Problem Chosen	2025 MCM/ICM Summary Sheet	Team Control Number
B		**2505364**

Sustainable Tourism: A Delicate Balance

Summary

Tourism can drive economic growth but also strain fragile ecosystems, as seen in Juneau, where dramatic glacier retreat and overcrowding highlight urgent management needs. This city of about 30,000 residents hosts up to 1.6 million cruise-ship visitors per season. To advance sustainable tourism in Juneau and beyond, we develop a three-part model addressing both short-term planning and long-term dynamics.

In **Model I**, we construct a multi-objective framework integrating the interests of **residents, tourists, the environment, and the government**. Objective functions quantify well-being, satisfaction, and ecological impact. For residents, the model **balances tourism income with the negative effects of overcrowding**. Tourist satisfaction employs **Monod functions** to link **ecosystem value (E)**, **capital investment (C)**, and **tourist congestion**, treating tourism as a social-ecological system (SES). Ecological value tracks **natural growth and degradation**, while government objectives **balance fiscal, environmental, and economic factors**.

In **Model II**, we develop an **Efficient Pareto Improvement Model (EPI)** that uses the **Non-dominated Sorting Genetic Algorithm II (NSGA-II)** to reconcile stakeholder goals under infrastructural, sustainability, and budget constraints. The framework applies **distance reduction** to a Pareto frontier and emphasizes cost-effectiveness by balancing **marginal utility (MU)** and **marginal cost (MC)**. Applied to Juneau, it finds that environment-friendly policies, such as cruise port electrification, yield the highest initial **MU/MC ratio** (about 0.26), followed by tourist-friendly measures (about 0.18). Locals-friendly policies (about 0.075) gain effectiveness later due to slower cost growth. Overall, the model suggests early investments in environment- and tourist-focused measures, shifting to locals-friendly initiatives over time to ensure equity and sustainability.

Model III employs a **difference equations system** to simulate population, tourist flows, tourism investment, and ecological value. Without intervention, environmental degradation reduces tourist arrivals and revenue. To reverse these trends, we propose strict environmental regulations early on to prevent irreversible damage, followed by staged infrastructure expansion and local economic support. Every five years, adaptive taxation and visitor flow management balance ecological preservation and fiscal needs. Under these measures, we project a 50% increase in tourist numbers, a slower rate of ecological decline, and a 60% rise in resident well-being, though initial government deficits may grow. Regular evaluations and policy adjustments will be essential to maintain long-term sustainability.

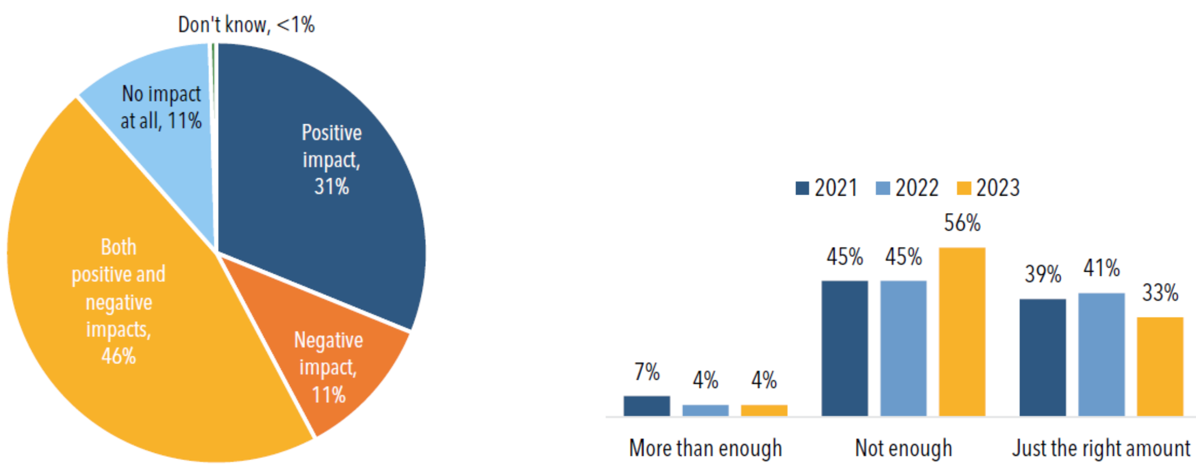
Keywords: NSGA-II; Multi-objective Model, Equi-marginal Principle, Difference Equation System

Contents

1 Introduction

1.1 Background

Tourism can invigorate local economies while simultaneously threatening the very environments that make destinations appealingnowhere is this tension more evident than in Juneau, Alaska, where dramatic glacier retreat and concerns about overcrowding underscore the need for better tourism management. In recent years, this city of about 30,000 residents has hosted up to 1.6 million cruise-ship visitors per season, sometimes seeing seven vessels docked on a single busy day. The revenue generated is considerable, yet it arrives hand in hand with infrastructure strain, ecological degradation, and quality-of-life concerns for residents. Currently, Such a predicament highlights a universal challenge facing many popular destinations worldwide: how to balance economic gain with environmental preservation and social welfare.



(a) Perceived Influence of Tourism in Jeunea

(b) Perceived Government Management of Tourism in Jeunea

Figure 1: Comparison of Perceived Influence and Government Management of Tourism in Jeunea

1.2 Problem Restatement

The goal is to develop a sustainable tourism model for Juneau, adaptable to other over-visited destinations. This model must balance economic returns, visitor capacity, local well-being, and ecological sustainability.

- **Problem 1: Sustainable Tourism Modeling**: Develop a mathematical model that defines the boundaries and objectives of sustainable tourism, ensuring it captures key economic, social, and environmental factors.
- **Problem 2: Policy Evaluation and Optimization**: Evaluate management strategies (e.g., visitor caps, hotel taxes, fees) to support sustainable tourism and effectively allocate surplus revenue.
- **Problem 3: Generalization and Stability Testing**: Extend the model to regions beyond Juneau, conducting sensitivity analyses and comparative assessments to test key factors and

ensure model robustness.

