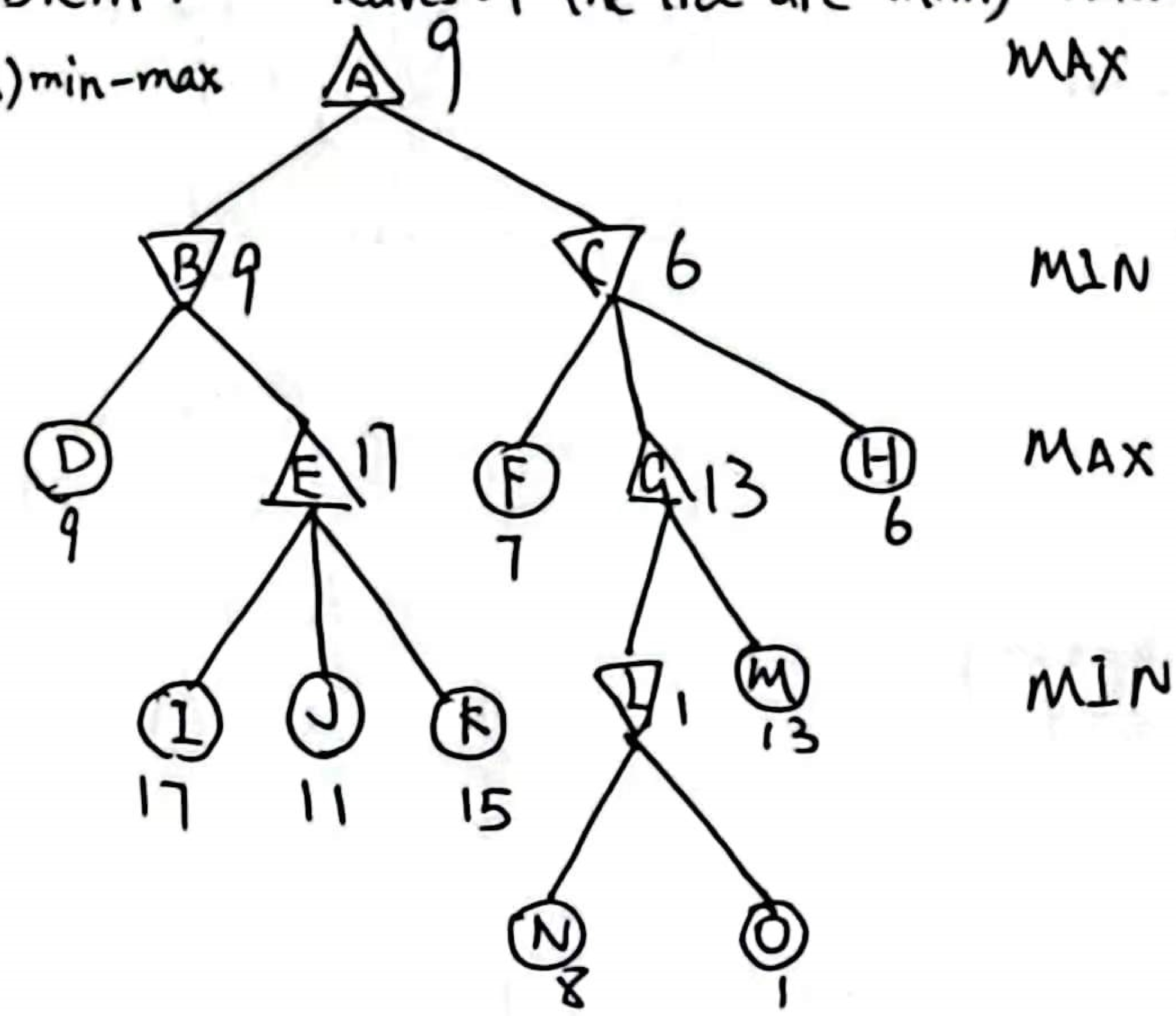
Foundations of AI HW2 学号: t11902210 姓名: 張一凡

Problem 1 Consider the MAX-MIN game tree shown below the numbers underneath the leaves of the tree are utility values from the first player's point of view(max)

