

ISSN: 0256-307X

# 中国物理快报

# Chinese Physics Letters

Volume 31 Number 12 December 2014

A Series Journal of the Chinese Physical Society  
Distributed by IOP Publishing

Online: <http://iopscience.iop.org/0256-307X>  
<http://cpl.iphy.ac.cn>

CHINESE PHYSICAL SOCIETY  
**IOP** Publishing

JUST FOR AUTHORS  
— CHINESE PHYSICS LETTERS

# Darboux Transformation with a Double Spectral Parameter for the Myrzakulov-I Equation \*

CHEN Hai(陈海), ZHOU Zi-Xiang(周子翔)\*\*

School of Mathematical Sciences, Fudan University, Shanghai 200433

(Received 2 September 2014)

The Darboux transformation with a double spectral parameter for the Myrzakulov-I equation is obtained by taking a suitable limit of the parameters. The globalness of the derived solutions is proved.

PACS: 05.45.Yv, 02.30.Jr

DOI: 10.1088/0256-307X/31/12/120504

The Myrzakulov-I equation is a 2+1-dimensional generalization of the Heisenberg ferromagnetic equation and has a non-isospectral Lax pair.<sup>[1,2]</sup> The explicit solutions to the Myrzakulov-I equation have been discussed by many researchers.<sup>[3,4]</sup> Darboux transformation is one of the useful methods to obtain explicit solutions to the nonlinear partial differential equation.<sup>[5]</sup> The Darboux transformation of degree 1 for this equation has been constructed and exact global ‘one-soliton’ solutions are derived.<sup>[6]</sup>

On the other hand, the Darboux transformation of degree 2 usually relates to a double soliton.<sup>[5,7,8]</sup> Its limit solution is more important in certain cases. Sometimes they can describe a rogue wave.<sup>[9,10,11,12,13]</sup> However, lots of limit solutions have singularity. Hence searching for a global solution is important in both mathematics and applications.

In this Letter, we construct the Darboux transformation of degree 2 and reduce it to the Darboux transformation with a double spectral parameter by taking suitable limits. Globalness of the limit solutions is proved.

The Myrzakulov-I equation is in the form

$$\begin{aligned} S_t &= \frac{i}{2}[S, S_y]_x + (uS)_x, \\ u_x &= -\frac{i}{4}\text{tr}(S[S_x, S_y]), \end{aligned} \quad (1)$$

where  $u$  is an unknown scalar function,

$$S = \sum_{j=1}^3 S_j(x, y, t) \sigma_j \quad (2)$$

is an unknown matrix with  $S^2 = \sum_{j=1}^3 S_j^2 = 1$ , and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Equation (1) has the Lax pair

$$\begin{aligned} \Phi_x &= M(\lambda) \Phi = \frac{i}{2} \lambda S \Phi, \\ \Phi_t &= N(\partial_y, \lambda) \Phi \\ &= -\lambda \Phi_y + \frac{i}{2} \lambda u S \Phi - \frac{1}{2} \lambda S S_y \Phi, \end{aligned} \quad (3)$$

where the ‘spectral parameter’  $\lambda$  satisfies

$$\lambda_x = 0, \quad \lambda_t = -\lambda \lambda_y.$$

From Ref. [6] we know that the Darboux matrix of degree 1 for Eq. (3) is in the form

$$\begin{aligned} G(x, y, t) &= \sqrt{y^2 + \sigma^2} (\lambda N - I), \\ N(x, y, t) &= H \Lambda^{-1} H^{-1}, \end{aligned} \quad (4)$$

where

$$\Lambda = \begin{pmatrix} \mu & 0 \\ 0 & \bar{\mu} \end{pmatrix}, \quad H = \begin{pmatrix} h_1 & -\bar{h}_2 \\ h_2 & \bar{h}_1 \end{pmatrix},$$

and  $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$  is a column solution to Eq. (3) with

$$\begin{aligned} \lambda = \mu(y, t) &= \frac{yt + \sigma \tau \cos \theta}{t^2 + \tau^2} \\ &\pm i \frac{\sqrt{\tau^2 y^2 - 2\sigma \tau y t \cos \theta + \sigma^2 t^2 + \sigma^2 \tau^2 \sin^2 \theta}}{t^2 + \tau^2}, \end{aligned} \quad (5)$$

where  $\sigma, \theta, \tau$  are real constants with  $\tau \neq 0$ . It is easy to prove that  $\begin{pmatrix} -\bar{h}_2 \\ \bar{h}_1 \end{pmatrix}$  is a solution to Eq. (3) with  $\lambda = \bar{\mu}(y, t)$ . The authors of Ref. [7] gave the following result.

