

ASSIGNMENT: SAMPLING, DISTRIBUTIONS AND INTERVALS

Read and comprehend the topics on chapters 6, 7 and 8 of the book

Applied Statistics and Probability for Engineers

CHAPTER 6 Page 202 : 6-18, 19, 20, 26

6-18 Find the median and the quartiles for the motor fuel octane data

In Exercise 6-14

| NO. OF DATA | MEDIAN / SECOND QUANTILE | LOWER QUANTILE | UPPER QUANTILE |
|-------------|--|-------------------------------------|----------------------------------|
| $n = 82$ | $Q_2 = (82+1)/2 = 41.5\text{th value}$ | $Q_1 = 83/4 = 20.75\text{th value}$ | $Q_3 = 3(83/4) = 62.25\text{th}$ |
| | $Q_2 = (90.4 + 90.4) / 2 = 90.4$ | $Q_1 = 88.58$ | $Q_3 = 92.20$ |
| | 41st = 90.4 ; 42nd = 90.4 | 20th = 88.5 ; 21st = 88.6 | 62nd = 92.2 ; 63rd = 92.2 |

6-19 Find the median and the quartiles for the motor failure data in

Exercise 6-15

| NO. OF DATA : | MEDIAN / SECOND QUANTILE : | LOWER QUANTILE : | UPPER QUANTILE : |
|---------------|----------------------------------|---|---|
| $n = 70$ | $Q_2 = (70+1)/2 = 35.5\text{th}$ | $Q_1 = 71/4 = 17.75\text{th}$ | $Q_3 = 3(71/4) = 53.25\text{th}$ |
| | $Q_2 = (1421 + 1452) / 2$ | $17\text{th} = 1085 ; 18\text{th} = 1102$ | $53\text{rd} = 1730 ; 54\text{th} = 1750$ |
| | $Q_2 = 1436.50$ | $Q_1 = 1097.5$ | $Q_3 = 1735.00$ |
| | 35th = 1421 36th = 1452 | | |

6-20 Find the median, mode, and sample average of the data in Exercise 6-16.

Explain how these measures of location describe different features in the data.

| NO. OF DATA : | MEDIAN : | MODE : | SAMPLE AVERAGE |
|---------------|----------------------------------|--------|--|
| $n = 64$ | $Q_2 = (64+1)/2 = 32.5\text{th}$ | 34.7 | $\bar{x} = \frac{\sum x_i}{n} = \frac{2227.1}{64}$ |
| | $Q_2 = (34.7 + 34.7) / 2$ | | $\bar{x} = 34.79844$ |
| | $Q_2 = 34.7$ | | $\bar{x} = 34.$ |

6-26 A semiconductor manufacturer produces devices used as central processing units in personal computers. The speed of the device (in megahertz) is important because it determines the price that the manufacturer can charge for the devices. The following table contains measurements on 120 devices. Construct a stem-and-leaf diagram for this data and comment on any important features that you notice. Compute the sample mean, sample standard deviation, and the sample median. What percentage of the devices has a speed exceeding 700 megahertz?

NO. OF DATA STEM-AND LEAF DIAGRAM

$n = 120$

STEM

LEAF

| | |
|----|---------------------------------------|
| 63 | 21 7 |
| 64 | 2 4 8 9 9 |
| 65 | 2 2 3 5 6 6 8 9 9 |
| 66 | 6 0 0 0 0 0 1 2 3 3 4 5 5 7 8 8 8 9 9 |
| 67 | 0 0 2 2 4 5 5 5 6 7 8 9 9 |
| 68 | 0 0 0 0 1 1 1 1 2 3 3 3 3 3 4 5 8 |
| 69 | 0 0 0 0 1 1 2 3 4 5 5 5 5 6 7 7 8 8 9 |
| 70 | 0 1 1 2 2 3 4 4 4 5 5 6 |
| 71 | 0 0 5 7 8 8 9 |
| 72 | 0 0 0 0 1 2 2 3 4 4 4 7 |
| 73 | 5 9 |
| 74 | 6 8 |
| 75 | 3 |

SAMPLE MEAN

$$\bar{x} = \frac{\sum x_i}{n} = \frac{82413}{120} = 686.775 \text{ mhz}$$

660 is the most frequent value.

