

# A Novel Unequal Lumped-Element Coupler With Arbitrary Phase Differences and Arbitrary Impedance Matching

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**Abstract**—In this brief, for the first time, a novel compact lumped-element coupler is proposed, which can implement arbitrary power-dividing ratios, arbitrary phase differences, and arbitrary impedance matching. In the design, the proposed coupler is composed of eight lumped LC elements, which achieves a compact circuit size and avoids the complexity of implementation. In the section of circuit analysis, explicit close-formed design equations are derived. For demonstration, three cases of normal couplers and two cases of rat-race couplers extracted from the design procedure are simulated and fabricated. Good agreements between the measurements and simulations verify the design theory of the multifunctional LC coupler.

**Index Terms**—Arbitrary phase differences, arbitrary power-dividing ratios, arbitrary terminated impedances, coupler, lumped elements, rat-race coupler.

## I. INTRODUCTION

COUPLER is one of the most important components in radio frequency (RF)/microwave systems, which can be applied in mixers, power amplifiers, phase shifters, and beam-forming antenna arrays. Extensive researches about coupler have been proposed with multifunction in order to meet new requirements, including arbitrary output power division [1]–[9], multiband or wideband [6], [9]–[19], size reduction [3]–[8], [16], impedance transforming [1]–[3], [5], and arbitrary output phase differences [1], [3], [7]. By function, coupler can be classified into many types, such as directional couplers [1], [3]–[9], rat-race couplers [1], [2], [10], and crossovers [18]. According to the division of components of the circuit model, it can be divided into distributed-element couplers [1], [2], [6]–[9], and lumped-element couplers [3]–[5], [16]–[20]. The distributed couplers have been studied extensively. An unequal branch-line coupler with arbitrary phase differences, and arbitrary I/O impedances is proposed in [1], and the supplementary explanation of the

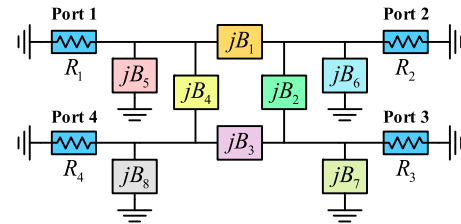


Fig. 1. The circuit model of the proposed lumped-element coupler.

multifunctional rat-race coupler is in [2]. In addition, coupler can be fabricated with PCB [1], [8]–[13], SMD [4], [5], CMOS [16], MMIC [3], [17], and low-temperature co-fired ceramic (LTCC) [19] technology.

However, the distributed components occupy a large area on the circuit board, which hinder the size reduction, especially in microwave integrated circuits at low frequencies (below 20 GHz). The use of lumped-element components can reduce the size and decrease manufacturing complexity, in line with the trend of miniaturization. There is no specific topology structural limitation for designing the lumped-element couplers. In [3], the design of lumped-element coupler is described with arbitrary power division and impedance transformation between the input and output in MIC and MMIC applications. In [5], an unequal lumped-element coupler is presented with impedance transformation based on the replacement of branch-line coupler using Pi-type network. Designing a compact lumped-element coupler with arbitrary power-dividing ratios and arbitrary phase difference is proposed [4]. Using LTCC technology, a wideband lumped-element rat-race coupler is designed [19]. However, a lumped-element coupler with arbitrary power-dividing ratios, arbitrary phase differences, and arbitrary terminated impedances has not been proposed, and the corresponding design method using lumped elements has not been presented in the previous work.

In this brief, a lumped-element coupler with arbitrary power-dividing ratios (between through and coupled ports), arbitrary phase differences (including rat-race case), and arbitrary impedance matching is proposed for the first time, which only uses eight LC components. The basic topology is inspired by the branch-line coupler [1] and lumped-element directional couplers [3]. Eight susceptance modules constitute a new lumped-element coupler, which provide sufficient design flexibility. The closed-form design equations are derived. The design equations are appropriated for the phase difference ranging from  $0^\circ$  to  $360^\circ$  ( $-180^\circ$  to  $+180^\circ$ ), including

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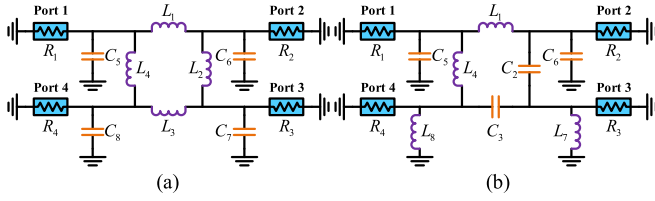


Fig. 2. Schematics of the LC couplers. (a) Case A &amp; Case C. (b) Case B.

the case of the rat-race coupler. Finally, compared to the previous designs [1], [4], the proposed coupler has the advantages of the simple circuit structure and design procedure, easy implementation, arbitrary power-dividing ratios, arbitrary phase differences, and good arbitrary terminated impedance matching.

