Supplementary Information for the paper entitled "Network Controllability Robustness Learning via Spatial Graph Neural Networks"

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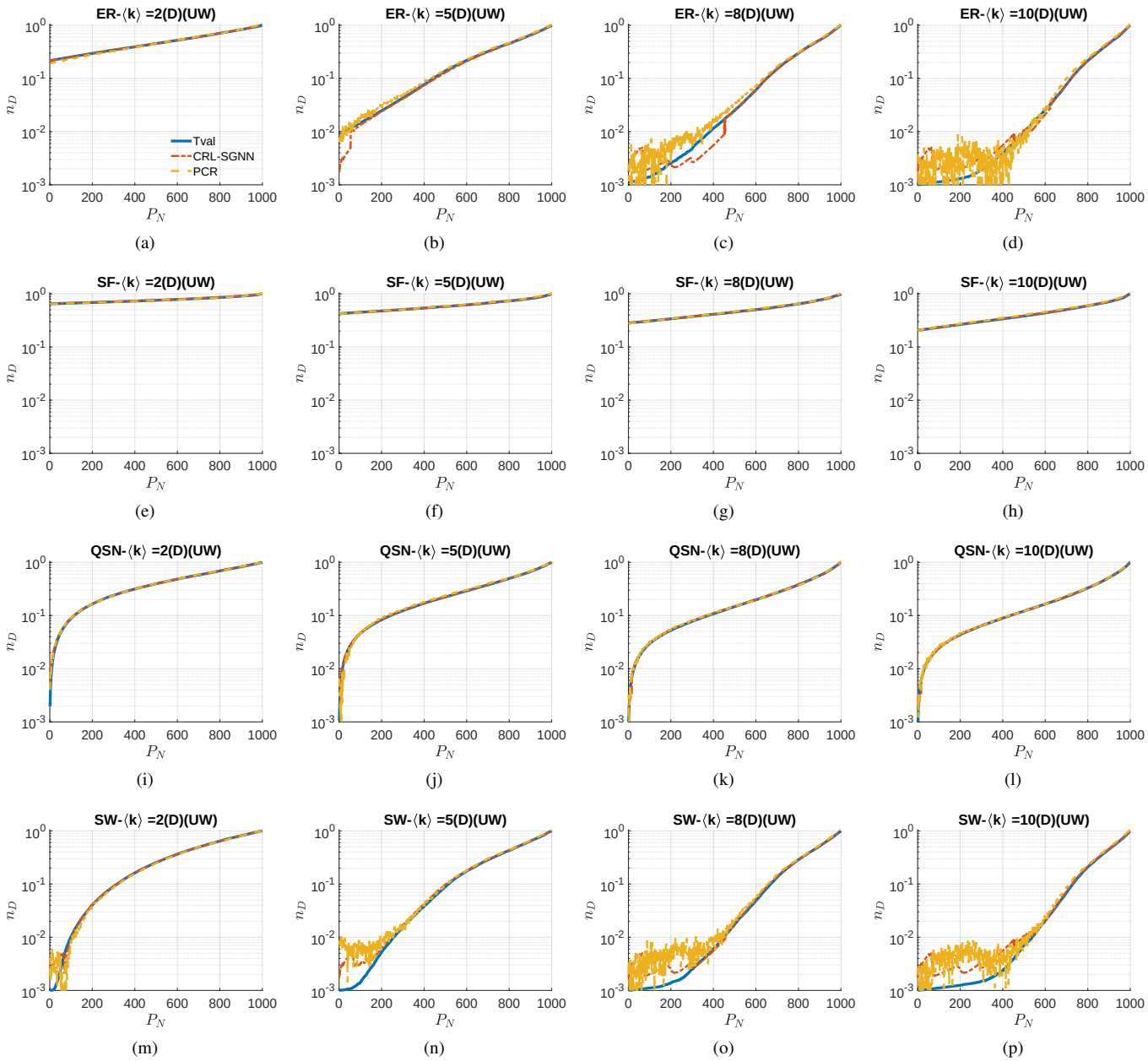

Fig. S1: Performance comparison of CRL-SGNN and PCR on directed unweighted networks.

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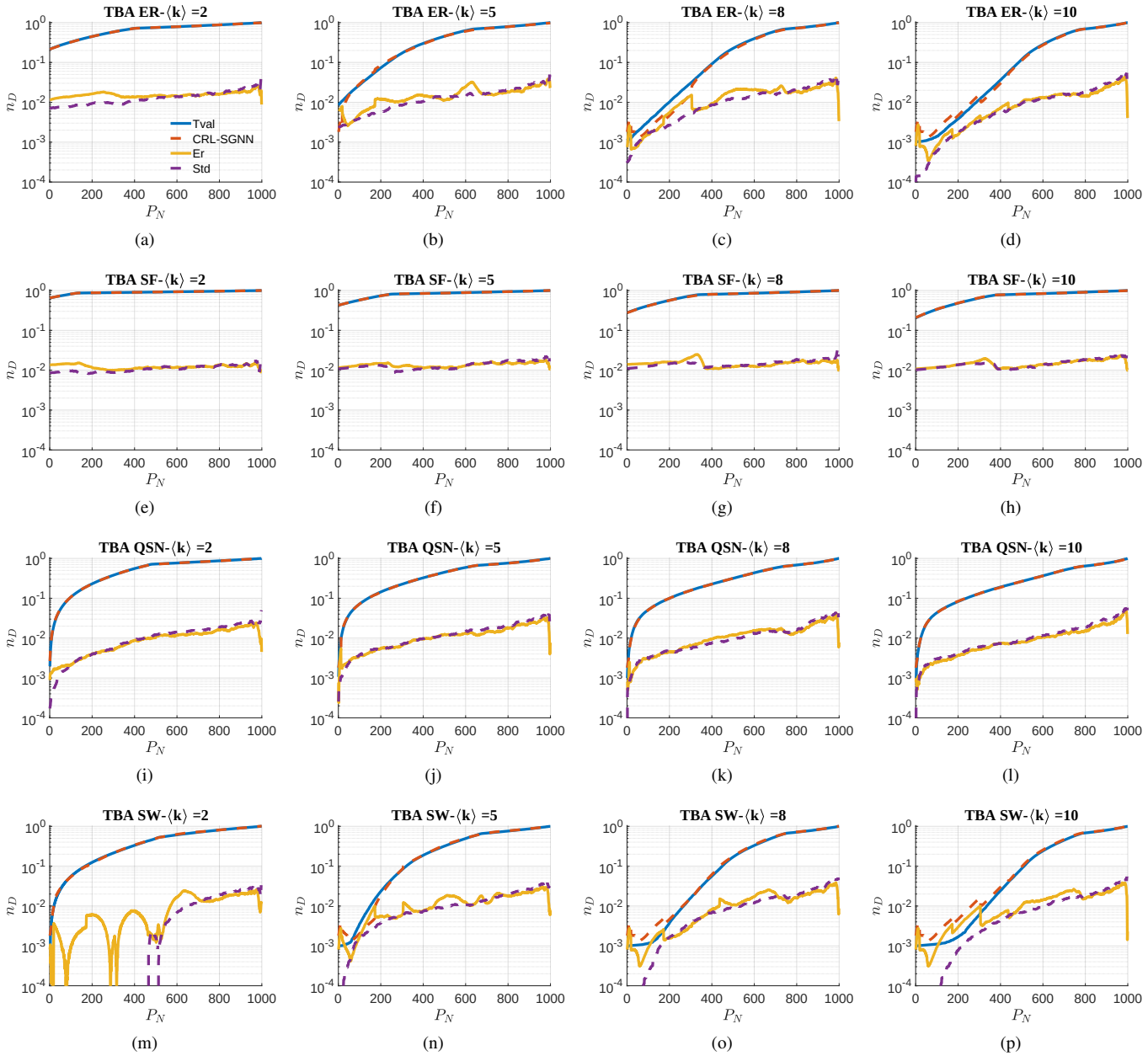

Fig. S2: Results of CRL-SGNN for controllability robustness learning under TBA.

Table S1. Comparison of the mean standard deviation of test data and mean error of learning values of CRL-SGNN under TBA.

TBA		$\langle k \rangle = 2$	$\langle k \rangle = 5$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.014	0.014	0.013	0.012
	CRL-SGNN	0.016	0.015	0.014	0.011
SF	$\overline{Std}$	0.011	0.013	0.015	0.015
	CRL-SGNN	0.012	0.014	0.015	0.015
QSN	$\overline{Std}$	0.012	0.014	0.013	0.013
	CRL-SGNN	0.01	0.012	0.012	0.011
SW	$\overline{Std}$	0.008	0.012	0.012	0.011
	CRL-SGNN	0.01	0.012	0.012	0.012