(a) min-max



Performing process:

Step 1: (4th layer)

L min

8(N) > ~~0(1)~~ 1(O)

$\Rightarrow$ L = 1

Step 2: (3rd layer)

① E max

17(I) > 15(K) > J(11)

$\Rightarrow$ E = 17

② G max

1(L) < 13(M)

$\Rightarrow$ G = 13

Step 3: (2nd layer)

① B min

9(D) < 17(E)

$\Rightarrow$ B = 9

② C min

6(H) < 7(F) < 13(G)

$\Rightarrow$ C = 6

Step 4: (1st layer)

A max

9(B) > 6(C)

$\Rightarrow$ A = 9

Resulting: A: 9

B: 9 C: 6

E: 17 G: 13

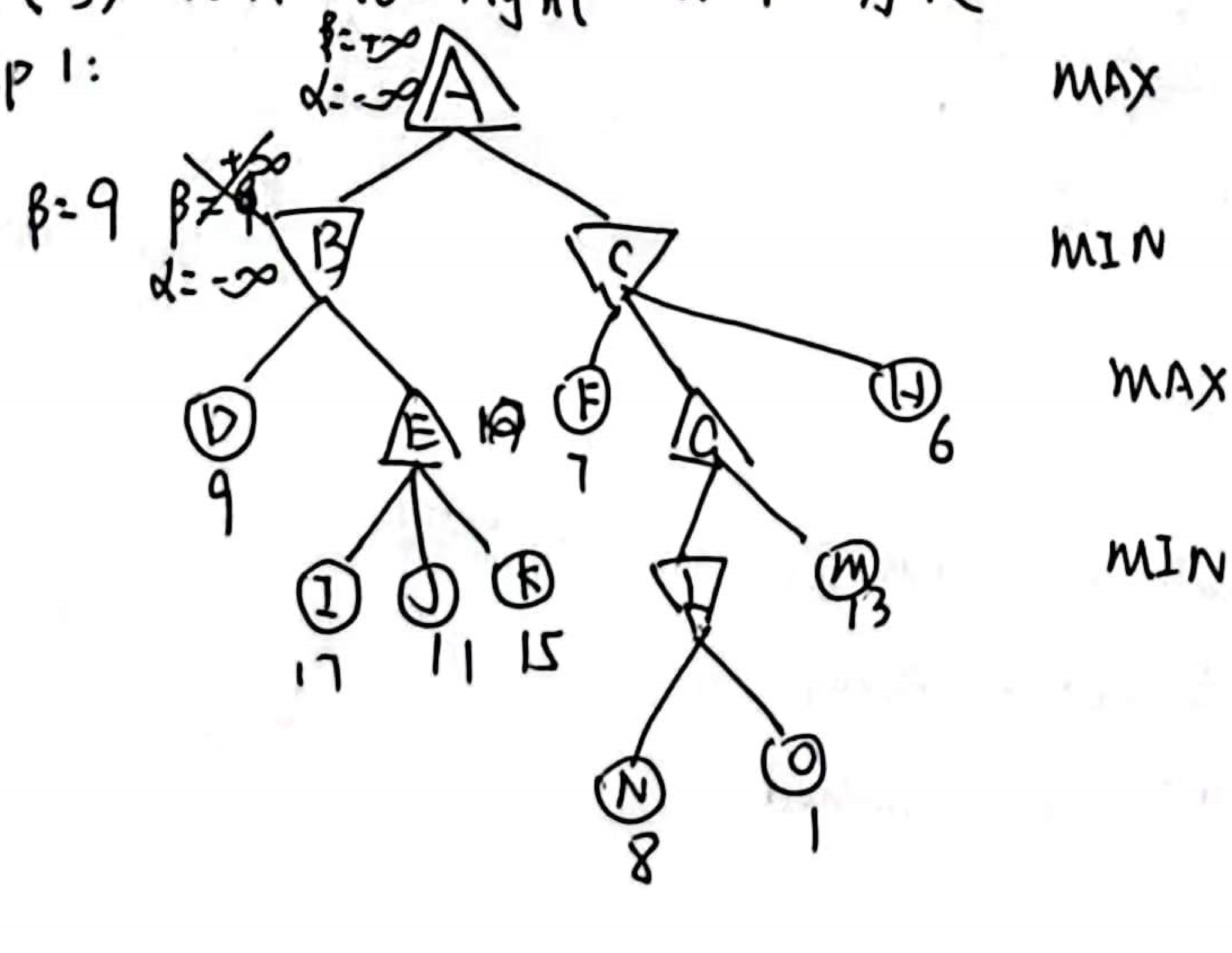
L: 1

question (a):
Draw a copy of the tree on paper and perform the minmax algorithm algorithm on it by hand.
Write the resulting minmax value next to every node

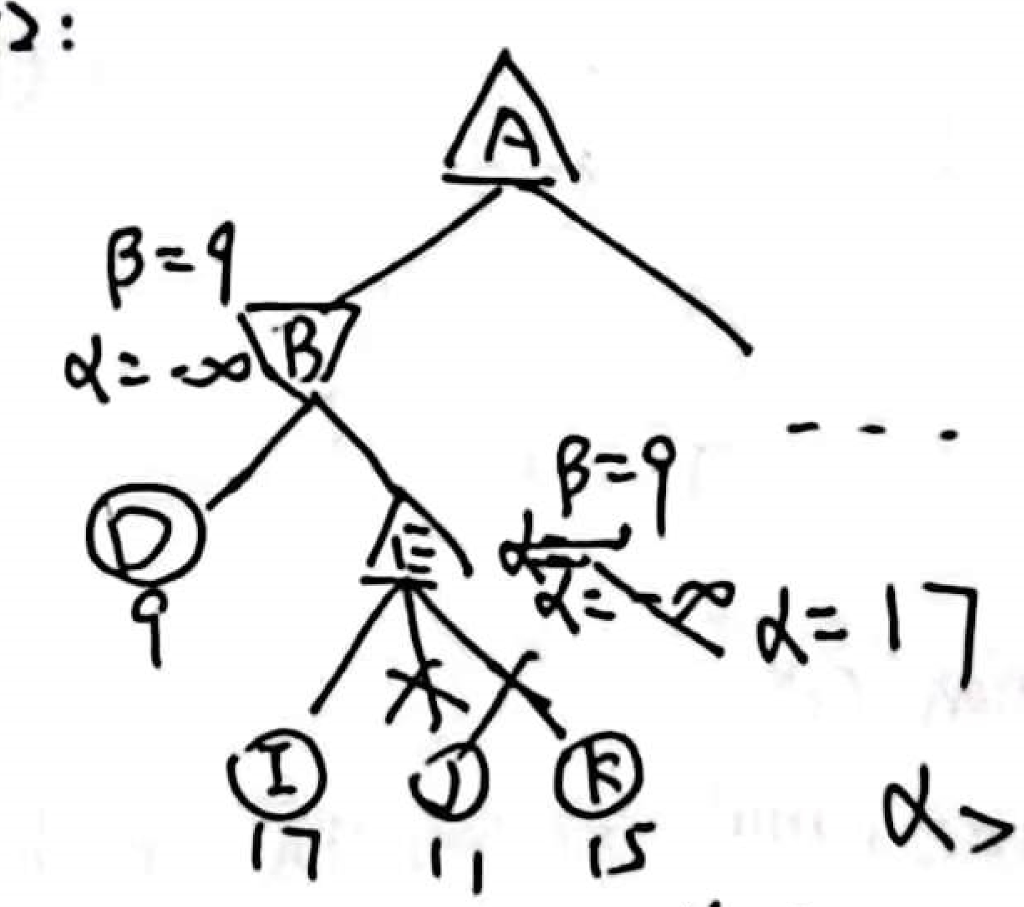
question (b):
Do the same. but with left-to-right $\alpha-\beta$ pruning.
Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

(b) left-to-right $\alpha-\beta$ 剪枝 (Performing Process)

Step 1:



step 2:



$\alpha > \beta$

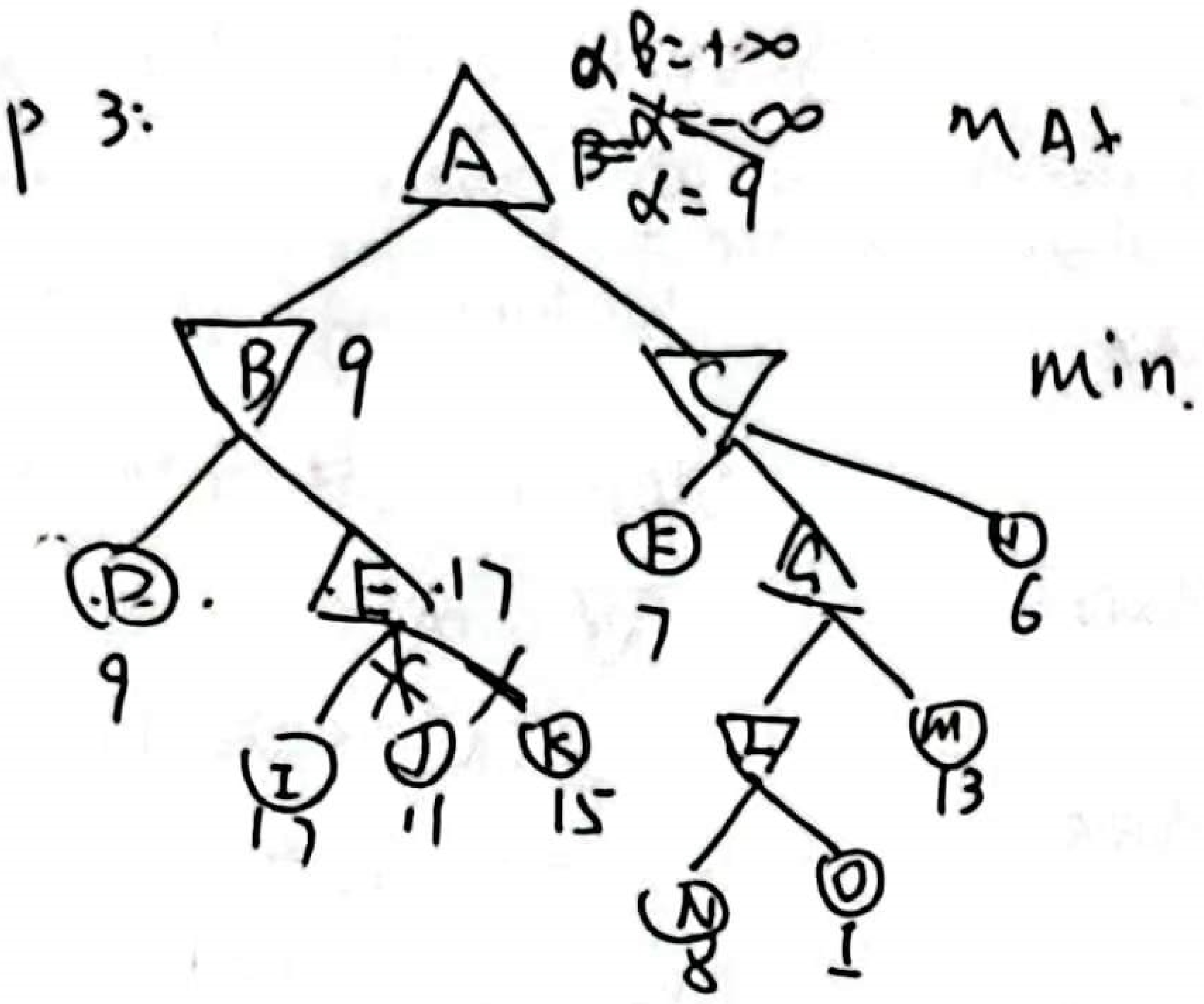
剪掉 J. K

E = 17

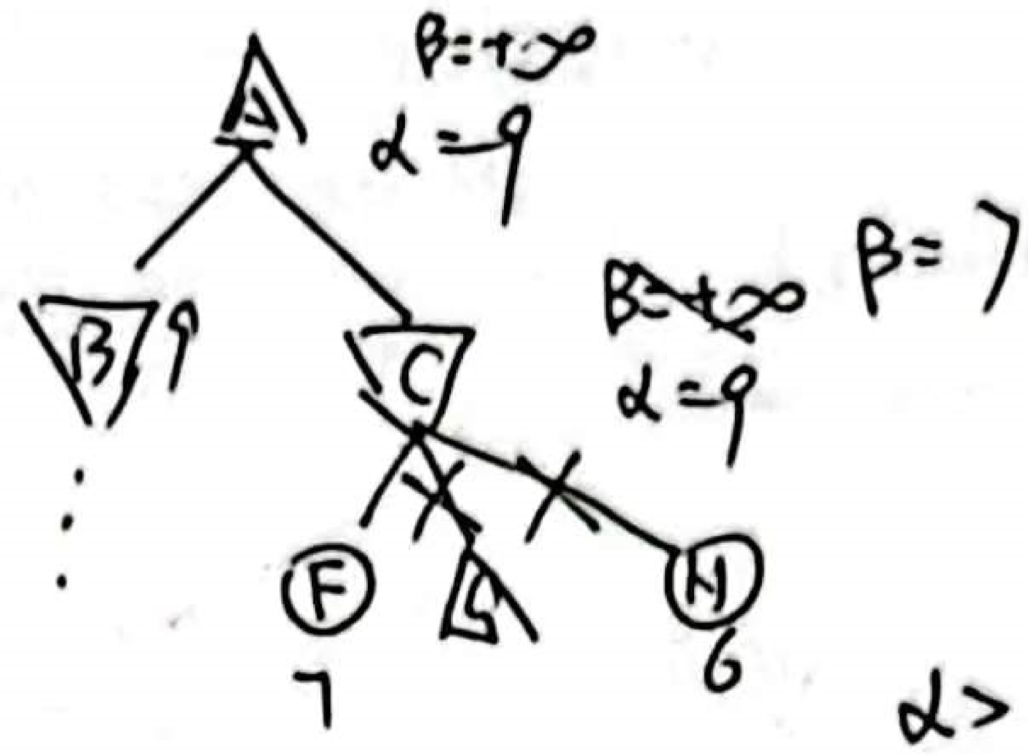
$\rightarrow$ B min

B = 9

step 3:



step 4:



$\alpha>\beta$

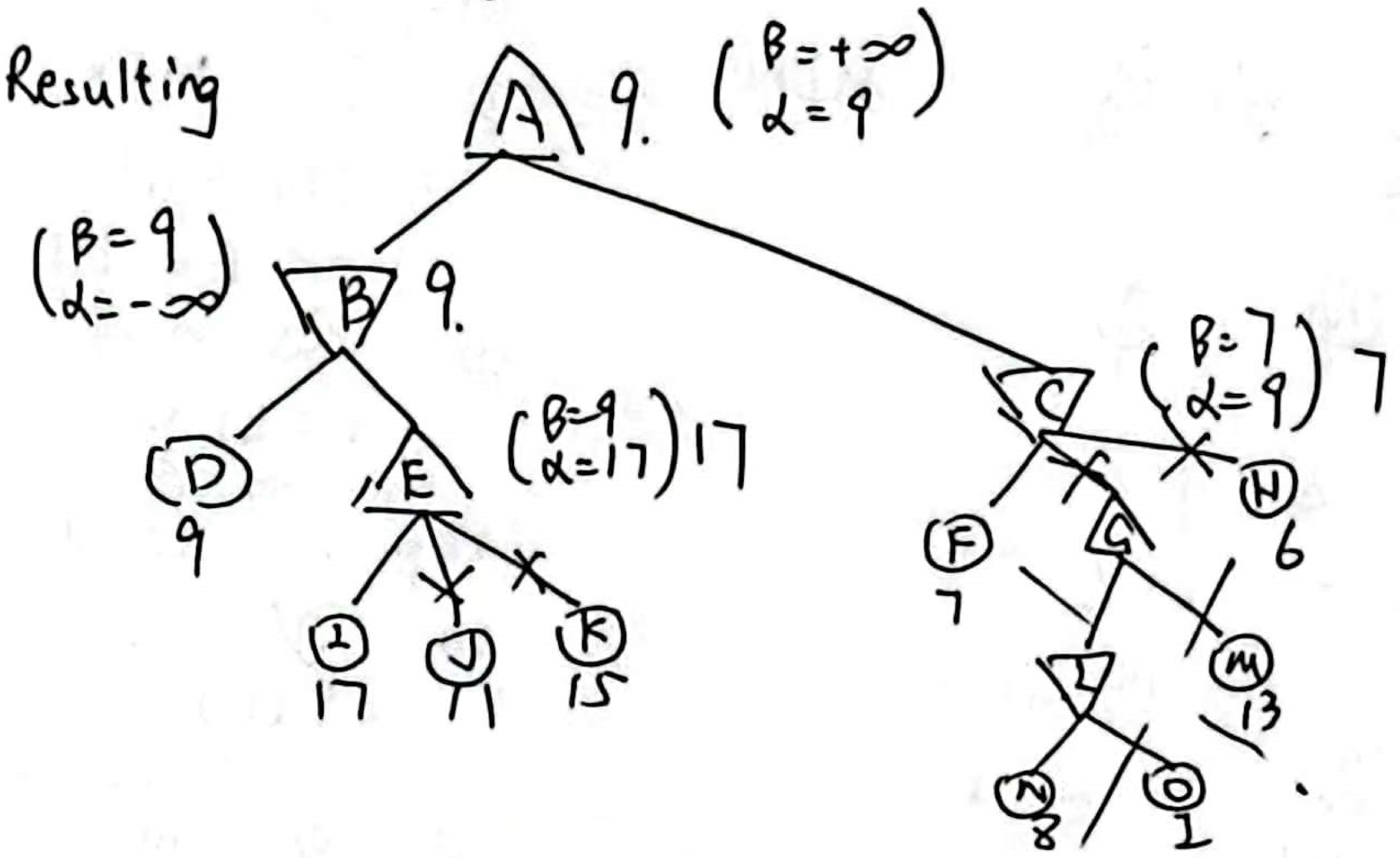
剪掉 G. H

故 C=7