## II. CIRCUIT STRUCTURE & DESIGN EQUATIONS

The circuit model of the proposed unequal LC coupler with arbitrary phase differences and arbitrary impedances matching is shown in Fig. 1. In this model, there is a lumped component connection between port 1 and 2, 2 and 3, 3 and 4, 4 and 1, which can be expressed as  $B_1$ ,  $B_2$ ,  $B_3$ , and  $B_4$ . Each port and ground are connected with a lumped component, which is  $B_5$ ,  $B_6$ ,  $B_7$ , and  $B_8$ . Terminated impedance of the four ports can be expressed as  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ , which are the real numbers. The eight admittance components can be expressed as  $Y_i = jB_i$ , and the susceptance  $B_i$  can be defined as

$$B_i = \begin{cases} \omega C_i, & 0 < B_i < \infty \\ \text{open}, & B_i = 0 \\ -\frac{1}{\omega L_i}, & -\infty < B_i < 0 \\ \text{short}, & B_i = \pm\infty, \end{cases} \quad (1)$$

where  $i = 1, 2, \dots, 8$ .

Starting from the  $Y$ -parameter definition, according to the circuit model,  $Y$ -matrix of the coupler can be expressed as

$$Y_{\text{circuit}} = \begin{pmatrix} jB_5 + jB_4 + jB_1 & -jB_1 & 0 & -jB_4 \\ -jB_1 & jB_6 + jB_1 + jB_2 & -jB_2 & 0 \\ 0 & -jB_2 & jB_7 + jB_2 + jB_3 & -jB_3 \\ -jB_4 & 0 & -jB_3 & jB_8 + jB_3 + jB_4 \end{pmatrix}. \quad (2)$$

### A. Phase Difference Not Equal to $0^\circ$ or $180^\circ$ (Normal Case)

With the normal coupler definition, port 1 is the input port, port 2 is the through port, port 3 is the coupled port, and port 4 is the isolated port. Therefore, the expected  $S$ -matrix can be discussed. Assuming that the phases of  $S_{12}$  and  $S_{34}$  are  $\psi_{1,2}$ , the phase difference is  $\Phi$ , and the power-dividing ratio is  $k^2 = |S_{21}| / |S_{31}|^2$ , the expected  $S$ -matrix can be expressed as

$$S_{\text{expected}} = \begin{pmatrix} 0 & \frac{ke^{j\psi_1}}{\sqrt{k^2+1}} & \frac{e^{j(\psi_1-\phi)}}{\sqrt{k^2+1}} & 0 \\ \frac{ke^{j\psi_1}}{\sqrt{k^2+1}} & 0 & 0 & \frac{e^{j(\pi+\psi_2+\phi)}}{\sqrt{k^2+1}} \\ \frac{e^{j(\psi_1-\phi)}}{\sqrt{k^2+1}} & 0 & 0 & \frac{ke^{j\psi_2}}{\sqrt{k^2+1}} \\ 0 & \frac{e^{j(\pi+\psi_2+\phi)}}{\sqrt{k^2+1}} & \frac{ke^{j\psi_2}}{\sqrt{k^2+1}} & 0 \end{pmatrix}. \quad (3)$$

Due to the circuit characteristic is denoted by  $Y$ -matrix, the expected  $S$ -matrix needs to be converted to  $Y$ -matrix. The

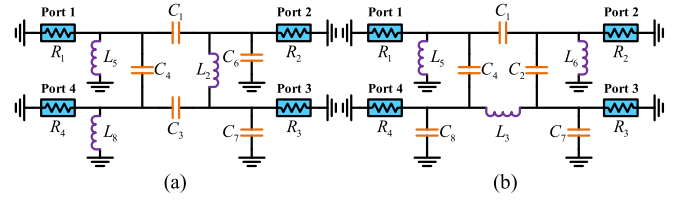


Fig. 3. Schematics of the LC rat-race couplers. (a) Case D. (b) Case E.

expected  $Y$ -matrix is calculated as [1]

$$Y_{\text{expected}} = \begin{pmatrix} \frac{j \cot \phi}{R_1} & \mp \frac{j\sqrt{k^2+1} \csc \phi}{k\sqrt{R_1 R_2}} & 0 & \mp \frac{j \csc \phi}{k\sqrt{R_1 R_4}} \\ \mp \frac{j\sqrt{k^2+1} \csc \phi}{k\sqrt{R_1 R_2}} & \frac{j \cot \phi}{R_2} & \frac{j \csc \phi}{k\sqrt{R_2 R_3}} & 0 \\ 0 & \frac{j \csc \phi}{k\sqrt{R_2 R_3}} & -\frac{j \cot \phi}{R_3} & \frac{j\sqrt{k^2+1} \csc \phi}{k\sqrt{R_3 R_4}} \\ \mp \frac{j \csc \phi}{k\sqrt{R_1 R_4}} & 0 & \frac{j\sqrt{k^2+1} \csc \phi}{k\sqrt{R_3 R_4}} & -\frac{j \cot \phi}{R_4} \end{pmatrix}, \quad (4)$$

where the upper and lower signs represent  $\psi_1 = \Phi$  and  $\psi_1 = \Phi + \pi$ , respectively.

