**1** Single-Choice Questions:

1) For the entropy of a single symbol discrete source , the entropy of the N extended source  must satisfy\_\_\_\_\_\_\_\_\_\_\_.

A.  B. 

C.  D. 

2) What is right in the following statement \_\_\_\_\_\_\_\_.

A. The average mutual information can be negative.

B. The mutual information can be negative in certain condition.

C. For the entropy of certain source, it can increase to infinity.

D. The channel capacity must be larger than any rate distortion function.

3) The ultimate entropy  satisfies \_\_\_\_\_\_\_\_\_.

A. It must be smaller than any single entropy.

B. It must be larger than  for the independent extended source.

C. It should be equal to  for the Markov source.

D. It would be larger than  for the Markov source.

**2** Completion.

a) The components for the Shannon communication system includes\_\_\_\_\_\_\_

, \_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_.

b) As the probability of a certain event increases, its self-information would \_\_\_\_\_\_\_\_\_\_\_.

c) For the rate distortion function, if there is only one minimum element in each row of the distortion matrix, then R(Dmin)=\_\_\_\_, while R(Dmax)=\_\_\_\_\_\_\_\_

(H(X) is the entropy for the source X).

d) For the channel capacity C from the source X to the destination Y, the range for C should be among \_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_ .

**3** Assume the probability space of one Discrete Memoryless Source (DMS) is:



If there is a noise channel that the source X gets through, the destination symbol set is Y=[b1,b2] and the transition probabilities matrix is:



Then please calculate:

a) I(a1) and I(a2); (2 scores) b) H(X); (4 scores) c) H(Y); (4 scores)

d) H(X,Y); (4 scores) e) I(X.Y). (6 scores)

**3** There exists a binary 2-step Markov Chain, where the source symbol set is {0,1}. The symbol transition probabilities are:

P(0|00)=P(1|11)=0.6 P(1|00)=P(0|11)=0.4

P(0|01)=P(0|10)=P(1|01)=P(1|10)=0.5.

a) Draft the state transition graph of this source; (4 scores)

b) Give the probability transition matrix; (2 scores)

c) Calculate the probability distribution of stationary state; (6 scores)

d) Please give the ultimate distribution of symbols 0 and 1. (2 scores)

**4** If the discrete no memory channel matrix is

A)  B) 

a) please give the types of channel in A) and B), respectively. (2 scores)

b) Please calculate the channel capacities of A) and B), respectively. (16 scores)

**5** If the probability distribution of one DMS is:



a) Please provide the Fenno coding that making sensible and the unique Huffman coding. (10 scores)

b) Please calculate both average code lengths. (4 scores)

c) Please calculate the coding efficiency of both coding methods, respectively. (4 scores)

**I**. (9 scores) Single-Choice Questions:

1) For the entropy of a single symbol discrete source , the entropy of the N extended source  must satisfy\_\_\_\_B\_\_\_\_\_\_\_.

A.  B. 

C.  D. 

2) What is right in the following statement \_\_\_B\_\_\_\_

A. The average mutual information can be negative.

B. The mutual information can be negative in certain condition.

C. For the entropy of certain source, it can increase to infinity.

D. The channel capacity must be larger than any rate distortion function.

3) The ultimate entropy  satisfies \_\_\_\_A\_\_\_\_.

A. It must be smaller than any single entropy.

B. It must be larger than  for the independent extended source.

C. It should be equal to  for the Markov source.

D. It would be larger than  for the Markov source.

**II.** (12 scores) Completion.

a) The components for the Shannon communication system includes\_source\_

, channel\_\_ and \_\_\_destination\_\_.

b) As the probability of a certain event increases, its self-information would \_\_\_decrease\_\_\_.

c) For the rate distortion function, R(Dmin)=\_\_ H(X) \_, while R(Dmax)=\_\_ 0\_\_

(H(X) is the entropy for the source X).

d) For the channel capacity C from the source X to the destination Y, the range for C should be among \_\_\_0\_\_\_ and \_\_min{H(X),H(Y)}\_\_\_ .

**III**. (20 scores) Assume the probability space of one Discrete Memoryless Source (DMS) is:



If there is a noise channel that the source X gets through, the destination symbol set is Y=[b1,b2] and the transition probabilities matrix is:



Then please calculate:

a) I(a1) and I(a2); (2 scores) b) H(X); (4 scores) c) H(Y); (4 scores)

d) H(X,Y); (4 scores) e) I(X.Y). (6 scores)

解：a) I(a1)=-log P(X=a1)=1 bit

I(a2)=-log P(X=a2)=1 bit

b) H(X)=P(X=a1)\* I(a1)+ P(X=a2)\* I(a2)

=0.5\*1+0.5\*1 bit/symbol = 1 bit/symbol

c) P(Y=b1)=0.6\* P(X=a1)+0.2\* P(X=a2)

