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Link Travel Time and Delay Estimation Using Transit AVL Data

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***Abstract*—Estimating arterial link travel times and traffic delays using vehicular positioning data, such as automatic vehicle location (AVL) data, is still a challenging subject. The difficulties exist in allocating the travel time between two consecutive AVL reports of a vehicle to each traversed link, especially when the data sampling frequency is low, and identifying the proportion of traffic delay in link travel time. In this paper, transit buses with 30-second sampling interval AVL data were applied as probes to estimate link travel time and traffic delay caused by intersections or alighting and boarding at bus stops. The estimation model proposed in this paper decomposed travel time into three components: free flow travel time, congestion time, and stopping time at signalized intersections and bus stops, then allocated them to each road link. Unlike existing deterministic methods, the proposed solution defined a likelihood function that is maximized to solve for the most likely traffic delays for each road segment on the route. Field tests were conducted on a typical arterial corridor in Edmonton, Canada for data collection and algorithm performance evaluation. The results suggested that the proposed model provides effective and accurate estimation of traffic delay, which can be further applied to transit based or probe vehicle based traffic applications, such as travel time estimation and travel speed estimation.**

Keywords—link travel time estimation; traffic delay estimation; transit; automatic vehicle location data

I. Introduction

Travel time estimation is a challenging subject in the urban intelligent transportation system. This is mainly because that urban traffic delay is usually uncertain due to traffic signal control, congestion effects, stochastic incidents, etc. Accurate estimation of travel time has an important significance to improve the operation efficiency of urban road network.

Traditionally, loop detectors are used to collect traffic data including traffic flow, speed, occupancy, and many models have been developed to estimate traffic delay or travel times based on loop detector data (*1,2,3*). However, the deployment of loop detectors is often not well balanced in urban road network, hence it can only support certain road segments. Besides, installing and maintaining loop detectors is quite costly and time-consuming. With the development of technology, mobile traffic sensors such as GPS-equipped probe vehicles are more and more used to collect traffic data for the whole urban road network. Probe vehicles can collect various information, such as geographical positions, instantaneous travel speeds, and timestamps at any network location without the requirement of roadside equipment. Liu et al.(*4*) proved the feasibility of using data from taxi dispatching systems to collect reliable traffic information and conducted that real-time detection of congestion on link is possible. Researches on travel time estimation and traffic delay estimation using probe vehicle data has grown in recent years as the data collection technology has become more available.

Most previous researchers about using probe vehicle data to estimate traffic delay or travel time made use of taxi GPS data (*5,6*) or simulated data (*7,8,9*), few focused on such data from transit vehicles. Transit automatic vehicle location (AVL) data has special advantages in traffic state estimation. Firstly, bus is the transportation mean in the most common use, usually, transit routes cover most urban arterials, so the data collected from transit vehicles can cover most major urban roads. Secondly, because a transit vehicle always travels along a fixed route repeatedly every day, it's driving route is more certainty comparing with taxi, so the path inference work can be easily accomplished during data processing. Besides, transit has a regular cycle of departure, AVL data can be collected under all conditions (e.g. working days and weekdays, rush hours and peak hours, different weather conditions), which provides rich data resource for research. To sum up, transit vehicles GPS data has a superiority in temporal and spatial coverage.

In order to get a good travel time estimation result, the probe vehicles GPS data is desired to collected with highly reported frequencies (e.g. 1 second) (*10*). Ko et al. proved that in the process of speed smoothing, the sampling interval of the GPS data from 1 s to 10s has little effect on the calculation results of the control delay, so the best sampling interval of the GPS data should be 10 second (*11*). However, such high-frequency probe data is hard to obtain in reality due to transmission and latency delay, generally, probe vehicles GPS data is recorded on time intervals from 30 second to 60 second (*7*), which causes the problem of inaccurate estimation result. Another challenge for using transit probes to estimate travel time is that the transit vehicles may stop at bus stops due to passenger alighting and boarding, which causes additional traffic delays. This paper is aimed to address this problem.