A max => 9>7

A=9

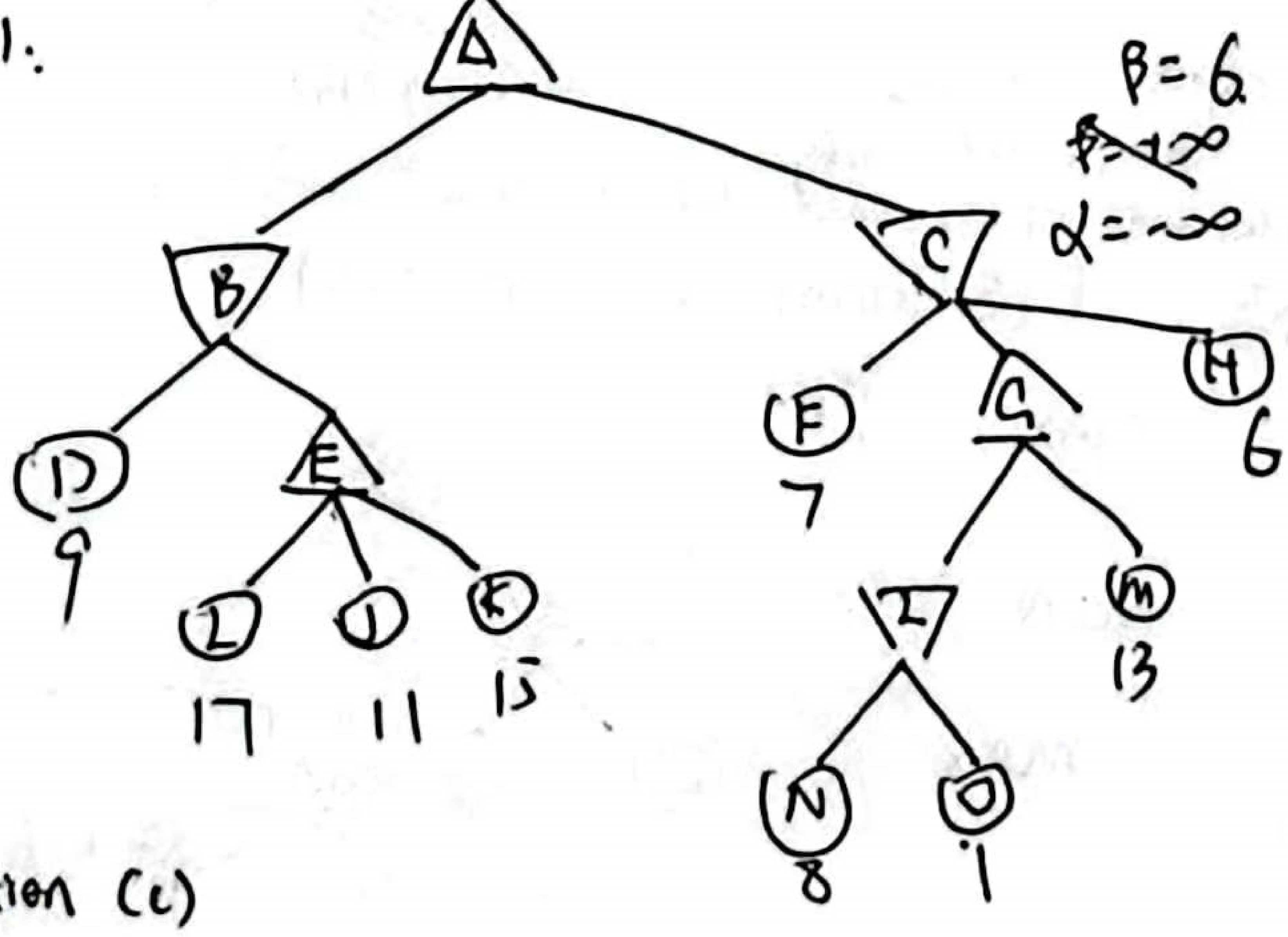
Resulting



