 \\ -1 & 1 & \cdots & 1 & 1 \end{vmatrix}$   $\begin{vmatrix} -1 & 1 & \cdots & 1 & -n \\ 0 & 0 & \cdots & -n-1 & 1+n \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & -n-1 & \cdots & 0 & 1+n \\ 0 & 0 & \cdots & 0 & 1+n \end{vmatrix}$ 

原式 
$$=\frac{r_1+r_2+\cdots+r_n}{-1}$$
  $\begin{vmatrix} -1 & 1 & \cdots & 1 & -n \\ -1 & 1 & \cdots & -n & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ -1 & -n & \cdots & 1 & 1 \\ -1 & 1 & \cdots & 1 & 1 \end{vmatrix}$   $=\frac{r_i-r_1}{i=2,\cdots,n}$   $\begin{vmatrix} -1 & 1 & \cdots & 1 & -n \\ 0 & 0 & \cdots & -n-1 & 1+n \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & -n-1 & \cdots & 0 & 1+n \\ 0 & 0 & \cdots & 0 & 1+n \end{vmatrix}$ 

原式 
$$= - \begin{vmatrix} 0 & \cdots & -n-1 & 1+n \\ \vdots & \vdots & \vdots \\ -n-1 & \cdots & 0 & 1+n \\ 0 & \cdots & 0 & 1+n \end{vmatrix}_{n-1}$$

$$= -(n+1) \begin{vmatrix} 0 & \cdots & -n-1 \\ \vdots & & \vdots \\ -n-1 & \cdots & 0 \end{vmatrix}_{n-2}$$

$$= -(n+1)(-1)^{\frac{(n-2)(n-3)}{2}}(-n-1)^{n-2}$$

$$= (-1)^{\frac{(n-2)(n-3)}{2}+(n-1)}(n+1)^{n-1}$$

$$= (-1)^{\frac{n^2-5n+6+2n-2}{2}}(n+1)^{n-1}$$

$$= (-1)^{\frac{n^2-3n+4}{2}}(n+1)^{n-1}$$

$$= (-1)^{\frac{n^2+n-4n+4}{2}}(n+1)^{n-1}$$

$$= (-1)^{\frac{n^2+n-4n+4}{2}}(n+1)^{n-1}$$

$$\begin{vmatrix} a_1 + \lambda_1 & a_2 & a_3 & \cdots & a_n \\ a_1 & a_2 + \lambda_2 & a_3 & \cdots & a_n \\ a_1 & a_2 & a_3 + \lambda_3 & \cdots & a_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \cdots & a_n + \lambda_n \end{vmatrix}$$

原式 = 
$$\begin{bmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 + \lambda_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 + \lambda_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 & a_3 + \lambda_3 & \cdots & a_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_1 & a_2 & a_3 & \cdots & a_2 + \lambda_2 \end{bmatrix}$$

原式 = 
$$\begin{vmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 + \lambda_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 + \lambda_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 & a_3 + \lambda_3 & \cdots & a_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & a_1 & a_2 & a_3 & \cdots & a_n + \lambda_n \end{vmatrix}$$

原式 = 
$$\begin{vmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 + \lambda_1 & a_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 + \lambda_2 & a_3 & \cdots & a_n \\ 0 & a_1 & a_2 & a_3 + \lambda_3 & \cdots & a_n \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & a_1 & a_2 & a_3 & \cdots & a_n + \lambda_n \end{vmatrix}_{n+1}$$

$$= \begin{vmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_n \\ -1 & \lambda_1 & 0 & 0 & \cdots & 0 \\ -1 & 0 & \lambda_2 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \lambda_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \cdots & \lambda_n \end{vmatrix}_{n+1}$$

$$= \begin{vmatrix} 1 + \sum_{i=1}^n \frac{a_i}{\lambda_i} & 0 & 0 & 0 & \cdots & 0 \\ -1 & \lambda_1 & 0 & 0 & \cdots & 0 \\ -1 & 0 & \lambda_2 & 0 & \cdots & 0 \\ -1 & 0 & \lambda_2 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \lambda_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0 & 0 & 0 & \lambda_3 & \cdots & 0 \end{vmatrix} = \left(1 + \sum_{i=1}^n \frac{a_i}{\lambda_i}\right) \prod_{i=1}^n a_i.$$

