CS 161
HW5
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1.

(a). The original propositional logic sentence can be converted to:

$$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire) \equiv \neg(\neg Smoke \lor Fire) \lor (Smoke \lor \neg Fire)$$
$$\equiv (Smoke \land \neg Fire) \lor Smoke \lor \neg Fire \equiv Smoke \lor \neg Fire$$

The truth table is:

$Smoke$	$Fire$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
T	T	T
T	F	T
F	T	F
F	F	T

From the truth table, we find out that this propositional logic sentence is neither valid nor unsatisfiable.

(b). The original propositional logic sentence can be converted to:

$$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \lor Heat) \Rightarrow Fire)$$
$$\equiv \neg(\neg Smoke \lor Fire) \lor (\neg(Smoke \lor Heat) \lor Fire)$$
$$\equiv (Smoke \land \neg Fire) \lor (\neg Smoke \land \neg Heat) \lor Fire$$

The truth table is:

$Smoke$	$Fire$	$Heat$	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \lor Heat) \Rightarrow Fire)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

From the truth table, we find out that this propositional logic sentence is neither valid nor unsatisfiable.

(c). The original propositional logic sentence can be converted to:

$$
\begin{aligned}
&\big((Smoke \wedge Heat) \Rightarrow Fire\big) \Leftrightarrow \big((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\big) \\
&\quad \equiv \Big(\big((Smoke \wedge Heat) \Rightarrow Fire\big) \Rightarrow \big((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\big)\Big) \\
&\quad \wedge \Big(\big((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\big) \Rightarrow \big((Smoke \wedge Heat) \Rightarrow Fire\big)\Big) \\
&\quad \equiv \neg(\neg(Smoke \wedge Heat) \vee Fire) \vee \big((\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire)\big) \\
&\quad \wedge \neg\big((\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire)\big) \vee (\neg(Smoke \wedge Heat) \vee Fire) \\
&\quad \equiv \Big(\big((Smoke \wedge Heat) \wedge \neg Fire\big) \vee (\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire)\Big) \\
&\quad \wedge \Big(\big(\neg(\neg Smoke \vee Fire) \wedge \neg(\neg Heat \vee Fire)\big) \vee (\neg Smoke \vee \neg Heat) \vee Fire\Big) \\
&\quad \equiv \big((Smoke \wedge Heat \wedge \neg Fire) \vee \neg Smoke \vee Fire \vee \neg Heat \vee Fire\big) \\
&\quad \wedge \Big(\big((Smoke \wedge \neg Fire) \wedge (Heat \wedge \neg Fire)\big) \vee \neg Smoke \vee \neg Heat \vee Fire\Big) \\
&\quad \equiv \big((Smoke \wedge Heat \wedge \neg Fire) \vee \neg Smoke \vee Fire \vee \neg Heat\big) \\
&\quad \wedge \big((Smoke \wedge \neg Fire \wedge Heat) \vee \neg Smoke \vee \neg Heat \vee Fire\big) \\
&\quad \equiv (Smoke \wedge Heat \wedge \neg Fire) \vee \neg Smoke \vee Fire \vee \neg Heat \\
&\quad \equiv \neg(\neg Smoke \vee Fire \vee \neg Heat) \vee (\neg Smoke \vee Fire \vee \neg Heat) \equiv T
\end{aligned}
$$

The truth table is:

$Smoke$	$Fire$	$Heat$	$\big((Smoke \wedge Heat) \Rightarrow Fire\big) \Leftrightarrow \big((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire)\big)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

From the truth table, we find out that this propositional logic sentence is valid.

2.

Variables defined for this question:

$Mythical$, $Immortal$, $Mammal$, $Horned$ and $Magical$

Their semantics are:

$Mythical = the\ unicorn\ is\ mythical$

$Immortal = the\ unicorn\ is\ immortal$

$Mammal = the\ unicorn\ isa\ mammal$

$Horned = the\ unicorn\ is\ horned$

$Magical = the\ unicorn\ is\ magical$

(a). The knowledge base is:

- (I). $Mythical \Rightarrow Immortal$
- (II). $\neg Mythical \Rightarrow (\neg Immortal \wedge Mammal)$
- (III). $(Immortal \vee Mammal) \Rightarrow Horned$

- (IV). $Horned \Rightarrow Magical$

(b). With the help of logic equivalences, we can convert each entry in the knowledge base to a clause or a conjunction.