In reality, the reported positions of probe vehicles on links are randomly distributed, which means that travel times collected by probe vehicles do not originate from a single complete link but distributed from a certain position on one link to a certain position on another link. It is necessary to allocate the travel time between two consecutive AVL reports into individual links. Helligna et al. (*12*) developed an analytical model for solving the problem of decomposing sampling interval time to individual road segments. The algorithm is evaluated using simulated data and the results suggest that the method on average improves the accuracy of estimated link travel time by up to 90%. Zheng et al. (*7*) conducted a three-layer neural network model to estimate complete link travel time for individual probe vehicle traversing the link and compared it with the model developed by Helligna. The author found that when congestion occurs, stopping time and congestion time are the main components of the estimated link travel time, which also suggests that stopping probability and congestion probability should be properly calibrated, especially when dealing with road segments with signalized intersection. Helligna's model has been well proved of its feasibility and advantage by simulated general vehicle trajectory data, yet so far it has not been evaluated by transit AVL data. Accurate computation of all kinds of traffic delay helps to show vehicle travel process more clearly, which is useful for traffic management, planning, and forecasting. Although link stopping delay can be estimated by Helligna's model, the estimation accuracy has not been discussed yet. This paper make some exploration about these problems.

It is mentioned by Helligna that inferring traffic conditions from position data requires five steps as follows: (1). Map matching; (2). Path identification; (3). Probe filtering; (4). Travel time allocation; (5). Travel time aggregation. Helligna's paper focuses only on step 4 – travel time allocation and compares performance of two travel time allocation schemes (benchmark travel time decomposition method and the proposed model in Helligna's paper). This paper also focus on the issue of travel time allocation, but uses transit vehicles as probes for link travel time estimation and link delay estimation on arterial. Field test were conducted on a typical arterial corridor in Edmonton, Canada for data collection and algorithm performance evaluation. The original collected transit AVL data is recorded on polling interval duration of approximately 1 second. However, the AVL data with polling interval duration of 30 second or 60 second is more often used in reality.

This paper aims to explore the feasibility and potential of using sparse transit probe data to estimate link travel time and delay. Hence, a re-sampling procedure with 30 second interval was taken to extract data from the original collected data set. The extracted data were used for experiment and the original data (with 1 second polling interval duration) were used as ground truth to verify the estimation results. The proposed algorithm assumes minimal information of the network topography (i.e. links and nodes), and estimate travel time and traffic delay at link level using transit AVL data. Unlike existing deterministic methods, the proposed solution defines a likelihood function that is maximized to solve for the most likely traffic delays for each road segment on the route. In order to evaluate the accuracy of the model, three different link partition methods are adopted for experiments owing to the particularity of bus operation.

II. METHODOLOGY

A. Link travel time estimation model

A transit operation route consists of serval road segments, bus stops, and intersections geographically. In the route model, nodes are used to represent bus stops and intersection along the route (which are denoted by points) and link is used to represent the road segment connecting two nodes (which are denoted by line segment). Therefore, a transit route can be represented by n links and $n+1$ nodes.

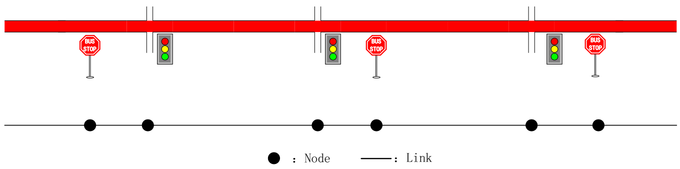


FIG.1. TRANSIT ROUTE MODEL.

It is obvious that the travel time collected by a probe vehicle (i.e. sampling interval) usually do not originate from a single complete link but distributed from a certain position on one link to a certain position on another link. Each sampling interval can be categorized into three cases as illustrated in Fig.2. P_1, P_2, P_3, P_4 are reported positions on the corresponding links and t_1, t_2, t_3, t_4 are timestamps. Thus t_2-t_1, t_3-t_2, t_4-t_3 accordingly represent the sampling interval between two consecutive reported positions. Serval consecutive nodes is denoted by n_a, n_b, n_c, n_d and the link connecting two nodes is denoted by $link_{ab}$, $link_{bc}$, $link_{cd}$. The complete link travel time here is defined as the time difference between the time instant when the vehicle passes the upstream node and the time instant when the vehicle passes the downstream node. $t_{12,ab}$, $t_{12,bc}$, $t_{23,bc}$, $t_{23,cd}$, etc. represent the decomposed link travel time of each partial link based on the sampling interval.