解:

- (ロ) (個) (E) (E) (9(0)

$$D_{n} = \frac{r_{i} - r_{i-1}}{i = n, \dots, 2} \begin{vmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ 1 & 1 & 1 & \dots & 1 & 1-n \\ 1 & 1 & 1 & \dots & 1-n & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 - n & 1 & \dots & 1 & 1 \end{vmatrix}$$

$$= \frac{c_{i} - c_{1}}{i = 2, \dots, n} \begin{vmatrix} 1 & 1 & 2 & \dots & n-2 & n-1 \\ 1 & 0 & 0 & \dots & 0 & -n \\ 1 & 0 & 0 & \dots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 0 & 0 & \dots & -n & 0 \end{vmatrix}$$

$$D_{n} = \frac{r_{i} - r_{i-1}}{\stackrel{i=n, \dots, 2}{=}} \begin{bmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ 1 & 1 & 1 & \dots & 1 & 1-n \\ 1 & 1 & 1 & \dots & 1-n & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1-n & 1 & \dots & 1 & 1 \end{bmatrix}$$

$$\frac{c_{i} - c_{1}}{\stackrel{i=2, \dots, n}{=}} \begin{bmatrix} 1 & 1 & 2 & \dots & n-2 & n-1 \\ 1 & 0 & 0 & \dots & 0 & -n \\ 1 & 0 & 0 & \dots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$D_{n} = \frac{r_{i} - r_{i-1}}{\stackrel{i=n,\dots,2}{=}} \begin{vmatrix} 1 & 2 & 3 & \dots & n-1 & n \\ 1 & 1 & 1 & \dots & 1 & 1-n \\ 1 & 1 & 1 & \dots & 1-n & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1-n & 1 & \dots & 1 & 1 \end{vmatrix}$$

$$= \frac{c_{i} - c_{1}}{\stackrel{i=2,\dots,n}{=}} \begin{vmatrix} 1 & 1 & 2 & \dots & n-2 & n-1 \\ 1 & 0 & 0 & \dots & 0 & -n \\ 1 & 0 & 0 & \dots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \end{vmatrix}$$

$$D_{n} = \frac{r_{i} - r_{i-1}}{\stackrel{r_{i} - r_{i-1}}{= n, \cdots, 2}} \begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 1 & 1 & 1 & \cdots & 1 & 1-n \\ 1 & 1 & 1 & \cdots & 1-n & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 - n & 1 & \cdots & 1 & 1 \end{vmatrix}$$

$$= \frac{c_{i} - c_{1}}{\stackrel{r_{i} - r_{i-1}}{= 2, \cdots, n}} \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{1}+c_{2}+\cdots+c_{n}}{i=2,\cdots,n} n^{n-1} \begin{bmatrix} 1 & \frac{1}{n} & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$= c_{n-1} \begin{bmatrix} 1 & 1 & n(n-1) \end{bmatrix} \begin{pmatrix} 1 & \frac{(n-1)(n-2)}{(n-1)(n-2)} & 1 & \frac{(n-1)(n-2)}{(n-2)(n-2)} & 1 & \frac{(n-1)(n-2)(n-2)}{(n-2)(n-2)} & 1 & \frac{(n-1)(n-$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\frac{c_{1}+c_{2}+\cdots+c_{n}}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{n-1} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{n} n^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$\frac{c_{1}+c_{2}+\cdots+c_{n}}{n} n^{n-1} \begin{vmatrix} 1 + \sum_{i=1}^{n-1} \frac{i}{n} & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

4□ > 4□ > 4 = > 4 = > = 90

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 & 0 \end{vmatrix}$$

$$\frac{c_{i} \div n}{i=2,\cdots,n} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 & 0 \end{vmatrix}$$

$$\frac{c_{1}+c_{2}+\cdots+c_{n}}{i} n^{n-1} \begin{vmatrix} 1 + \sum_{i=1}^{n-1} \frac{i}{n} & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