1.3 Our work

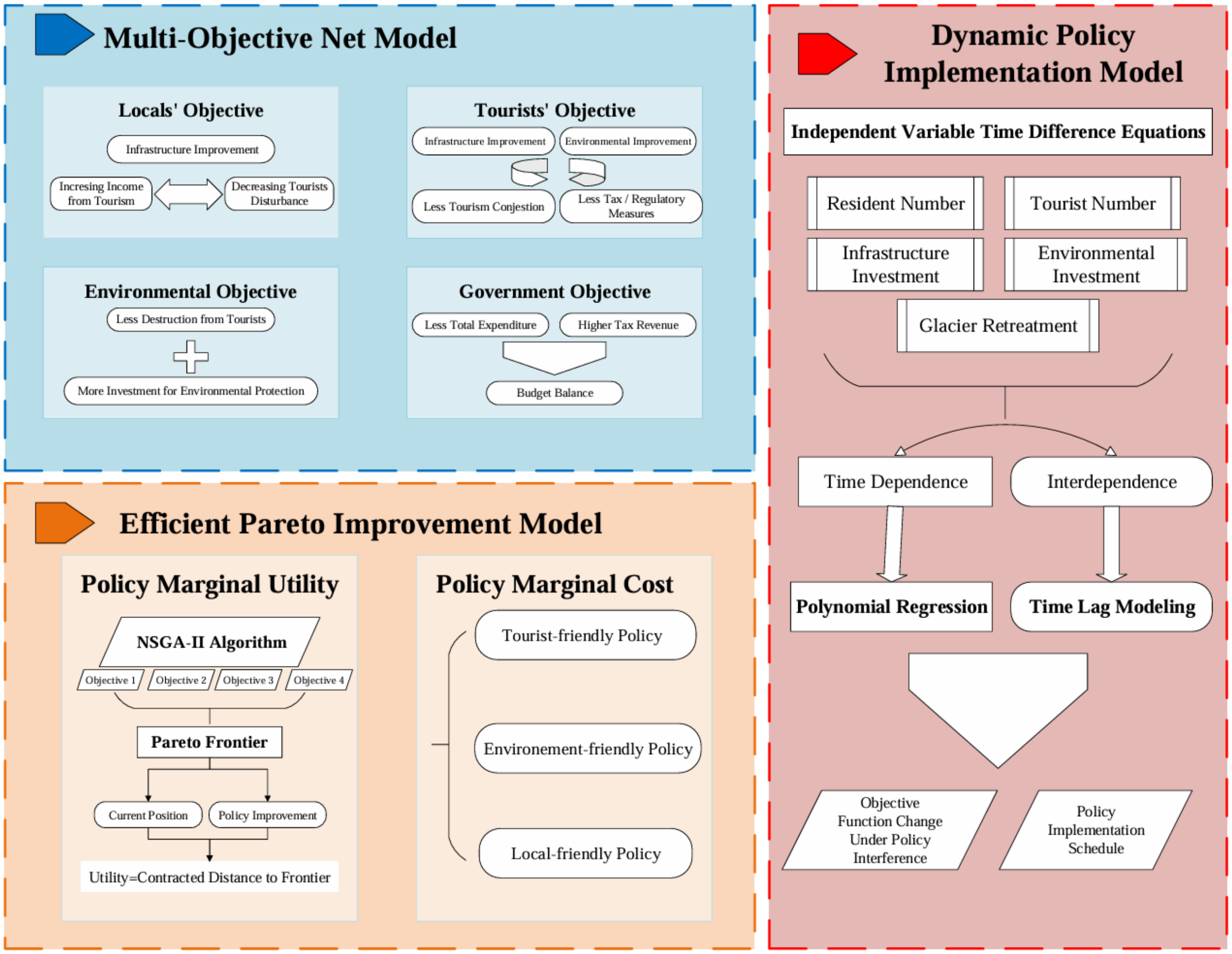


Figure 2: Ourwork

2 Assumptions

Assumption 1: Each interest group only considers it's own benefits

Justification 1: We suppose on an average scale, no matter it's tourists, local residents, environmentalists, or the government, they all prioritize their interest rationally. Their interests are entangled: i.e environment is the most important concern for environmentalists, yet it's also in the consideration of tourists for better scenary

Assumption 2: The marginal utility of each stakeholder group is diminishing

Justification 2: According to economic theory, the marginal benefit of an increase in any positively related variable for tourists, the government, local residents, and the environment exhibits diminishing returns. For instance, tourists may experience a significant benefit from an initial reduction in hotel taxes. However, as the tax cut deepens, they become less responsive to the corresponding decrease in tourism costs.

Assumption 3: Interdependence of variables only exhibits in objective functions

Justification 3: Sustainable tourism is indeed a dynamic problem, which will makes variables in our objetive functions:i.e tourists,and residents number interrelated. However, in our first model, we consider the static case, in which all variables are considered as fixed. The interdependence of variables is considered into our objective function as one interest group's objective include several variables

3 Models

3.1 Model I: Multi-Objective Evaluation Model (MEO)

3.1.1 Problem Analysis

For a tourist city like Juneau, the preservation of local natural resources, such as glaciers, often conflicts with economic development. In this model, we conceptualize the key stakeholders as four distinct groups: the government, tourists, local residents, and the environment. Each of these groups has unique objective functions, formulated using variables that are closely tied to their respective interests.

3.1.2 Model Overview and Notation

Through a thorough literature review, we identify key variables that influence the interests of each stakeholder group. The objective functions are then constructed based on a combination of these variables. Given that the interactions among stakeholders resemble a classic partial game theory model, we define a separate objective function for each group (i.e., local residents, tourists, the environment, and the government), with each group seeking to maximize its respective function.

In reality, all sustainability-related variables are time-dependent, evolving over time and exhibiting intricate interdependencies. However, for the sake of tractability, we adopt a static framework in this model, assuming all variables to be fixed. This simplification allows us to determine the optimal policy mix in the subsequent section. Temporal and interdependent effects will be incorporated into Model III, where dynamic simulations and policy implementations will be addressed using difference equations.

3.1.3 Goal 1: Local Residents

Based on the labor supply model in economics, we assume that the impact of tourism on local residents is primarily reflected in the following two aspects:

- Substitution Effect: Tourism is an important source of income for local residents. An increase in the number of tourists brings higher income, making the tourism sector more economically attractive.
- Income Effect: However, excessive tourism activities can disrupt daily life and strain local infrastructure, causing congestion and overload. As income grows, residents place greater emphasis on quality of life and become less tolerant of the negative externalities associated with tourism.

These two effects offset each other. In the early stages, the substitution effect dominates, but as tourism develops, the income effect gradually becomes more significant. Therefore, we model this relationship as an inverted parabola, where the marginal utility of tourism for local residents first increases and then decreases:

$$O_{\text{residents}} = -\gamma_1 \cdot N^2 + \gamma_2 \cdot N$$

where N denotes the number of tourists, and γ_1 and γ_2 are constants.

Government investment in infrastructure (G_1)for example, improving transportation, sanitation, and public spacesdirectly enhances residents quality of life. Thus, we introduce $\ln(G_1 + 1)$ to capture the diminishing marginal effect of infrastructure investment on residents utility:

$$O_{\text{residents}} = -\gamma_1 \cdot N^2 + \gamma_2 \cdot N + \gamma_3 \cdot \ln(G_1 + 1)$$

where γ_3 is a constant representing the sensitivity of residents well-being to government spending.

Drawing on the predator-prey model, we assume that ill-mannered (or uncivil) tourists impose additional pressure on residents, which grows with the product of the number of tourists and residents. The term $-\gamma_4 \cdot N \cdot R$ describes this negative externality:

$$O_{\text{residents}} = -\gamma_1 \cdot N^2 + \gamma_2 \cdot N + \gamma_3 \cdot \ln(G_1 + 1) - \gamma_4 \cdot N \cdot R$$

where γ_4 is a constant.

Finally, we divide the objective function by the number of local residents R to obtain the per capita objective function for residents:

$$\boxed{\text{obj(a resident)} = \frac{-\gamma_1 \cdot N^2 + \gamma_2 \cdot N + \gamma_3 \cdot \ln(G_1 + 1) - \gamma_4 \cdot N}{R}}$$

3.1.4 Goal 2: Tourists

Assume that the tourist satisfaction function $A(N, E, C)$ (where C is the capital for tourist accommodation and entertainment, and E is the ecological value) includes three factors: the attractiveness associated with environmental quality $a_E(N, E, C)$, the attractiveness associated with accommodation and entertainment facilities $a_C(N, E, C)$, and the attractiveness related to crowding $a_N(N, E, C)$. Therefore:

$$A(N, E, C) = a_E(N, E, C) + a_C(N, E, C) + a_N(N, E, C)$$

Although Lacitignola and Casagrandi [9, 16] categorize tourists into eco-tourists (who indirectly enhance the ecosystem) and mass tourists (the main drivers of ecological degradation via their interaction with capital C), the proportion of eco-tourists in Juneau is negligible [17]. Hence, we exclude their influence. Mass tourists still seek high ecological value but gradually deplete resources without reacting to quality thresholds. Consequently, the environmental attractiveness a_E is represented by a Monod function of E:

$$a_E(E) = \frac{\omega_1 E}{E + \phi_E}$$

where ω_1 and ϕ_E are constants.

Mass tourists actively seek accommodation and entertainment facilities. We assume that the attractiveness a_C is given by an increasing but saturating Monod function of per-capita facility availability. This assumption is reasonable because capital C is made up of entertainment investments (e.g., hotels, swimming pools, discos), which need to be sufficiently abundant to attract mass tourists, but cannot exceed a certain threshold. Therefore:

$$a_C(N, C) = \frac{\omega_2 \frac{C}{N+1}}{\frac{C}{N+1} + \phi_C}$$

The attractiveness a_T is related to crowding:

$$a_T(N) = -\omega_3 N - \omega_4 N^2$$

indicating that the crowding-related attractiveness is negatively correlated not only with the presence of tourists but also with the number of interactions among them.