Case 1: Two reported positions (P_1, P_2) are on the same link (e.g. $link_{bc}$) as shown in Fig.2a, the estimated travel time of $link_{bc}$ is composed of three parts:

$$t_{linkbc} = t_{12,bc} + t_3 - t_2 + t_{34,bc}$$

For this case, the link is long or the traffic condition on the target link is likely to be congested or vehicles waste the time on red light since the probe vehicle experiences long travel time on this link which are longer than sampling interval.

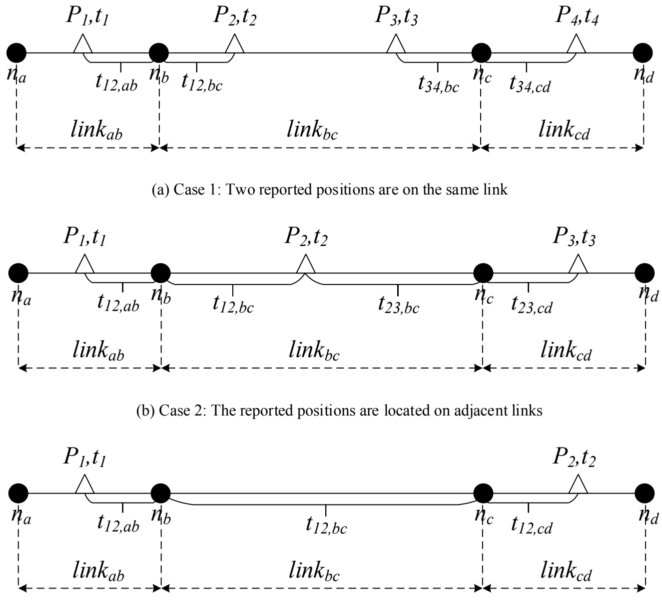
Case 2: The reported positions (P_1, P_2, P_3) are located on adjacent links as shown in Fig.2b, then the travel time of $link_{bc}$ is estimated as:

$t_{linkbc}= t_{12,bc}+ t_{23,bc}$

Case 3: At least one full link is existing between two consecutive reported positions illustrated in Fig.2c, the estimated travel time of $link_{bc}$ is:

$t_{linkbc}= t_{12,bc}$

For this case, the traffic condition on the target link is likely to be free flow or under free flow since the probe vehicle short travel time.



(a) Case 1: Two reported positions are on the same link

(b) Case 2: The reported positions are located on adjacent links

(c) Case 3: At least one full link is existing between two consecutive reported positions

FIG.2. ASSIGNMENT OF TRAVEL TIMES BETWEEN REPORTED POSITIONS TO THE LINK IN THE MIDDLE.

In the following, an analytical travel time estimation model developed by Hellinga was applied to solve the problem of decomposing sampling interval into partial links traversed by probe vehicle *(12)*. This method is evaluated using simulated data and compared to a benchmark deterministic method. The evaluation results suggest that Hellinga's model outperforms the bench mark method and on average improves the accuracy of the estimated link travel times by up to 90%. In this paper, the sampling interval is set to be 30 second. According to Hellinga's evaluation results, the travel time estimation error is very small for polling interval duration of 30 second, meanwhile, the relative improvement in estimation accuracy is about 35%, which suggest sampling interval of 30 second is suitable for research. Besides, Hellinga's algorithm was to estimate travel time for simulated general vehicles, but has not yet been proved available for transit vehicles. This paper is aimed to solve this issue.

