

## 18.726, Homework 2

Due Thursday March 3, 11.00 am

1. Recall we showed in class that, given a reduced scheme  $X$  over a base scheme  $S$  and a separated morphism  $Y \rightarrow S$ , then any morphism  $U \rightarrow Y$  from an open dense subscheme  $U \subset X$  has at most one extension to a morphism  $X \rightarrow Y$ . Give a counterexample if we weaken the constraint that  $X$  is reduced, or if we weaken the constraint that  $Y$  is separated over  $S$ .
2. Given a rational map from a nonsingular, reduced, irreducible variety  $X$  over  $k$  to a proper variety  $Y$ , show that there exists a maximal open set  $U \subset X$  such that the rational map comes from an actual morphism  $U \rightarrow Y$  and show that the complement of  $U$  is codimension at least two everywhere. In the case of the rational map  $\mathbb{P}_k^2 \dashrightarrow \mathbb{P}_k^2$  defined by  $[x : y : z] \rightarrow [1/x, 1/y, 1/z]$  what is the maximal domain of definition?
3. Exercises from Hartshorne II.3:
  1. Closed immersions: 3.11 (assume exercises from section 2 mentioned)
  2. Chevalley's theorem: 3.19 (I know you covered this last semester, but the statement holds more generally)
4. Exercises from Hartshorne II.4:
  1. 4.1
  2. 4.7
  3. 4.10: Chow's Lemma.
5. Exercises from Hartshorne II.5:
  1. 5.8