◆□▶ ◆□▶ ◆□▶ ◆□▶ ◆□▶ ◆□▶

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{\stackrel{i}{=} 2, \cdots, n}} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{1} + c_{2} + \cdots + c_{n}}{n}}{n^{n-1}} n^{n-1} \begin{vmatrix} 1 + \sum_{i=1}^{n-1} \frac{i}{n} & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1} \right]$$

4□ > 4□ > 4 = > 4 = > = 9 < 0</p>

$$D_{n} = \begin{vmatrix} 1 & 1 & 2 & \cdots & n-2 & n-1 \\ 1 & 0 & 0 & \cdots & 0 & -n \\ 1 & 0 & 0 & \cdots & -n & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 1 & -n & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{i} \div n}{\stackrel{i}{i=2,\cdots,n}} n^{n-1} \begin{vmatrix} 1 & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ 1 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= \frac{c_{1} + c_{2} + \cdots + c_{n}}{\stackrel{i}{i=1}} n^{n-1} \begin{vmatrix} 1 + \sum_{i=1}^{n-1} \frac{i}{n} & \frac{1}{n} & \frac{2}{n} & \cdots & \frac{n-2}{n} & \frac{n-1}{n} \\ 0 & 0 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -1 & 0 & \cdots & 0 & 0 \end{vmatrix}$$

$$= n^{n-1} \left[ 1 + \frac{1}{n} \frac{n(n-1)}{2} \right] (-1)^{\frac{(n-1)(n-2)}{2}} (-1)^{n-1} = (-1)^{\frac{(n-1)n}{2}} \frac{n+1}{2} n^{n-1}.$$

◆ロト ◆団ト ◆豆ト ◆豆ト 豆 からぐ

证明

$$=\sum_{i=1}^n x_i \prod_{1\leq j< i\leq n} (x_i-x_j)$$

证明:考察行列式

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n & y \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 & y^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_1^{n-2} & x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} & y^{n-2} \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} & y^{n-1} \\ x_1^n & x_2^n & x_3^n & \cdots & x_n^n & y^n \end{vmatrix} = \prod_{i=1}^n (y - x_i)$$

$$= \prod_{i=1}^{n} (y - x_i) \prod_{1 \le i < i \le n} (x_i - x_j)$$

等式两端均为关于y的多项式,比较y"-1的系数便得结论

◆ロト ◆問ト ◆差ト ◆差ト 差 めなべ

证明

$$=\sum_{i=1}^n x_i \prod_{1\leq j< i\leq n} (x_i-x_j)$$

考察行列式 证明:

$$\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n & y \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 & y^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_1^{n-2} & x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} & y^{n-2} \\ x_1^{n_1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n_1} & y^{n-1} \\ x_1^n & x_2^n & x_3^n & \cdots & x_n^n & y^n \end{vmatrix} = \prod_{i=1}^n (y - x_i) \prod_{1 \le j < i \le n} (x_i - x_j)$$

$$=\prod_{i=1}^n(y-x_i)\prod_{1\leq j< i\leq n}(x_i-x_j)$$

等式两端均为关于y的多项式,比较y<sup>n-1</sup>的系数便得结论。

用数学归纳法证明:

$$\frac{d}{dt} \begin{vmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & & \vdots \\ a_{n1}(t) & a_{n2}(t) & \cdots & a_{nn}(t) \end{vmatrix} = \sum_{j=1}^{n} \begin{vmatrix} a_{11}(t) & \cdots & \frac{d}{dt}a_{1j}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & \cdots & \frac{d}{dt}a_{2j}(t) & \cdots & a_{2n}(t) \\ \vdots & & \vdots & & \vdots \\ a_{n1}(t) & \cdots & \frac{d}{dt}a_{nj}(t) & \cdots & a_{nn}(t) \end{vmatrix}$$