B. Travel time allocation model

Here are some definitions in Hellinga's model *(12)*: k represents a sampled probe vehicle, $k=1, 2, ..., K$. $t_{k,i}(i=0, 1, 2,...)$ represents the timestamp of the ith reported position. $m_k(t_{k,i})$ represents the matched reported position of proble vehicle k at time $t_{k,j}$. The road segments traveled by the probe vehicle between two consecutive reported positions, $m_k(t_{k,i})$ and $m_k(t_{k,i+1})$, is denoted by $r_k(t_{k,i}, t_{k,i+1})$, and it is defined as a sequence of links along the route:

$r_k(t_{k,i}, t_{k,i+1})=\{l_{k,i,0}, l_{k,i,1}, ..., l_{k,i,J(k,i)-1}, l_{k,i,J(k,i)}\}=\{l_{k,i,j}|0\leqslant j \leqslant J(k,i)\}$

where $l_{k,i,j}$ is a concise representation of complete or partial link j probe vehicle traveled.

According to the definition proposed by Hellinga et al. *(11)*, arterial link travel times can be decomposed into three parts:

(1) Free flow travel time: The free flow travel time of a link is calculated as the link length divided by the free flow speed, denoted as $t_f(l_{k,i,j})$. In this paper, the free flow speed of any link is set as *60 km/h*.
(2) Congestion time: delay caused by traffic congestion, denoted as $t_c(l_{k,i,j})$. Congestion time results when the probe vehicle travels at a speed less than the free flow speed due to the impedance of other vehicle.
(3) Stopping time: denoted as $t_s(l_{k,i,j})$, it reflects the stopped delay caused to the probe vehicle by stopping at nodes.

Besides, acceleration and deceleration time during vehicle operation is assumed to be included within $t_s(l_{k,i,j})$ when these delays are caused by nodes and within $t_f(l_{k,i,j})$ when caused by route geometry.

Based on the above definitions, the travel time between two consecutive reports of a probe vehicle k can then be described as:

$$t_{k,i+1}-t_{k,i}=\sum_{j=0}^{J(k,i)}\{t_f(l_{k,i,j})+t_s(l_{k,i,j})+t_c(l_{k,i,j})\} \quad (1)$$

The first part, free flow travel time, can be calculated by dividing the link length by the free flow speed:

$$t_f(l_{k,i,j})=\frac{L(l_j)}{v_f(l_j)} \quad (2)$$

where $L(l_j)$ is the length of link j and $v_f(l_j)$ is the free flow speed on link j. Although the free flow speed may vary with driving behavior, weather condition, vehicle type, etc., in this paper the speed limit *60 km/h* is used to represent the free flow speed.

To calculate the congestion time and stopping time, a term named congestion index is introduced as:

$$w=\frac{\sum_{j=0}^{J(k,i)}\{t_c(l_{k,i,j})\}}{\sum_{j=0}^{J(k,i)}\{t_c(l_{k,i,j})+t_f(l_{k,i,j})\}} \quad (3)$$

The congestion index w is used to refer the congestion level. By assuming that the degree of congestion experienced by the vehicle probe during the most recent sampling interval is not substantially different from the degree of congestion experienced by this same probe during the previous sampling interval, the likelihood that a certain degree of congestion is experienced by probe k is expressed as:

$$P_w(k,i,w) = \min\left(1, \frac{T_c(k,i-1)+T_c(k,i)}{(t_{k,i}-t_{k,i-1})+(t_{k,i+1}-t_{k,i})}\frac{1}{w}\right) \quad (4)$$

$$T_c(k,i) = t_{k,i+1} - t_{k,i} - \sum_{j=0}^{J(k,i)} t_f(l_{k,i,j}) \quad (5)$$

where $T_c(k,i)$ is the maximum congestion time the probe vehicle can experience.

