# CMVTF:

*Yipeng Liu,Sixing Zeng*

School of Information and Communication Engineering / Center for robotics,
University of Electronic Science and Technology of China (UESTC), Chengdu, China

**ABSTRACT**

Coupled Matrix-Vector Tensor Factorization(CMVTF) is proposed for fusing a low-spatial-resolution hyperspectral image(HSI) and a high-spatial-resolution multispectral image(MSI) to produce a super-resolution image (SRI) with high spatial and spectral resolutions.In the proposed CMVTF method,We consider the HSI and MSI as a three-dimensional tensor and can be decomposed into a sum of several component tensor , and each component tensor is from the outer produt of a vector(endmember) and a matrix(corresponding abundances). This coupled tensor factorization model is consistent with linear spectral mixture model and experiments demonstrate the superiority of the proposed CMVTF method over current state-of-the-art HSI-MSI fusion approaches.

***Index Terms*—** One, two, three, four, five

## 1. INTRODUCTION

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## 2. NOTATIONS AND PRELIMINARIES

Firstly we would like to give the notations to be used. A vector, a matrix, and a tensor are written as $\mathbf{a}$, $\mathbf{A}$, and $\mathcal{A}$, respectively. A $N$-order tensor $\mathcal{A} \in \mathbb{R}^{I_1\times\cdots\times I_N}$, and $I_n$ is the dimensional size. The vectorization of tensor $\mathcal{A}$ is denoted as $\mathrm{Vec}(\mathcal{A})$. $\langle\mathcal{A},\mathcal{B}\rangle = \langle\mathrm{Vec}(\mathcal{A}),\mathrm{Vec}(\mathcal{B})\rangle$ denotes the tensor inner product. The Frobenius norm of $\mathcal{A}$ is denoted as $\|\mathcal{A}\|_{\mathrm{F}} = \langle\mathcal{A},\mathcal{A}\rangle^{\frac{1}{2}}$. The Kronecker product of two tensors can be denoted as $\mathcal{C} = \mathcal{A}\otimes\mathcal{B} \in \mathbb{R}^{I_1J_1\times\cdots\times I_NJ_N}$, where $\mathcal{A}\in\mathbb{R}^{I_1\times\cdots\times I_N}$, $\mathcal{B}\in\mathbb{R}^{J_1\times\cdots\times J_N}$. For $M$ same tensors, the Kronecker product can be defined as $\mathcal{C} = \mathcal{A}_1\otimes\cdots\otimes\mathcal{A}_M = \otimes_{m=1}^{M}\mathcal{A}_m \in \mathbb{R}^{I_1^M\times\cdots\times I_N^M}$, with $\mathcal{A}\in\mathbb{R}^{I_1\times\cdots\times I_N}$. The Hadamard product of two tensors is defined as $(\mathcal{A}\odot\mathcal{B})_{i_1,\cdots,i_N} = \mathcal{A}_{i_1,\cdots,i_N}\mathcal{B}_{i_1,\cdots,i_N}$, where $\mathcal{A}_{i_1,\cdots,i_N}$ and $\mathcal{B}_{i_1,\cdots,i_N}$ are the entries of $\mathcal{A}$ and $\mathcal{B}$. The operator $\mathrm{reshape}$ turns a matrix or a vector to a tensor, e.g., we can obtain a tensor $\mathcal{X}$ by the operating $\mathrm{reshape}(\mathbf{X}, I_1, I_2, I_3)$ from a matrix $\mathbf{X}\in\mathbb{R}^{I_1I_2\times I_3}$ or $\mathrm{reshape}(\mathbf{x}, I_1, I_2, I_3)$ from a vector $\mathbf{x}\in\mathbb{R}^{I_1I_2I_3}$. The mide-$n$ tensor-matrix product can be presented as $\mathcal{C} = \mathcal{A}\times_n\mathbf{B}$, where $\mathcal{A}\in\mathbb{R}^{I_1\times\cdots\times I_N}$, $\mathbf{B}\in\mathbb{R}^{J\times I_n}$, $\mathcal{C}\in\mathbb{R}^{I_1\times\cdots\times I_{n-1}\times J\times I_{n+1}\times I_N}$.

**Definition 1** (***Mode-k unfolding***) **[?]** *For an $N$-order tensor $\mathcal{A}\in\mathbb{R}^{\mathrm{I}_1\times\cdots\times\mathrm{I}_N}$, its mode-k unfolding is defined as a matrix*

$$\mathbf{A}_{[k]}\in\mathbb{R}^{I_k\times I_1\cdots I_{k-1}I_{k+1}\cdots I_N}$$

*with entries*

$$a_{i_1i_2\cdots i_N} = \mathbf{A}_{[k]}(i_k, \overline{i_1\cdots i_{k-1}i_{k+1}\cdots i_N}).$$

**Definition 2** (***Tensor contracted product***) **[?]** *For multi-way arrays $\mathcal{A}\in\mathbb{R}^{I_1\times\cdots\times I_N}$ and $\mathcal{B}\in\mathbb{R}^{J_1\times\cdots\times J_L}$*, with some common mode such as $I_1 = J_1,\cdots I_n = J_n$*. The contracted product $\mathcal{C}\in\mathbb{R}^{I_{n+1}\times\cdots\times I_N\times J_{n+1}\times\cdots\times J_L}$ can be written as*

$$\mathcal{C} =< \mathcal{A},\mathcal{B} >_n$$

*with entries*

$$c_{i_{n+1},\cdots,i_N,j_{n+1},\cdots,j_L} = \sum_{i_1=1,\cdots,i_n=1}^{I_1,\cdots,I_n} a_{i_1,\cdots,i_N}b_{j_1,\cdots,j_L}.$$