- $1^{\circ}$  当n=1时,结论显然成立。
- $2^{\circ}$  假设结论对阶数 $\leq n-1$ 的行列式成立,考虑阶数为n的行列式,对第一列展开得

$$D = a_{11}A_{11} + a_{21}A_{21} + \cdots + a_{n1}A_{n1},$$
  

$$D' = a'_{11}A_{11} + a'_{21}A_{21} + \cdots + a'_{n1}A_{n1} +$$
  

$$a_{11}A'_{11} + a_{21}A'_{21} + \cdots + a_{n1}A'_{n1},$$

其中

$$a_{11}'(t)A_{11}(t)+a_{21}'(t)A_{21}(t)+\cdots+a_{n1}'(t)A_{n1}(t)=\left|egin{array}{cccc} a_{11}'(t) & a_{12}(t) & \cdots & a_{1n}(t) \ a_{21}'(t) & a_{22}(t) & \cdots & a_{2n}(t) \ dots & dots & dots \ a_{2n}'(t) & a_{2n}(t) & \cdots & a_{nn}(t) \end{array}
ight|$$

4□ > 4□ > 4 = > 4 = > = 90

$$a_{11}A'_{11} + a_{21}A'_{21} + \dots + a_{n1}A'_{n1} = a_{11}\sum_{j=2}^{n} \begin{vmatrix} a_{22}(t) & \cdots & a'_{2j}(t) & \cdots & a_{2n}(t) \\ a_{32}(t) & \cdots & a'_{3j}(t) & \cdots & a_{3n}(t) \\ \vdots & & \vdots & & \vdots \\ a_{n2}(t) & \cdots & a'_{nj}(t) & \cdots & a_{nn}(t) \end{vmatrix}$$

$$-a_{21}\sum_{j=2}^{n} \begin{vmatrix} a_{12}(t) & \cdots & a'_{1j}(t) & \cdots & a_{1n}(t) \\ a_{32}(t) & \cdots & a'_{3j}(t) & \cdots & a_{3n}(t) \\ \vdots & & \vdots & & \vdots \\ a_{n2}(t) & \cdots & a'_{nj}(t) & \cdots & a_{nn}(t) \end{vmatrix} + \cdots$$

$$+(-1)^{n+1}a_{n1}\sum_{j=2}^{n} \begin{vmatrix} a_{12}(t) & \cdots & a'_{1j}(t) & \cdots & a_{1n}(t) \\ a_{22}(t) & \cdots & a'_{2j}(t) & \cdots & a_{2n}(t) \\ \vdots & & \vdots & & \vdots \\ a_{n-1,2}(t) & \cdots & a'_{1j}(t) & \cdots & a_{1n}(t) \end{vmatrix}$$

$$= \sum_{j=2}^{n} \begin{vmatrix} a_{12}(t) & a_{12}(t) & \cdots & a'_{1j}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1}(t) & a_{n2}(t) & \cdots & a'_{n,j}(t) & \cdots & a_{nn}(t) \end{vmatrix}$$

设3个点 $P_1(x_1,y_1), P_2(x_2,y_2), P_3(x_3,y_3)$ 不在一条直线上,求过点 $P_1, P_2, P_3$ 的圆的方程。

解: 圆的一般方程为

$$a(x^2 + y^2) + bx + cy + d = 0, \quad a \neq 0$$

因P1, P2, P3在圆上, 故

$$\begin{cases} a(x^2 + y^2) + bx + cy + d = 0, \\ a(x_1^2 + y_1^2) + bx_1 + cy_1 + d = 0, \\ a(x_2^2 + y_2^2) + bx_2 + cy_2 + d = 0, \\ a(x_3^2 + y_3^2) + bx_3 + cy_3 + d = 0, \end{cases}$$

该齐次线性方程组有非零解的充分必要条件是系数行列式为零,即

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0.$$

- 4 ロ ト 4 昼 ト 4 種 ト 4 種 ト ■ 9 Q (C)