Combining the above factors, the total attractiveness of the destination to tourists is:

$$A(N, E, C) = \frac{\omega_1 E}{E + \phi_E} + \frac{\omega_2 \frac{C}{N+1}}{\frac{C}{N+1} + \phi_C} - \omega_3 N - \omega_4 N^2$$

Next, we incorporate the negative effect of tourism taxes and behavioral restrictions (T_0) on tourist experience, and the positive effect of infrastructure investments (G_1) on tourist facilities and services:

$$A(N,E,C) = \frac{\omega_1 E}{E+\phi_E} + \frac{\omega_2 \frac{C}{N+1}}{\frac{C}{N+1}+\phi_C} - \omega_3 N - \omega_4 N^2$$
$$- \omega_5 \cdot N \cdot T_0 + \omega_6 \cdot \ln(G_1+1),$$

where ω_5 and ω_6 are constants. Since individual tourist experience is our focus, we divide by N, yielding:

$$\text{obj(a tourist)} = \omega_1 \frac{E}{N(E+\phi_E)} + \omega_2 \frac{C}{CN+\phi_C N(N+1)} - \omega_3 - \omega_4 N - \omega_5 T_0 + \omega_6 \frac{\ln(G_1+1)}{N}.$$

3.1.5 Goal 3: Ecological Value

We define the ecological value objective as:

$$\text{obj(environment)} = E = \delta_1 E_0 + \delta_2 G_2 - \delta_3 N^2 - \delta_4 N^2,$$

where E_0 is the ecological value without tourism and capital interaction. Here, $\delta_2 G_2$ reflects the governments capacity to improve the environment, $-\delta_3 N^2$ captures damage from tourism activities, and $-\delta_4 N^2$ represents the potential positive impact of tourist activities. Unlike Casagrandi and Rinaldi [9], we emphasize that tourism can also yield ecological benefits (e.g., soft tourism [10, 11, 12]), although in Juneau the negative impacts of mass tourism dominate, given the small proportion of eco-tourists.

Hence, we simplify the ecological value objective function to:

$$\boxed{\text{obj(environment)} = E = \delta_1 \cdot E_0 + \delta_2 \cdot G_2 - \delta_3 \cdot N^2}$$

3.1.6 Goal 4: Government

The governments objective function combines fiscal balance ($T_0 \cdot N - G_1 - G_2$) and residents well-being ($O_{\text{residents}}$). This design logic reflects the trade-off the government makes between promoting economic development and ensuring a satisfactory quality of life for residents. At the same time, the government must balance tourism income with expenditures for improving infrastructure and protecting the environment:

$$\boxed{\text{obj(government)} = \beta_1 \cdot (T_0 \cdot N - G_1 - G_2) + \beta_2 \cdot O_{\text{residents}}}$$

where β_1 and β_2 are weighting coefficients that adjust the priority given to economic benefits vs. residents well-being in government decision-making. This flexible setting allows the model to be adapted to different regional policy goals.

3.1.7 Summary

We set the objective functions for the four main stakeholders as follows:

$$\text{obj(a resident)} = \frac{-\gamma_1 N^2 + \gamma_2 N + \gamma_3 \ln(G_1 + 1) - \gamma_4 N}{R} \tag{1a}$$

$$\text{obj(a tourist)} = \omega_1 \cdot \frac{E}{N(E + \phi_E)} + \omega_2 \cdot \frac{C}{C \cdot N + \phi_C N(N+1)} - \omega_3 - \omega_4 N - \omega_5 T_0 + \omega_6 \cdot \frac{\ln(G_1 + 1)}{N} \tag{1b}$$

$$\text{obj(environment)} = \delta_1 \cdot E_0 + \delta_2 \cdot G_2 - \delta_3 \cdot N^2 \tag{1c}$$

$$\text{obj(government)} = \beta_1 \cdot (T_0 \cdot N - G_1 - G_2) + \beta_2 \cdot O_{\text{residents}} \tag{1d}$$

In another simplified formulation, we write:

$$\text{obj(a resident)} = \frac{-\gamma_1 \cdot N^2 + \gamma_2 \cdot N + \gamma_3 \cdot \ln(G_1 + 1) - \gamma_4 \cdot N}{R} \tag{2a}$$

$$\text{obj(a tourist)} = \omega_1 \cdot \frac{\text{obj(environment)}}{N} - \omega_2 \cdot N - \omega_3 \cdot T_0 + \omega_4 \cdot \frac{\ln(G_1 + 1)}{N} \tag{2b}$$

$$\text{obj(environment)} = \delta_1 \cdot E + \delta_2 \cdot G_2 - \delta_3 \cdot N^2 \tag{2c}$$

$$\text{obj(government)} = \beta_1 \cdot (T_0 \cdot N - G_1 - G_2) + \beta_2 \cdot O_{\text{residents}} \tag{2d}$$

3.2 Modeling II: Efficient Pareto Improvement Model (EPI)

3.2.1 Problem Analysis

After defining the various objective functions, we explore how best to balance different stakeholders' interests and enhance overall welfare. We focus on government policies to support sustainable tourism, recognizing that no single policy benefits everyone: for instance, environmental protection may reduce local income, while tourism-friendly measures may threaten glaciers and whales. Literature reviews indicate that simply assigning subjective weights to combine objectives is suboptimal, especially amid ongoing human-versus-nature debates. Even among locals, preferences vary: lower-income individuals may accept more tourists (tolerating reduced environmental quality) for higher earnings, whereas wealthier citizens prefer fewer tourists.

3.2.2 Model Overview

We adopt the Pareto Improvement principle, aiming to improve at least one group's outcomes without harming another. Accordingly, we propose a range of policy interventions and identify those that yield the greatest overall Pareto Improvement. Given Juneau's small population and limited budget, we also emphasize cost-effectiveness in the chosen policy mix.

3.2.3 Supplemental Assumptions

- **Assumption 1: Five-year policy horizon.** Tourism-related policies (e.g., sales and hotel taxes) in Juneau typically last 3–7 years. We use five years as a representative planning interval.

- **Assumption 2: Scalable policy effects.** Although annual impacts vary, we assume an average effect over five years, enhancing reliability in identifying optimal policy sets.

- **Assumption 3: Convex first derivatives of cost functions.** As marginal cost typically rises over time (e.g., more expensive measures are needed to sustain benefits when tourism or environmental demands grow), cost functions follow standard economic principles of increasing marginal costs.

- **Assumption 4: Discrete policy implementation.** We update policy impacts annually, aligning with Model IIIs difference equations and avoiding the complexity of mid-year policy changes, which are impractical for local governance in Juneau.

3.2.4 NSGA-II Algorithm

To quantify Pareto Improvement, we first apply the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to obtain the Pareto Frontier under static conditions. We then measure Pareto Improvement by how much each policy set reduces the distance to that Frontier.

Difinition of the Optimization Problem The Multi-Objective optimization problem can be defined as the equations below:

$$\text{Maximize} \quad \begin{cases} \text{obj}_1(\boldsymbol{x}) \\ \text{obj}_2(\boldsymbol{x}) \\ \vdots \\ \text{obj}_n(\boldsymbol{x}) \end{cases}$$

$$\boldsymbol{x} = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n.$$

Several constraints are listed below

- Infrastructure Constraint: The pressure on infrastructure cannot exceed the maximum sustainable limit, or the basic livelihood of the locals and the experience of tourists will plummet. We suppose the pressure on infrastructure is positively related with tourist amount and negatively related with the government investment on infrastructure

$$\frac{aN}{bG_1} \geq L_1 \tag{3}$$

- Tourist Constraint: According to histrical tourism data, the tourists amount in Jeunue is always larger than zero and will keep be positive

$$N \geq 0 \tag{4}$$

- Environment Sustainable Constraint: the blight on environment must be larger than an amount that will not let the ecosytem be instantly unsustainble and deteriorite fastly.

$$E \geq L_2 \tag{5}$$

- Government Investment Constraint: Government's investment on local infrastructure and environment protection must be non-negative. Notely, in our model, the government is allowed to hold debt and create budget deficit, as long as the deficit will not accumulate continuously in the long-term

$$G_1, G_2 \geq 0 \tag{6}$$

- Government Budget Constraint: Considering Juenea government is seeking for balancing government budget these years, the sum of investment for environment and infrastructure must not exceed the 5% larger than the budget assigned for the task

Initialization of NSGA-II Algorithm In NSGA-II, each possible combination of control variables in multiple objective functions is expressed as a vector, which is the individual that represents a set of solutions. For the initialization, we set each variable randomly within our predefined constraint.

$$\boldsymbol{x} \;=\; [C,\ E,\ G,\ G_1,\ G_2,\ N,\ NP,\ T,\ \text{tax}].$$

According to our database, we set these variables, here G1 is estimated as the General expense plus the infrastructure expense.

$$R = 31,555, \quad \text{Tourist} = 1,600,000, \quad G_1 = 53,062,900, \quad G_2 = 5,720,000, \quad T = 0.09$$

Undominated Sorting and Crowding NSGA-II ranks and selects individuals based on non-dominated sorting and crowding distance. Each non-dominated frontier contains individuals that do not dominate each other. Within the same frontier, individuals with lower crowding distance (i.e., fewer neighbors) are preferred to maintain diversity.