The coupler circuit needs to be worked as expected, so the equation is  $Y_{\text{circuit}} = Y_{\text{expected}}$ . Through this equation, all eight admittance parameters are calculated as (5), where the upper and lower signs represent  $\psi_1 = \Phi$  and  $\psi_1 = \Phi + \pi$ , respectively. The choice of  $\psi_1$  requires consideration of circuit performance, which will be discussed in Section III-A.

$$B_1 = \pm \frac{\sqrt{k^2+1} \csc \phi}{k\sqrt{R_1 R_2}}, B_2 = -\frac{\csc \phi}{k\sqrt{R_2 R_3}}, \quad (5a)$$

$$B_3 = -\frac{\sqrt{k^2+1} \csc \phi}{k\sqrt{R_3 R_4}}, B_4 = \pm \frac{\csc \phi}{k\sqrt{R_1 R_4}}, \quad (5b)$$

$$B_5 = \frac{\cot \phi}{R_1} \mp \frac{\csc \phi}{k\sqrt{R_1 R_4}} \mp \frac{\sqrt{k^2+1} \csc \phi}{k\sqrt{R_1 R_2}}, \quad (5c)$$

$$B_6 = \frac{\cot \phi}{R_2} \mp \frac{\sqrt{k^2+1} \csc \phi}{k\sqrt{R_1 R_2}} + \frac{\csc \phi}{k\sqrt{R_2 R_3}}, \quad (5d)$$

$$B_7 = -\frac{\cot \phi}{R_3} + \frac{\csc \phi}{k\sqrt{R_2 R_3}} + \frac{\sqrt{k^2+1} \csc \phi}{k\sqrt{R_3 R_4}}, \quad (5e)$$

$$B_8 = -\frac{\cot \phi}{R_4} + \frac{\sqrt{k^2+1} \csc \phi}{k\sqrt{R_3 R_4}} \mp \frac{\csc \phi}{k\sqrt{R_1 R_4}}. \quad (5f)$$

### B. Phase Difference Equal to $0^\circ$ or $180^\circ$ (the Rat-Race Case)

With the rat-race coupler definition, port 1 is the input port, port 2 is the through port, port 4 is the coupled port, and port 3 is the isolated port. The output ports will be in-phase when port 1 is excited, the output ports will be the anti-phase when port 3 is excited. Similar to the above, the expected  $S$ -matrix and  $Y$ -matrix can be expressed as (6) and (7), where the upper and lower signs represent  $\psi_2 = \psi_1$  and  $\psi_2 = \psi_1 + \pi$ , respectively.

$$S_{\text{expected}} = \begin{pmatrix} 0 & \frac{ke^{j\psi_1}}{\sqrt{k^2+1}} & 0 & \frac{e^{j\psi_1}}{\sqrt{k^2+1}} \\ \frac{ke^{j\psi_1}}{\sqrt{k^2+1}} & 0 & \frac{e^{j(\psi_2+\pi)}}{\sqrt{k^2+1}} & 0 \\ 0 & \frac{e^{j(\psi_2+\pi)}}{\sqrt{k^2+1}} & 0 & \frac{ke^{j\psi_2}}{\sqrt{k^2+1}} \\ \frac{e^{j\psi_1}}{\sqrt{k^2+1}} & 0 & \frac{ke^{j\psi_2}}{\sqrt{k^2+1}} & 0 \end{pmatrix}, \quad (6)$$