**Definition 3** (***Tensor in Graph formats***) **[?]** *Given any graph $G = (\mathbb{V},\mathbb{E})$, where $\mathbb{V} = \{1,\cdots,N\}$ and $\mathbb{E} = \{\{i_1,j_1\},\cdots,\{i_M,j_M\}\}$ are the set of $N$ vertices and $M$ edges, respectively. $\hat{\mathbb{E}} = \{(i_1,j_1),\cdots,(i_M,j_M)\}$ are the edges for arbitrary directions. For each $n\in\mathbb{V}$, the tensor product space is defined as:*

$$(\otimes_{m\in\mathrm{IN}(n)}\mathbb{E}_m)\otimes\mathbb{V}_n\otimes(\otimes_{m\in\mathrm{OUT}(n)}\mathbb{E}_m^*)$$

*where $\mathbb{E}_m^*$ is the dual space of $\mathbb{E}_m$, $\mathrm{IN}(i) = \{j\in\{1,\cdots,M\} : (j,i)\in\hat{\mathbb{E}}\}$, and $\mathrm{OUT}(i) = \{j\in\{1,\cdots,M\} : (i,j)\in\hat{\mathbb{E}}\}$.*

*The contraction map for tensor can be defined as:*

$$\begin{aligned} K_G : \quad & \otimes_{n=1}^{N}((\otimes_{m\in\mathrm{IN}(n)}\mathbb{E}_m)\otimes\mathbb{V}_n\otimes(\otimes_{m\in\mathrm{OUT}(n)}\mathbb{E}_m^*)) \\ & \to \otimes_{n=1}^{N}\mathbb{V}_n \end{aligned}$$

*Contracting along all edges, we can obtain a tensor in $\mathbb{V}_1\otimes\cdots\otimes\mathbb{V}_N$.*

Thanks to XYZ agency for funding.

**Definition 4** (***Complete-graph tensor network***) *For a multiway array $\mathcal{A} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$, the complete-graph tensor network in graph is defined as*

$$\mathcal{A} = K_G(\otimes_{n=1}^{N}(\mathcal{U}_n)).$$

*where all edges $\mathbb{E}$ for complete graph are assigned to* IN, *and $\mathcal{U}_n = ((\otimes_{m \in \mathrm{IN}_{(n)}} \mathbb{E}_m) \otimes \mathbb{V}_n) \in \mathbb{R}^{r^m \times I_n}$, $n = 1, \cdots, N$ are the cores, and the rank of complete graph tensor network is defined by the weigh of edges. The graphical of 4-order complete graph tensor network can be seen in Fig.* **??**.

**Proposition 1** (***Edge removal***) [**?**] *For an N-order tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_N}$, which can be represented by the $G = (\mathbb{V}, \mathbb{E})$ with $N$ vertices and $M$ edges. Let $G_r = (r_1, \cdots, r_M)$ are the weight of edges. Suppose that the edge $e \in \mathbb{E}$ has weight $r_1 = 1$ and $G^{'} = (\mathbb{V}, \mathbb{E} \setminus \{e\})$ is the graph obtained by removing the edge $e$* from *$G$ and suppose $G^{'}$ has no isolated vertices. Then*

$$\mathcal{A} = K_G \simeq K_{G'}.$$

With edge removal, tensor train decomposition and tensor ring decomposition are special cases of complete-graph tensor network. An example of the equal conversion can be seen as Fig. (**??**).

## 3. CMVTF FOR DATA FUSION

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In this paper we consider hyperspectral image and multispectral image as a three-dimension tensor. Hyperspectral imae($\mathcal{X} \in \mathbb{R}^{I_H \times J_H \times K}$) is low-spatial-resolution but high-spectral-resolution and multispectral($\mathcal{Y} \in \mathbb{R}^{I \times J \times K_M}$) image is high-spatial-resolution but low-spectral-resolution . The aim of data fusion is to estimate a super-resolution image($\mathcal{Z} \in \mathbb{R}^{I \times J \times K}$) that with high resolution both in spatial and spectral. $I$, $J$ and $K$ denote the dimensions of two spatial and one spectral coordinates,respectively.Obviously,$I_H \ll I, J_H \ll J$ and $K_M \ll K$ are satisfied by the spcetral and spatial resloutions of the two images.

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**Algorithm** : CNMVF algorithm

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**Input** : **HSI**, **MSI**, **D1**, **D2**, **D3**, $\lambda$
**While** not converged do
1. $A = \arg\min_A ||\mathrm{Y_h}^{(1)} - S_{11}(D_1 A)^T||_F^2 + \lambda||\mathrm{Y_m}^{(1)} - S_{12}A^T||_F^2$
2. $B = \arg\min_B ||\mathrm{Y_h}^{(2)} - S_{21}(D_2 B)^T||_F^2 + \lambda||\mathrm{Y_m}^{(2)} - S_{22}B^T||_F^2$
3. $C = \arg\min_C ||\mathrm{Y_h}^{(3)} - S_{31}C^T||_F^2 + \lambda||\mathrm{Y_m}^{(3)} - S_{32}(D_3 C)^T||_F^2$
**End while**

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