**Lemma 1:** Suppose that  $(S, u)$  is a solution to Eq. (1). After the Darboux transformation of  $G(x, y, t)$ , the derived solution to Eq. (1) is

$$\tilde{S} = N S N^{-1}, \quad \tilde{u} = u - i \text{tr}(N^{-1} N_y S). \quad (6)$$

Now we consider the Darboux matrix of degree 2 for Eq. (3).

\*Supported by the National Natural Science Foundation of China under Grant No 11171073, and the Key Laboratory of Mathematics for Nonlinear Sciences of the Ministry of Education of China.

\*\*Corresponding author. Email: zxzhou@fudan.edu.cn

© 2014 Chinese Physical Society and IOP Publishing Ltd

According to Ref. [5], let

$$A_1 = \begin{pmatrix} \mu_1 & \\ & \bar{\mu}_1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} \mu_2 & \\ & \bar{\mu}_2 \end{pmatrix},$$

where  $\mu_i$  is the function in the form of Eq. (5) with  $\sigma_i, \tau_i, \theta_i, i = 1, 2$ . Let

$$H_1^{(i)} = \begin{pmatrix} h_1^{(i)} \\ h_2^{(i)} \end{pmatrix}, \quad H_2^{(i)} = \begin{pmatrix} -\bar{h}_2^{(i)} \\ \bar{h}_1^{(i)} \end{pmatrix}$$

be solutions to Eq. (3) with  $\lambda = \mu_i$  and  $\lambda = \bar{\mu}_i$ , respectively, and  $H_i = (H_1^{(i)}, H_2^{(i)})$ .

Let

$$N_1 = H_1 A_1^{-1} H_1^{-1}, \quad G_1(\lambda) = \sqrt{y^2 + \sigma^2}(\lambda N_1 - I), \quad (7)$$

then  $G_1(\lambda)$  is a Darboux matrix for Eq. (3).

Next we take

$$\tilde{H}_1^{(2)} = G_1(\mu_2) H_1^{(2)}, \quad \tilde{H}_2^{(2)} = G_1(\bar{\mu}_2) H_2^{(2)},$$

then  $\tilde{H}_1^{(1)}$  and  $\tilde{H}_2^{(2)}$  are new solutions to the Lax pair of Eq. (3) with  $\lambda = \mu_2$  and  $\lambda = \bar{\mu}_2$ , respectively, where  $(S, u)$  is replaced by  $(\tilde{S}, \tilde{u})$ . As a result,

$$\tilde{H}_2 = (\tilde{H}_2^{(1)}, \tilde{H}_2^{(2)}) = \sqrt{y^2 + \sigma^2}(N_1 - N_2)N_2^{-1}H_2,$$

and

$$\tilde{N}_2 = \tilde{H}_2 A_2^{-1} \tilde{H}_2^{-1} = (N_2 - N_1)N_2(N_2 - N_1)^{-1}. \quad (8)$$

This leads to another Darboux matrix

$$G_2(\lambda) = \sqrt{y^2 + \sigma^2}(\lambda \tilde{N}_2 - I) \\ = \sqrt{y^2 + \sigma^2}(\lambda(N_2 - N_1)N_2(N_2 - N_1)^{-1} - I).$$

The composition  $G(\lambda) = G_2(\lambda)G_1(\lambda)$  is a Darboux matrix  $G$  of degree 2. It is

$$G = \sqrt{y^2 + \sigma_2^2}(\lambda \tilde{N}_2 - I) \sqrt{y^2 + \sigma_1^2}(\lambda N_1 - I) \\ = \sqrt{(y^2 + \sigma_2^2)(y^2 + \sigma_1^2)} \\ \cdot (\lambda^2(N_2 - N_1)N_2(N_2 - N_1)^{-1}N_1 \\ - \lambda(N_2^2 - N_1^2)(N_2 - N_1)^{-1} + I). \quad (9)$$

Suppose that  $(S, u)$  is a solution to Eq. (1). After the Darboux transformation  $G$  of degree 2, the derived solution to Eq. (1) is

$$\tilde{\tilde{u}} = u - i \text{tr}(N_1^{-1}N_{1y}S) - i \text{tr}(\tilde{N}_2^{-1}\tilde{N}_{2y}\tilde{S}), \\ \tilde{\tilde{S}} = \tilde{N}_2 \tilde{S} \tilde{N}_2^{-1} = (N_2 - N_1)(N_1^{-1} - N_2^{-1})^{-1} \\ \cdot S(N_1^{-1} - N_2^{-1})(N_2 - N_1)^{-1}, \quad (10)$$

where  $\tilde{S} = N_1 S N_1^{-1}$ , and  $\tilde{N}_2 = (N_2 - N_1)N_2(N_2 - N_1)^{-1}$ .

