Travel Time Estimation in Vehicle Routing Problem

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Abstract - **Vehicle routing is a well-known problem in operations research. The problem aims to find an optimal route for vehicles touring all customer locations. Travel time is one of objectives in vehicle routing problem(VRP). The travel time of a route is the sum of travel times between two customers in the route. In reality, there are multiple road segments between two customer sites. Thus, probability distribution of travel time between two customers can be derived by the convolution of distributions of road segments within the arc of two locations. This paper suggests a method of estimation of travel time of the arc in the network of VRP. An example addresses how the method applies to estimate the distribution of the travel time.**

Keywords - **Vehicle Routing Problem, Travel Time, City Vehicle Routing**

I. INTRODUCTION

Vehicle routing problem (VRP) in urban area, so called city VRP, has uncertainties including traffic congestions, road conditions, accidents, events, and so on. The uncertainty in VRP drives us to pay more attention to dynamic vehicle routing problem (DVRP). An important factor of dynamics in DVRP is travel time between locations [1]. Travel times are varying from the fluctuations of vehicle speed due to traffic congestions.

Since the travel time plays an important role in city VRP, the solution of DVRP enables to improve significantly when the travel time is correctly predicted. Therefore, the estimation of travel time is essential in city VRP. This paper proposes an estimation method for travel times in the transportation network, specifically in urban area.

Probability distribution function of travel time has been assumed to follow normal distribution and to be time dependent [2]. A vehicle travels more than one customer sites in vehicle routing problem, which is a route of the vehicle. In a route, all arcs between two customers have their different distances and travel times. Each arc consists of several road segments. In most studies in VRP, the road segment within an arc has been ignored when estimating the distribution of travel time of the arc. However, road segments have their own travel time distributions and are different each other.

Accordingly, the probability distribution of travel time of an arc is supposed to be obtained by convolution of distributions of road segments within the arc [3]. In city, the travel time distributions of road segments are different road by road or street by street. Thus, once we estimate the probability distribution functions of travel times for road segments correctly, the distribution of travel time for the arc can be quite accurate. Variations of travel times of an arc also consists of the fluctuations of travel times of road segments in the arc. Based on the variation of travel time, we may change road segments in an arc adaptively, which can be plugged in DVRP solution procedure.

The contributions of this paper has two folds: Firstly, a novel method of estimation of arc travel time in a route is proposed. The convolution of road segment distributions addresses the travel time distribution of the arc. Secondly, the dynamics of travel times of road segments are considered when estimating the travel times of the arc. The dynamics of travel time originally starts from road segment level.

The issues we explore in this paper provide accurate distribution of travel time in the road network and enable to obtain better solution of dynamic vehicle routing problem in city area. This research aims to estimate the travel time distribution which is more close to real road network and provide good input parameters for DVRP solution process.

The remainder of the paper is as follows. Related works are briefly reviewed in section II. Section III presents the estimation of travel time for an arc and shows how to plug in DVRP solution procedure. Section IV discusses conclusions and possible future research.

II. RELATED WORKS

The distribution of travel time has been assumed with a variety of different distributions. Gamma distribution has been used to represent the travel time. Tas et al. [4] addressed travel times of the arcs follow in dependent identical Gamma distributions. Schilde et al. [5] assumed the request of demand as Poisson distribution and the travel times in links as Gamma distribution. They derived the distribution of travel time from the distribution of vehicle speed. An independent integer valued discrete distribution was used to address the distribution of travel times [6, 7]. Groß et al. [11] suggested the usage of interval travel time for cost efficient and reliable routing in city VRP.

In many studies, the travel time has been assumed with normal distribution [8. 9. 10]. Since the normal distribution has an advantage computationally when we derive the convolution of random variables or calculate

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the mean and variance, many DVRP studies assumed the travel times as normal distributions.

In DVRP, the travel time has been assumed time dependent or stochastic or both [12]. Time dependent model denotes the dynamic model. Since two types of dynamic and stochastic model are hard to solve respectively, many studies only adopted one type of the model.