TABLE I  
PARAMETER VALUES OF THE GENERALIZED CASES

|                             | Case A            | Case B       | Case C             | Case D        | Case E              |
|-----------------------------|-------------------|--------------|--------------------|---------------|---------------------|
| $k$ (dB)                    | 4                 | 0            | 3                  | 0             | 3                   |
| $\psi_1$ (°) / $\psi_2$ (°) | $\Phi$ (60) + 180 | $\Phi$ (250) | $\Phi$ (130) + 180 | $\psi_1$ (80) | $\psi_1$ (85) + 180 |
| $R_1$ ( $\Omega$ )          | 50                | 42           | 50                 | 50            | 50                  |
| $R_2$ ( $\Omega$ )          | 50                | 50           | 75                 | 45            | 50                  |
| $R_3$ ( $\Omega$ )          | 50                | 60           | 60                 | 42            | 50                  |
| $R_4$ ( $\Omega$ )          | 50                | 75           | 100                | 60            | 50                  |
| $L_1$ (nH) / $C_1$ (pF)     | 5.83 (nH)         | 4.85 (nH)    | 6.09 (nH)          | 2.41 (pF)     | 2.61 (pF)           |
| $L_2$ (nH) / $C_2$ (pF)     | 10.92 (nH)        | 3.09 (pF)    | 11.55 (nH)         | 9.64 (nH)     | 1.85 (pF)           |
| $L_3$ (nH) / $C_3$ (pF)     | 5.83 (nH)         | 3.57 (pF)    | 7.71 (nH)          | 2.28 (pF)     | 9.71 (nH)           |
| $L_4$ (nH) / $C_4$ (pF)     | 10.92 (nH)        | 8.39 (nH)    | 12.18 (nH)         | 2.09 (pF)     | 1.85 (pF)           |
| $L_5$ (nH) / $C_5$ (pF)     | 8.50 (pF)         | 9.62 (pF)    | 3.57 (pF)          | 6.44 (nH)     | 6.07 (nH)           |
| $L_6$ (nH) / $C_6$ (pF)     | 8.50 (pF)         | 3.29 (pF)    | 4.57 (pF)          | 0.84 (pF)     | 6.07 (nH)           |
| $L_7$ (nH) / $C_7$ (pF)     | 4.83 (pF)         | 3.32 (nH)    | 7.70 (pF)          | 1.02 (pF)     | 1.04 (pF)           |
| $L_8$ (nH) / $C_8$ (pF)     | 4.83 (pF)         | 19.11 (nH)   | 6.70 (pF)          | 6.50 (nH)     | 1.04 (pF)           |

The rat-race coupler needs to be worked as expected, which is  $Y_{\text{circuit}} = Y_{\text{expected}}$ . Through this equation, all eight admittance parameters are calculated as (8), where the upper and lower signs represent  $\psi_2 = \psi_1$  and  $\psi_2 = \psi_1 + \pi$ , respectively. The choice of  $\psi_2$  requires consideration of circuit performance.

$$Y_{\text{expected}} = \begin{pmatrix} \frac{j \cot \psi_1}{R_1} & -\frac{jk \csc \psi_1}{\sqrt{R_1 R_2 \sqrt{k^2 + 1}}} & 0 & -\frac{j \csc \psi_1}{\sqrt{R_1 R_4 \sqrt{k^2 + 1}}} \\ -\frac{jk \csc \psi_1}{\sqrt{R_1 R_2 \sqrt{k^2 + 1}}} & \frac{j \cot \psi_1}{R_2} & \pm \frac{j \csc \psi_1}{\sqrt{R_2 R_3 \sqrt{k^2 + 1}}} & 0 \\ 0 & \pm \frac{j \csc \psi_1}{\sqrt{R_2 R_3 \sqrt{k^2 + 1}}} & \frac{j \cot \psi_1}{R_3} & \mp \frac{jk \csc \psi_1}{\sqrt{R_3 R_4 \sqrt{k^2 + 1}}} \\ -\frac{j \csc \psi_1}{\sqrt{R_1 R_4 \sqrt{k^2 + 1}}} & 0 & \mp \frac{jk \csc \psi_1}{\sqrt{R_3 R_4 \sqrt{k^2 + 1}}} & \frac{j \cot \psi_1}{R_4} \end{pmatrix} \quad (7)$$

$$B_1 = \frac{k \csc \psi_1}{\sqrt{R_1 R_2 \sqrt{k^2 + 1}}}, B_2 = \mp \frac{\csc \psi_1}{\sqrt{R_2 R_3 \sqrt{k^2 + 1}}}, \quad (8a)$$

$$B_3 = \pm \frac{k \csc \psi_1}{\sqrt{R_3 R_4 \sqrt{k^2 + 1}}}, B_4 = \frac{\csc \psi_1}{\sqrt{R_1 R_4 \sqrt{k^2 + 1}}}, \quad (8b)$$

$$B_5 = \frac{\cot \psi_1}{R_1} - \frac{\csc \psi_1}{\sqrt{R_1 R_4 \sqrt{k^2 + 1}}} - \frac{k \csc \psi_1}{\sqrt{R_1 R_2 \sqrt{k^2 + 1}}}, \quad (8c)$$

$$B_6 = \frac{\cot \psi_1}{R_2} - \frac{k \csc \psi_1}{\sqrt{R_1 R_2 \sqrt{k^2 + 1}}} \pm \frac{\csc \psi_1}{\sqrt{R_2 R_3 \sqrt{k^2 + 1}}}, \quad (8d)$$

$$B_7 = \frac{\cot \psi_1}{R_3} \pm \frac{\csc \psi_1}{\sqrt{R_2 R_3 \sqrt{k^2 + 1}}} \mp \frac{k \csc \psi_1}{\sqrt{R_3 R_4 \sqrt{k^2 + 1}}}, \quad (8e)$$

$$B_8 = \frac{\cot \psi_1}{R_4} \mp \frac{k \csc \psi_1}{\sqrt{R_3 R_4 \sqrt{k^2 + 1}}} - \frac{\csc \psi_1}{\sqrt{R_1 R_4 \sqrt{k^2 + 1}}}. \quad (8f)$$

### III. DESIGN PROCEDURE

#### A. Normal Coupler

According to (5), the value of  $\psi_1$  can be chosen by  $\psi_1 = \Phi$  and  $\psi_1 = \Phi + \pi$ . The choice of  $\psi_1$  requires consideration of circuit performance. Considering that there are different performances, a set of termination conditions are presented as follows.