We will discuss the limit of the solution (10) when one parameter tends to another.

For given  $\mu(x, y, t)$  and  $H(x, y, t)$ , let  $\mu^{(\epsilon)}(x, y, t)$  and  $H^{(\epsilon)}(x, y, t)$  be a smooth function and a smooth matrix-valued function such that  $\mu^{(0)}(x, y, t) = \mu(x, y, t)$  and  $H^{(0)}(x, y, t) = H(x, y, t)$ , respectively.

Now  $\mu_1 = \mu$ ,  $H_1 = H$ ,  $\mu_2 = \mu^{(\epsilon)}$ , and  $H_2 = H^{(\epsilon)}$  are adopted, and  $N$  and  $N^{(\epsilon)}$  are defined as in Eq. (4).

We can construct Darboux matrix  $G(x, y, t, \epsilon)$  by Eq. (9). The limit of  $G(x, y, t, \epsilon)$  as  $\epsilon$  tends to zero will give a Darboux transformation with a double spectral parameter  $\mu$ .

According to Eq. (4),

$$N = \frac{1}{|h_1|^2 + |h_2|^2} \\ \cdot \begin{pmatrix} \mu^{-1}|h_1|^2 + \bar{\mu}^{-1}|h_2|^2 & (\mu^{-1} - \bar{\mu}^{-1})h_1\bar{h}_2 \\ (\mu^{-1} - \bar{\mu}^{-1})\bar{h}_1h_2 & \bar{\mu}^{-1}|h_1|^2 + \mu^{-1}|h_2|^2 \end{pmatrix}. \quad (11)$$

Denote  $U = \left\{ \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} \mid a, b \in \mathbb{C} \right\}$ , then  $AA^* = \det(A)I$  for any  $A \in U$ . Clearly,  $N, N^{(\epsilon)} \in U$ , and  $N^{(\epsilon)} - N \in U$ . Equation (8) becomes

$$\tilde{N} = \frac{(N^{(\epsilon)} - N)N^{(\epsilon)}(N^{(\epsilon)} - N)^*}{\det(N^{(\epsilon)} - N)}. \quad (12)$$

For any smooth function  $f^{(\epsilon)}(x, y, t)$ , denote

$$Df(x, y, t) = \frac{\partial}{\partial \epsilon} f^{(\epsilon)}(x, y, t)|_{\epsilon=0}.$$

Suppose  $DN \neq 0$ , then Eq. (12) becomes

$$\tilde{N} = \frac{(DN\epsilon + O(\epsilon^2))N^{(\epsilon)}(DN\epsilon + O(\epsilon^2))^*}{\det(DN\epsilon + O(\epsilon^2))} \\ = \frac{(DN + O(\epsilon))N^{(\epsilon)}(DN + O(\epsilon))^*}{\det(DN + O(\epsilon))}. \quad (13)$$

Thus as  $\epsilon \rightarrow 0$ ,

$$\tilde{N} \rightarrow \frac{(DN)N(DN)^*}{\det(DN)} = (DN)N(DN)^{-1}.$$

*Lemma 2:* If  $DN \neq 0$ , the Darboux transformation with a double spectral parameter is

$$\bar{G} = (y^2 + \sigma^2) \{ \lambda^2 (DN)N(DN)^{-1}N \\ - \lambda [N + (DN)N(DN)^{-1}] + I \} \quad (14)$$

and the solutions (10) become

$$\bar{u} = u - i \text{tr}(N^{-1}N_y S) \\ - i \text{tr} \{ (DN)N^{-1}(DN)^{-1} \\ \cdot [(DN)N(DN)^{-1}]_y N S N^{-1} \}, \\ \bar{S} = (DN)N(DN)^{-1}N S N^{-1}(DN)N^{-1}(DN)^{-1}. \quad (15)$$

What we care about in the following is whether Eq. (15) is defined globally. The key point is to find out whether  $(DN)^{-1}$  exists globally.

**Theorem 1:** Let  $(S, u)$  be a solution to Eq. (1) such that the Lax pair Eq. (3) is uniquely solvable. Let  $(h_1, h_2)^T$  be a non-zero global solution to Eq. (3) and  $N$  is given by Eq. (11). Then  $\bar{G}$  given by Eq. (14) is globally defined if  $D\mu \neq 0$ .