In Hellinga's model, it is assumed that a probe vehicle stops at most once during the sampling interval, therefore the probability of stopping on link $l_{k,i,j}$ is given by:

$$P_s(l_{k,i,j},w) = \begin{cases} H_s(l_{k,i,0},w) & \text{if } J(k,i)=0 \\ H_s(l_{k,i,J},w)\prod_{j\neq J}(1-H_s(l_{k,i,j},w)) & \text{otherwise} \end{cases} \quad (6)$$

$$\begin{aligned} H_s(l_{k,i,j},w) &= \frac{1}{\lambda_2-\lambda_1}\int_{\lambda_1}^{\lambda_2}\{(1-w)e^{p(\lambda-1)}+C_2 w\}d\lambda \\ &= \frac{1-w}{p(\lambda_2-\lambda_1)}(e^{p(\lambda_2-1)}-e^{p(\lambda_1-1)})+C_2 w \end{aligned} \quad (7)$$

where $p=\frac{c_1}{w}$, C_1 and C_2 are model parameters that are used to reflect the stopping likelihood pattern of a link. λ is a location parameter that is used to identify any location on the link, i.e. a link between two consecutive nodes n_a and n_b can be described as $link_{ab}=\{(1-\lambda)n_a+\lambda n_b \mid 0\leq \lambda \leq 1\}$.

Finally, the congestion time can be calculated based on the probability function as:

$$t_c(l_{k,i,J}) = \int_0^{w_{\max}} \delta_{k,i,J} t_c \frac{\sum_{j=0}^{J(k,i)} P_w(k,i,w)P_s(l_{k,i,J},w)}{Q_s(k,i)}dw \quad (8)$$

where

$$\delta_{k,i,J} = \frac{t_f(l_{k,i,J})}{\sum_{j=0}^{J(k,i)} t_f(l_{k,i,j})} \quad (9)$$

$$t_c = \frac{w}{1-w}\sum_{j=0}^{J(k,i)} t_f(l_{k,i,j}) \quad (10)$$

And the stopping time can be expressed as:

$$t_s(l_{k,i,j}) = \int_0^{w_{\max}} t_s \frac{P_w(k,i,w)P_s(l_{k,i,J},w)}{Q_s(k,i)}dw \quad (11)$$

where

$$t_s = t_{k,i+1} - t_{k,i} - \sum_{j=0}^{J(k,i)} t_f(l_{k,i,j}) - t_c \quad (12)$$

$$Q_s(k,i) = \int_0^{w_{\max}} P_w(k,i,w)\sum_{j=0}^{J(k,i)} P_s(l_{k,i,j},w)dw \quad (13)$$

III. Model Evaluation

A. *Field test description*

As shown in Fig. 3, there is a typical arterial corridor called "23 Avenue" in Edmonton, Canada. A section of 23 Avenue, from Legar transit center to Century Park transit center, was applied in the field test. This section is a typical urban bus operation route with signalized intersections. It is about 3.6 *km* long and contains 6 signalized intersections and 8 bus stops.

FIG.3. SELECTED CORRIDOR FOR FIELD TEST AND THE LOCATIONS OF BUS STOPS AND SIGNALIZED INTERSECTIONS.

Field test was conducted during the evening peak period in July 28th, 2016. Transit AVL data was collected on 9 buses running along the test route. A smart phone APP which can collect moving object's GPS data was used for transit AVL data collection. Finally, 9 GPS trajectory data of 9 bus trips (4 westbound trips and 5 eastbound trips) was effectively collected. All the GPS positions and timestamps were recorded every 1 second.

B. *Re-sampling procedure*

As mentioned above, every second, positions and timestamps of probe transit vehicles were recorded by the GPS collection APP. However, in the real world, the AVL data polling interval duration of transit is much longer than 1s. Instead, the polling interval duration of 30 second or 60 second is more often used in reality. Hence, a re-sampling procedure with 30 second interval was taken to extract data from the original collected data set. The 30 second re-sampling strategy is applied as follows: if the initial sampling instant time is i, then the next instant time is $i+30$, and then $i+60$, $i+90$, etc. The extracted 30 second interval data were used for experiment and the original data (with 1 second polling interval duration) were used as ground truth to verify the estimation results.

C. *Link partition methods*

Nodes and links are determined according to their geographically location on the test route. The original teat route map is simplified to a diagram by points and line segments. In order to evaluate model estimation result specifically, three link partition types are applied in model evaluation.

The names and definitions of three link partition types are as follow:

Type "All": nodes consists of both bus stops and intersections on the teat route.

Type "Bus stop": only bus stops constitute the whole nodes.