1) Parameter values of the capacitor and inductor. The values of the capacitance and inductance need to be selected at a reasonable order of magnitude.

2) Performance requirement. The fractional bandwidth of the S-parameters, the smoothness of the phase difference,

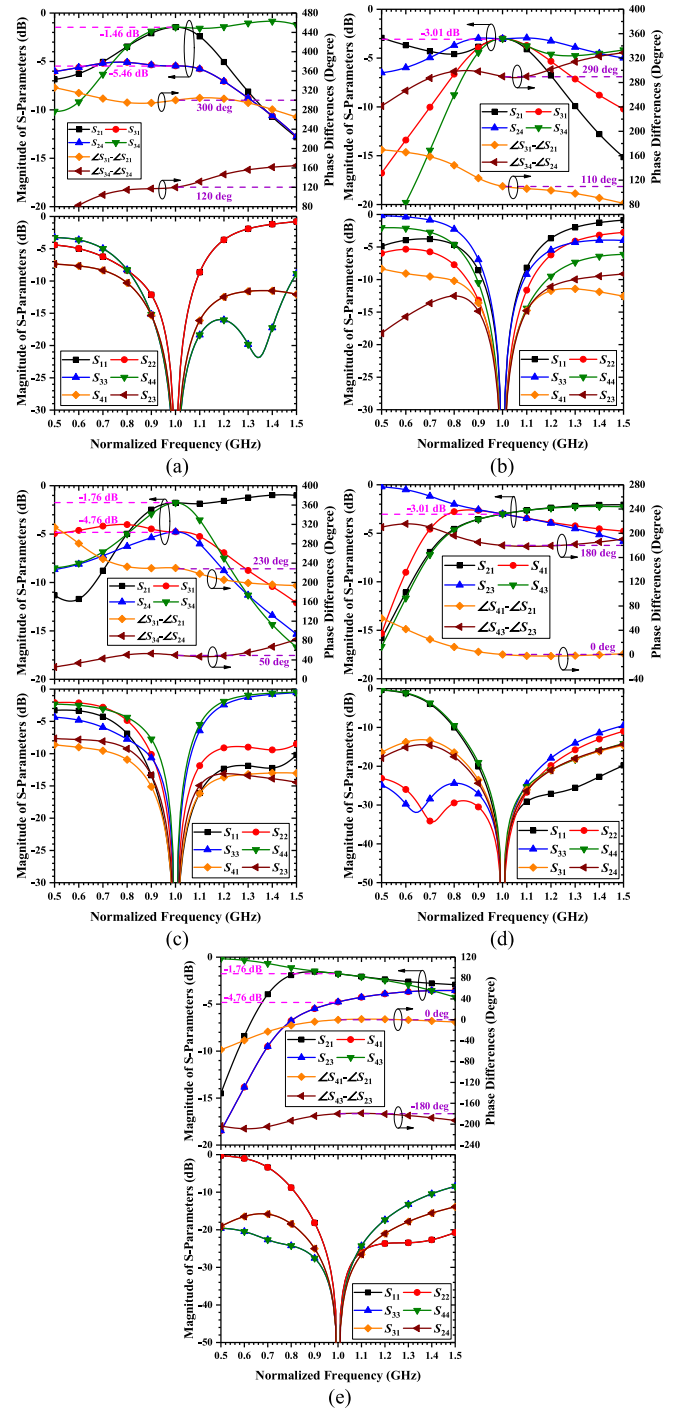


Fig. 4. Calculated ideal S-parameters and phase differences. (a) Case A. (b) Case B. (c) Case C. (d) Case D. (e) Case E.

and the four port impedances should be suitable for the specific needs.

In summary, the design procedure of the LC normal coupler (phase difference not equal to 0° or 180°) is listed as follows.

1) *Step 1*: Designate the values of  $\Phi$ ,  $k$ , and  $R_{1,2,3,4}$  according to the requirements.

2) *Step 2*: Value  $\psi_1$  has two choices,  $\psi_1 = \Phi$  and  $\psi_1 = \Phi + \pi$ . Based on the choice of  $\psi_1$ , choose either group of signs for (5) to calculate  $B_{1,...,8}$  under different values of  $\psi_1$ .

3) *Step 3*: Choose one of the results which is more consistent with the requirements, and then extract the final  $B_{1,...,8}$ .