Selection, Crossover, and Mutation Selection uses a tournament mechanism to choose parents for crossover and mutation, generating new offspring.

New Population Generation Parents and offspring are combined, then processed again through non-dominated sorting and crowding distance calculation to form the next generation.

Termination Condition These steps repeat until a preset iteration limit is reached or solutions no longer improve significantly.

3.2.5 Interference Policies and Cost

Policies are divided into locals-friendly, tourist-friendly, and environment-friendly, aligning with each stakeholder group. To identify the optimal mix under Juneaus limited budget, we apply the equi-marginal principle:

$$\frac{\mathrm{MU}_{P_1}}{\mathrm{MC}_{P_1}} = \frac{\mathrm{MU}_{P_2}}{\mathrm{MC}_{P_2}} = \cdots = \frac{\mathrm{MU}_{P_n}}{\mathrm{MC}_{P_n}},$$

where MU is the marginal utility (the first derivative of total utility under policy changes) and MC is the marginal cost (the first derivative of the cost function). We measure MU by the reduction in distance to the Pareto Frontier per unit of policy implementation.

Locals-Friendly Policy

1. **Seasonal Employment Incentive Program**
   To raise ε_1 (government focus on residents welfare), offer subsidies or financial incentives for locals to undertake seasonal tourism jobs (e.g., guides, hotel staff, transportation). This boosts residents share of tourism revenue, thereby enhancing their well-being.

The goal is to improve residents welfare via infrastructure and revenue sharing. The cost function is:

$$\text{Cost}_{\text{Locals}} = a_1 \cdot M^2 + a_2 \cdot \varepsilon_1,$$

where M is infrastructure spending, ε_1 is the emphasis on residents welfare, and a_1, a_2 balance these factors. This policy lowers γ_2, granting locals a larger share of tourism revenue, and increases G_1 (government infrastructure investment) by 5% annually beyond current trends.

Tourist-Friendly Policy

1. **Reduction of Administrative Burden on Tourists**
   To reduce M (the negative impact of government measures), streamline entry regulations and minimize restrictive policies (e.g., simpler attraction access, clear guidelines). This lowers the burden on tourists and improves their experience. The government can also hire more attraction staff and operate more public transit for added convenience.

This policy optimizes tourist management, simplifying administrative processes. Its cost function is:

$$\text{Cost}_{\text{Tourist-Friendly}} = b_1 \cdot N + b_2 \cdot \delta_2 \cdot \frac{1}{T_0^2},$$

where N is the number of tourists, T_0 is the total tax rate, δ_2 represents travelers sensitivity to taxes, and b_1, b_2 are coefficients. The policy lowers sales and hotel taxes by 2% over five years and increases tourist numbers by 5% annually beyond current trends.

Environment-Friendly Policy

1. **Cruise Port Electrification**
   To reduce C (negative environmental impact), accelerate shore power facilities in cruise ports. This cuts emissions from docked cruise ships and mitigates tourisms impact on air quality and the local environment.

The policy preserves natural resources and reduces tourism-related environmental damage. Its cost function is:

$$\text{Cost}_{\text{Environment}} = c_1 \cdot \text{obj}(\text{environment})^2 + c_2 \cdot N^2,$$

where obj(environment) is the environmental quality (requiring higher maintenance costs if already improved), and N is the number of tourists, increasing costs (e.g., cruise port electrification). The policy raises environmental investment growth by 5% yearly beyond existing trends, while ticket price hikes and regulatory measures reduce tourist numbers by 8% annually beyond the current trajectory.

3.2.6 Result and Policy Evaluation

By scaling each policys effect, we derive the maximum values of purely positive independent variables. The resulting Pareto Frontier over the next five years is shown below. Since it is a four-dimensional optimization problem, we visualize objective combinations in three dimensions. Three different 3D plots illustrate the fitness distribution among the three primary objectives.

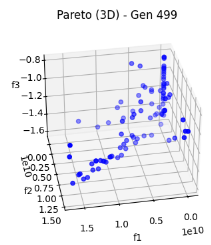


Figure 3: Pareto Front-1

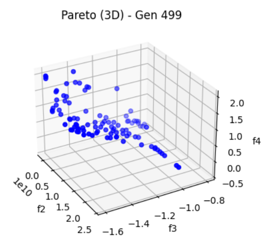


Figure 4: Pareto Front-2

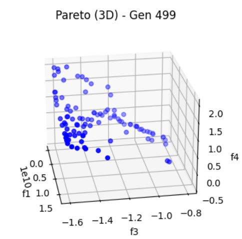


Figure 5: Pareto Front-d3

To calculate the distance to Pareto Frontier, we calculate the value of objectives of current situation before and after policy interference, then finding its distance to the closest point non-domonated point on Pareto Frontier and then calculate it's distance. The relationship of each policy implementation relative to the distance is shown as graph below

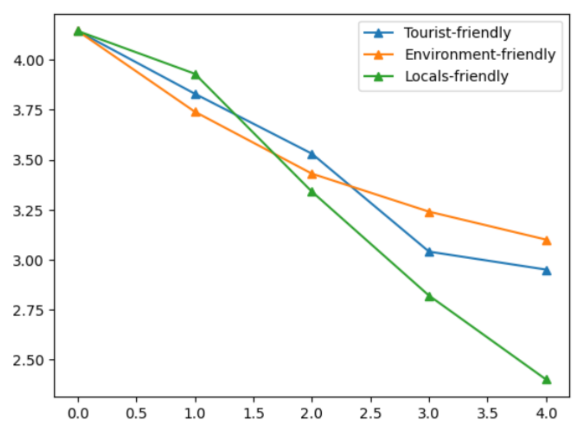


Figure 6: Contracted Distance of Each Policy

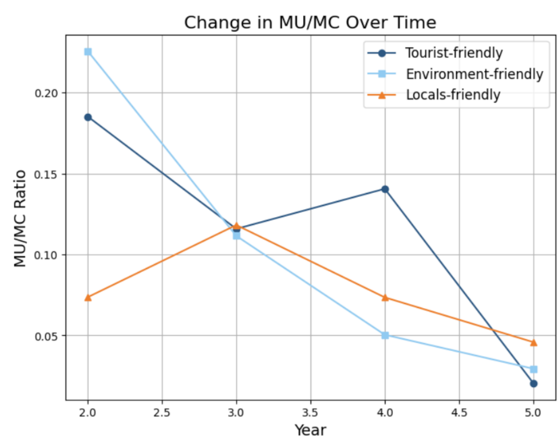


Figure 7: Cost Efficiency

By calculate the increased total cost of implementation of each policy each year, we approximate marginal cost in our discrete setting as below:

Considering both cost and Contracted distance to Pareto Fronteir, the cost-efficiency of each policy $\frac{MU_{P_i}}{MC_{P_i}}$ is shown as below

So we'll then calculate the cost-effectiveness of these policies so as to decide our final prioritization.