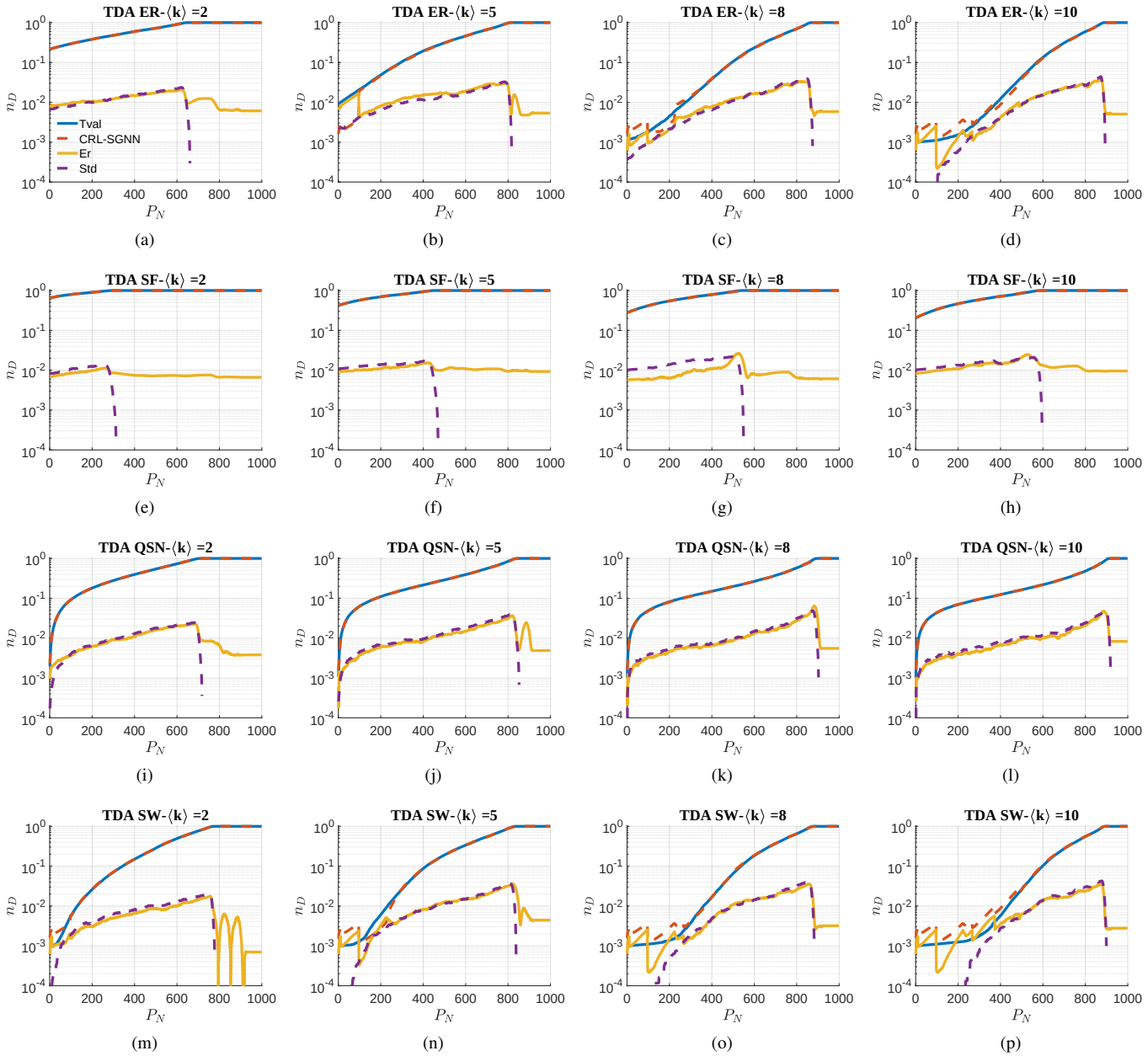

Fig. S3: Results of CRL-SGNN for controllability robustness learning under TDA.

Table S2. Comparison of the mean standard deviation of test data and mean error of learning values of CRL-SGNN under TDA.

TDA		$\langle k \rangle = 2$	$\langle k \rangle = 5$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.009	0.011	0.01	0.01
	CRL-SGNN	0.012	0.014	0.011	0.01
SF	$\overline{Std}$	0.003	0.006	0.008	0.009
	CRL-SGNN	0.008	0.011	0.009	0.013
QSN	$\overline{Std}$	0.008	0.012	0.012	0.011
	CRL-SGNN	0.009	0.012	0.012	0.01
SW	$\overline{Std}$	0.006	0.009	0.009	0.009
	CRL-SGNN	0.006	0.01	0.009	0.009

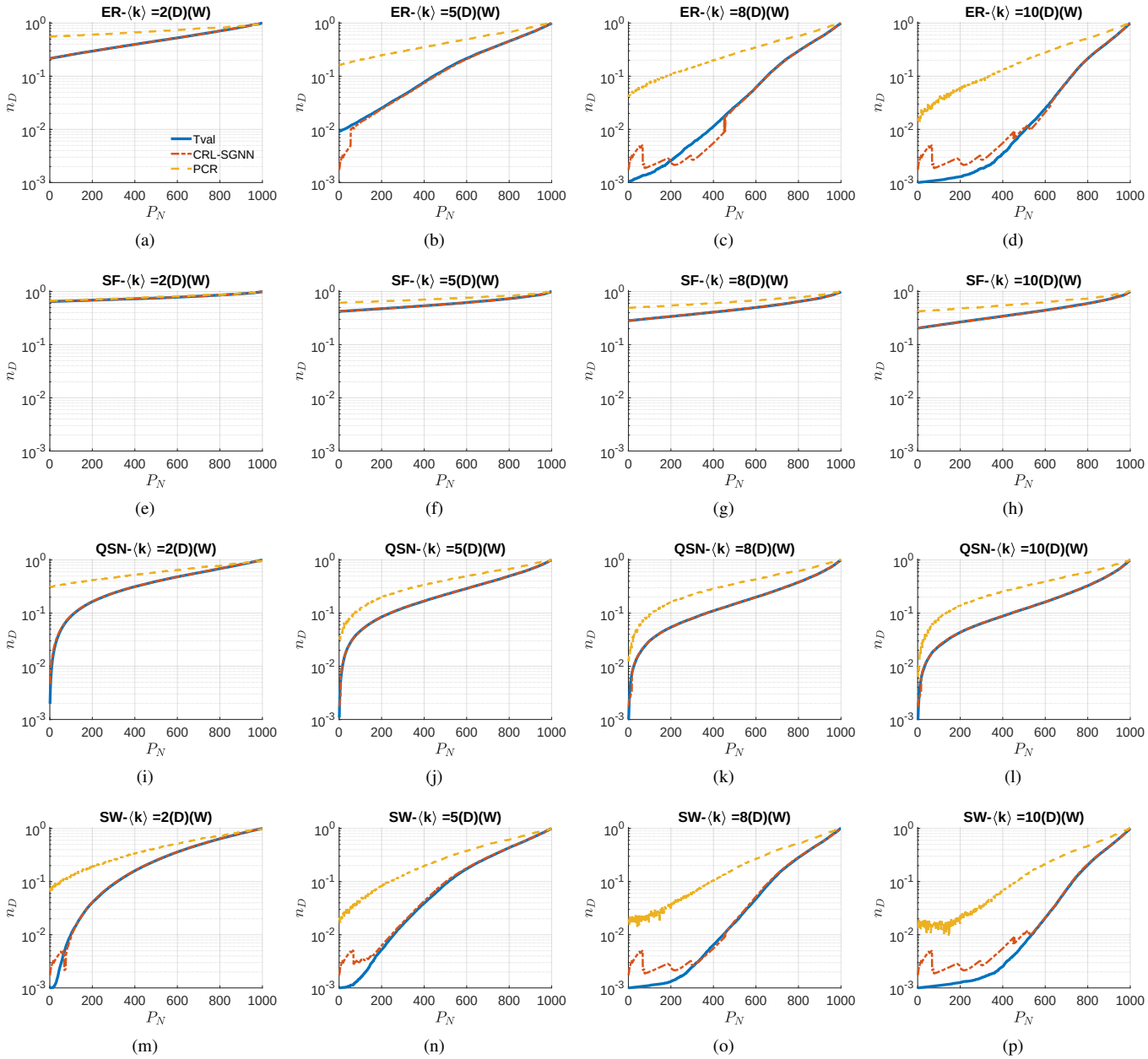

Fig. S4: Comparison of generalization ability of CRL-SGNN and PCR on directed weighted networks.

Table S3. Comparison of the mean standard deviation of the test set and mean errors of learning values of CRL-SGNN and PCR on directed weighted networks.

Directed weighted networks		$\langle k \rangle = 2$	$\langle k \rangle = 5$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.019	0.019	0.015	0.013
	CRL-SGNN	0.017	0.016	0.012	0.011
	PCR	0.218	0.229	0.191	0.168
SF	$\overline{Std}$	0.021	0.025	0.026	0.026
	CRL-SGNN	0.021	0.023	0.022	0.023
	PCR	0.029	0.142	0.172	0.181
QSN	$\overline{Std}$	0.017	0.016	0.014	0.014
	CRL-SGNN	0.014	0.013	0.012	0.011
	PCR	0.179	0.151	0.171	0.169
SW	$\overline{Std}$	0.015	0.016	0.013	0.012
	CRL-SGNN	0.012	0.014	0.011	0.010
	PCR	0.129	0.137	0.133	0.125