*Proof:* By the unique solvability of Eq. (3),  $(h_1, h_2)^T$  is non-zero everywhere if it is non-zero at one point. According to Eq. (11),

$$DN = \begin{pmatrix} Da & -D\bar{c} \\ Dc & D\bar{a} \end{pmatrix},$$

where

$$Da = -\frac{\mu^{-2}D\mu|h_1|^2 + \overline{\mu^{-2}D\mu}|h_2|^2}{|h_1|^2 + |h_2|^2} - \frac{\mu^{-1} - \bar{\mu}^{-1}}{(|h_1|^2 + |h_2|^2)^2}(Y + \bar{Y}), \quad (16)$$

$$Dc = \frac{(-\mu^{-2}D\mu + \overline{\mu^{-2}D\mu})\bar{h}_1h_2}{|h_1|^2 + |h_2|^2} + \frac{(\mu^{-1} - \bar{\mu}^{-1})}{(|h_1|^2 + |h_2|^2)^2}M, \quad (17)$$

$$Y = |h_1|^2\bar{h}_2Dh_2 - |h_2|^2\bar{h}_1Dh_1, \quad (18)$$

$$M = |h_1|^2\bar{h}_1Dh_2 - \bar{h}_1^2Dh_1 + |h_1|^2h_2D\bar{h}_1 - \bar{h}_1h_2^2D\bar{h}_2, \quad (19)$$

or

$$h_1\bar{h}_2M = |h_1|^2Y - |h_2|^2\bar{Y}. \quad (20)$$

Here  $\det(DN) = 0$  if and only if  $Da = Dc = 0$ . Considering the real and imaginary parts of Eqs. (16) and (17), we have

$$\operatorname{Re}\left(\frac{D\mu}{\mu^2}\right) = 0, \quad (21)$$

$$\left(\operatorname{Im}\frac{D\mu}{\mu^2}\right)(|h_1|^4 - |h_2|^4) + 4\left(\operatorname{Im}\frac{1}{\mu}\right)(\operatorname{Re}Y) = 0, \quad (22)$$

$$\operatorname{Im}Y = 0, \quad (23)$$

$$|h_1|^2|h_2|^2\operatorname{Im}\frac{D\mu}{\mu^2}(|h_1|^2 + |h_2|^2) - \left(\operatorname{Im}\frac{1}{\mu}\right)\operatorname{Re}Y(|h_1|^2 - |h_2|^2) = 0 \quad (24)$$

when  $h_1 \neq 0$  and  $h_2 \neq 0$ . It is easy to verify that these equations also hold when  $h_1 = 0$  or  $h_2 = 0$ . Eliminating  $\operatorname{Re}Y$  in Eqs. (22) and (24), we obtain

$$\operatorname{Im}\frac{D\mu}{\mu^2}(|h_1|^2 + |h_2|^2)^2 = 0.$$

Considering Eq. (21), we have  $D\mu = 0$ . This contradicts the assumption  $D\mu \neq 0$ . The theorem is proved.

**Remark 1:** The condition  $D\mu \neq 0$  holds in quite general cases, which will be shown in the following examples.

To obtain an explicit expression of the solution, we take  $S = \sigma_3$ ,  $u = 0$  as a seed solution to Eq. (1). Let

$\mu(y, t)$  be given by Eq. (5), then the solution to the Lax pair in Eq. (3) with  $\lambda = \mu$  is

$$\Phi = e^{\frac{i}{2}\mu x \sigma_3} \Phi_0(\mu),$$

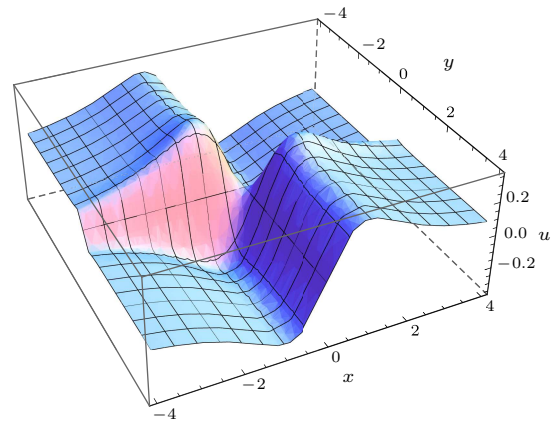
where  $\Phi_0(\mu)$  is a matrix whose entries are holomorphic functions of  $\mu$ . Since  $\mu$  is globally defined, so is  $\Phi$ .

Since  $\mu$  depends on three parameters  $\sigma$ ,  $\tau$ ,  $\theta$ , we can discuss the change of  $\sigma$ ,  $\tau$ ,  $\theta$ , respectively.