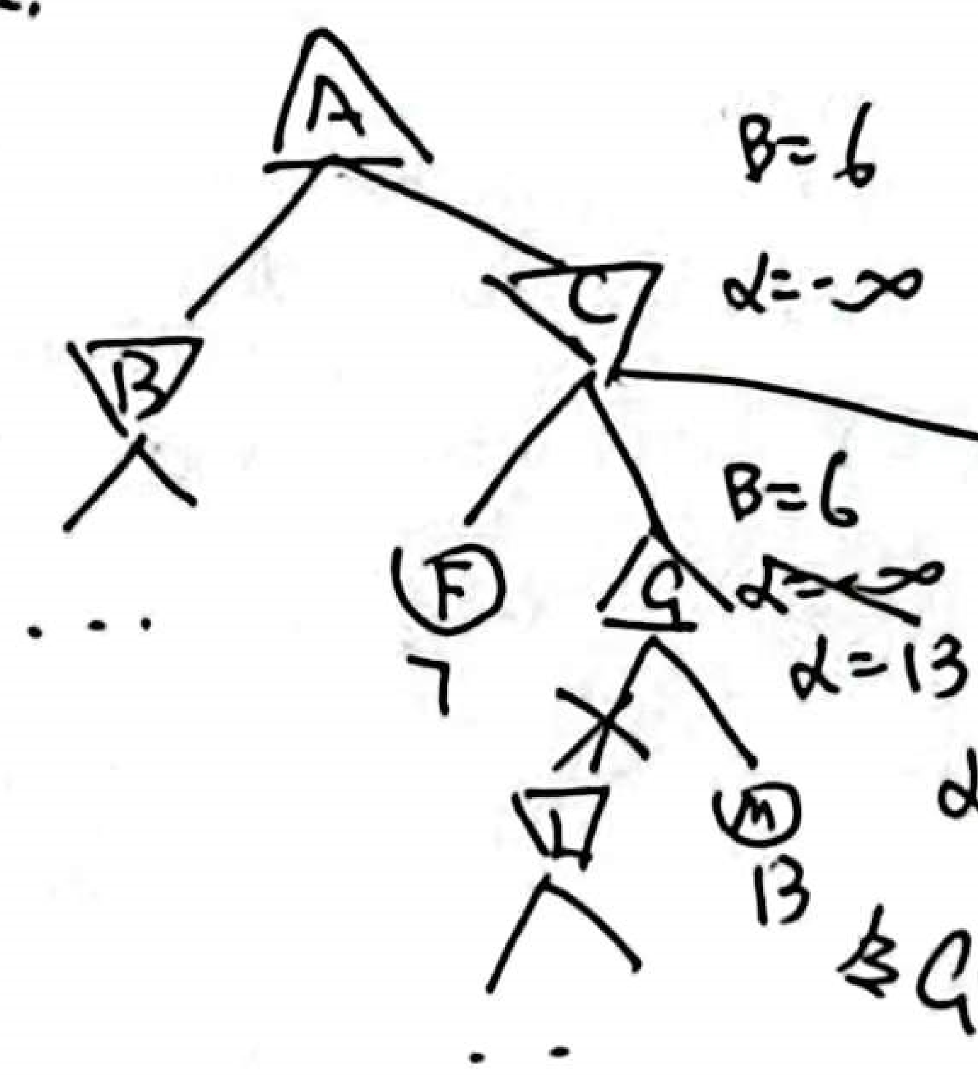
not examined node: J, K, G. H, L. M. N. O