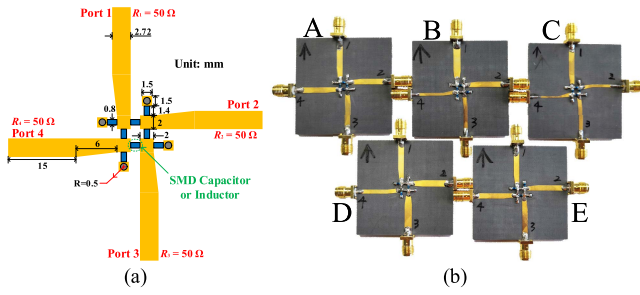


Fig. 5. (a) Layout of the proposed coupler (*Case A*). (b) Photographs of the five fabricated LC couplers.

TABLE II  
SIMULATED AND MEASURED PARAMETER  
VALUES (SIM. / MEA.)

|                         | Case A        | Case B        | Case C        | Case D        | Case E        |
|-------------------------|---------------|---------------|---------------|---------------|---------------|
| $L_1$ (nH) / $C_1$ (pF) | 3.7 / 4.7(nH) | 3.4 / 4.3(nH) | 4.8 / 4.7(nH) | 2.2 / 2.2(pF) | 2.4 / 2.4(pF) |
| $L_2$ (nH) / $C_2$ (pF) | 8.5 / 8.7(nH) | 2.1 / 3.0(pF) | 10.4/10.0(nH) | 8.3 / 9.5(nH) | 1.7 / 1.8(pF) |
| $L_3$ (nH) / $C_3$ (pF) | 3.7 / 4.3(nH) | 2.8 / 3.3(pF) | 6.3 / 6.2(nH) | 2.1 / 2.2(pF) | 8.2 / 9.1(nH) |
| $L_4$ (nH) / $C_4$ (pF) | 8.5 / 8.7(nH) | 6.0 / 7.5(nH) | 11.0/11.0(nH) | 1.9 / 2.0(pF) | 1.7 / 1.8(pF) |
| $L_5$ (nH) / $C_5$ (pF) | 6.4 / 6.0(pF) | 6.7 / 6.0(pF) | 2.9 / 3.0(pF) | 5.4 / 6.8(nH) | 4.8 / 5.6(nH) |
| $L_6$ (nH) / $C_6$ (pF) | 6.4 / 7.0(pF) | 2.8 / 3.0(pF) | 3.6 / 3.6(pF) | 0.8/0.75(pF)  | 4.6 / 5.6(nH) |
| $L_7$ (nH) / $C_7$ (pF) | 3.8 / 3.9(pF) | 2.3 / 2.2(nH) | 5.6 / 5.6(pF) | 1.2 / 1.0(pF) | 0.8 / 1.0(pF) |
| $L_8$ (nH) / $C_8$ (pF) | 3.8 / 3.9(pF) | 14.6/15.0(nH) | 5.0 / 5.0(pF) | 5.1 / 5.6(nH) | 1.0 / 1.0(pF) |

### B. Rat-Race Coupler

According to (8), the value of  $\psi_2$  can be chosen by  $\psi_2 = \psi_1$  and  $\psi_2 = \psi_1 + \pi$ , and the choice of values for  $\psi_1$  is arbitrary. The choice of  $\psi_1$  requires consideration of circuit performance. The termination conditions are the same as the above. The design procedure of the LC rat-race coupler is listed as follows.

- 1) *Step 1*: Designate the values of  $\psi_2$  ( $\psi_1$  or  $\psi_1 + \pi$ ),  $k$ , and  $R_{1,2,3,4}$  according to the requirements.
- 2) *Step 2*: Select the value of  $\psi_1$ . Choose either group of signs for (8) to calculate  $B_{1,\dots,8}$  based on the choice of  $\psi_1$ .
- 3) *Step 3*: If the result based on  $B_{1,\dots,8}$  does not fit the termination conditions, then repeat Steps 2 and 3.

## IV. DISCUSSION & EXPERIMENTAL VERIFICATION

To validate the design theory above, three examples of normal couplers (*Case A*, *B*, and *C*) and two examples of rat-race couplers (*Case D* and *E*) are given. The center frequency of all examples is 1 GHz. The configurations of the circuit parameters are listed in Table I. Fig. 2 and Fig. 3 shows the schematics. Fig. 4 show the ideal performances.

The simulations and optimizations are performed by ANSYS HFSS. The designed prototypes are fabricated and measured on F4B substrate ( $\epsilon_r = 2.65$ ,  $h = 1$  mm). MuRata capacitors and inductors are used for building the circuits. The simulated and measured values of the capacitors and inductors are listed in Table II. Fig. 5 shows the corresponding circuit layout and photographs of the fabricated LC coupler. The core circuit size of the five couplers is about  $11.2 \text{ mm} \times 11.2 \text{ mm}$  ( $0.061\lambda_g \times 0.061\lambda_g$ ). The measured results are obtained by the four-port vector network analyzer of Rohde & Schwarz ZVA8.