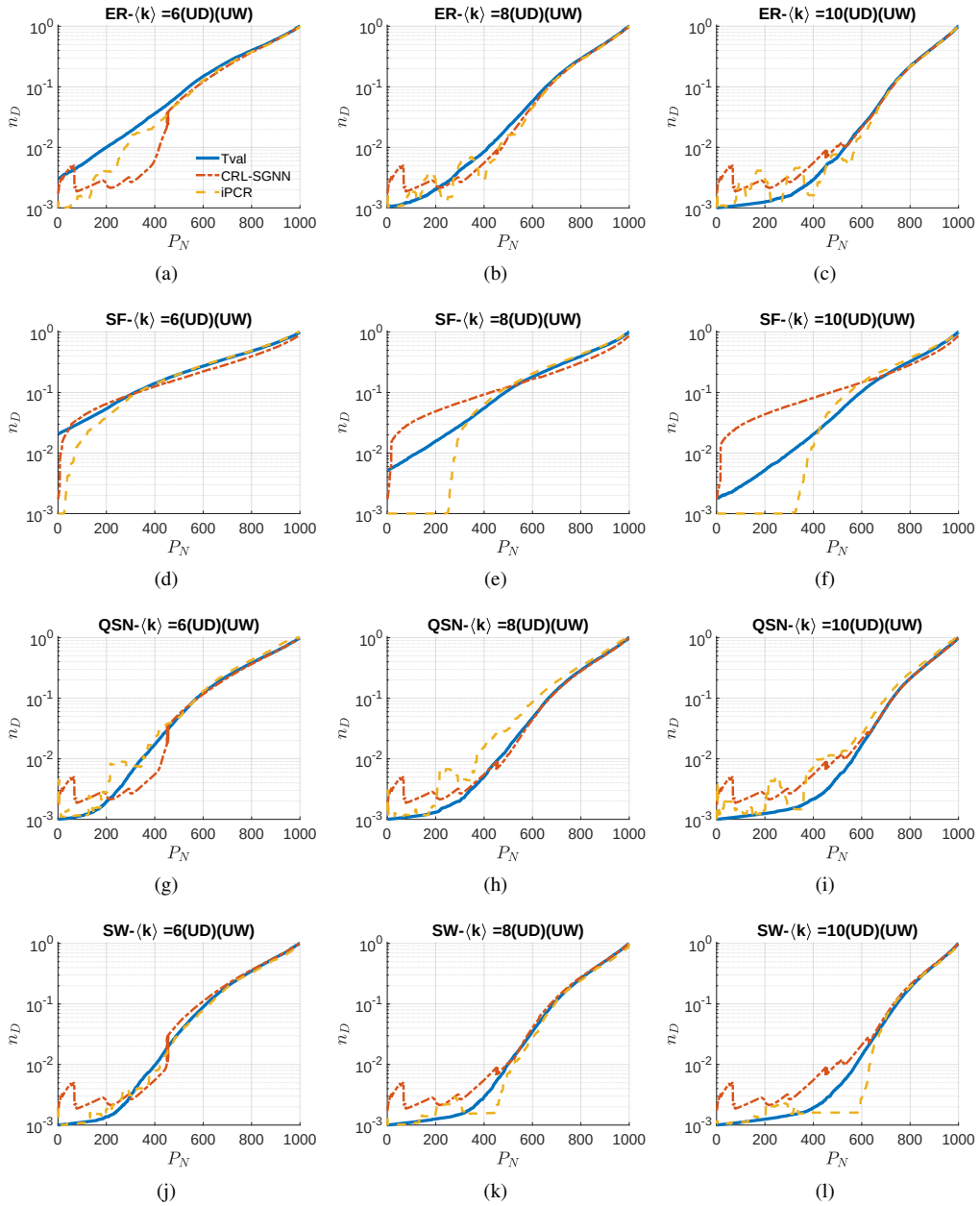

Fig. S5: Performance comparison of CRL-SGNN and iPCR on undirected unweighted networks

Table S4. Comparison of the mean standard deviation of the test set and mean errors of learning values of CRL-SGNN and iPCR on undirected unweighted networks

Undirected unweighted networks		$\langle k \rangle = 6$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.023	0.020	0.017
	CRL-SGNN	0.026	0.017	0.015
	iPCR	0.024	0.018	0.014
SF	$\overline{Std}$	0.029	0.027	0.024
	CRL-SGNN	0.047	0.052	0.052
	iPCR	0.026	0.027	0.029
QSN	$\overline{Std}$	0.021	0.019	0.017
	CRL-SGNN	0.019	0.016	0.015
	iPCR	0.027	0.030	0.023
SW	$\overline{Std}$	0.019	0.018	0.016
	CRL-SGNN	0.019	0.016	0.015
	iPCR	0.019	0.019	0.015

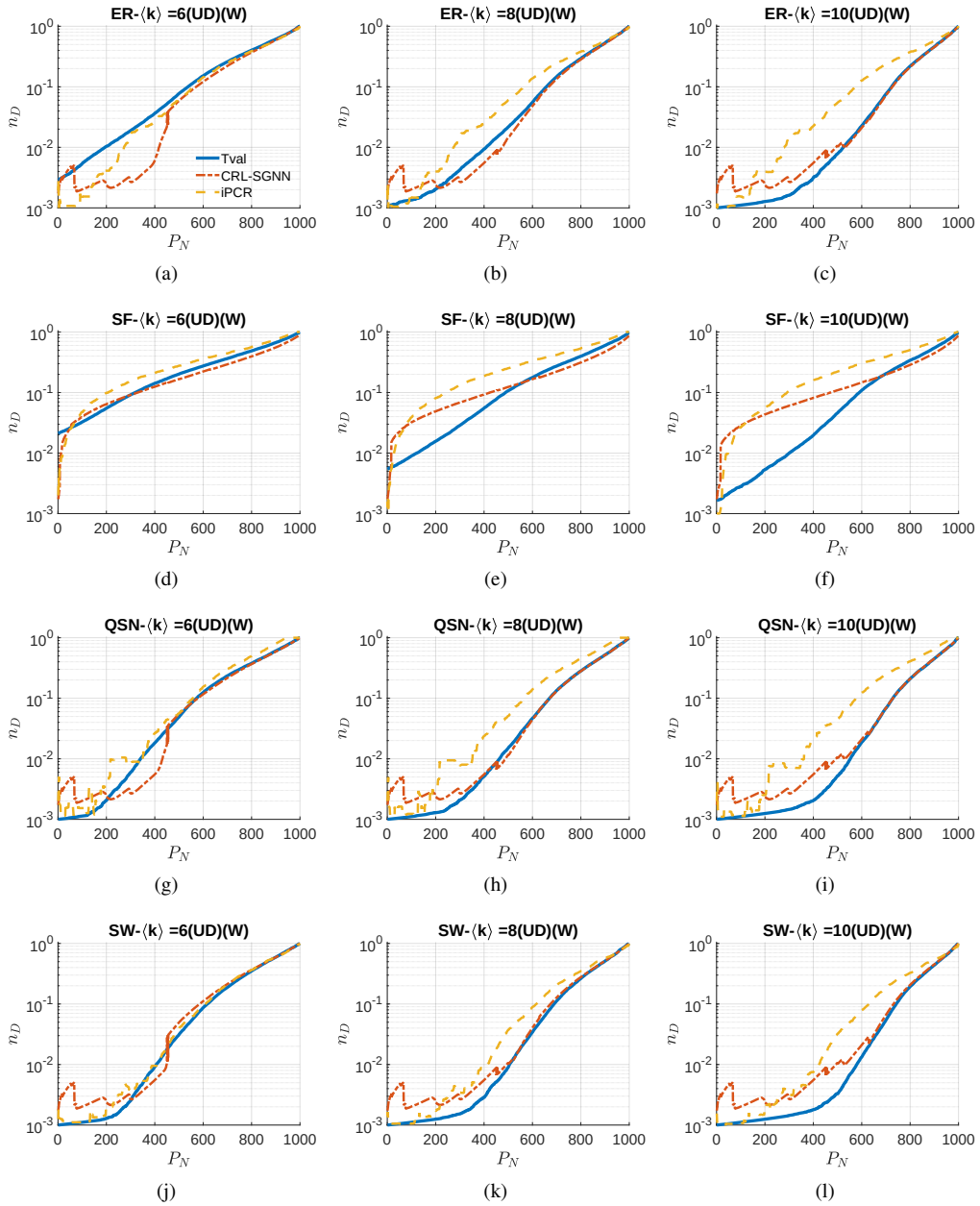

Fig. S6. Comparison of generalization ability of CRL-SGNN and iPCR on undirected weighted networks.

Table S5: Comparison of the mean standard deviation of the test set and mean errors of learning values of CRL-SGNN and iPCR on undirected weighted networks.

Undirected weighted networks		$\langle k \rangle = 6$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.024	0.019	0.018
	CRL-SGNN	0.028	0.017	0.016
	iPCR	0.022	0.041	0.065
SF	$\overline{Std}$	0.028	0.025	0.024
	CRL-SGNN	0.049	0.052	0.054
	iPCR	0.064	0.109	0.123
QSN	$\overline{Std}$	0.021	0.020	0.017
	CRL-SGNN	0.019	0.016	0.015
	iPCR	0.049	0.074	0.083
SW	$\overline{Std}$	0.018	0.018	0.016
	CRL-SGNN	0.019	0.015	0.015
	iPCR	0.017	0.037	0.051

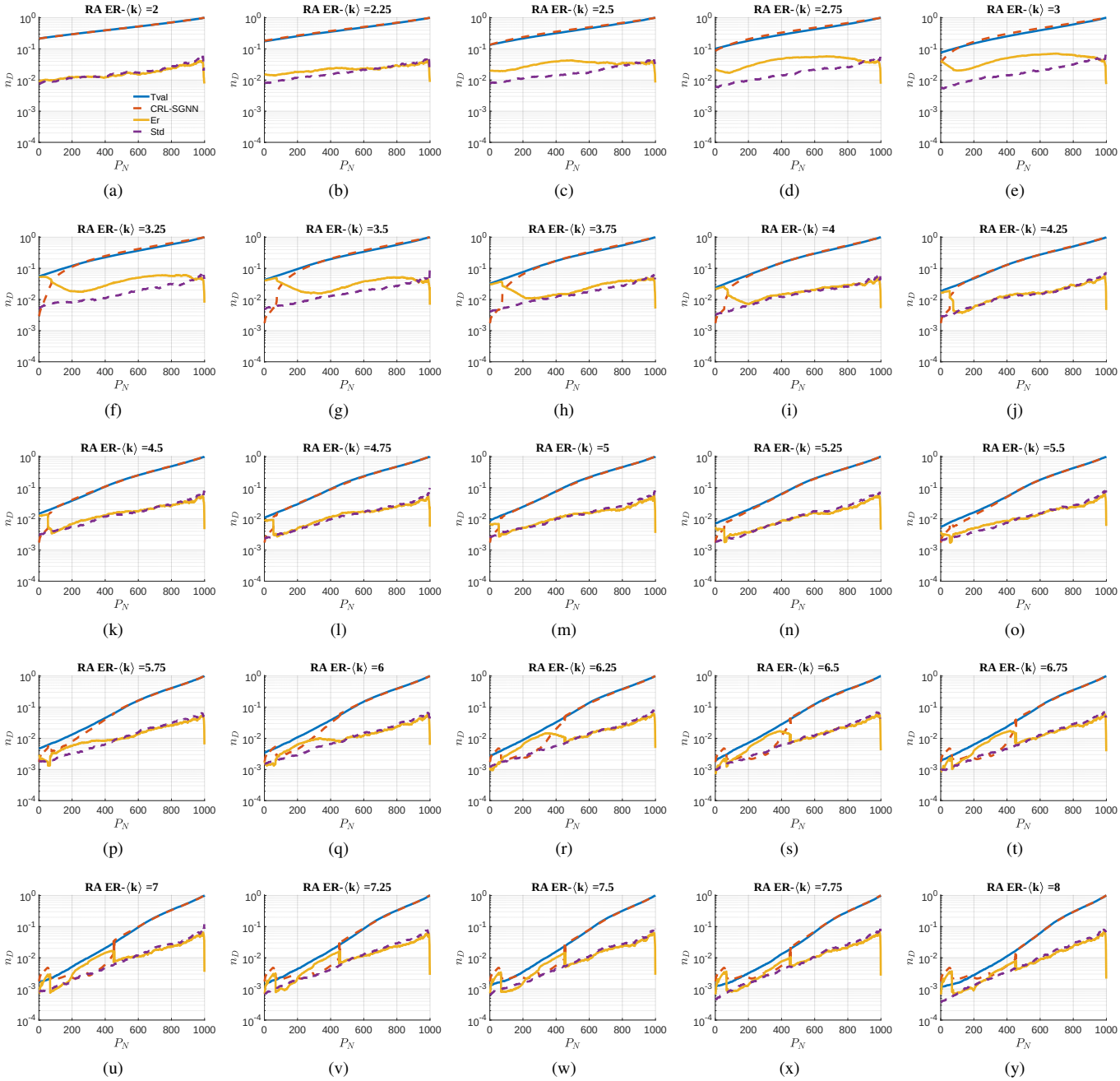

Fig. S7: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on ER networks.