First we consider the limit of  $\sigma$ . Let

$$\mu = \mu(y, t, \theta, \tau, \sigma), \quad \mu^{(\epsilon)} = \mu(y, t, \theta, \tau, \sigma + \epsilon),$$

then  $DN = \partial_\sigma N$ . The solution is given by Eq. (15) where  $DN$  is replaced by  $\partial_\sigma N$ .



**Fig. 1.** The value of  $u$  of the solution with single spectral parameter.

Similarly, replacing  $DN$  by  $\partial_\tau N$  or  $\partial_\theta N$  respectively in Eq. (15), we obtain the limit solution for  $\tau$  or  $\theta$ .

**Remark 2:** For the limit of  $\theta$ , since

$$\det N = \det N^{(\epsilon)} = \frac{y^2 + \sigma^2}{t^2 + \tau^2},$$

is independent of  $\theta$ , the expression (15) for  $\bar{S}$  can be simplified. According to Eq. (10),

$$\begin{aligned} \tilde{\bar{S}} &= (N^{(\epsilon)} - N)(N^{-1} - (N^{(\epsilon)})^{-1})^{-1}S \\ &\quad \cdot (N^{-1} - (N^{(\epsilon)})^{-1})(N^{(\epsilon)} - N)^{-1} \\ &= (N^{(\epsilon)} - N)(N^* - (N^{(\epsilon)})^*)^{-1}S \\ &\quad \cdot (N^* - (N^{(\epsilon)})^*)(N^{(\epsilon)} - N)^{-1} \\ &= (N^{(\epsilon)} - N)^2 S (N^{(\epsilon)} - N)^{-2}, \end{aligned} \quad (25)$$

hence the limit solution is

$$\bar{S} = (\partial_\theta N)^2 S (\partial_\theta N)^{-2}. \quad (26)$$

For the limit of  $\theta$ ,

$$\operatorname{Re}(D\mu) = \operatorname{Re}(\partial_\theta \mu) = -\frac{\sigma\tau \sin \theta}{t^2 + \tau^2}.$$

According to theorem 1, Eq. (15) can be defined globally if  $\sigma \neq 0$ ,  $\tau \neq 0$  and  $\sin \theta \neq 0$ .

For the limit of  $\sigma$ ,

$$\operatorname{Re}(D\mu) = \operatorname{Re}(\partial_\sigma \mu) = \frac{\tau \cos \theta}{t^2 + \tau^2},$$

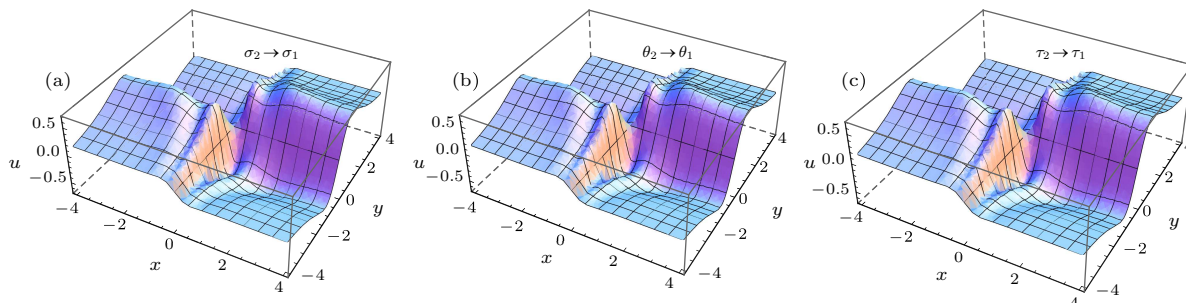
Eq. (15) is defined globally if  $\tau \neq 0$  and  $\cos \theta \neq 0$ .

For the limit of  $\tau$ , it is easier to compute  $\frac{D\mu}{\mu^2}$  than

$D\mu$ . Hence

$$\operatorname{Re}\left(\frac{D\mu}{\mu^2}\right) = -\operatorname{Re}(\partial_\tau \mu^{-1}) = -\frac{\sigma \cos \theta}{y^2 + \tau^2}.$$

The solution is defined globally if we choose  $\sigma \neq 0$  and  $\cos \theta \neq 0$ .



**Fig. 2.** The value of  $u$  of the solution with a double spectral parameter

Taking  $\sigma = 1$ ,  $\tau = 1$ ,  $\theta = \pi/4$ ,  $t = 0$  and  $\Phi(\mu) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ , here we only plot the  $u$  part of the solutions.