The Mu/MC cost curves evaluate the most efficient cost for Pareto Improvement overtime. The graph represents the **Marginal Utility per Marginal Cost (MU/MC)** ratio for three policy types**Tourist-friendly, Environment-friendly, and Locals-friendly**over time. According to the **equi-marginal principle in economics**, the most efficient allocation of resources is achieved when the **MU/MC ratios across all policies are equalized**:

$$\frac{MU_{P_1}}{MC_{P_1}} = \frac{MU_{P_2}}{MC_{P_2}} = ... = \frac{MU_{P_n}}{MC_{P_n}}$$

where MU represents the marginal utility of each policy, and MC is the marginal cost.

Key Observations from the Graph

3.2.7 Declining MU/MC Ratios Over Time

These three policies are interdependent. The environment-friendly policy has the highest cost due to large-scale infrastructure and port electrification. The tourist-friendly policy ranks second, driven by streamlined administration and tax cuts to boost tourism. The locals-friendly policy grows most slowly in cost because of its long-term investment and slower returns.

Policy Efficiency Over Time In **Year 2**, the environment-friendly policy shows the highest marginal utility-to-marginal cost (MU/MC) ratio, making it initially most efficient, followed by the tourist-friendly policy. The locals-friendly policy starts with the lowest MU/MC but declines more slowly, becoming advantageous in later years. By **Year 3**, all three converge; after that, tourist-friendly and environment-friendly policies drop sharply, while locals-friendly remains relatively stable.

Recommendations for Sustainable Development A **dynamic policy mix** is recommended to optimize Juneaus limited budget. We first prioritize the environment-friendly and tourist-friendly policies to exploit their high initial MU/MC values, then shift more resources to locals-friendly policies from Year 3 to Year 5.

3.3 Model III: Time-Difference Simulation and Policy Interference

To ensure long-term viability, we schedule policy implementation based on time-dependent growth and interdependencies of key variables. In the previous models, variables were treated as static; here, we expand to include dynamic interactions and extended policy practices from Model II. Through multivariate regression and differential-equation fitting, we predict the long-term evolution of control variables.

By taking the difference of Eqs. (1a)–(2d), we incorporate time dependence:

- For parameters T_0, G_1, and G_2, potentially shaped by policies, we express them as difference terms, aiding numerical analysis of policy effects.
- We first examine the autonomous trends of resident population and ecological value, then embed these time dependencies into the objective functions, linking them via schematic relationships.

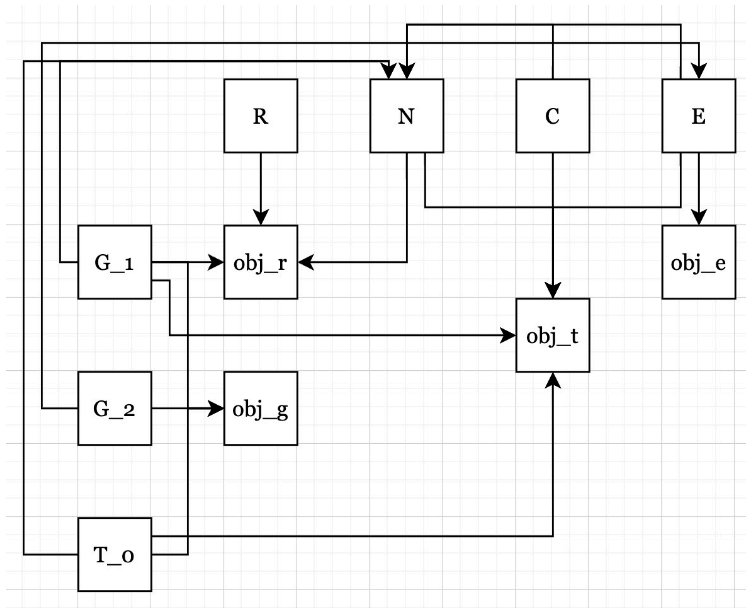


Figure 8: Schematic of Difference Equation Relationships

3.3.1 Difference Form of Resident Population Over Time

A population growth model that takes into account the difference of resource constraints can be written:

$$R_{t+1} = \frac{R_t\, r}{1 + \dfrac{R_t(r-1)}{K}}.$$

This equation is known as the Beverton-Holt model.

To simplify, define $a' = (r-1)/K$. The equation becomes:

$$R_{t+1} = \frac{R_t\, r}{1 + a'\, R_t}.$$

For a clearer understanding of density dependence, one can distinguish between the density-independent growth parameter r_I and the actual growth parameter r_A, yielding:

$$R_{t+1} = R_t\, r_A,$$

where

$$r_A = \frac{r_I}{1 + a'\, R_t} = \frac{r_I}{1 + \dfrac{R_t(r_I - 1)}{K}}.$$

Which simplifies to:

$$r_A = r_I\left(1 + \frac{R_t\, r_I - R_t}{K}\right)^{-1}.$$

The above derivation is based on two points $(0, 1/r)$ and $(K, 1)$ in a linear relationship, referred to as exactly compensating density dependence. Since this assumption may be unrealistic, Hassell

[7] proposed relaxing it by raising the formula to the power $-b^*$, yielding:

$$r_A = r_I \left[1 + \left(\frac{R_t\, r_I - R_t}{K} \right) \right]^{-b^*}.$$

When $b^* = 1$, this corresponds to exact compensation (linear density dependence). If $b^* > 1$, it indicates overcompensation (e.g., plant yield decreases faster than expected with increasing density). If $b^* < 1$, it signifies undercompensation, meaning population decline is slower than expected when density increases (compared to full compensation).

In our context, the difference equation for the resident population is:

$$\boxed{R[t] = R[t-1] \cdot r_I \left[1 + \left(\frac{R[t-1]\, r_I - R[t-1]}{K} \right) \right]^{-b^*}}.$$

3.3.2 Difference Form of Tourist Population Over Time

By appropriately modifying the previously mentioned tourist objective function, we obtain the change in the number of tourists:

$$\frac{dN}{dt} = \Big(\frac{\omega_1 E}{E + \phi_E} + \frac{\omega_2 \dfrac{C}{N+1}}{\dfrac{C}{N+1} + \phi_C} - \omega_3 N - \omega_4 N^2 - \omega_5 \cdot N \cdot T_0 + \omega_6 \cdot \ln(G_1 + 1) \Big) \cdot N.$$

Hence, the difference equation for tourist population can be written as:

$$\boxed{N[t] = N[t-1] + \Omega[t] \cdot N[t-1],}$$

where

$$\boxed{\begin{aligned} \Omega[t] = \omega_1 \cdot \frac{E[t]}{E[t] + \phi_E} + \omega_2 \cdot \frac{C[t]}{C[t] + \phi_C(N[t-1]+1)} - \omega_3 \cdot N[t-1] - \omega_4 \cdot N[t-1]^2 \\ - \omega_5 \cdot T_0\, N[t-1] + \omega_6 \cdot \ln(G_1 + 1). \end{aligned}}$$

When estimating the number of tourists, we collected data from Juneaus annual statistics. However, since these data are based on surveys conducted only once every one or two years and contain missing values, we assume an exponential growth trend for tourists and perform interpolation for fitting. To highlight uncertainties in tourism, we add a disturbance term based on residual size to the final fitted exponential function.

3.3.3 Difference Form of Tourism Capital Over Time

The dynamics of capital C reflect a balance between the decay of tourism facilities and investment inflows. According to Casagrandi and Rinaldi [9], the total investment is assumed to be a fixed proportion of total income generated by tourism, and income is assumed proportional to the number of tourists. Therefore:

$$\frac{dC}{dt} = -\epsilon_1\, C + \epsilon_2\, N,$$

where ϵ_1 is the facility decay rate and ϵ_2 is the investment rate. The difference equation for capital becomes:

$$\boxed{C[t] = C[t-1] - \epsilon_1 C[t] + \epsilon_2 \cdot \Delta N[t].}$$

3.3.4 Difference Form of Ecological Value Over Time

The dynamics of ecological value E can be seen as a balance of:

$$\frac{dE}{dt} = \xi_1(E) - \xi_2(N, E, C),$$

where $\xi_1(E)$ represents ecological value change without tourist or capital interactions, and $\xi_2(N, E, C)$ represents damage to the ecosystem caused by tourist activities.