Type "Intersection": only intersections constitute the whole nodes.

Nodes are numbered for convenience and the number of each node is fixed under all link partition types. Links are also numbered but the number changes when link partition type changes, this is because link length differs in different link partition type.

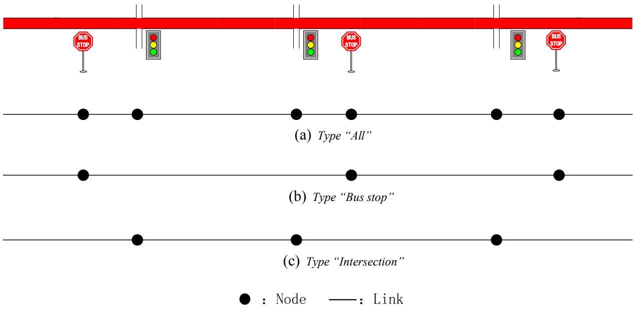


FIG.4. SKETCH OF THREE LINK PARTITION TYPES.

IV. RESULTS

A. Link travel time estimation results

In order to evaluate the Hellinga's model performance of link travel time estimation, two performance indicators: Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) are used to quantify the performance:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|t_{estimated,i} - t_{ground,i}\right| \tag{14}$$

$$MAPE = 100\% * \frac{1}{n}\sum_{i=1}^{n}\left|\frac{t_{estimated,i} - t_{ground,i}}{t_{ground,i}}\right| \tag{15}$$

where $t_{estimated,i}$ is the estimated link travel time of link i; $t_{ground,i}$ is the actual link travel time of link i. All link travel time estimation results of 9 probe vehicles under each link partition type are considered to evaluate estimation accuracy comprehensively. Table 1 shows the three link partition types' MAPE results.

TABLE I. LINK TRAVEL TIME ESTIMATION RESULTS EVALUATION

	Link partition type		
	All	Bus stop	Intersection
MAE(s)	5.08	4.23	6.65
MAPE(%)	26.83	12.04	10.35

Table I. shows the MAEs of link travel times under three link partition type are all under 7s, which indicates that estimation results error are within acceptable range. The difference of three link partition types' MAE is about 1s. *Type "Bus stop"* gets lowest MAE and its MAPE decreases a lot compared with *Type "All"*. Notice that the link partition *Type "All"* has a low MAE but highest MAPE. This is because there are some short links exist in this link partition type, the actual link travel times are originally short in these links. In this circumstance, the effect of errors are amplified and high MAPE is got.

B. Stopping time estimation results

Due to the specialty of transit, we made the stop delay estimation evaluation. The stopping time estimation evaluation results of both bus stops and intersections are shown from Fig.5 to Fig.8. In addition to MAE, Standard Deviation (S.E.) is also used to evaluate estimation result:

$$S.E. = \sqrt{\frac{\sum_{i=1}^{n}\left(t_{estimated,i} - \bar{t}_{estimated}\right)^2}{n}} \tag{16}$$

First is the bus stops delay. In the Fig.5 and Fig.6, the MAE shows the estimation MAE of all probe vehicles passing through the corresponding bus stops, and S.E. shows the corresponding estimation standard deviation. For most bus stops, the value of MAE and Standard deviation in *Type "Bus stop"* is less than it in *Type "All"*. This indicates that the estimated stopping times at bus stops would be more accurate if links are divided by bus stops.

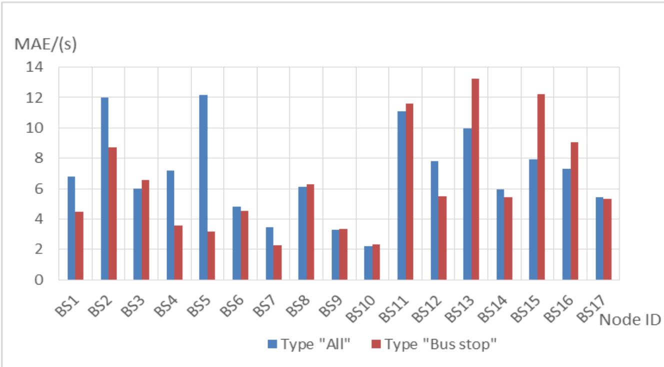


FIG.5. MAE OF BUS STOP DELAY ESTIMATION RESULTS.