However, there are lack of studies to deal with the estimation issue of travel time for links. Kim et al. [3] suggested the estimation method for the travel time of the arc using convolution of road segment distributions. This paper extends their suggestions and proposes the dynamic issues from road segments.

III. ESTIMATION OF TRAVEL TIMES

A. Route Problem in Arc

In city VRP, the complexity of road network is higher than rural area. There are numerous roads, streets, and avenues when a vehicle move one location to another. Figure 1 presents the road segments in an arc.

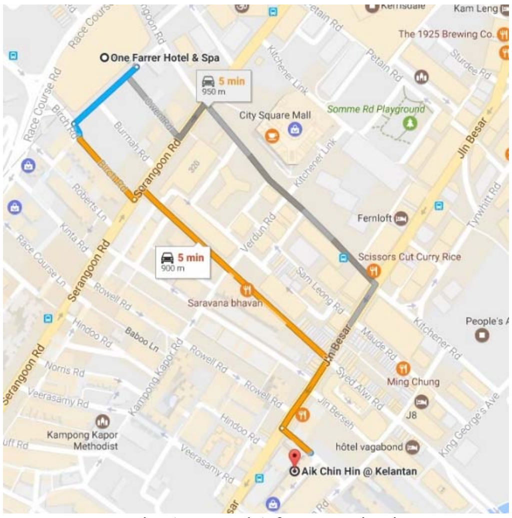

Fig. 1. Travel 1 for an arc in city area

Figure 1 shows the travel between two locations in city area. There are multiple roads when a vehicle moves the arc as follows.

Head southwest on Farrer Park Station Rd toward Owen Rd
130 m

Turn left onto Birch Rd
150 m

Turn left onto Serangoon Rd
22 m

Turn right onto Syed Alwi Rd
400 m

Turn right onto Jln Besar
130 m

Turn left onto Kelantan Ln
Destination will be on the right
60 m

Following the route in Figure 1, the vehicle goes through five road segments such as Farrer Park Station Rd, Birch Rd, Serangoon Rd, Jln Besar, Kelantan Ln.

There is an alternative route for the travel as follows.

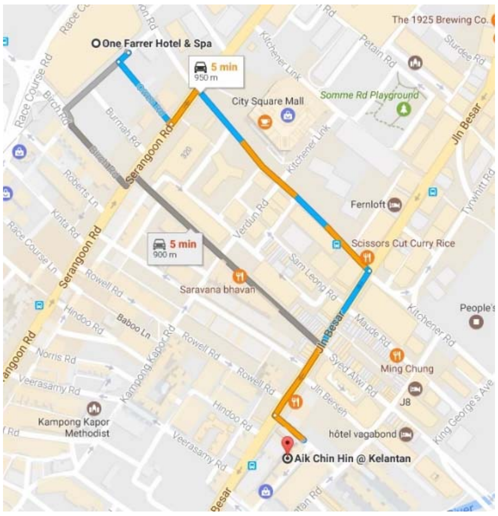

Fig. 2. Travel 2 for an arc in city area

The road segments within alternative route in Figure 2 are as follows.

Head southwest on Farrer Park Station Rd toward Owen Rd
21 m

Turn left onto Owen Rd
130 m

Turn left onto Serangoon Rd
69 m

Turn right onto Kitchener Rd
400 m

Turn right onto Jln Besar
270 m

Turn left onto Kelantan Ln
Destination will be on the right
60 m

Six road segments including Farrer Park Station Rd, Owen Rd, Serangoon Rd, Kitchener Rd, Jln Besar, Kelantan Ln are used to travel in the alternative route shown in Figure 2.

As we have seen is Figure 1 and 2, there are various road segments in an arc which belongs to the set of arcs in a route. Road segments have different distances and travel time distributions because traffic congestions are different. Thus, we have to consider the distributions of road segments to estimate the distribution of the travel time in the arc.

B. Estimation of Travel Time

In some cities, there are sensors on the road to collect the speed of vehicles with certain intervals. Therefore, the data for vehicle speed can be obtained from those sensors. The data are utilized to calculate the distribution of vehicle speed. The distribution of vehicle speed is used to estimate the travel time of the road which is a road segment.