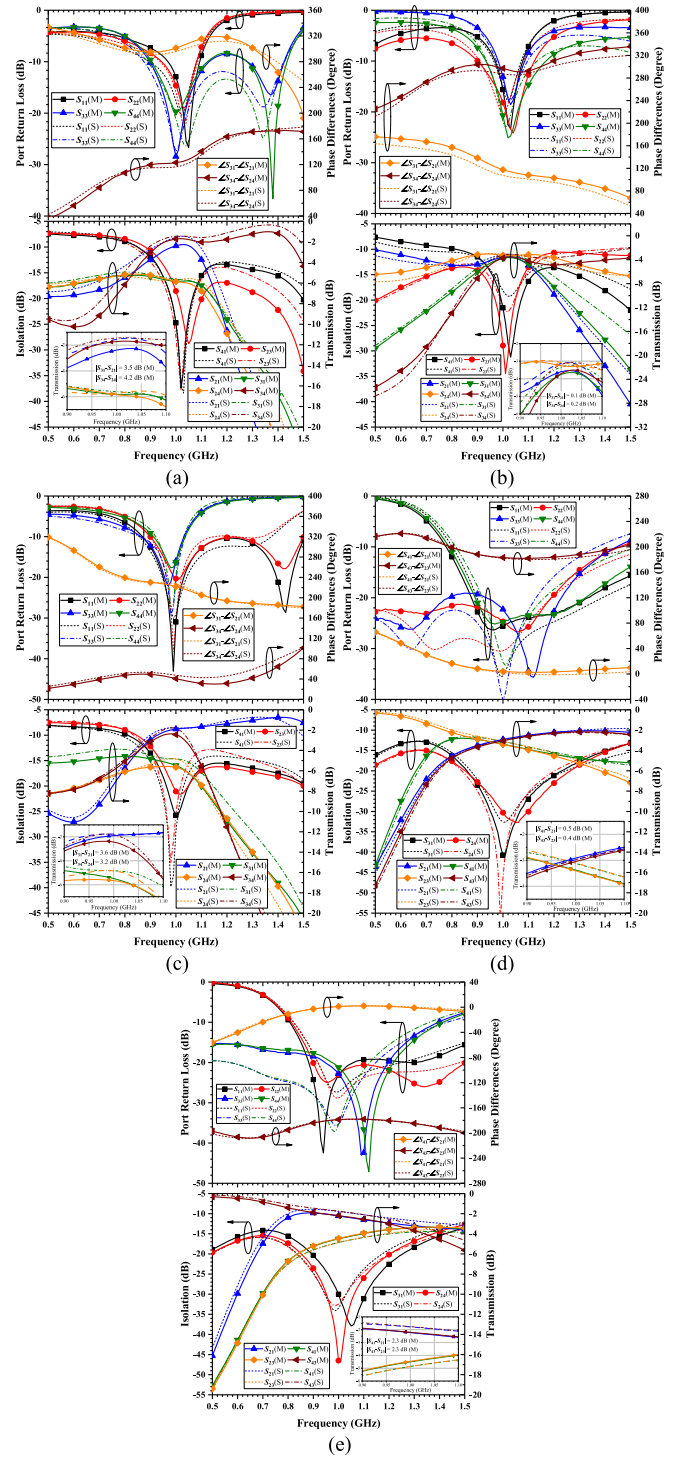


Fig. 6. Measured (M) and EM-simulated (S) S-parameters and phase differences. (a) *Case A*. (b) *Case B*. (c) *Case C*. (d) *Case D*. (e) *Case E*.

Fig. 6 shows the EM simulated and measured results of the five cases. Table III shows the bandwidth performances of the three cases (*A*, *B*, and *C*), where the measured data are in parentheses. The measured matching is all greater than 15 dB, and the isolation is all greater than 20 dB. Most of the deviations of the phase differences are less than  $\pm 5^\circ$  between the simulated and measured results. In *Case A* (4-dB), the power distributions at the two output ports are measured, 5.8 & 2.3 dB ( $|S_{31}|$ ,  $|S_{21}|$ ), 5.9 & 1.7 dB ( $|S_{24}|$ ,  $|S_{34}|$ ). Similarly, the