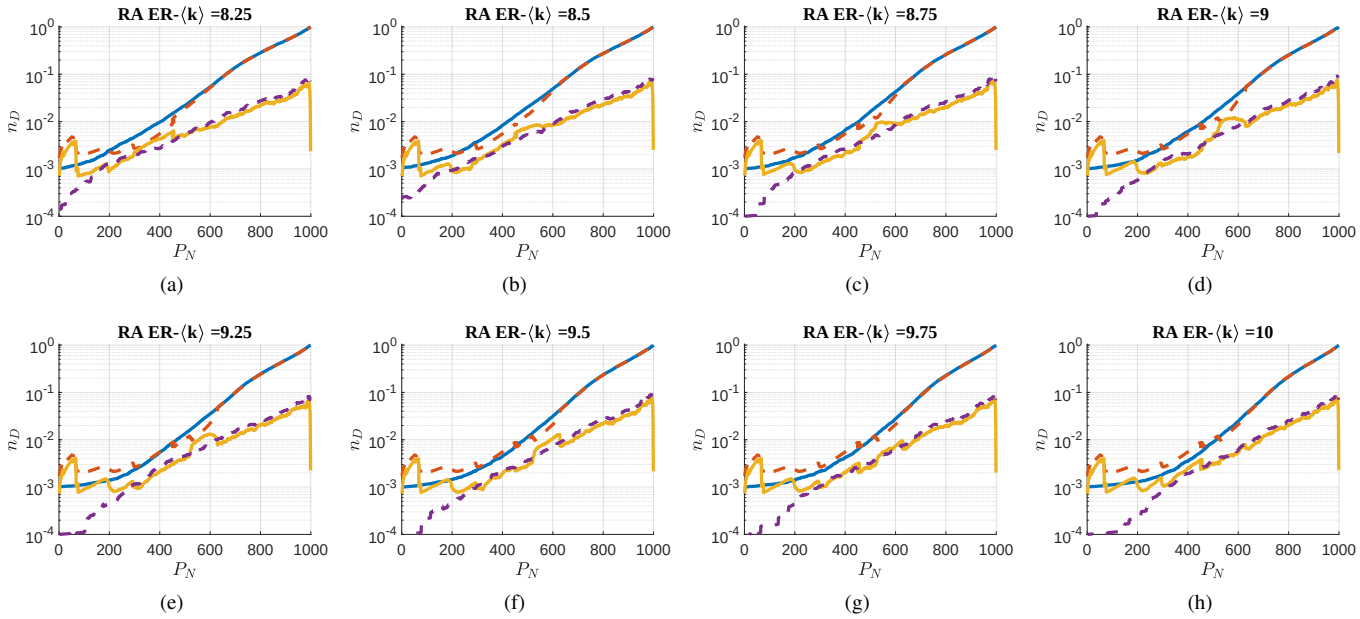

Fig. S7: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on ER networks.

Table S6. Comparison of the mean standard deviation of test data and mean error of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on ER networks.

ER	$\langle k\rangle \in [2,3]$	$\langle k\rangle = 2$	$\langle k\rangle = 2.25$	$\langle k\rangle = 2.5$	$\langle k\rangle = 2.75$	$\langle k\rangle = 3$
	$\overline{Std}$	0.019	0.020	0.020	0.020	0.021
	CRL-SGNN	0.017	0.023	0.033	0.042	0.049
	$\langle k\rangle \in [3.25,4.25]$	$\langle k\rangle = 3.25$	$\langle k\rangle = 3.5$	$\langle k\rangle = 3.75$	$\langle k\rangle = 4$	$\langle k\rangle = 4.25$
	$\overline{Std}$	0.019	0.019	0.019	0.018	0.018
	CRL-SGNN	0.041	0.034	0.026	0.020	0.018
	$\langle k\rangle \in [4.5,5.5]$	$\langle k\rangle = 4.5$	$\langle k\rangle = 4.75$	$\langle k\rangle = 5$	$\langle k\rangle = 5.25$	$\langle k\rangle = 5.5$
	$\overline{Std}$	0.018	0.017	0.017	0.017	0.017
	CRL-SGNN	0.018	0.016	0.015	0.014	0.015
	$\langle k\rangle \in [5.75,6.75]$	$\langle k\rangle = 5.75$	$\langle k\rangle = 6$	$\langle k\rangle = 6.25$	$\langle k\rangle = 6.5$	$\langle k\rangle = 6.75$
	$\overline{Std}$	0.017	0.016	0.016	0.015	0.015
	CRL-SGNN	0.015	0.015	0.016	0.016	0.015
	$\langle k\rangle \in [7,8]$	$\langle k\rangle = 7$	$\langle k\rangle = 7.25$	$\langle k\rangle = 7.5$	$\langle k\rangle = 7.75$	$\langle k\rangle = 8$
	$\overline{Std}$	0.015	0.015	0.015	0.015	0.015
	CRL-SGNN	0.015	0.014	0.013	0.013	0.013
	$\langle k\rangle \in [8.25,9.25]$	$\langle k\rangle = 8.25$	$\langle k\rangle = 8.5$	$\langle k\rangle = 8.75$	$\langle k\rangle = 9$	$\langle k\rangle = 9.25$
	$\overline{Std}$	0.014	0.015	0.014	0.014	0.013
	CRL-SGNN	0.012	0.012	0.012	0.012	0.012
	$\langle k\rangle \in [9.5,10]$	$\langle k\rangle = 9.5$	$\langle k\rangle = 9.75$	$\langle k\rangle = 10$		
	$\overline{Std}$	0.014	0.013	0.013		
	CRL-SGNN	0.012	0.011	0.011		

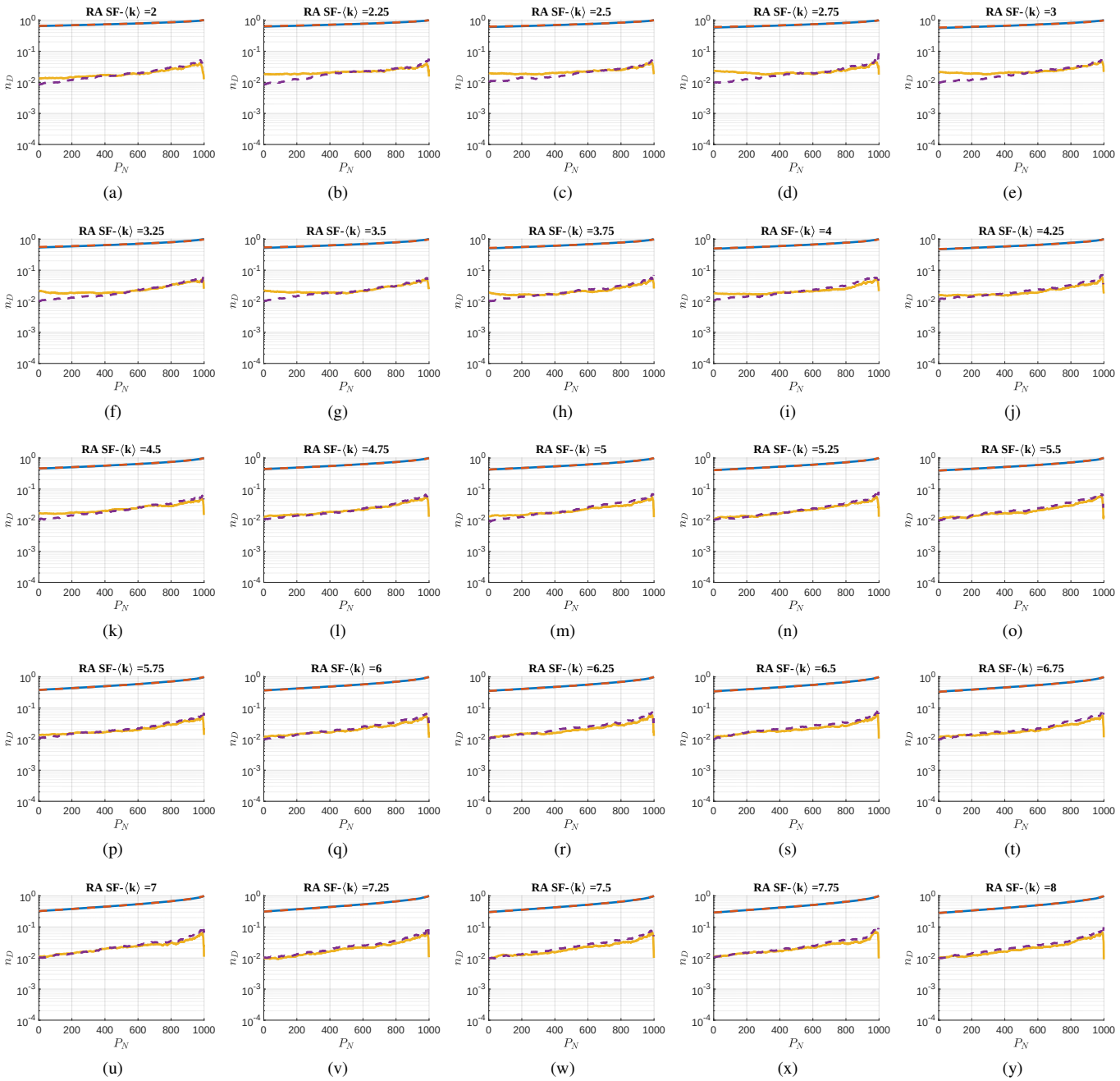

Fig. S8: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on SF networks.

