1.
   - To prove $f^{-1}(W)$ is a closed submanifold of M, we can check it locally on M by the fact that $f^{-1}(W)$ is a closed in M.
   - Here we choose V to be an open neiborhood of N, with coordinate x_i, $i = \{1, 2, \dots, dim(N)\}$, and $V \cap W = \{w \in V | x_i(w) = 0, 1 \leq i \leq codim(W, N)$, and let $U = f^{-1}(V)$, we only need to prove $f^{-1}(V \cap W)$ is a close submanifold of U.
   - We define map ϕ from U to $R^{codim(W,N)}$ by functions y_i, for $1 \leq i \leq codim(W, N)$ s.t $y_i = f \circ x_i$. Then $f^{-1}(W \cap V) = \phi^{-1}(0)$ and 0 is a regular value of ϕ by the assumtion. Hence $f^{-1}(W \cap V)$ is a closed submanifold of U.

2.
   - $T_g A$ is a compact operator, since A is compact. Hence T_f is fredholm every where.
   - For $y \in H$, since g is bounded, $f^{-1}(y)$ is bounded.
   - For a regular value y, if $f^{-1}(y)$ is not finite, find a sequence x_i s.t $Ax_i \to z_0$. Then $x_i = y + g(Ax_i) \to y + g(z_0)$, hence $x = y + g(z_0)$ also belongs to $f^{-1}(y)$ since f is a smooth map. Thus by submersion theorem, $f^{-1}(y)$ is a locally dimension 0 manifold and hence discrete, which contradicts to the fact $x_i \to x$.