Let $X(t)$ be the random variable of travel time at time t on an arc. Assume that the arc consists of k road segments and denote $X_1(t), X_2(t), \dots, X_k(t)$ be the random variables of travel time at time t on the road segments $i = 1,2,\dots,k$, respectively. We assume that each random variable for the travel time on a road segment follows normal distribution; $X_i(t) \sim N(\mu_{x_i}, \sigma_{x_i}^2),\ i = 1,\dots,k$. Since the travel time of each road segment is obtained by dividing the distance by the speed, the random variable of travel time for an arc is the sum of random variables of the travel times of the road segments. The random variable of the arc is represented as follows.

$$X(t) = X_1(t) + X_2(t) + \cdots + X_k(t). \qquad (1)$$

If we assume that the travel times of the road segments are independent of each other, the mean and variance of the travel time for the arc can be calculated as follows:

$$\mu_{X(t)} = E[X(t)]$$
$$= E[X_1(t)] + E[X_2(t)] + \cdots + E[X_k(t)], \qquad (2)$$

$$\sigma_{X(t)}^2 = Var[X(t)]$$
$$= Var[X_1(t)] + Var[X_2(t)] + \cdots + Var[X_k(t)]. \ (3)$$

From Figure 1 and 2, the arc has two routes which have different road segments with different distributions. In the context of the solution procedure of DVRP, one of routes for the arc is supposed to be chosen. In the previous studies, the route between the two locations was chosen with the shortest route and fixed in the solution procedure. However, when we consider the states of dynamics, the shortest route would be changed depending on the state of traffic congestions.

To send the dynamic properties or genes from road segment level to the arc, we have to monitor the states of traffic congestions in road segments which belongs to the routes of the arc. The candidates of the routes for the arc can be one or more than two.

With the idea of the transfer of dynamic states from road segments to the arc, we propose a procedure to take into account the dynamics of road segment in DVRP solution procedure.

Step 1. Initialization
- Define start node and end node (for the arc)

Step 2. Find Alternatives
- Find the shortest and second or third shortest path between start and end nodes
- Make the list of candidate routes

Step 3. Road segments
- Check the road segments in the candidate routes

Step 4. Monitoring
- Monitor the state of traffic congestions of all road segments

Step 5. Calculate the travel times of routes
- Calculate the travel times of all candidate routes dynamically (time dependent)

Step 6. Chose routes
- Choose one of routes out of candidate routes

Step 7. Estimate the travel time of the arc
- Estimate the travel time distribution of the arc with the chosen candidate route.

Step 8. Apply DVRP
- Plug the travel time of the arc in the DVRP solution procedure

In dynamic routing, a vehicle makes a decision sequentially at every node where to go next. Once the next destination is determined, the vehicle should go from the current location to the next node. There are multiple road segments in the path between the current location and the next target node. Those road segments have different travel time distribution both dynamically and stochastically. Thus, we consider those travel time distributions of road segments.

Like travel 1 and travel2 in previous section, there are multiple paths from the start node to the end node in an arc. In step 2, the algorithm makes the list of paths that have minimum travel time. Road segments in the candidate routes are identified and monitored in step 3 and 4. Based on the traffic conditions, the travel times of candidate routes are calculated. The algorithm chooses the shortest route in step 6. The travel time distribution of the route becomes the travel time distribution of the arc. We then plug the travel time distribution of the arc in the DVRP solution process.

Consider a complete graph $\mathrm{G} = (\mathrm{V}, \mathrm{A})$ where V is the set of nodes and A is the set of arcs. Let $s_a(t)$ be the state of traffic congestion for arc a at time t. Assume that $\mathrm{c}(v_1, t, s_a(t), v_2, t')$ is the travel cost from node v_1 at time t to node v_2 at time t' with traffic congestion state $s_a(t)$. The probability of travel time is defined as follows:

$$\mathrm{P}(X = x | v_1, t, s_a(t), v_2, t'). \quad (4)$$

The probability denotes that a vehicle starts from node v_1 at time t to node v_2 at time t' under the traffic congestion state $s_a(t)$ and the duration time is x. That means $\mathrm{x} = \mathrm{t}' - \mathrm{t}$. Thus, the expected travel cost between v_1 and v_2 is defined as follows:

$$\mathrm{g}(x, t, s_a(t)) =$$
$$\textstyle\sum_x \mathrm{P}(x | v_1, t, s_a(t), v_2, t')\mathrm{c}(v_1, t, s_a(t), v_2, t') \qquad (5)$$

There are many routes to get from node v_1 to node v_2. Each route consists of multiple road segments. Each road segment has its own travel time distribution that is dependent on the traffic congestion state.