设3个点 $P_1(x_1,y_1),P_2(x_2,y_2),P_3(x_3,y_3)$ 不在一条直线上,求过点 $P_1,P_2,P_3$ 的圆的方程。

解: 圆的一般方程为

$$a(x^2 + y^2) + bx + cy + d = 0, \quad a \neq 0$$

因 $P_1, P_2, P_3$ 在圆上,故

$$\begin{cases} a(x^2 + y^2) + bx + cy + d = 0, \\ a(x_1^2 + y_1^2) + bx_1 + cy_1 + d = 0, \\ a(x_2^2 + y_2^2) + bx_2 + cy_2 + d = 0, \\ a(x_3^2 + y_3^2) + bx_3 + cy_3 + d = 0, \end{cases}$$

该齐次线性方程组有非零解的充分必要条件是系数行列式为零,即

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0.$$

# 求使3点 $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ 位于一直线上的充分必要条件。

解: 三点位于一直线上的充分必要条件是

$$\frac{y_1-y_2}{x_1-x_2}=\frac{y_1-y_3}{x_1-x_3},$$

即

$$(x_1-x_3)(y_1-y_2)=(x_1-x_2)(y_1-y_3)$$

亦即

$$x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2) = 0$$

其行列式形式为

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

求使3点 $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ 位于一直线上的充分必要条件。

解: 三点位于一直线上的充分必要条件是

$$\frac{y_1-y_2}{x_1-x_2}=\frac{y_1-y_3}{x_1-x_3},$$

即

$$(x_1-x_3)(y_1-y_2)=(x_1-x_2)(y_1-y_3)$$

亦即

$$x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2) = 0$$

其行列式形式为

$$\left|\begin{array}{ccc} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{array}\right| = 0.$$

求过3点(1,1,1), (2,3,-1), (3,-1,-1)的平面方程。

解: 平面方程为

$$ax + by + cz + d = 0,$$

因3点位于平面上,故

$$\begin{cases} ax + by + cz + d = 0 \\ a + b + c + d = 0 \\ 2a + 3b - c + d = 0 \\ 3a - b - c + d = 0 \end{cases}$$

该齐次线性方程组有非零解, 故其系数行列式为零, 即

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 1 \\ 3 & -1 & -1 & 1 \end{vmatrix} = 0.$$

即

$$-8x - 2y - 6z + 16 = 0.$$

亦即

$$4x + y + 3z - 8 = 0.$$

求过3点(1,1,1),(2,3,-1),(3,-1,-1)的平面方程。

### 解: 平面方程为

$$ax + by + cz + d = 0,$$

因3点位于平面上,故

$$\begin{cases} ax + by + cz + d = 0, \\ a + b + c + d = 0, \\ 2a + 3b - c + d = 0, \\ 3a - b - c + d = 0 \end{cases}$$

该齐次线性方程组有非零解,故其系数行列式为零,即

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 1 \\ 3 & -1 & -1 & 1 \end{vmatrix} = 0.$$

即

$$-8x - 2y - 6z + 16 = 0.$$

亦即

$$4x + y + 3z - 8 = 0.$$

### 求过点(1,1,1),(1,1,-1),(1,-1,1),(-1,0,0)的球面方程,并求其中心与半径。

解: 球面的一般方程为

$$a(x^{2} + y^{2} + z^{2}) + bx + cy + dz + e = 0.$$

过该四点的球面方程为

$$\begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ 1^2 + 1^2 + 1^2 & 1 & 1 & 1 & 1 \\ 1^2 + 1^2 + (-1)^2 & 1 & 1 & -1 & 1 \\ 1^2 + (-1)^2 + 1^2 & 1 & -1 & 1 & 1 \\ (-1)^2 + 0^2 + 0^2 & -1 & 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ 3 & 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 \end{vmatrix} = 0$$

按第一行展开可知

$$-8(x^2 + y^2 + z^2) + 8x + 16 = 0$$

即

$$x^{2} + y^{2} + z^{2} - x - 2 = 0, \Rightarrow (x - \frac{1}{2})^{2} + y^{2} + z^{2} = (\frac{3}{2})^{2}$$