$$\bar{x} = 686.78 \text{ mhz}$$

SAMPLE MEDIAN

$$\tilde{x} = (120 + 1) / 2 = 60.5 \text{th}$$

$$\tilde{x} = (683 + 683) / 2 = 683$$

$$\tilde{x} = 683 \text{ mhz}$$

SAMPLE STANDARD DEVIATION

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{78462.93}{119}$$

$$s^2 = 658.85 \text{ mhz}^2$$

$$s = 25.67 \text{ mhz}$$

PERCENTAGE OF DEVICES WITH SPEED > 700 mhz

NO. OF DEVICES : 35

$$\% \text{ OF DEVICES : } 35 / 120 \cdot 100\% = 29.166\% = 29\%$$

29% OF DEVICES HAS A SPEED EXCEEDING 700 mhz

For CHAPTER 6 Page 208 : 6-42

6-42 Exercise 6-13 presented the joint temperatures of the O-rings ($^{\circ}\text{F}$) for each test firing of actual launch of the space shuttle rocket motor. In that exercise you were asked to find the sample mean and sample standard deviation of temperature.

SAMPLE MEAN $\bar{x} = 63.08^{\circ}\text{F}$ SAMPLE STANDARD DEVIATION $s = 14.05^{\circ}\text{F}$

(a) Find the upper and lower quantiles of temperature $n = 36$

UPPER QUANTILE

LOWER QUANTILE

$$Q_3 = (37/4) = 27.75\text{th}$$

$$Q_1 = (36+1)/4 = 9.25\text{th}$$

$$Q_3 = 74.50^{\circ}\text{F}$$

$$Q_1 = 48.25^{\circ}\text{F}$$

(b) Find the median

MEDIAN

$$Q_2 = 37/2 = 18.5\text{th}$$

$$Q_2 = 64.50^{\circ}\text{F}$$

(c) set aside the smallest observation and recompute the quantities in parts (a) and (b). Comment on your findings. How "different" are the other temperatures from this smallest value?