In Casagrandi and Rinaldi [9], ecological value in the absence of tourists or capital interaction is assumed to follow logistic growth. We use the function:

$$\xi_1(E) = \frac{r_E \cdot E^2}{K_E} - r_E \cdot E.$$

This form shows an unstable equilibrium at $E = K_E$, given that real-world social-ecological systems inherently remain far from equilibrium [13]. Both theoretical and observational evidence supports this, since intensified human activity reduces the quality and quantity of ecosystem goods and services [14], inevitably lowering ecosystem quality [15].

Regarding the interaction term $\xi_2(N, E, C)$, mass tourists are responsible for degrading ecological value, while accommodation and entertainment capital C alters the natural assets. Thus:

$$\xi_2(N, E, C) = E(\varphi_1 C + \varphi_2 N).$$

Hence, the dynamics of ecological value are:

$$\frac{dE}{dt} = \frac{r_E \cdot E^2}{K_E} - r_E \cdot E - E(\varphi_1 C + \varphi_2 N),$$

leading to the difference equation:

$$\boxed{E[t] = E[t-1] + \frac{r_E \cdot E[t-1]^2}{K_E} - r_E \cdot E[t-1] - E[t-1](\varphi_1 C[t] + \varphi_2 N[t]).}$$

3.3.5 Difference Form of Government Investment Over Time

Although government investment is typically decided based on the current years budget, overall, the City and Borough of Juneau has shown a steadily increasing trend in both environmental and infrastructure investments. We obtained all data from the Juneau Adopted Budget. For infrastructure investment, we approximated it using the capital investment category, and for environmental investment, we summed the relevant items from general spendingcity. Since the data are sufficiently robust, we used multiple regression for fitting.

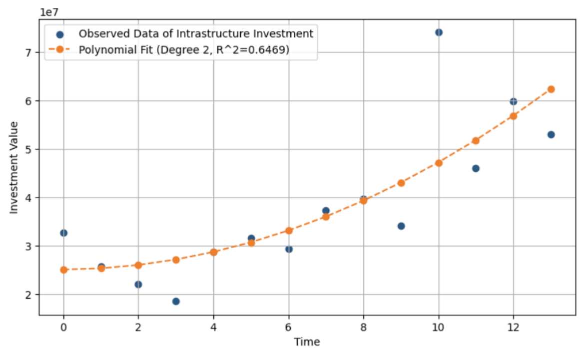


Figure 9: Fitting of Government Infrastructure Investment

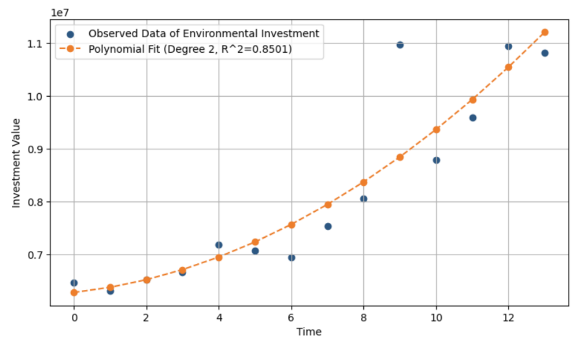


Figure 10: Fitting of Government Environmental Investment

A fitted polynomial function of Infrastructure Investment is shown below:

$$y = 25145586.07 + 39226.52 \cdot x + 217248.70 \cdot x^2.$$

A fitted polynomial function of Environmental Investment is shown below:

$$y = 6270656.96 + 73622.89 \cdot x + 23579.61 \cdot x^2.$$

Overall, the government is progressively increasing its investments in both infrastructure and environmental projects, following a quadratic growth trend. The environmental sector exhibits a more stable and predictable growth pattern, while infrastructure investment appears more volatile.

3.3.6 Comprehensive Model

The system of difference equations to be computed in code is as follows:

$$R[t] = R[t-1] \cdot r_I \left[1 + \left(\frac{R[t-1]\, r_I - R[t-1]}{K} \right) \right]^{-b^*}, \tag{7a}$$

$$\begin{aligned}\Omega[t] = {} & \omega_1 \cdot \frac{E[t]}{E[t] + \phi_E} + \omega_2 \cdot \frac{C[t]}{C[t] + \phi_C (N[t-1] + 1)} - \omega_3 N[t-1] - \omega_4 N[t-1]^2 - \omega_5 T_0 N[t-1] \\ & + \omega_6 \ln(G_1 + 1),\end{aligned} \tag{7b}$$

$$N[t] = N[t-1] + \Omega[t] \cdot N[t-1], \tag{7c}$$

$$C[t] = C[t-1] - \epsilon_1 C[t] + \epsilon_2 \Delta N[t], \tag{7d}$$

$$E[t] = E[t-1] + \frac{r_E E[t-1]^2}{K_E} - r_E E[t-1] - E[t-1](\varphi_1 C[t] + \varphi_2 N[t]), \tag{7e}$$

$$\mathrm{obj}_r[t] = \mathrm{obj}_r[t-1] - \gamma_1 \frac{N[t]^2}{R[t]} - \gamma_2 \frac{\Delta N[t]}{R[t]} + \gamma_3 \frac{\ln(G_1 + 1)}{R[t]} - \gamma_4 N[t], \tag{7f}$$

$$\mathrm{obj}_t[t] = \frac{\Omega[t]}{N[t-1]}, \tag{7g}$$

$$\mathrm{obj}_e[t] = E[t], \tag{7h}$$

$$\mathrm{obj}_g[t] = \mathrm{obj}_g[t-1] - \beta_1 (G_1 + G_2 - T_0 \cdot N[t]) + \beta_2 (\mathrm{obj}_r[t] - \mathrm{obj}_r[t-1])\, R[t], \tag{7i}$$

where $\Delta N[t] = N[t] - N[t-1]$ and $\Delta E[t] = E[t] - E[t-1]$.

3.3.7 Model Results

In the absence of policy interventions, the time-dependent difference equations in Model III evolve over the years as shown below:

Phenomena and Explanations

1. **Resident population growth:** Continues to rise, driven primarily by its own growth equation, with minimal impact from tourists or ecological value.
2. **Tourist numbers declining:** Sizable initial drop due to a sudden decrease in ecological value, which suppresses tourism; eventually, both ecological value and tourists decline together.
3. **Degradation of ecological value:** Continues to decrease because slow natural growth cannot offset tourism-related impacts, making ecologically-based tourism difficult to sustain.
4. **Residents objective function decreasing:** Initially stressed by a large tourist influx, followed by a reduction in tourism revenue, it ultimately levels off.
5. **Tourists objective function declining:** Primarily due to disappointment from worsening ecological conditions.
6. **Governments objective function diminishing:** Driven by reduced tax revenue from fewer tourists and a drop in residents welfare.