圆心为 $(\frac{1}{2},0,0)$ ,半径为 $\frac{3}{2}$ 

求过点(1,1,1),(1,1,-1),(1,-1,1),(-1,0,0)的球面方程,并求其中心与半径。

解: 球面的一般方程为

$$a(x^2 + y^2 + z^2) + bx + cy + dz + e = 0.$$

过该四点的球面方程为

$$\begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ 1^2 + 1^2 + 1^2 & 1 & 1 & 1 & 1 \\ 1^2 + 1^2 + (-1)^2 & 1 & 1 & -1 & 1 \\ 1^2 + (-1)^2 + 1^2 & 1 & -1 & 1 & 1 \\ (-1)^2 + 0^2 + 0^2 & -1 & 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ 3 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & -1 & 1 \\ 3 & 1 & -1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 \end{vmatrix} = 0$$

按第一行展开可知

$$-8(x^2 + y^2 + z^2) + 8x + 16 = 0$$

即

$$x^{2} + y^{2} + z^{2} - x - 2 = 0, \quad \Rightarrow (x - \frac{1}{2})^{2} + y^{2} + z^{2} = (\frac{3}{2})^{2}$$

圆心为 $(\frac{1}{2},0,0)$ ,半径为 $\frac{3}{2}$ 

求过点
$$(1,1,1),(1,1,-1),(1,-1,1),(-1,0,0)$$
的球面方程,并求其中心与半径。

解: 球面的一般方程为

$$a(x^2 + y^2 + z^2) + bx + cy + dz + e = 0.$$

过该四点的球面方程为

$$\begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ 1^2 + 1^2 + 1^2 & 1 & 1 & 1 & 1 \\ 1^2 + 1^2 + (-1)^2 & 1 & 1 & -1 & 1 \\ 1^2 + (-1)^2 + 1^2 & 1 & -1 & 1 & 1 \\ (-1)^2 + 0^2 + 0^2 & -1 & 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ 3 & 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 1 & 1 \\ 3 & 1 & -1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 \end{vmatrix} = 0$$

按第一行展开可知

$$-8(x^2+y^2+z^2)+8x+16=0,$$

即

$$x^{2} + y^{2} + z^{2} - x - 2 = 0$$
,  $\Rightarrow (x - \frac{1}{2})^{2} + y^{2} + z^{2} = (\frac{3}{2})^{2}$ 

圆心为 $(\frac{1}{2},0,0)$ ,半径为 $\frac{3}{2}$ .

4□ > 4ⓓ > 4≧ > 4≧ > ½ 9 9 9

张晓平 线性代数

已知 $a^2 \neq b^2$ ,证明方程组

## 有唯一解,并求解。

**解**: 其系数行列式为 D<sub>2n</sub> =

a b ... ... b

→ 4回 → 4 呈 → 4 呈 → 9 Q @

已知 $a^2 \neq b^2$ ,证明方程组

有唯一解, 并求解。

**解**: 其系数行列式为 D<sub>2n</sub> =

a ... ... b ... b

把D<sub>2n</sub>中的第2n行依次与第2n-1行、...、第2行对调(共2n-2次相邻对换), 再把 第2n列依次与第2n-1列、...、第2列对调,得

于是

$$D_{2n} = D_2 D_{2(n-1)}$$

$$= (a^2 - b^2) D_{2(n-1)} = (a^2 - b^2)^2 D_{2(n-2)}$$

$$= \dots = (a^2 - b^2)^{n-1} D_2$$

$$= (a^2 - b^2)^n.$$

 $= (a-b)(a^2-b^2)^{n-1}.$ 

同理可证

$$D_{2n}^{(i)} = (a-b)(a^2-b^2)^{n-1}, \quad i=2,\cdots,2n.$$

于是

$$x_i = \frac{D_{2n}^{(i)}}{D_{2n}} = \frac{(a-b)(a^2-b^2)^{n-1}}{(a^2-b^2)^n} = \frac{1}{a+b}, \quad i=1,\cdots,2n.$$