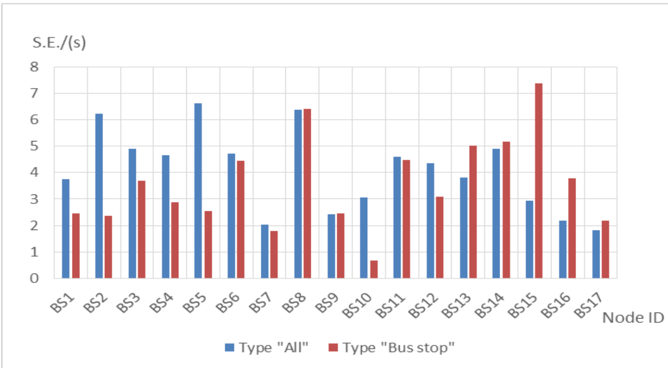


FIG.6. S.E. OF BUS STOP DELAY ESTIMATION RESULTS.

As for intersections, Fig.7 and Fig.8 show that the MAE and S.E. of link partition *Type "Intersection"* are usually bigger than *Type "All"*. This means it doesn't work well when links are divided by intersections. The reason of this problem is : it is because the stopping time of bus stop at downstream is mistakenly calculated in the intersection at upstream due to the defectiveness of Helligna's algorithm.

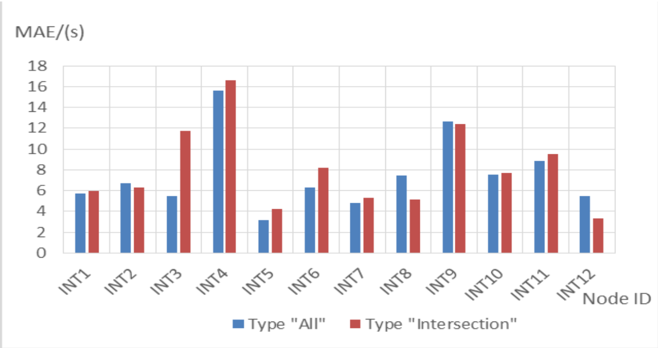


FIG.7. MAE OF INTERSECTION DELAY ESTIMATION RESULTS.

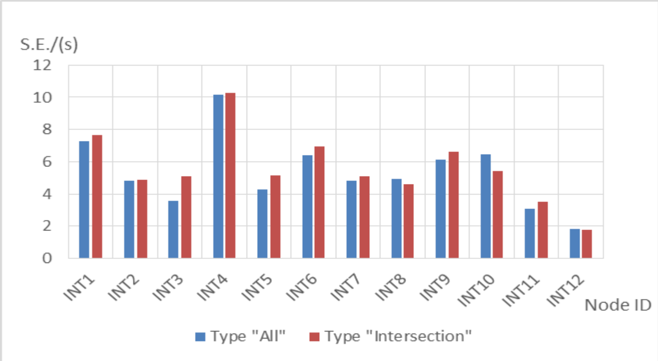


FIG.8. MAE OF INTERSECTION DELAY ESTIMATION RESULTS.

V. CONCLUSIONS

This paper investigated the use of transit AVL data for link travel time estimation and delay estimation. An analytical model proposed by Helligna et al. *(12)* was used to estimate complete link travel time and traffic delay at bus stops and intersections. The input information included individual transit probe vehicle's positions, timestamps and link IDs.

A re-sampling procedure with 30 second interval was taken to extract data from the original collected data set. These data were used for experiment and the original data (with 1 second polling interval duration) were used as ground truth to verify the estimation results. The sampling frequency is assumed to be low because the AVL data with polling interval duration of 30 second or 60 second is more often used in reality due to cost of data transmission and storage and functional requirements. This paper aims to investigate the feasibility and potential of using sparse transit probe data to estimate link travel time and delay without the availability of classical traffic data such as flows. Besides, according to Hellinga's evaluation results, the travel time estimation error is very small for polling interval duration of 30 second, meanwhile, the relative improvement in estimation accuracy is about 35%, which suggest sampling interval of 30 second is suitable for research.