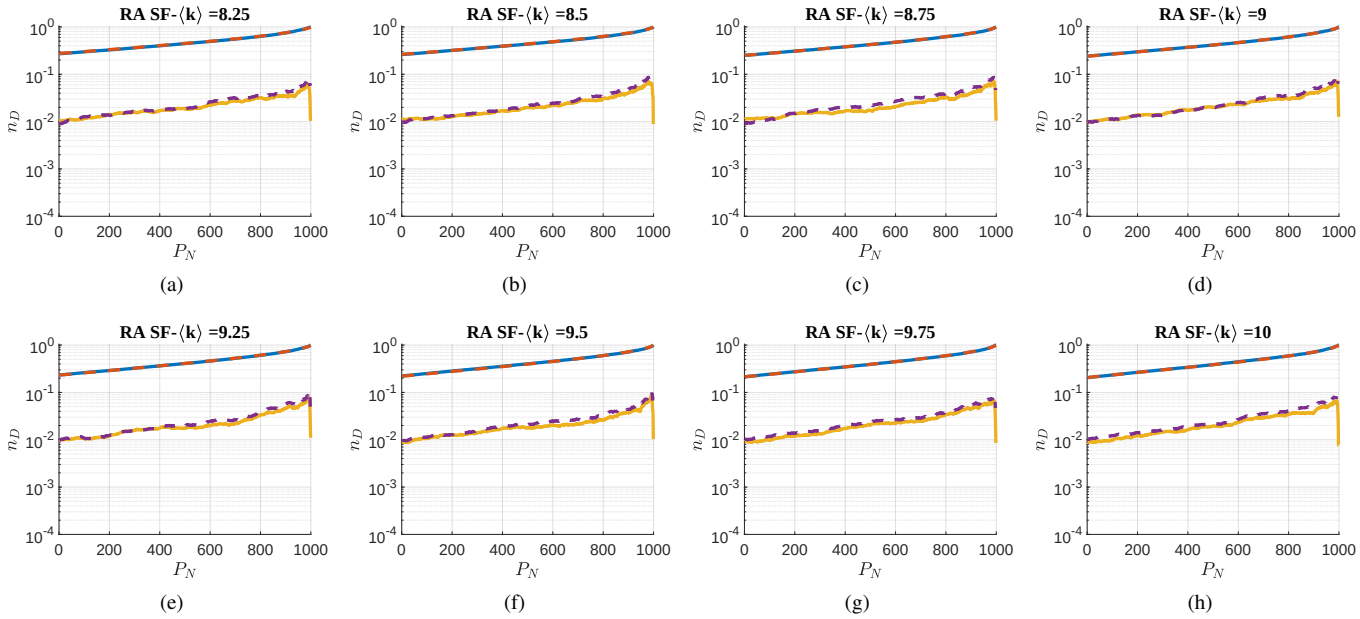

Fig. S8: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on SF networks.

Table S7. Comparison of the mean standard deviation of test data and mean error of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on SF networks.

SF	$\langle k\rangle \in [2,3]$	$\langle k\rangle = 2$	$\langle k\rangle = 2.25$	$\langle k\rangle = 2.5$	$\langle k\rangle = 2.75$	$\langle k\rangle = 3$
	$\overline{Std}$	0.021	0.021	0.020	0.021	0.022
	CRL-SGNN	0.020	0.023	0.023	0.023	0.024
	$\langle k\rangle \in [3.25,4.25]$	$\langle k\rangle = 3.25$	$\langle k\rangle = 3.5$	$\langle k\rangle = 3.75$	$\langle k\rangle = 4$	$\langle k\rangle = 4.25$
	$\overline{Std}$	0.023	0.023	0.023	0.024	0.024
	CRL-SGNN	0.025	0.025	0.022	0.023	0.022
	$\langle k\rangle \in [4.5,5.5]$	$\langle k\rangle = 4.5$	$\langle k\rangle = 4.75$	$\langle k\rangle = 5$	$\langle k\rangle = 5.25$	$\langle k\rangle = 5.5$
	$\overline{Std}$	0.025	0.024	0.025	0.024	0.026
	CRL-SGNN	0.024	0.023	0.023	0.022	0.022
	$\langle k\rangle \in [5.75,6.75]$	$\langle k\rangle = 5.75$	$\langle k\rangle = 6$	$\langle k\rangle = 6.25$	$\langle k\rangle = 6.5$	$\langle k\rangle = 6.75$
	$\overline{Std}$	0.024	0.025	0.025	0.027	0.025
	CRL-SGNN	0.022	0.022	0.022	0.023	0.021
	$\langle k\rangle \in [7,8]$	$\langle k\rangle = 7$	$\langle k\rangle = 7.25$	$\langle k\rangle = 7.5$	$\langle k\rangle = 7.75$	$\langle k\rangle = 8$
	$\overline{Std}$	0.026	0.026	0.027	0.026	0.027
	CRL-SGNN	0.023	0.022	0.022	0.023	0.023
	$\langle k\rangle \in [8.25,9.25]$	$\langle k\rangle = 8.25$	$\langle k\rangle = 8.5$	$\langle k\rangle = 8.75$	$\langle k\rangle = 9$	$\langle k\rangle = 9.25$
	$\overline{Std}$	0.025	0.026	0.027	0.026	0.026
	CRL-SGNN	0.022	0.023	0.023	0.023	0.023
	$\langle k\rangle \in [9.5,10]$	$\langle k\rangle = 9.5$	$\langle k\rangle = 9.75$	$\langle k\rangle = 10$		
	$\overline{Std}$	0.025	0.027	0.028		
	CRL-SGNN	0.021	0.023	0.023		

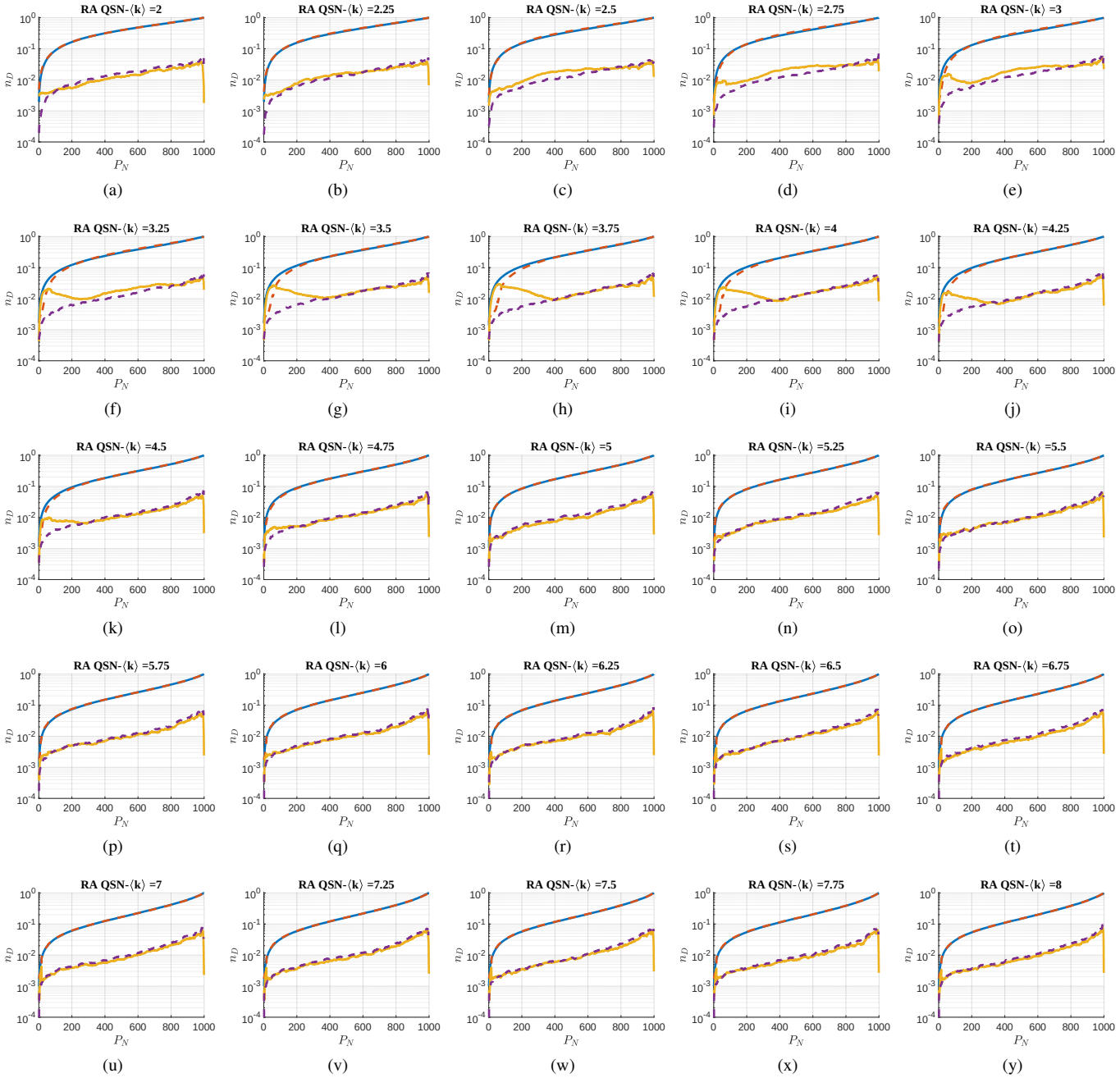

Fig. S9: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on QSN networks.