Using the static policy-planning model from Model II and replanning the best policy combination every five years, we obtain an optimal long-term policy mix. Figure 15 illustrates this schedule in a Gantt chart:

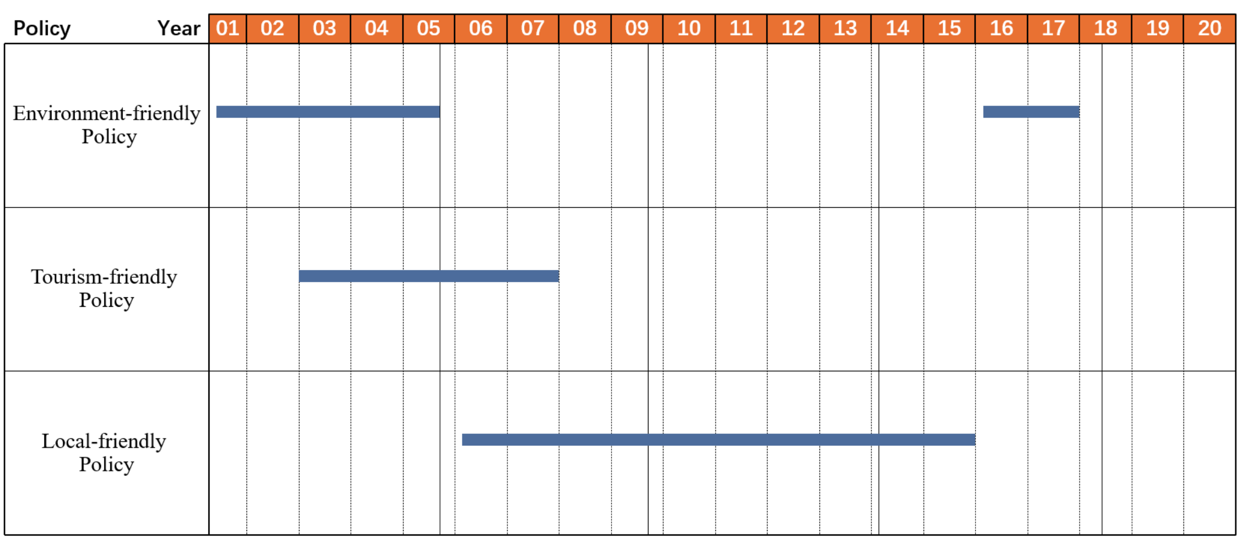


Figure 11: Sample Gantt Chart of Long-term Policy Implementation

3.4 Post-Interference Dynamics

Figure 13 shows the dynamic systems evolution after policy interference:

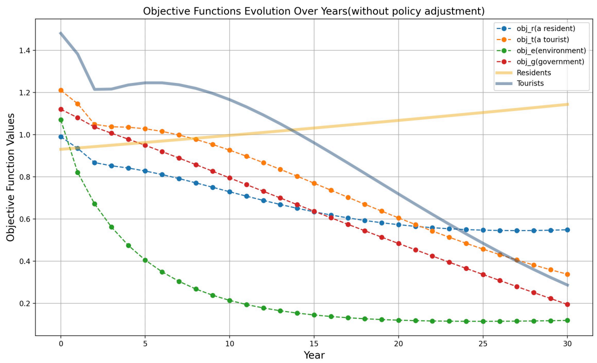


Figure 12: Objective Evolution Without Policy Interference

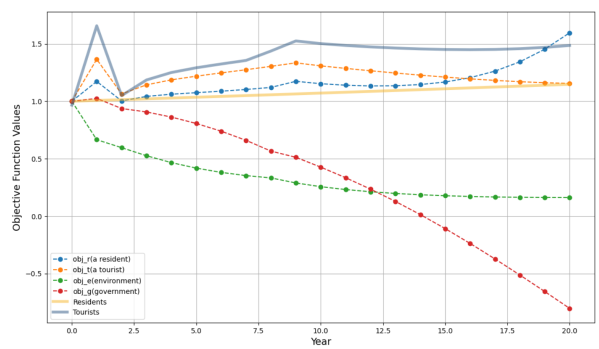


Figure 13: Objective Evolution With Policy Interference

Without policy adjustments, residents (obj_r), tourists (obj_t), and governments (obj_g) objectives all decline steadily, and the environment (obj_e) deteriorates fastest. Tourist numbers fall sharply, while the resident population grows slowly, pushing the system toward unsustainability. In contrast, with targeted policy interference, obj_r and obj_t improve significantly, the environmental decline slows, and tourist numbers stabilize before eventually increasing. Although obj_g continues to decrease, it does so more slowly, reflecting better fiscal and governance strategies. Overall, these results indicate that policy adjustments can balance economic growth, environmental sustainability, and social welfare, placing all stakeholders on a more stable and sustainable path.Under these measures, we project a 50% increase in tourist numbers, a slower rate of ecological decline, and a 60% rise in resident well-being. We acknowledge that government deficits may initially grow under these active policy interventions, so regular evaluations and adjustments will be necessary.

4 Sensitivity Analysis and Generalization

Our model is self-adjustable because, within its objective functions, it incorporates increasing marginal costs and decreasing marginal utility to reflect the interests of different groups. In locations with a high volume of tourists and relatively few residentssuch as Jeunuathese features lead to stronger negative effects from additional tourism, which in turn makes the model favor more environmentally and locally friendly policies. Conversely, for less-visited attractions, the model tends to select policy sets that benefit tourists. Nonetheless, adjusting the coefficients in both the objective function and the cost function can better capture trade-offs in different contexts and account for the varying costs that arise.

4.1 Generalization in Heated Tourism Attractions

For Heated Tourism Attractions, we find Venice as an ideal spot for the generalization of our model; For less visited Tourism Attractions, we choose Yulin Grottos, a less visited attraction compared it's neighbor Mogao grottos. There are similarities between Venice, Yulin and Juenea that makes our generalization possible and can allow us to test stability of our model.

To generalize the sustainable tourism model developed for **Juneau**, we compare it with **Venice, Italy** and **Yulin Grottoes, China**. These destinations face different tourism challenges but share similar concerns regarding overtourism, environmental impact, and sustainability strategies. By comparing various coefficients in our model, we change several key coefficients of each objective to see their effects:

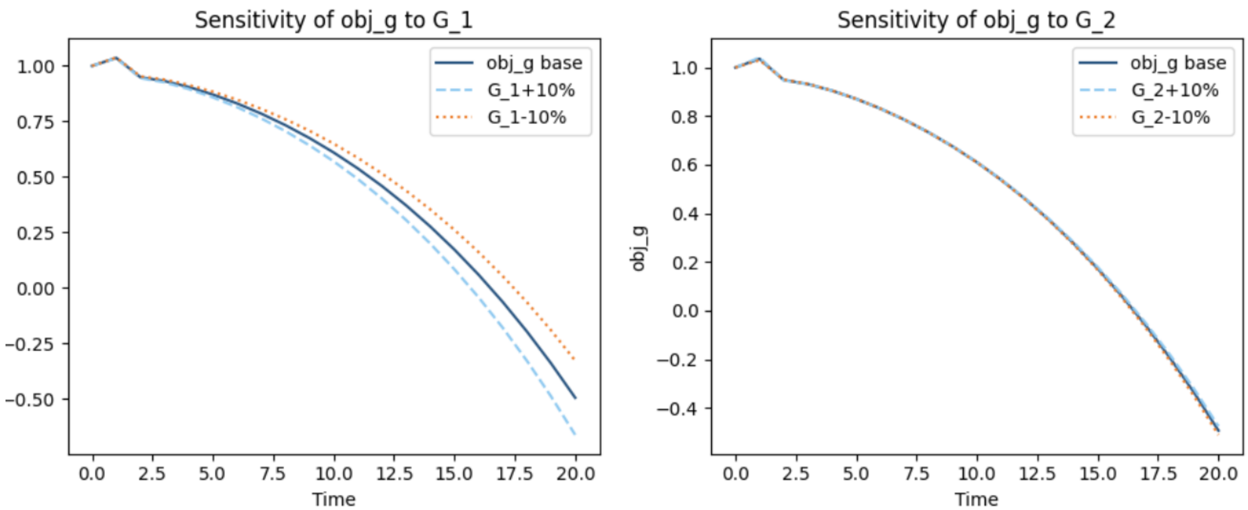


Figure 14: Sensitivity Analysis of Government Objective

For the government objective, the sensitivity analysis (Figure **??**) shows that G_1, which controls the tolerance for budget deficits and deviations from balance, has a greater impact than G_2, which represents the government's perceived importance of local residents' welfare. This suggests that fiscal policies play a more decisive role in shaping government objectives compared to direct welfare considerations.

