18.726, Homework 1

Due Tuesday February 16th, 11.00 am

1. Let X be a topological space and $\mathcal{B}$ a basis of open sets (so $\mathcal{B}$ is closed under finite intersections and every open set is covered by elements of $\mathcal{B}$). Recall from class the notion of a sheaf of groups with respect to $\mathcal{B}$. Prove that a sheaf with respect to $\mathcal{B}$ extends uniquely to a sheaf on X. Given two sheaves F, F' on X in the usual sense, show that a map between the restrictions of F and F' with respect to $\mathcal{B}$ extends uniquely to a map between F and F'.

2. Find an example of rings R and S, a continuous map of topological spaces $f : X = \mathrm{Spec}R \to Y = \mathrm{Spec}S$, and a map $\mathcal{O}_Y \to f_*\mathcal{O}_X$ of sheaves of rings, which is not a morphism of schemes.

3. Exercises from Hartshorne II.2:

    1. 2.4 (and then deduce that every scheme admits a unique morphism to $\mathrm{Spec}\mathbb{Z}$.)
    2. 2.8
    3. 2.12
    4. 2.16
    5. 2.17

4. Exercises from Hartshorne II.3:

    1. 3.1
    2. 3.5 a) and c)
    3. 3.10