Let $R_a = \{r_1, r_2, \ldots, r_l\}$ be the set of routes for arc a between two nodes. Denote $\varepsilon(r_i(t))$ the cost function of route i at time t. Let $o = arg \min_{r_i \in R_a} \{\varepsilon(r_i(t))\}$ and $r_o(t)$ be the route with minimal cost at time t and traffic congestion state $s_a(t)$.

Road segments in the route $r_o(t)$ have different travel time distributions at time t.

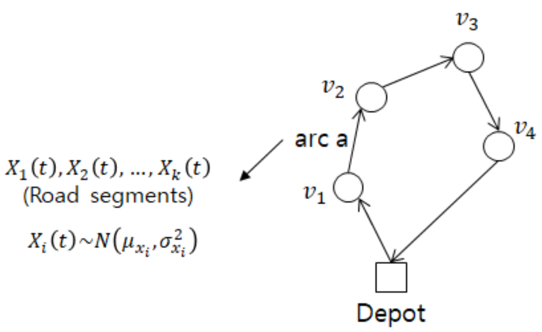

Fig 3. Arc and its road segments

Figure 3 presents an example of an arc and its road segments for one vehicle route. The travel times of the road segment $X_1(t), X_2(t), \ldots, X_k(t)$ is in the $r_o(t)$ that is the minimum route out of the set R_a.

Denote $c_{x_1}(x_1, t, s_a(t))$ be the travel cost for the road segment x_1 at time t under the traffic congestion state $s_a(t)$. The probability of travel time for arc a can be defined as follows:

$$\mathrm{P}(\textstyle\sum_{conv} x_i \,|v_1, t, s_a(t), v_2, t') \qquad (6)$$

where $\sum_{conv} x_i$ denotes the convolution of random variables $x_i, i = 1, \ldots, k$.

Therefore, the expected cost of travel time (5) between two nodes can be replaced as follows:

$$\mathrm{g}(x, t, s_a(t)) = \sum_x \sum_{\forall x_i} \mathrm{P}(\textstyle\sum_{conv} x_i \,|v_1, t, s_a(t), v_2, t') c_{x_i}(x_i, t, s_a(t)) \quad (7)$$

The expected travel cost defined in (7) is obtained by combining all travel time distributions for road segments in $r_o(t)$. The travel cost of the arc is simply calculated by summation of all costs of road segments. On the other hand, the probability of travel time is from the convolution of multiple random variables.

If all road segments have the same probability distributions, the convolution is easy to calculate. For example, if all road segments follow the normal distribution, we can simply calculate the distribution of the convolution. On the other hand, if some road segment follow Gamma distributions, some have normal distributions, and others have different distributions, the convolution is hard to obtain.

Weather the road segments have the same distribution or not, the travel time distribution of an arc in the road network in dynamic routing is supposed to be obtained by the convolution of road segments in the minimum route between two nodes.

IV. CONCLUSIONS

Dynamc vehicle routing problem in urban area has obtained keen attention by practitioners and researchers recent decades. Travel time is an important factor to solve DVRP in city. However, the estimation of travel time depending on time in a route is different from real road network and too simplified.

In this paper, we propose an estimation method for travel time in an arc which belongs to the solution of DVRP. The distributions of road segments within the arc have been used to calculate the travel time distribution. Furthermore, the dynamics of road segment are considered in the solution procedure of DVRP.

Travel times among road segments are assumed independent random variables. However, they may have dependency. So, finding the estimation model assuming there are correlations among travel times of road segments is a challenging issue in the future work.

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