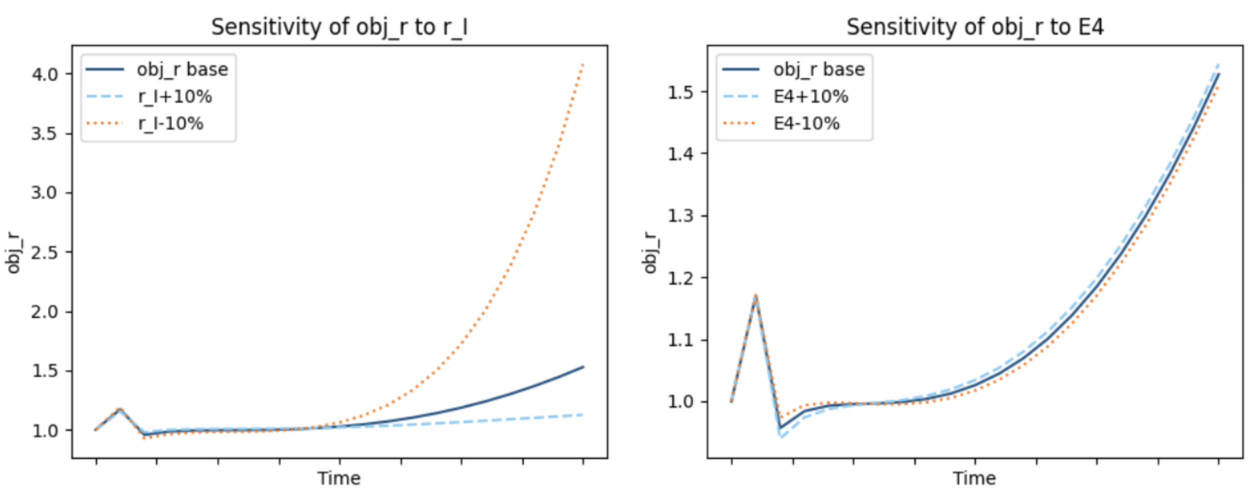


Figure 15: Sensitivity Analysis of Resident Objective

For the resident objective, the analysis (Figure **??**) indicates that r_1, which determines the local population growth rate, has a stronger effect than E_4, which represents residents' influence on local policies. This finding suggests that rather than the extent of tourism itself, it is more crucial to ensure that the number of tourists and government investments align with the growth of the local population.

4.2 Generalization Conclusion

The comparison of these tourist destinations reveals three distinct yet interconnected challenges in tourism:

- **Juneau** faces issues related to seasonal mass tourism and the impacts of climate change, requiring the implementation of visitor limits and infrastructure investments.
- **Venice** suffers from year-round overtourism and environmental degradation, necessitating visitor flow control and the promotion of alternative forms of tourism.
- **Yulin Grottoes** is threatened by cultural heritage degradation caused by tourism, calling for controlled access and increased funding for preservation.

5 Model Evaluation and Promotion

5.1 Model Evaluation

5.1.1 Advantages

1. By adopting multiple objective functions, such as those for residents, tourists, and the environment, the model provides a comprehensive assessment of the impacts of tourism. Furthermore, incorporating time-dependent effects in Model III enhances the models capability to evaluate long-term impacts.
2. The use of Pareto improvements for policy evaluation in the second model effectively assesses different policy combinations, ensuring that improvements in one stakeholders welfare do not harm others.
3. By introducing time-varying government investment, the model better evaluates the long-term impacts of government funding on society and the environment while optimizing resource allocation.
4. The model incorporates economic theories such as substitution and income effects, allowing it to more accurately reflect the actual needs of different groups, which aligns with the complexity of sustainable tourism issues.

5.1.2 Limitations

1. The interactions among stakeholders, including residents, tourists, the environment, and the government, are highly complex and dynamic. Our model simplifies these relationships by assuming static preferences, which may not fully capture the behavioral changes and interdependencies among stakeholders.

2. The model relies on estimated data to derive parameters such as tourist numbers, resident impact, and environmental effects. Due to the limited or unstable availability of such data, the accuracy of the initial values may be compromised, affecting the models predictive accuracy, particularly in addressing volatile or uncertain tourism trends.

3. The assumption that certain policies are mutually exclusive may overlook potential synergies between policies. Future iterations could explore more flexible policy combinations to achieve better outcomes.

6 Conclusions

References

[1] Prandtl L. Fluid motions with very small friction. *Proceedings of the 3rd International Mathematical Congress*. Heidelberg: H. Schlichting; 1904:484-491.

[2] McKinley Research Group. Juneau Tourism Survey 2023. Prepared for the City and Borough of Juneau; December 2023.

[3] Visitor Industry Task Force. Final Report. January 2021.

[4] Author unknown. Study: Southeast tourism thrives, seafood suffers during record year in 2023 for both industries. *Juneau Empire*.

[5] Juneau Economic Development Council (JEDC). 2024 Juneau SEAK Economic Indicators and Outlook Report. 2024.

[6] Southeast Conference. Southeast Alaska Tourism Impacts. 2021.

[7] Hassell M P. Density-dependence in single-species populations[J]. The Journal of animal ecology, 1975: 283-295.

[8] Vacquié-Garcia J, Lydersen C, Ims R A, et al. Habitats and movement patterns of white whales Delphinapterus leucas in Svalbard, Norway in a changing climate[J]. Movement Ecology, 2018, 6: 1-12.

[9] Casagrandi R, Rinaldi S. A theoretical approach to tourism sustainability[J]. Conservation ecology, 2002, 6(1).

[10] Al-Sayed, M. Ecotourism seminar[J]. Kuwait foundation for advancement of science. Kuwait, Publisher, 1999.

[11] Kuraibet, A. Ecotourism seminar[J]. Kuwait foundation for advancement of science. Kuwait, Publisher, 1999. Publisher.

[12] Clark B. Ecotourism seminar[J]. Kuwait foundation for advancement of science. Kuwait, Publisher, 1999.

[13] Gunderson L H, Holling C S. Panarchy: Understanding transformations in human and natural systems[M]. 2002.

[14] Daily G C. Introduction: what are ecosystem services[J]. Natures services: Societal dependence on natural ecosystems, 1997, 1(1).

[15] Dobson A P, Bradshaw A D, Baker A J M. Hopes for the future: restoration ecology and conservation biology[J]. Science, 1997, 277(5325): 515-522.

[16] Lacitignola D, Petrosillo I, Cataldi M, et al. Modelling socio-ecological tourism-based systems for sustainability[J]. Ecological modelling, 2007, 206(1-2): 191-204.

[17] Juneau Economic Development Council. Juneau and Southeast Alaska Economic Indicators and Outlook Report [M]. Juneau: Juneau Economic Development Council, 2024.

A Letter To Juenea Tourism Commette

We, the assigned MCM team, analyzed the interests of government, residents, tourists, and environmentalists to develop immediate and long-term sustainable tourism policies for Juneau. We propose a short-term plan evaluated for cost-effectiveness and Pareto improvement (benefiting all groups without harming any), followed by a 20-year strategy in five-year cycles.

In the short term, we recommend installing shore power at cruise ports to reduce emissions, enforcing stricter environmental regulations, streamlining entry requirements, improving transportation services, offering targeted tax cuts (e.g., hotel or sales tax reductions), subsidizing seasonal employment programs, and redistributing tourism tax revenue to enhance local infrastructure and community development. Environmental measures show a high initial impact but may decline over time due to infrastructure limits; tourist-friendly approaches yield quick gains but can plateau as congestion grows; and resident-focused strategies accumulate long-term economic benefits. Therefore, we advise prioritizing environmental and tourist-friendly measures during the first two years, then gradually shifting to resident-focused initiatives from year three onward to ensure sustained gains for all stakeholders.

For long-term stability, we developed a time-dependent difference equation model incorporating resident population, tourist numbers, capital investment, and ecological value. Without policy intervention, environmental degradation lowers tourist arrivals and government revenue, leading to insufficient infrastructure investment and diminished resident well-being. To counter these trends, we propose strict environmental regulations in the early years to prevent irreversible damage, followed by gradual infrastructure investment and local economic support to stabilize growth. Every five years, adaptive taxation and visitor flow management will balance ecological preservation and economic needs. Under these measures, we project a 50% increase in tourist numbers, a slower rate of ecological decline, and a 60% rise in resident well-being. We acknowledge that government deficits may initially grow under these active policy interventions, so regular evaluations and adjustments will be necessary.

Sincerely,
The MCM Team