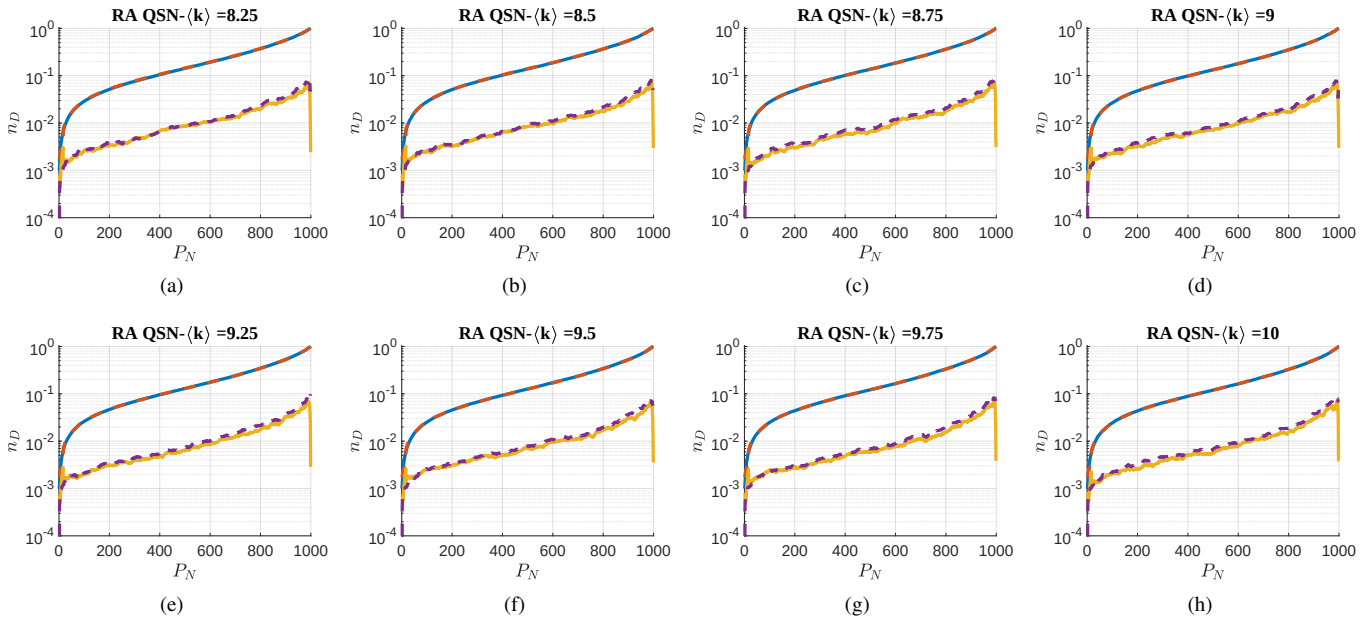

Fig. S9: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on QSN networks.

Table S8. Comparison of the mean standard deviation of test data and mean error of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on QSN networks.

QSN	$\langle k\rangle \in [2,3]$	$\langle k\rangle = 2$	$\langle k\rangle = 2.25$	$\langle k\rangle = 2.5$	$\langle k\rangle = 2.75$	$\langle k\rangle = 3$
	$\overline{Std}$	0.018	0.017	0.016	0.018	0.018
	CRL-SGNN	0.014	0.016	0.018	0.022	0.021
	$\langle k\rangle \in [3.25,4.25]$	$\langle k\rangle = 3.25$	$\langle k\rangle = 3.5$	$\langle k\rangle = 3.75$	$\langle k\rangle = 4$	$\langle k\rangle = 4.25$
	$\overline{Std}$	0.016	0.017	0.017	0.016	0.016
	CRL-SGNN	0.021	0.019	0.020	0.018	0.016
	$\langle k\rangle \in [4.5,5.5]$	$\langle k\rangle = 4.5$	$\langle k\rangle = 4.75$	$\langle k\rangle = 5$	$\langle k\rangle = 5.25$	$\langle k\rangle = 5.5$
	$\overline{Std}$	0.016	0.016	0.016	0.016	0.014
	CRL-SGNN	0.015	0.014	0.013	0.014	0.012
	$\langle k\rangle \in [5.75,6.75]$	$\langle k\rangle = 5.75$	$\langle k\rangle = 6$	$\langle k\rangle = 6.25$	$\langle k\rangle = 6.5$	$\langle k\rangle = 6.75$
	$\overline{Std}$	0.015	0.016	0.015	0.015	0.015
	CRL-SGNN	0.013	0.013	0.012	0.013	0.012
	$\langle k\rangle \in [7,8]$	$\langle k\rangle = 7$	$\langle k\rangle = 7.25$	$\langle k\rangle = 7.5$	$\langle k\rangle = 7.75$	$\langle k\rangle = 8$
	$\overline{Std}$	0.015	0.015	0.015	0.014	0.015
	CRL-SGNN	0.013	0.013	0.012	0.012	0.012
	$\langle k\rangle \in [8.25,9.25]$	$\langle k\rangle = 8.25$	$\langle k\rangle = 8.5$	$\langle k\rangle = 8.75$	$\langle k\rangle = 9$	$\langle k\rangle = 9.25$
	$\overline{Std}$	0.015	0.014	0.014	0.014	0.014
	CRL-SGNN	0.013	0.012	0.012	0.012	0.012
	$\langle k\rangle \in [9.5,10]$	$\langle k\rangle = 9.5$	$\langle k\rangle = 9.75$	$\langle k\rangle = 10$		
	$\overline{Std}$	0.013	0.014	0.014		
	CRL-SGNN	0.011	0.011	0.011		

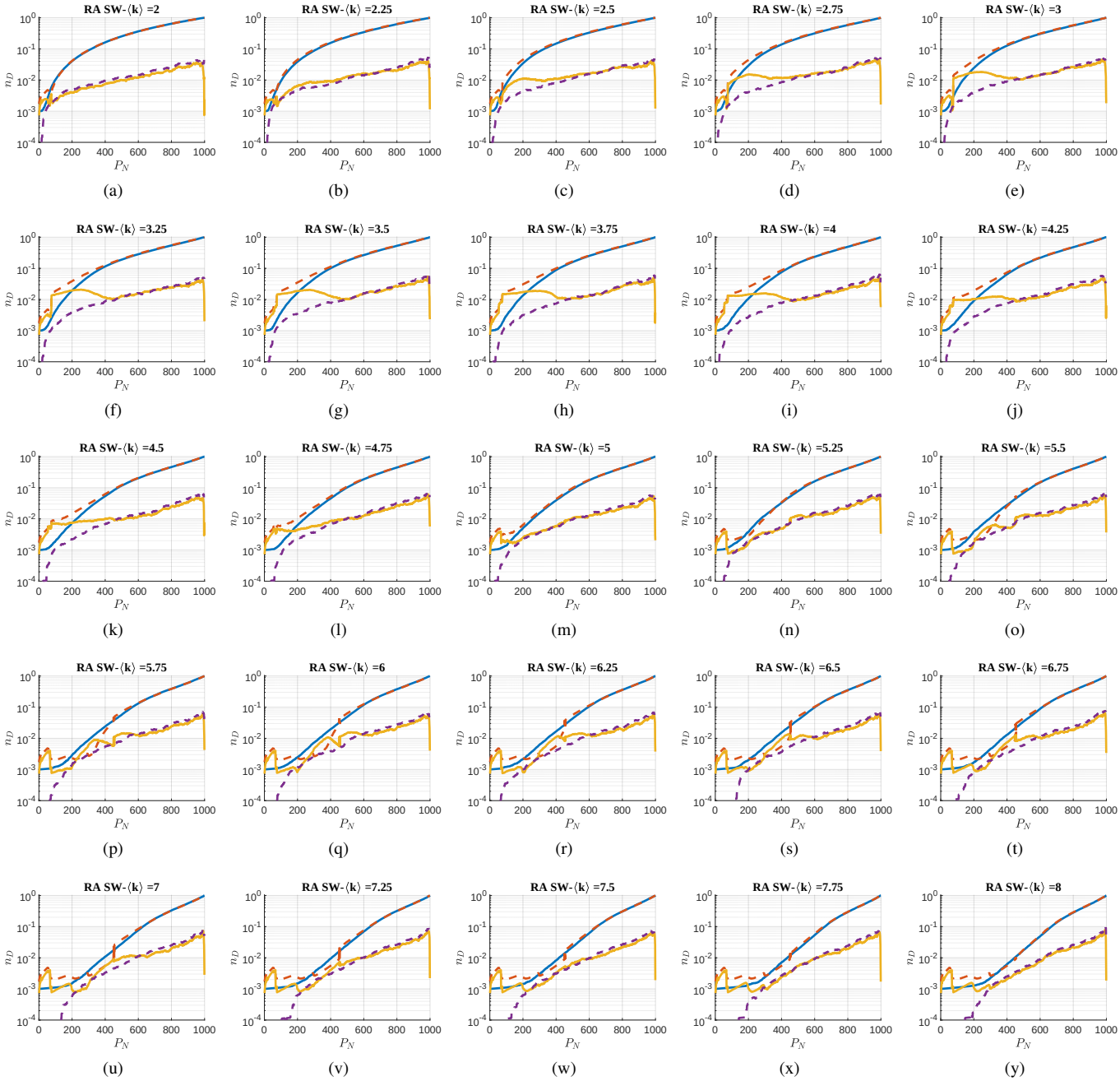

Fig. S10: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on SW networks.

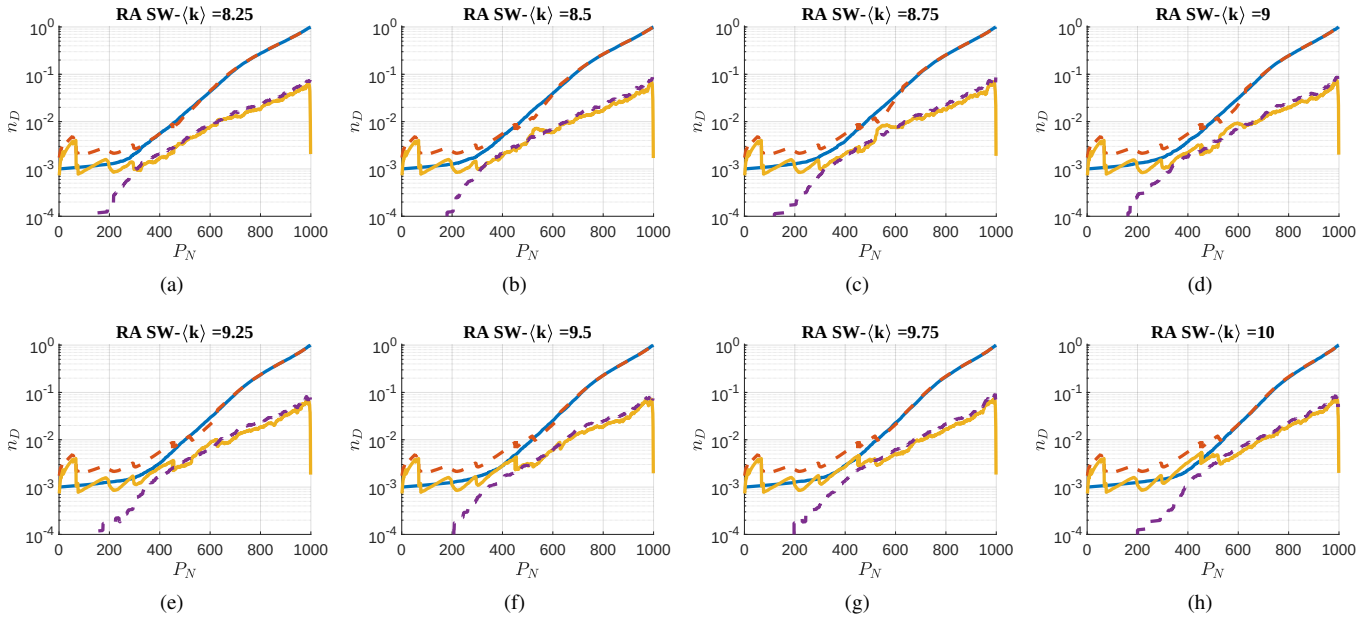

Fig. S10: Results of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on SW networks.