TABLE III  
FRACTIONAL BANDWIDTH OF THE IDEAL CASE  
AND MEASURED MODEL (UNIT: %)

|                                 |       | Case A*   | Case B* | Case C*    |
|---------------------------------|-------|-----------|---------|------------|
| Matching <sup>1</sup>           | 10 dB | 23 (12)   | 18 (11) | 13 (12)    |
|                                 | 15 dB | 11 (6)    | 10 (4)  | 7 (5)      |
| Isolation: $ S_{41} $           | 15 dB | 22 (17)   | 18 (16) | 23 (24)    |
|                                 | 20 dB | 12 (8)    | 10 (8)  | 11 (9)     |
| Isolation: $ S_{23} $           | 15 dB | 22 (19**) | 20 (16) | 18 (21***) |
|                                 | 20 dB | 12 (9)    | 10 (8)  | 9 (6)      |
| Power Division 1 <sup>2</sup>   | 1 dB  | 25 (16)   | 37 (28) | 24 (13)    |
| Power Division 2 <sup>3</sup>   | 1 dB  | 30 (18)   | 21 (14) | 28 (20)    |
| Phase Difference 1 <sup>4</sup> | 5°    | 21 (6)    | 21 (10) | 28 (8)     |
|                                 | 2°    | 6 (4)     | 7 (4)   | 22 (3)     |
| Phase Difference 2 <sup>5</sup> | 5°    | 25 (15)   | 23 (14) | 57 (9)     |
|                                 | 2°    | 7 (5)     | 16 (5)  | 10 (4)     |

<sup>1</sup>:  $|S_{11}|$  &  $|S_{22}|$  &  $|S_{33}|$  &  $|S_{44}|$ ; <sup>2</sup>:  $|S_{31}|$  &  $|S_{21}|$ ; <sup>3</sup>:  $|S_{34}|$  &  $|S_{24}|$ ; <sup>4</sup>:  $\angle S_{31} - \angle S_{21}$ ; <sup>5</sup>:  $\angle S_{34} - \angle S_{24}$

\*: Values in parentheses are the measured data

\*\* : 17-dB fractional bandwidth \*\*\* : 16-dB fractional bandwidth

TABLE IV  
PERFORMANCE COMPARISON FOR THE COUPLERS

| Ref.  | [1] (ex. 3)                   | [4] (Fig. 6)                   | [9] (Coupler 1)             | [19]          | This work (Case A)            |
|---|-------------------------------|--------------------------------|-----------------------------|---------------|-------------------------------|
| PD.   | Arbitrary                     | Arbitrary                      | Arbitrary                   | Equal         | Arbitrary                     |
| Load Impedances                                 | I/O arbitrary                 | Identical                      | Identical                   | Not arbitrary | Arbitrary                     |
| Phase Difference                                | -180°-180°, rat-race included | -180°-180°, except 0° and 180° | -180°-180°, except rat-race | 90°           | -180°-180°, rat-race included |
| Band  | Single                        | Single                         | Dual                        | Single        | Single                        |
| Tech.   | PCB                           | SMD                            | PCB                         | GaAs          | SMD                           |
| Matching FBW. (%/dB)*                           | 2 (20)                        | 7.5 (15)                       | 8.3/13.5 (20)               | 30.4 (20)     | 6 (15)                        |
| Iso. FBW. (%/dB)*                               | 11 (20)                       | 7 (20)                         | 54.2/7.7 (20)               | 44.5 (20)     | 8 (20)                        |
| Phase Diff. Imbalance FBW. (%/°)*               | 18 (±2)                       | N/A                            | 7.8/2.9 (±5)                | 17.4 (±2.5)   | 4 (±2)                        |
| PD. Deviation FBW. (%/dB)*                      | 42 (±1)                       | N/A                            | 14.1/7.4 (±1)               | N/A           | 16 (±1)                       |
| Effective Size ( $\lambda_g \times \lambda_g$ ) | 0.31 × 0.13                   | 0.03 × 0.04                    | 0.42 × 0.16                 | 0.04 × 0.02** | 0.06 × 0.06                   |

PD.: Power Division. Tech.: Technology. FBW.: Fractional Bandwidth. Iso.: Isolation.

\*: only when port 1 is excited. \*\*: when  $\epsilon_r = 12.9$ . All values are measured results.

unequal power divisions at the two output ports are measured in *Case C* (3-dB), 5.5 & 1.9 dB, 5.6 & 2.4 dB, and *Case E* (3-dB), 4.5 & 2.2 dB ( $|S_{41}|$ ,  $|S_{21}|$ ), 4.5 & 2.2 dB ( $|S_{23}|$ ,  $|S_{43}|$ ). The equal power division is realized in *Case B* (3.7 & 3.6 dB, 3.5 & 3.3 dB) and *Case D* (3.4 & 2.9 dB, 3.0 & 3.4 dB). The deviation of the power-dividing ratio in these five cases is negligible, compared with the EM simulation results.

Overall, all the measured results agree well with the EM simulated results. The performance comparison for the couplers are shown in Table IV.

## V. CONCLUSION

This brief proposed a compact coupler using eight LC elements which can implement arbitrary power-dividing ratios, arbitrary phase differences, and arbitrary terminated impedance matching. The close-formed design equations are derived and design procedure is listed including rat-race case. For verification, five instances are designed, simulated and

fabricated. Good agreement between EM simulation and measurement validates the multifunctional coupler is of great advantageous in miniaturization and configuration simplicity.

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