The solution with a single spectral parameter is plotted in Fig. 1 and the solutions with double spectral parameters (by the limit of  $\theta$ ,  $\sigma$ ,  $\tau$  respectively) are plotted in Fig. 2. From these figures, we can see that the solutions with a double spectral parameter look like perturbations of the solutions with a single spectral parameter, and the shapes of solutions with a double spectral parameter by different limits of  $\theta$ ,  $\sigma$  or  $\tau$  are similar.

## References

- [1] Anco S C and Myrzakulov R 2010 *J. Geom. Phys.* **60** 1576
- [2] Myrzakulov R, Nugmanova G N and Syzdykova R N 1998 *J. Phys. A* **31** 9535
- [3] Martina L, Myrzakul K, Myrzakulov R and Soliani G 2001 *J. Math. Phys.* **42** 1397
- [4] Myrzakulov R, Vijayalakshmi S, Nugmanova G N and Lakshmanan M 1997 *Phys. Lett. A* **233** 391
- [5] Gu C H, Hu H S and Zhou Z X 2005 *Darboux Transformations in Integrable Systems* (Dordrecht: Springer)
- [6] Chen C and Zhou Z X 2009 *Chin. Phys. Lett.* **26** 080504
- [7] Drazin P G and Johnson R S 1989 *Solitons: An Introduction* (Cambridge: Cambridge University Press)
- [8] Song Q F and Zhou Z X 2005 *Commun. Theor. Phys.* **44** 977
- [9] Akhmediev N, Ankiewicz A and Taki M 2009 *Phys. Lett. A* **373** 675
- [10] Guo B L and Ling L M 2011 *Chin. Phys. Lett.* **28** 110202
- [11] He J S, Xu S W and Porseizan K 2012 *Phys. Rev. E* **86** 066603
- [12] Tao Y S, He J S and Porseizan K 2013 *Chin. Phys. B* **22** 074210
- [13] Zhang Y, Nie X J and Zha Q L 2014 *Chin. Phys. Lett.* **31** 060201

# Chinese Physics Letters

Volume 31

Number 12

December 2014

## GENERAL

- 120301 Eigen Spectra of the Dirac Equation for Deformed Woods–Saxon Potential via the Similarity Transformation**  
ALSADI Khalid S
- 120501 Multi-Scale Time Asymmetry for Detecting the Breakage of Slug Flow Structure**  
HAO Qing-Yang, JIN Ning-De, HAN Yun-Feng, GAO Zhong-Ke, ZHAI Lu-Sheng
- 120502 Harmonic Noise-Induced Resonant Passing in an Inverse Harmonic Potential**  
HAN Jie, BAO Jing-Dong
- 120503 Periodic States in Chaotic Rössler Oscillators with On-Off Coupling**  
CHU Shuang-Tian, LIANG Xiao-Ming, LÜ Hua-Ping
- 120504 Darboux Transformation with a Double Spectral Parameter for the Myrzakulov-I Equation**  
CHEN Hai, ZHOU Zi-Xiang
- 120601 Laser 728 nm Spectroscopy of Electrodeless Discharge Rb Lamp**  
LIU Zhong-Zheng, XUE Xiao-Bo, NIU Fu-Zeng, ZHANG Li-Guo, LING Li, CHEN Jing-Biao
- 120602 Cs 728 nm Laser Spectroscopy and Faraday Atomic Filter**  
LIU Zhong-Zheng, TAO Zhi-Ming, JIANG Zhao-Jie, CHEN Jing-Biao

## NUCLEAR PHYSICS

- 122801 Fabrication of Nano-Columnar Tungsten Films and Their Deuterium and Helium Ion Irradiation Effects**  
CAI Ya-Nan, HAN Wen-Jia, CHEN Zhe, YU Jian-Gang, FENG Hong-Li, ZHU Kai-Gui
- 122901 Ion Transportation Study for Thick Gas Electron Multipliers**  
WANG Bin-Long, LIU Qian, LIU Hong-Bang, ZHOU Xiao-Kang, CHEN Shi, GE Dong-Sheng, HUANG Wen-Qian, XIE Yi-Gang, ZHENG Yang-Heng, DONG Yang, ZHANG Qiang, JIAO Xin-Da, WANG Jing, LI Min, CHANG Jie
- 122902 High-Precision Calibration of Electron Beam Energy from the Hefei Light Source Using Spin Resonant Depolarization**  
LAN Jie-Qin, XU Hong-Liang