In order to study the estimation result of link travel time and delay at nodes specifically, three link partition types are applied in model evaluation: *Type "All"*, *Type "Bus stop"* and *Type "Intersection"*. As discussed in Results section, model performs better in the link partition *Type "Bus stop"* compared with *Type "All"*, which indicates this model can estimate the transit delay at bus stops very well. As for *Type "Intersection"*, the estimation result of link travel time is good than *Type "All"*, but it doesn't perform well in delay estimation. This is caused by the defectiveness of Helligna's algorithm.

This paper highlights the feasibility and potential of using sparse transit probe data to estimate link travel time and delay without the availability of classical traffic data such as flows. It's mainly focus on the issue of travel time allocation. In future, it is going to improve this model of estimating traffic delay at signalized intersections.

VI. ACKNOWLEDGEMENT

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REFERENCES

[1] J. Kwon and K Petty, "Travel time prediction algorithm scalable to freeway networks with many nodes with arbitrary travel routes," in Transportation Research Record: Journal of the Transportation Research Board, No. 01023234, Transportation Research Board of the National Academies, Washington, D.C., 2005, pp. 147-153.

[2] J.W.C. van Lint, S.P. Hoogendoorn, and H.J. van Zuylen, "Accurate freeway travel time prediction with state-space neural networks under missing data," Transportation Research Part C Emerging Technologies, Volume 13, Issues 5–6, 2005, pp. 347-369.

[3] J. W. C. van Lint and N. J. van der Zijpp, "Improving a travel-time estimation algorithm by using dual loop detectors," in Transportation Research Record: Journal of the Transportation Research Board, No. 00966707, Transportation Research Board of the National Academies, Washington, D.C., 2003, pp. 41-48.

[4] K Liu, T Yamamoto, and Taka Morikawa, "Feasibility of using taxi dispatch system as probes for collecting traffic information," Journal of Intelligent Transportation Systems, Volume 13, 2009, pp. 16-27.

[5] E Jenelius and H. N. Koutsopoulos, "Travel time estimation for urban road networks using low frequency probe vehicle data," Transportation Research Part B Methodological, Volume 53, 2013, pp. 64-81.

[6] M. Rahmani, E. Jenelius, and H. N. Koutsopoulos, "Non-parametric estimation of route travel time distributions from low-frequency floating car data," Transportation Research Part C Emerging Technologies, Volume 58, 2015, pp. 343-362.

[7] F. Zheng and H. V. Zuylen, "Urban link travel time estimation based on sparse probe vehicle data," Transportation Research Part C Emerging Technologies, Volume 31, 2013, pp. 145-157.

[8] A. Hofleitner and A. Bayen, "Optimal decomposition of travel times measured by probe vehicles using a statistical traffic flow model," in Intelligent Transportation Systems (ITSC), 2011 14th International IEEE Conference on, No 12385545, 2011, pp. 815-821.

[9] M. Ramezani and N. Geroliminis, "On the estimation of arterial route travel time distribution with Markov chains," Transportation Research Part B Methodological, Volume 46, 2012, pp. 1576-1590.

[10] L. Zou, J.M. Xu, and L.X. Zhu, "Arterial speed studies with taxi equipped with global positioning receivers as probe vehicle," in International Conference on Wireless Communications, Networking and Mobile Computing, 2005, Proceedings, Volume 2, 2005, pp. 1343-1347.

[11] J. Ko, M. Hunter, and R. Guensler, "Measuring control delay components using second-by-second GPS speed data," Journal of Transportation Engineering, Volume 134, 2007, pp. 338-346.

[12] B. Hellinga , P. Izadpanah, and H. Takada, "Decomposing travel times measured by probe-based traffic monitoring systems to individual road segments," Transportation Research Part C Emerging Technologies, Volume 16, Issue 6, 2008, pp. 768-782.