- (I). $(\neg Mythical \vee Immortal)$
- (II). $(Mythical \vee \neg Immortal) \wedge (Mythical \vee Mammal)$
- (III). $(\neg Immortal \vee Horned) \wedge (\neg Mammal \vee Horned)$
- (IV). $(\neg Horned \vee Magical)$

The CNF version of the knowledge base is:

$$\begin{aligned} KB = {} & (\neg Mythical \vee Immortal) \wedge (Mythical \vee \neg Immortal) \wedge (Mythical \vee Mammal) \\ & \wedge (\neg Immortal \vee Horned) \wedge (\neg Mammal \vee Horned) \\ & \wedge (\neg Horned \vee Magical) \end{aligned}$$

(c). Let's give index to each clause appeared in our knowledge base:

- (1). $(\neg Mythical \vee Immortal)$
- (2). $(Mythical \vee \neg Immortal)$
- (3). $(Mythical \vee Mammal)$
- (4). $(\neg Immortal \vee Horned)$
- (5). $(\neg Mammal \vee Horned)$
- (6). $(\neg Horned \vee Magical)$

To prove the statements mentioned in the question, I use "proof by refutation":

"$Unicorn\ is\ mythical$":

We need to add another entry in the knowledge base, forming KB', which is

- (7). $(\neg Mythical)$

Actually, KB' is satisfiable if we assign as the following:

$Mythical = false$, $Immortal = true$, $Mammal = true$, $Horned = true$ and $Magical = true$.

Therefore, the original knowledge base KB does not entail "$Unicorn\ is\ mythical$".

"$Unicorn\ is\ magical$":

We need to add another entry in the knowledge base, forming KB', which is

- (7). $(\neg Magical)$

Resolution on (6), (7):

- (8). $\frac{(\neg Magical),(\neg Horned \vee Magical)}{\neg Horned}$

Resolution on (4), (8):

- (9). $\frac{(\neg Horned),(\neg Immortal \vee Horned)}{\neg Immortal}$

Resolution on (1), (9):

- (10). $\frac{(\neg Immortal),(\neg Mythical \vee Immortal)}{\neg Mythical}$

Resolution on (5), (8):

- (11). $\frac{(\neg Horned),(\neg Mammal \vee Horned)}{\neg Mammal}$

Resolution on (3), (11):

- (12). $\frac{(\neg Mammal),(Mythical \vee Mammal)}{Mythical}$

Resolution on (10), (12):

- (13). $\frac{(\neg Mythical),(Mythical)}{\emptyset}$

Since we generate an empty clause, we conclude that KB' is inconsistent, which means the original knowledge base KB entails "$Unicorn\ is\ magical$".

"$Unicorn\ is\ horned$":

We need to add another entry in the knowledge base, forming KB', which is

- (7). $(\neg Horned)$

Resolution on (4), (7):

- (8). $\frac{(\neg Horned),(\neg Immortal \vee Horned)}{\neg Immortal}$

Resolution on (1), (8):

- (9). $\frac{(\neg Immortal),(\neg Mythical \vee Immortal)}{\neg Mythical}$

Resolution on (5), (7):

- (10). $\frac{(\neg Horned),(\neg Mammal \vee Horned)}{\neg Mammal}$

Resolution on (3), (10):

- (11). $\frac{(\neg Mammal),(Mythical \vee Mammal)}{Mythical}$

Resolution on (9), (11):

- (12). $\frac{(\neg Mythical),(Mythical)}{\emptyset}$

Since we generate an empty clause, we conclude that KB' is inconsistent, which means the original knowledge base KB entails "$Unicorn\ is\ horned$".

3.