Table S9. Comparison of the mean standard deviation of test data and mean error of CRL-SGNN for controllability robustness learning under equal interval sampling at 0.25 intervals on SW networks

SW	$\langle k\rangle \in [2,3]$	$\langle k\rangle = 2$	$\langle k\rangle = 2.25$	$\langle k\rangle = 2.5$	$\langle k\rangle = 2.75$	$\langle k\rangle = 3$
	$\overline{Std}$	0.015	0.015	0.015	0.016	0.015
	CRL-SGNN	0.012	0.014	0.016	0.017	0.018
	$\langle k\rangle \in [3.25,4.25]$	$\langle k\rangle = 3.25$	$\langle k\rangle = 3.5$	$\langle k\rangle = 3.75$	$\langle k\rangle = 4$	$\langle k\rangle = 4.25$
	$\overline{Std}$	0.015	0.016	0.015	0.015	0.016
	CRL-SGNN	0.018	0.019	0.018	0.017	0.016
	$\langle k\rangle \in [4.5,5.5]$	$\langle k\rangle = 4.5$	$\langle k\rangle = 4.75$	$\langle k\rangle = 5$	$\langle k\rangle = 5.25$	$\langle k\rangle = 5.5$
	$\overline{Std}$	0.016	0.016	0.015	0.016	0.015
	CRL-SGNN	0.015	0.015	0.013	0.014	0.013
	$\langle k\rangle \in [5.75,6.75]$	$\langle k\rangle = 5.75$	$\langle k\rangle = 6$	$\langle k\rangle = 6.25$	$\langle k\rangle = 6.5$	$\langle k\rangle = 6.75$
	$\overline{Std}$	0.015	0.014	0.014	0.014	0.014
	CRL-SGNN	0.014	0.014	0.013	0.013	0.013
	$\langle k\rangle \in [7,8]$	$\langle k\rangle = 7$	$\langle k\rangle = 7.25$	$\langle k\rangle = 7.5$	$\langle k\rangle = 7.75$	$\langle k\rangle = 8$
	$\overline{Std}$	0.014	0.015	0.014	0.013	0.013
	CRL-SGNN	0.012	0.013	0.012	0.012	0.011
	$\langle k\rangle \in [8.25,9.25]$	$\langle k\rangle = 8.25$	$\langle k\rangle = 8.5$	$\langle k\rangle = 8.75$	$\langle k\rangle = 9$	$\langle k\rangle = 9.25$
	$\overline{Std}$	0.013	0.012	0.013	0.013	0.012
	CRL-SGNN	0.011	0.010	0.011	0.011	0.011
	$\langle k\rangle \in [9.5,10]$	$\langle k\rangle = 9.5$	$\langle k\rangle = 9.75$	$\langle k\rangle = 10$		
	$\overline{Std}$	0.013	0.012	0.012		
	CRL-SGNN	0.011	0.010	0.011		

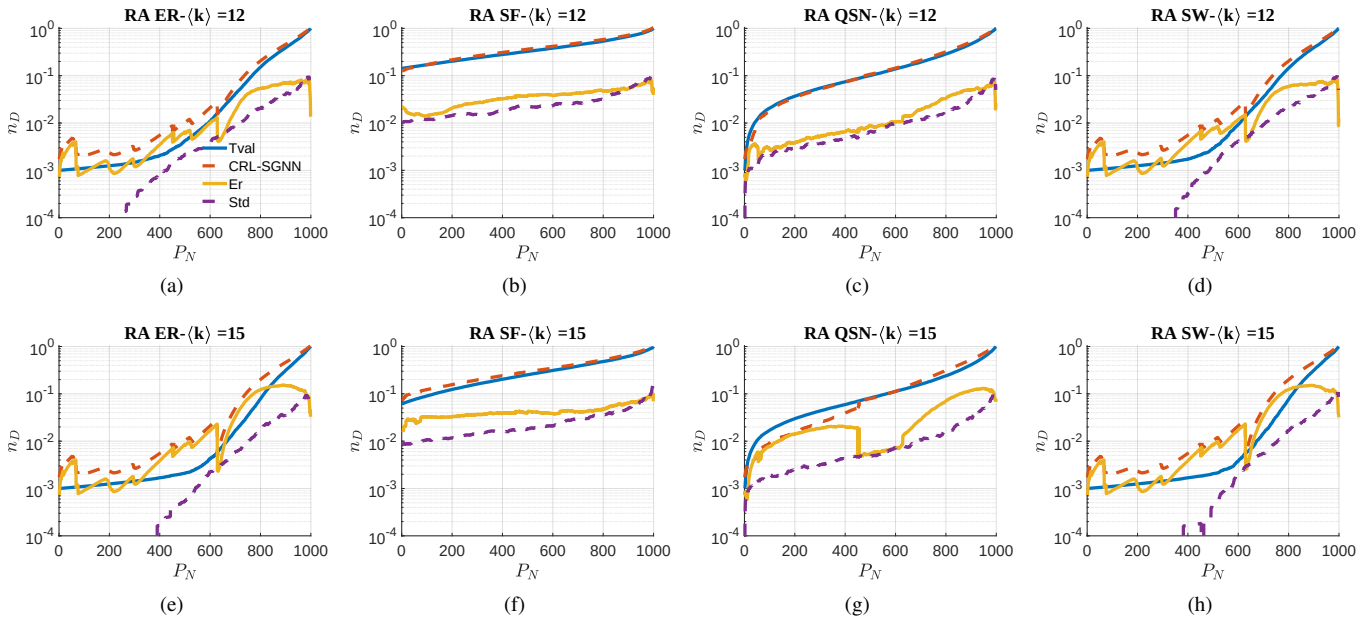

Fig. S11: Results of CRL-SGNN for controllability robustness learning with average degree $\langle k \rangle = 12, 15$.

Table S10. Comparison of the mean standard deviation of test data and mean error of CRL-SGNN for controllability robustness learning with average degree $\langle k \rangle = 12, 15$.

$\langle k \rangle = 12, 15$		$\langle k \rangle = 12$	$\langle k \rangle = 15$
ER	$\overline{Std}$	0.012	0.010
	CRL-SGNN	0.020	0.038
SF	$\overline{Std}$	0.027	0.026
	CRL-SGNN	0.036	0.043
QSN	$\overline{Std}$	0.012	0.012
	CRL-SGNN	0.017	0.035
SW	$\overline{Std}$	0.011	0.010
	CRL-SGNN	0.021	0.038

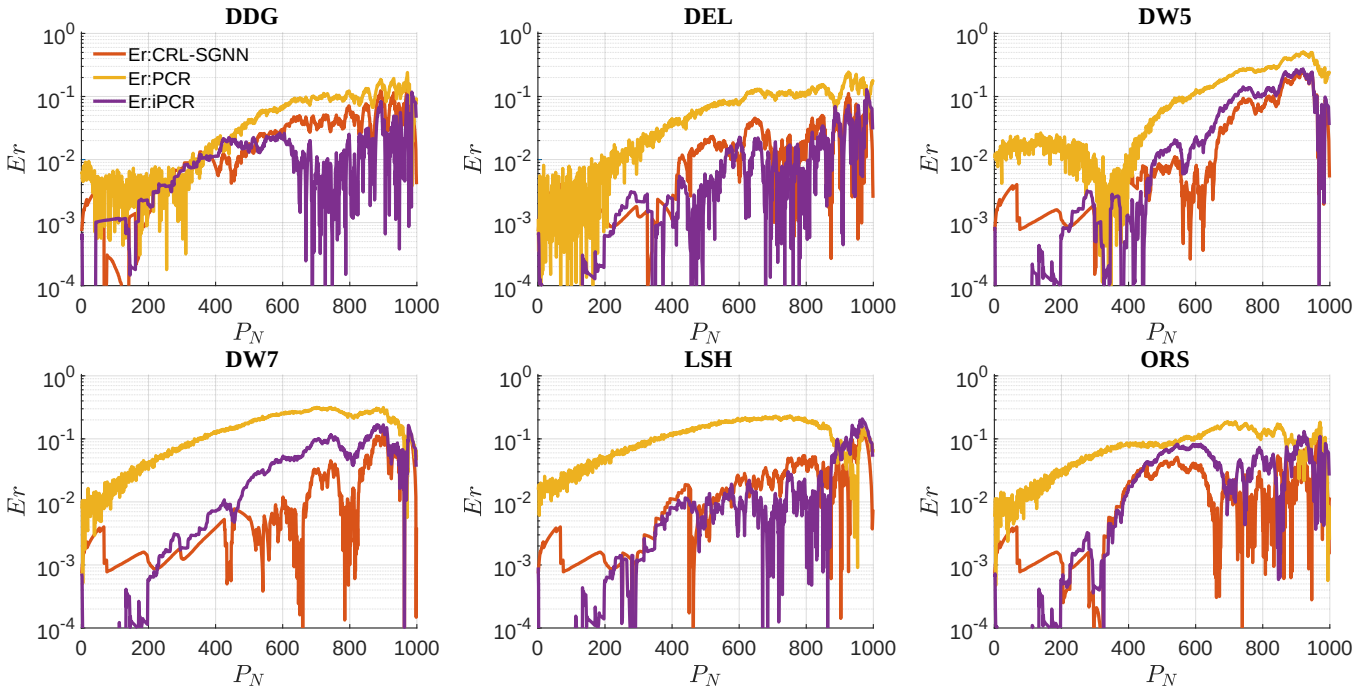

Fig. S12: Comparison of learning errors of CRL-SGNN, PCR, iPCR on real-world networks.