=0.6\*0.5+0.2\*0.5 = 0.4

P(Y=b2)=0.4\* P(X=a2)+0.8\* P(X=a1)

=0.4\*0.5+0.8\*0.5 = 0.6

or P(Y=b2)=1-P(Y=b1)=0.6

d) H(Y)= -(P(Y=b1)log P(Y=b1)+ P(Y=b2)log P(Y=b2))

= 0.6\* 0.7370+0. 4\*1.3219 bit/symbol

=0.971 bit/symbol

e) P(X=a1, Y=b1)= 0.5\*0.6=0.3

P(X=a1, Y=b2)= 0.5\*0.4=0.2

P(X=a2, Y=b1)= 0.5\*0.2=0.1

P(X=a2, Y=b2)= 0.5\*0.8=0.4

H(X,Y)=0.521+0.464+0.332+0.5288bit/symbol=1.846bit/symbol

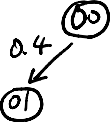
I(X,Y)=H(X)+H(Y)-H(X,Y)=0.125 bit/symbol

**III**. (14 scores) There exists a binary 2-step Markov Chain, where the source symbol set is {0,1}. The symbol transition probabilities are:



P(0|00)=P(1|11)=0.6 P(1|00)=P(0|11)=0.4

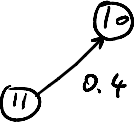
P(0|01)=P(0|10)=P(1|01)=P(1|10)=0.5.



a) Draft the state transition graph of this source; (4 scores)



b) Give the probability transition matrix; (2 scores)



c) Calculate the probability distribution of stationary state; (6 scores)

d) Please give the ultimate distribution of symbols 0 and 1. (2 scores)



解：a) 自己画

b) P=

c) 记w=( w1,w2,w3,w4)是(00,01,10,11)的极限分布概率,

由 w\*P=w 和 w1+w2+w3+w4=1,

得



解得：w=( w1,w2,w3,w4)=(5/18,2/9,2/9,5/18).

d) P(0)=P(0|00)w1+P(0|01)w2+P(0|10)w3+P(0|11)w4

=0.4\*5/18+0.5\*2/9+0.5\*2/9+0.6\*5/18

=1/2

P(1)=1-P(0)=1/2

**IV**. (18 scores) If the discrete no memory channel matrix is

A)  B) 

a) please give the types of channel in A) and B), respectively. (2 scores)

b) Please calculate the channel capacities of A) and B), respectively. (16 scores)

解：

a) A:对称信道 B: 准对称信道

b) A: C=log2m-H(4/5,1/10,1/10)

=log23-(0.2575+0.3322+0.3322)

=1.5850-0.9219bit/symbol=0.6631 bit/symbol

B: 分解成如下三个子对称信道矩阵：

B1=，B2= ，B2= 

其中 N1=1/3, M1=2/3; N2=1/6, M2=1/3; N3=1/2, M3=1.

C=log2n-H(1/6,1/3,1/2)-N1logM1-N2logM2 –N3logM3

=log22-(0.5+0.5283+0.4308)+1/3log3/2+2/3log3/2

=1-1.459+0.1950+0.39bit/symbol=0.126bit/symbol

**V**. (18 scores)If the probability distribution of one DMS is:



a) Please provide the Fenno coding that making sensible and the unique Huffman coding. (10 scores)

b) Please calculate both average code lengths. (4 scores)

c) Please calculate the coding efficiency of both coding methods, respectively. (4 scores)

解：

a) Fenno Coding:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | P |  |  |  | code |
| a1 | 0.4 | 0 |  |  | 0 |
| a2 | 0.2 | 1 | 0 | 0 | 100 |
| a3 | 0.2 | 1 | 101 |
| a4 | 0.1 | 1 | 0 | 110 |
| a5 | 0.1 | 1 | 111 |

Huffman Coding:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | P |  |  |  |  |  |  |  | code |
| a1 | 0.4 |  | 0.4 |  | 0.4 |  | 0.6 | 0 | 1 |
| a2 | 0.2 |  | 0.2 |  | 0.4 | 0 | 0.4 | 1 | 01 |
| a3 | 0.2 |  | 0.2 | 0 | 0.2 | 1 |  |  | 001 |
| a4 | 0.1 | 0 | 0.2 | 1 |  |  |  |  | 0000 |
| a5 | 0.1 | 1 |  |  |  |  |  |  | 0001 |

b) Fenno coding: K=1\*0.4+3\*0.2+3\*0.2+3\*0.1+3\*0.1=2.2bit/symbol

Huffman coding: K=1\*0.4+2\*0.2+3\*0.2+4\*0.1+4\*0.1=2.2bit/symbol

c) H(X)=-(0.4log0.4+0.4log0.2+2\*0.1log0.1)bit/symbol

=0.5288+0.9288+0.6644bit/symbol

=2.122bit/symbol

由于两种编码方法平均码长相同，其编码效率都为