## ATOMIC AND MOLECULAR PHYSICS

- 123201 Observation of Spin Polarized Clock Transition in  $^{87}\text{Sr}$  Optical Lattice Clock**  
WANG Qiang, LIN Yi-Ge, LI Ye, LIN Bai-Ke, MENG Fei, ZANG Er-Jun, LI Tian-Chu, FANG Zhan-Jun

## FUNDAMENTAL AREAS OF PHENOMENOLOGY(INCLUDING APPLICATIONS)

- 124201 Theoretical Investigation on THz Generation from Optical Rectification with Tilted-Pulse-Front Excitation**  
DU Hai-Wei, YANG Nan
- 124202 Passive Q-Switching Laser Performance of Yb:YVO<sub>4</sub> Crystal**  
LI Xiao-Hong, CHEN Xiao-Wen, HAN Wen-Juan, KONG Wei-Jin, LIU Jun-Hai
- 124203 Q-Switching Pulse Generation with Thulium-Doped Fiber Saturable Absorber**  
Zian Cheak Tiu, Arman Zarei, Sin Jin Tan, Harith Ahmad, Sulaiman Wadi Harun
- 124204 Diffraction Properties for 1000 Line/mm Free-Standing Quantum-Dot-Array Diffraction Grating Fabricated by Focused Ion Beam**  
ZHANG Ji-Cheng, LIU Yu-Wei, HUANG Cheng-Long, ZHANG Qiang-Qiang, YI Yong, ZENG Yong, ZHU Xiao-Li, FAN Quan-Ping, QIAN Feng, WEI Lai, WANG Hong-Bin, WU Wei-Dong, CAO Lei-Feng
- 124205 In-Band Pumped High Power Ho:YAG Ceramic Laser by a Tm:YLF Laser**  
YUAN Jin-He, YAO Bao-Quan, DUAN Xiao-Ming, SHEN Ying-Jie, CUI Zheng, YU Kuai-Kuai, LI Jiang, PAN Yu-Bai

- 124301 Low-Frequency Hydroacoustic Experiments on the Shelf Using the Data of Geoacoustic Sediment Model**  
SAMCHENKO A. N., KOSHELEVA A. V., SHVYREV A. N., PIVOVAROV A. A.
- 124302 A Multiple Resonant Mode Film Bulk Acoustic Resonator Based on Silicon-on-Insulator Structures**  
CHEN Xiao, YANG Yi, CAI Hua-Lin, ZHOU Chang-Jian, Mohammad Ali MOHAMMAD, REN Tian-Ling

### **PHYSICS OF GASES, PLASMAS, AND ELECTRIC DISCHARGES**

- 125201 Laser Heated Emissive Probes Design and Development under National Fusion Program and Potential Measurement**  
MEHTA Payal, SARMA Arun, GHOSH Joydeep
- 125202 Observation of Runaway Electrons with Soft X-Ray Camera on HT-7 Tokamak**  
CHEN Ye-Bin, CHEN Kai-Yun, XU Li-Qing, ZHOU Rui-Jie, HU Li-Qun

### **CONDENSED MATTER: STRUCTURE, MECHANICAL AND THERMAL PROPERTIES**

- 126101 Enhanced Total Ionizing Dose Hardness of Deep Sub-Micron Partially Depleted Silicon-on-Insulator n-Type Metal-Oxide-Semiconductor Field Effect Transistors by Applying Larger Back-Gate Voltage Stress**  
ZHENG Qi-Wen, CUI Jiang-Wei, YU Xue-Feng, GUO Qi, ZHOU Hang, REN Di-Yuan
- 126201 Electrical Resistivity of Silane Multiply Shock-Compressed to 106 GPa**  
ZHONG Xiao-Feng, LIU Fu-Sheng, CAI Ling-Cang, XI Feng, ZHANG Ming-Jian, LIU Qi-Jun, WANG Ya-Ping, HAO Bin-Bin
- 126601 The Impact of Shallow-Trench-Isolation Mechanical Stress on the Hysteresis Effect of Partially Depleted Silicon-on-Insulator n-Type Metal-Oxide-Semiconductor Field Effects**  
LUO Jie-Xin, CHEN Jing, CHAI Zhan, L Kai, HE Wei-Wei, YANG Yan, WANG Xi