Table S11. Basic information of the real-world networks

Abbreviated name	file name	N	M
DDG	DD_g79	1022	2889
DEL	delaunay_n10	1024	3056
DW5	dwt_1005	1005	3808
DW7	dwt_1007	1007	3784
LSH	lshp1009	1009	2928
ORS	orsirr_1	1030	2914

Table S12. Mean error of learning values of $N = 3000$ vs. mean standard deviation of test data.

$N = 3000$		$\langle k \rangle = 2$	$\langle k \rangle = 5$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.011	0.010	0.009	0.008
	CRL-SGNN	0.010	0.011	0.009	0.009
SF	$\overline{Std}$	0.012	0.014	0.014	0.016
	CRL-SGNN	0.013	0.013	0.013	0.014
QSN	$\overline{Std}$	0.010	0.009	0.008	0.008
	CRL-SGNN	0.013	0.008	0.008	0.009
SW	$\overline{Std}$	0.009	0.009	0.008	0.007
	CRL-SGNN	0.012	0.009	0.007	0.007

Table S13. Mean error of learning values of $N = 600$ vs. mean standard deviation of test data.

$N = 600$		$\langle k \rangle = 2$	$\langle k \rangle = 5$	$\langle k \rangle = 8$	$\langle k \rangle = 10$
ER	$\overline{Std}$	0.025	0.022	0.018	0.016
	CRL-SGNN	0.026	0.021	0.017	0.013
SF	$\overline{Std}$	0.027	0.032	0.035	0.033
	CRL-SGNN	0.030	0.030	0.035	0.029
QSN	$\overline{Std}$	0.023	0.021	0.019	0.018
	CRL-SGNN	0.021	0.020	0.017	0.016
SW	$\overline{Std}$	0.019	0.020	0.017	0.016
	CRL-SGNN	0.016	0.017	0.016	0.015

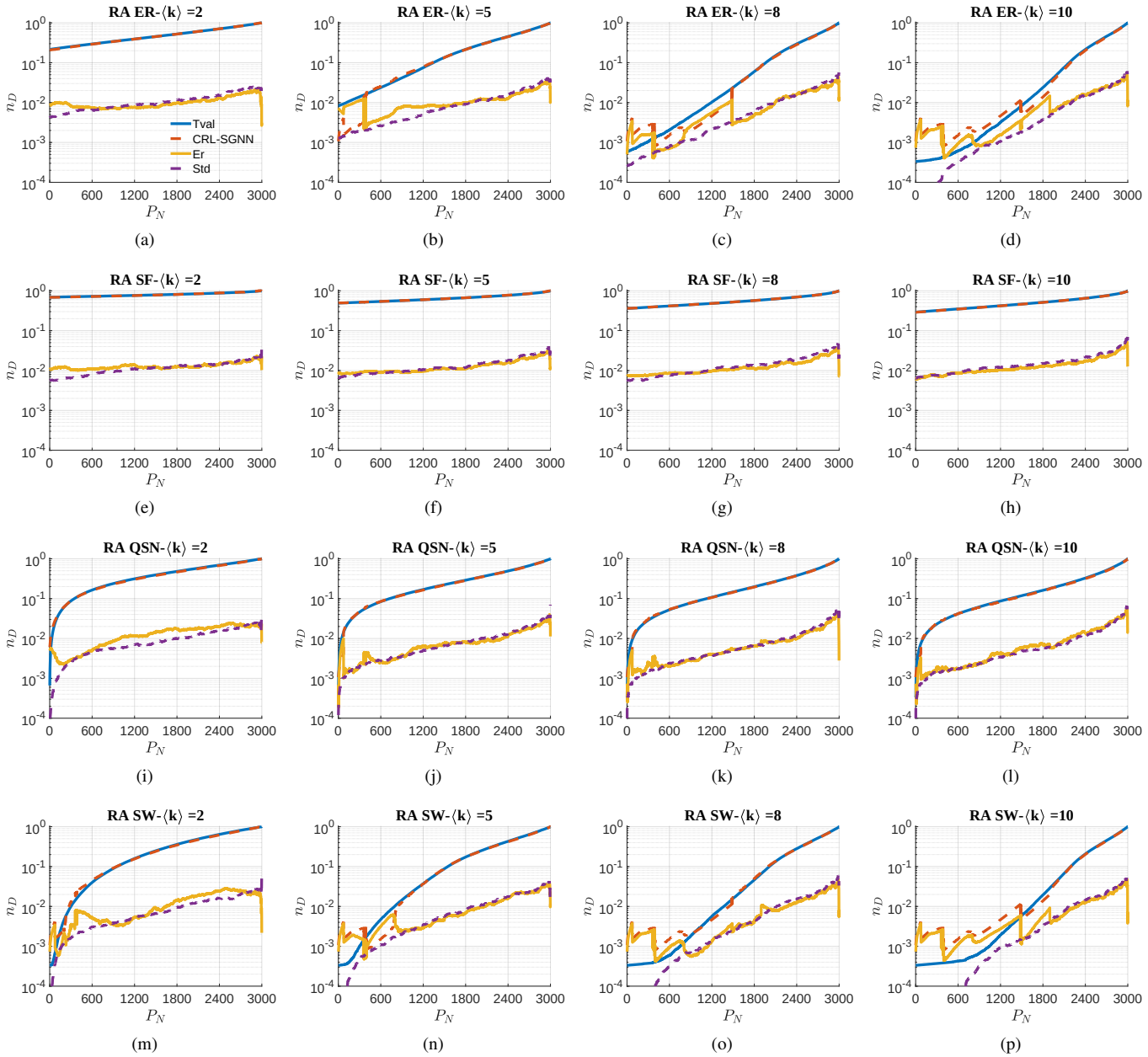

Fig. S13: Results of CRL-SGNN for controllability robustness learning with network size $N = 3000$.

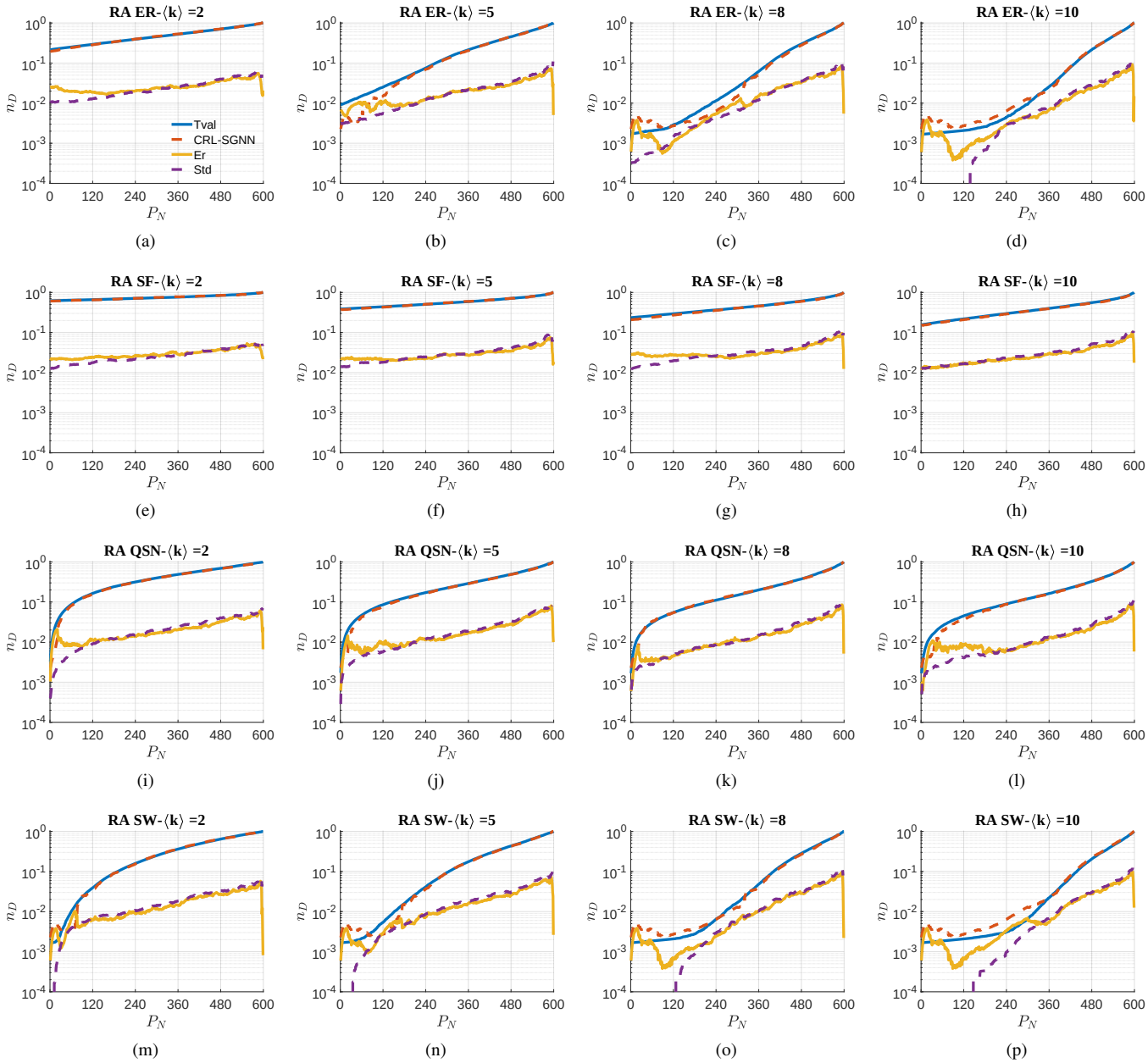

Fig. S14: Results of CRL-SGNN for controllability robustness learning with network size $N = 600$.