(a). $\{x/A, y/B, z/B\}$

Algorithm:

$P(\underline{A}, B, B), P(\underline{x}, y, z)$: $\{\underline{x/A}\}$

$P(A, \underline{B}, B), P(A, \underline{y}, z)$: $\{x/A, \underline{y/B}\}$

$P(A, B, \underline{B}), P(A, B, \underline{z})$: $\{x/A, y/B, \underline{z/B}\}$

(b). Unification fails

Algorithm:

$Q\left(\underline{y}, G(A,B)\right), Q(\underline{G(x,x)}, y)$: $\{\underline{y/G(x,x)}\}$

$Q\left(G(x,x), \underline{G(A,B)}\right), Q(G(x,x), \underline{G(x,x)})$: $\{\underline{y/G(x,x)}\} - recursion$

$Q\left(G(x,x), G(\underline{A}, B)\right), Q(G(x,x), G(\underline{x}, x))$: $\{y/G(x,x), \underline{x/A}\}$

$Q\left(G(A,A), G(A, \underline{B})\right), Q(G(A,A), G(A, \underline{A}))$: $cannot\ unify\ two\ different\ constants$

(c). $\{y/John, x/John\}$

Algorithm:

$Older\left(\underline{Father(y)}, y\right), Older\left(\underline{Father(x)}, John\right)$: $-recursion$

$Older\left(Father(\underline{y}), y\right), Older\left(Father(\underline{x}), John\right)$: $\{\underline{x/y}\}$

$Older\left(Father(y), \underline{y}\right), Older(Father(y), \underline{John})$: $\{x/y, y/John\} = \{x/John, y/John\}$

(d). Unification fails

Algorithms:

$Knows\left(\underline{Father(y)}, y\right), Knows(\underline{x}, x)$: $\{\underline{x/Father(y)}\}$

$Knows\left(Father(y), \underline{y}\right), Knows\left(Father(y), \underline{Father(y)}\right)$: $cannot\ unify\ y\ with\ Father(y)$

4.

(a). The knowledge base in first-order logic is:

- (1). $\forall x, Food(x) \Rightarrow Loves(John, x)$
- (2). $Food(Apple)$
- (3). $Food(Chicken)$
- (4). $\forall x \forall y, Eats(x, y) \wedge \neg KilledBy(x, y) \Rightarrow Food(y)$
- (5). $\forall x, [\exists y, KilledBy(x, y) \Rightarrow \neg Alive(x)]$
- (6). $Eats(Bill, Peanuts) \wedge Alive(Bill)$
- (7). $\forall x, Eats(Bill, x) \Rightarrow Eats(Sue, x)$

(b). The clausal form translated into is:

- (1). $\neg Food(x) \vee Loves(John, x)$
- (2). $Food(Apple)$
- (3). $Food(Chicken)$
- (4). $\neg Eats(x, y) \vee KilledBy(x, y) \vee Food(y)$
- (5). $\neg KilledBy(x, y) \vee \neg Alive(x)$
- (6a). $Eats(Bill, Peanuts)$
- (6b). $Alive(Bill)$
- (7). $\neg Eats(Bill, x) \vee Eats(Sue, x)$

(c). To prove "$John\ likes\ peanuts$", I use “proof by refutation”:

We need to add another entry in the knowledge base, forming KB', which is

- (8). $\neg Loves(John, Peanuts)$

Resolution on (1), (8):

- (9). $\neg Food(Peanuts), \theta = \{x/Peanuts\}$

Resolution on (4), (9):

- (10). $\neg Eats(x, Peanuts) \vee KilledBy(x, Peanuts), \theta = \{y/Peanuts\}$

Resolution on (6a), (10):

- (11). $KilledBy(Bill, Peanuts), \theta = \{x/Bill\}$

Resolution on (5), (11):

- (12). $\neg Alive(Bill)$

Resolution on (6b), (12):

- (13). $Empty\ clause$

Since we generate an empty clause, we conclude that KB' is inconsistent, which means the original knowledge base KB entails "$John\ likes\ peanuts$".