LOWER QUANTILE

UPPER QUANTILE

MEDIAN

$$Q_1 = (35+1)/4 = 9\text{th}$$

$$Q_3 = (36/4) = 27\text{th}$$

$$Q_2 = 36/2 = 18\text{th}$$

$$Q_1 = 48^{\circ}\text{F}$$

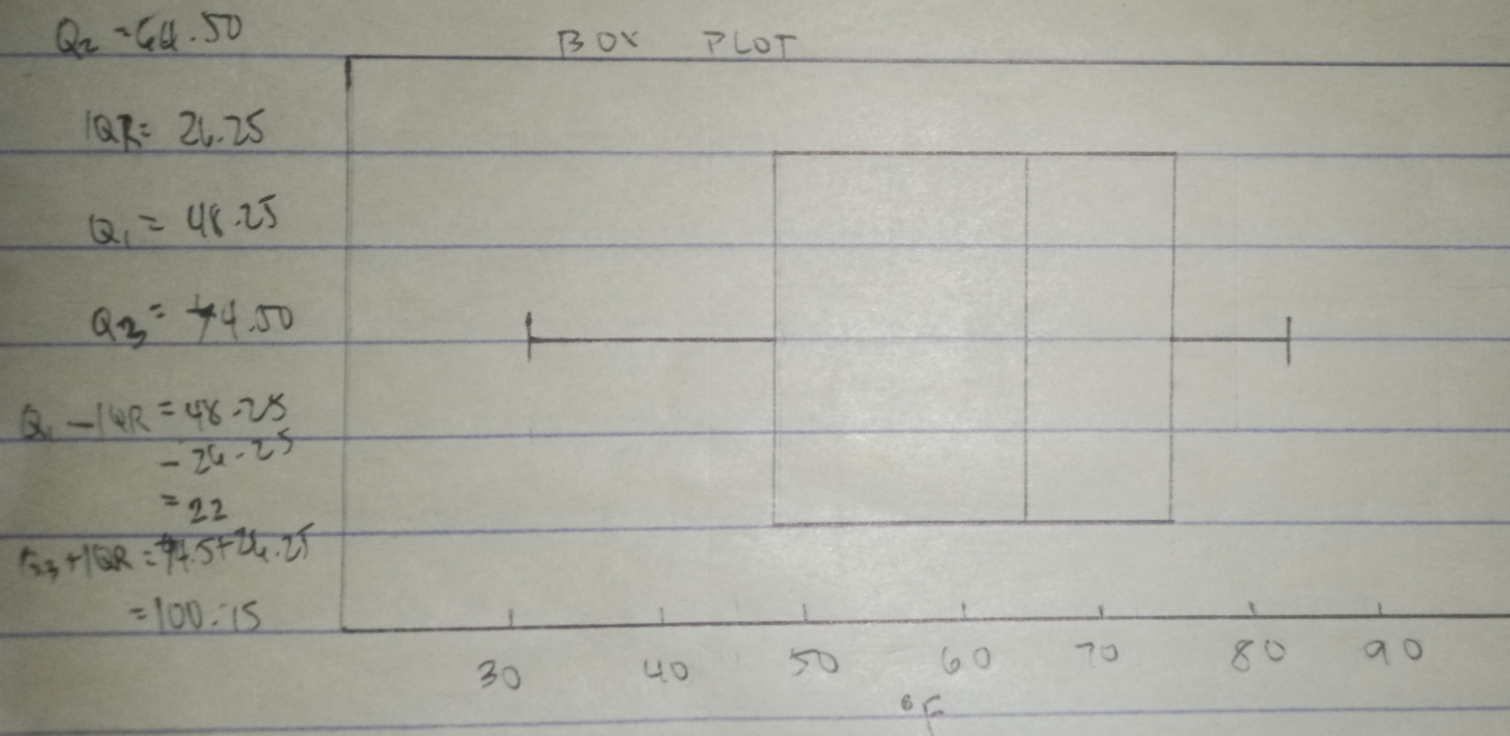
$$Q_3 = 73^{\circ}\text{F}$$

$$Q_2 = 63^{\circ}\text{F}$$

The new median and quantiles have a tiny difference from the original ones.

The smallest value is 9°F lower than the next value. This difference is higher than the differences in the other observations.

Q-42 (d). Construct a box plot of the data and comment on the possible presence of outliers.



No outliers are present since both minimum and maximum values are within $1.5IQR$ from the upper and lower quartiles.

P-4 A confidence interval estimate is desired for the gain in a circuit on a semiconductor device. Assume that gain is normally distributed with standard deviation of $\sigma = 20$

(a) Find a 95% CI for μ when $n=10$ and $\bar{x}=1000$

$$1000 - z_{0.025}(20)/\sqrt{10} \leq \mu \leq 1000 + z_{0.025}(20)/\sqrt{10} \quad z_{0.025} = 1.96$$

$$1000 - 1.96(20)/\sqrt{10} \leq \mu \leq 1000 + 1.96(20)/\sqrt{10}$$

$$\boxed{987.60 \leq \mu \leq 1012.40}$$

(b) Find a 95% CI for μ when $n=25$ and $\bar{x}=1000$

$$1000 - 1.96(20)/\sqrt{25} \leq \mu \leq 1000 + 1.96(20)/\sqrt{25}$$

$$1000 - 7.84 \leq \mu \leq 1000 + 7.84$$

$$\boxed{992.16 \leq \mu \leq 1007.84}$$

(c) Find a 99% CI for μ when $n=10$ and $\bar{x}=1000$

$$1000 - z_{0.005}(20)/\sqrt{10} \leq \mu \leq 1000 + z_{0.005}(20)/\sqrt{10}$$

$$1000 - 2.576(20)/\sqrt{10} \leq \mu \leq 1000 + 2.576(20)/\sqrt{10}$$

$$1000 - 16.29 \leq \mu \leq 1000 + 16.29$$

$$\boxed{983.71 \leq \mu \leq 1016.29}$$

(d) Find a 99% CI for μ when $n=25$ and $\bar{x}=1000$

$$1000 - 2.576(20)/\sqrt{25} \leq \mu \leq 1000 + 2.576(20)/\sqrt{25}$$

$$1000 - 10.304 \leq \mu \leq 1000 + 10.304$$

$$\boxed{989.694 \leq \mu \leq 1010.304}$$

8-9 The yield of a chemical process is being studied. From previous experience yield is known to be normally distributed and $\sigma = 3$. the past five days of plant operation have resulted in the following percent yields: 91.6, 88.75, 90.8, 89.95, and 91.3. Find a 95% two sided confidence interval on the true mean yield

GIVEN $n = 5$ $\bar{X} = \frac{90.48}{72.22} \%$ $\alpha = 0.05$ $\sigma = 3$

REQUIRED: CONFIDENCE INTERVAL (TWO-SIDED)

$$\bar{X} - Z_{\alpha/2} \sigma / \sqrt{n} \leq \mu \leq \bar{X} + Z_{\alpha/2} \sigma / \sqrt{n} \quad Z_{0.025} = 2.774$$

SOLUTION $\frac{90.48}{72.22} - 2.774(3) / \sqrt{5} \leq \mu \leq \frac{90.48}{72.22} + 2.774(3) / \sqrt{5}$

$$90.48 - 3.72 \leq \mu \leq 90.48 + 3.72$$

$$\boxed{86.76 \leq \mu \leq 94.20}$$