### **CONDENSED MATTER: ELECTRONIC STRUCTURE, ELECTRICAL, MAGNETIC, AND OPTICAL PROPERTIES**

- 127201 An Alternating-Current Voltage Modulated Thermal Probe Technique for Local Seebeck Coefficient Characterization**  
XU Kun-Qi, ZENG Hua-Rong, YU Hui-Zhu, ZHAO Kun-Yu, LI Guo-Rong, SONG Jun-Qiang, SHI Xun, CHEN Li-Dong
- 127301 Nonlinear Intersubband Transitions in Square and Graded Quantum Wells Modulated by Intense Laser Field**  
Emine Ozturk, Ismail Sokmen
- 127302 Electronic Transport of the Adsorbed Trigonal Graphene Flake: A First Principles Calculation**  
TAN Xun-Qiong
- 127303 Observation of a Flat Band in Silicene**  
FENG Ya, FENG Bao-Jie, XIE Zhuo-Jin, LI Wen-Bin, LIU Xu, LIU De-Fa, ZHAO Lin, CHEN Lan, ZHOU Xing-Jiang, WU Ke-Hui
- 127304 Current Fluctuations in a Semiconductor Quantum Dot with Large Energy Spacing**  
JEONG Heejun
- 127401 High-Pressure Single-Crystal Neutron Scattering Study of Magnetic and Fe Vacancy Orders in  $(\text{Tl,Rb})_2\text{Fe}_4\text{Se}_5$  Superconductor**  
YE Feng, BAO Wei, CHI Song-Xue, Antonio M. dos Santos, Jamie J. Molaison, FANG Ming-Hu, WANG Hang-Dong, MAO Qian-Hui, WANG Jin-Chen, LIU Juan-Juan, SHENG Jie-Ming
- 127501 Effect of Crystalline Quality on Magnetic Properties of Mn-Doped ZnO Nanowires**  
CHANG Yong-Qin, SUN Qing-Ling, LONG Yi, WANG Ming-Wen
- 127701 Electron Trap Energy Distribution in  $\text{HfO}_2$  by the Discharge-Based Pulse  $I-V$  Technique**  
ZHENG Xue-Feng, FAN Shuang, KANG Di, ZHANG Jian-Kun, CAO Yan-Rong, MA Xiao-Hua, HAO Yue

- 127702 Unique Charge Storage Characteristics of FEP/THV/FEP Sandwich Electret Membrane Polarized by Thermally Charging Technology**  
CHEN Gang-Jin, LEI Ming-Feng, XIAO Hui-Ming, WU Ling
- 127801 Detecting Cells with the Scatter Plot Pattern of an Orthogonal Scattering Mueller Matrix**  
WANG Qing-Hua, LI Zhen-Hua, LAI Jian-Cheng, HE An-Zhi
- 127802 Ultrafast Imaging of Electronic Relaxation in Ortho-xylene: New Features from Fragmentation-Ion Spectroscopy**  
LIU Yu-Zhu, KNOPP Gregor, XIAO Shao-Rong, GERBER Thomas
- 127901 Current Density-Dependent Thermal Stability of ZnSe Nanowire in M-S-M Nanostructure**  
TAN Yu, WANG Yan-Guo

## **CROSS-DISCIPLINARY PHYSICS AND RELATED AREAS OF SCIENCE AND TECHNOLOGY**

- 128101 Growth of High-Quality GaAs on Ge by Controlling the Thickness and Growth Temperature of Buffer Layer**  
ZHOU Xu-Liang, PAN Jiao-Qing, YU Hong-Yan, LI Shi-Yan, WANG Bao-Jun, BIAN Jing, WANG Wei
- 128102 Growth of Atomically Flat Ultra-Thin Ag Films on Si(111) by Introducing a  $\sqrt{3}\times\sqrt{3}$ -Ga Buffer Layer**  
HE Jie-Hui, JIANG Li-Qun, QIU Jing-Lan, CHEN Lan, WU Ke-Hui
- 128103 Synthesis and Optical Properties of InP Semiconductor Nanocombs**  
YU Yan-Long, ZHAO Yi-Song, GAO Fa-Ming
- 128501 Effects of Annealing on Schottky Characteristics in AlGaIn/GaN HEMT with Transparent Gate Electrode**  
WANG Chong, ZHANG Kun, HE Yun-Long, ZHENG Xue-Feng, MA Xiao-Hua, ZHANG Jin-Cheng, HAO Yue
- 128502 A Quasi-3D Threshold Voltage Model for Dual-Metal Quadruple-Gate MOSFETs**  
Visweswara Rao Samoju, Satyabrata Jit, Pramod Kumar Tiwari

## **GEOPHYSICS, ASTRONOMY, AND ASTROPHYSICS**

- 129701 Energy Extraction from a Black Hole and Its Influence on X-Ray Spectra**  
HUANG Chang-Yin, GONG Xiao-Long, WANG Ding-Xiong

NOT FOR AUTHORS  
— CHINESE PHYSICS LETTERS