(d). To show "$What\ food\ does\ Sue\ eat$", I use "proof by refutation":
We need to add another entry in the knowledge base, forming KB', which is

- $\neg(\exists x, Food(x) \wedge Eats(Sue, x))$

Its clausal form is:

- (8). $\neg Food(x) \vee \neg Eats(Sue, x)$

Resolution on (7), (8):

- (9). $\neg Eats(Bill, x) \vee \neg Food(x)$

Resolution on (6a), (9):

- (10). $\neg Food(Peanuts), \theta = \{x/Peanuts\}$

After this step, the rest of derivation is the same as part (c) after the second step. Therefore, we generate an empty clause at the end, which means KB' is inconsistent with the substitution $\theta = \{x/Peanuts\}$. That is, the original knowledge base KB entails "$Sue\ eats\ peanuts$".

(e). The replacing axioms are as follows:

- $\forall x(\neg\exists y, Eats(x, y)) \Rightarrow Dead(x)$
- $\forall x, Dead(x) \Rightarrow \neg Alive(x)$
- $Alive(Bill)$

The first statement can be translated into a clausal form as follows:

$$\forall x(\neg\exists y, Eats(x, y)) \Rightarrow Dead(x) \equiv \forall x \neg(\neg\exists y, Eats(x, y)) \vee Dead(x)$$
$$\equiv \forall x \exists y, Eats(x, y) \vee Dead(x) \equiv Eats(x, F(x)) \vee Dead(x)$$

In this expression, $F(x)$ is a Skolem function of x.
Then, the new knowledge base KB_{new} is:

- (1). $\neg Food(x) \vee Loves(John, x)$
- (2). $Food(Apple)$
- (3). $Food(Chicken)$
- (4). $\neg Eats(x, y) \vee KilledBy(x, y) \vee Food(y)$
- (5). $\neg KilledBy(x, y) \vee \neg Alive(x)$
- (6a). $Eats(x, F(x)) \vee Dead(x)$
- (6b). $\neg Dead(x) \vee \neg Alive(x)$
- (6c). $Alive(Bill)$
- (7). $\neg Eats(Bill, x) \vee Eats(Sue, x)$

To show "$What\ food\ does\ Sue\ eat$", I use "proof by refutation":
We need to add another entry in the knowledge base, forming KB_{new}', which is

- $\neg(\exists z, Food(z) \wedge Eats(Sue, z))$

Its clausal form is:

- (8). $\neg Food(z) \vee \neg Eats(Sue, z)$

Resolution on (7), (8):

- (9). $\neg Food(z) \vee \neg Eats(Bill, z), \theta = \{x/z\}$

Resolution on (6a), (9):

- (10). $\neg Food(F(Bill)) \vee Dead(Bill), \theta = \{x/Bill, z/F(Bill)\}$

Resolution on (6b), (10):

- (11). $\neg Food(F(Bill)) \vee \neg Alive(Bill), \theta = \{x/Bill\}$

Resolution on (6c), (11):

- (12). $\neg Food(F(Bill))$

Resolution on (4), (12):

- (13). $\neg Eats(x, F(Bill)) \vee KilledBy(x, F(Bill)), \theta = \{y/F(Bill)\}$

Resolution on (5), (13):

- (14). $\neg Eats(x, F(Bill)) \vee \neg Alive(x)$

Resolution on (6c), (14):

- (15). $\neg Eats(Bill, F(Bill)), \theta = \{x/Bill\}$

Resolution on (6a), (15):

- (16). $Dead(Bill), \theta = \{x/Bill\}$

Resolution on (6b), (16):

- (17). $\neg Alive(Bill), \theta = \{x/Bill\}$

Resolution on (6c), (17):

- (18). $Empty\ clause$

In (10), $F(Bill)$ means what Bill eats. Hence, we know that $KB_{\text{new}}{}'$ is inconsistent with the substitution $\theta = \{x/Bill, z/F(Bill)\}$. That is, the original knowledge base KB entails "$Sue\ eats\ what\ Bill\ eats$".