(c) right-to-left. αβ. 剪枝 (do the same)

step 1:



step 2:



$\alpha>\beta$

故 G=13

L 剪掉

C: min

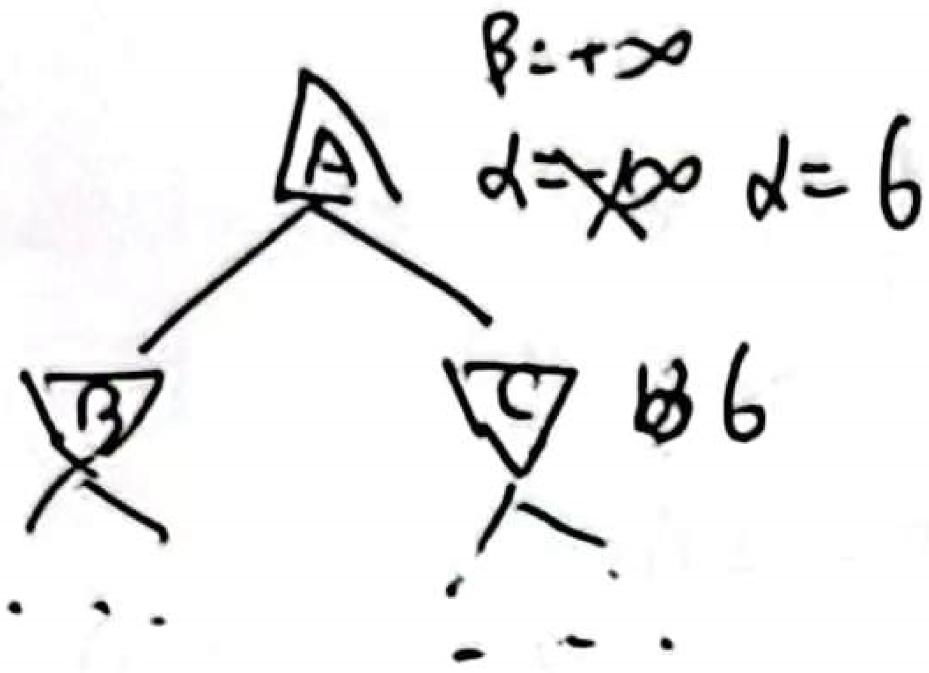
6<7<13

取 C= ~~13~~ 6

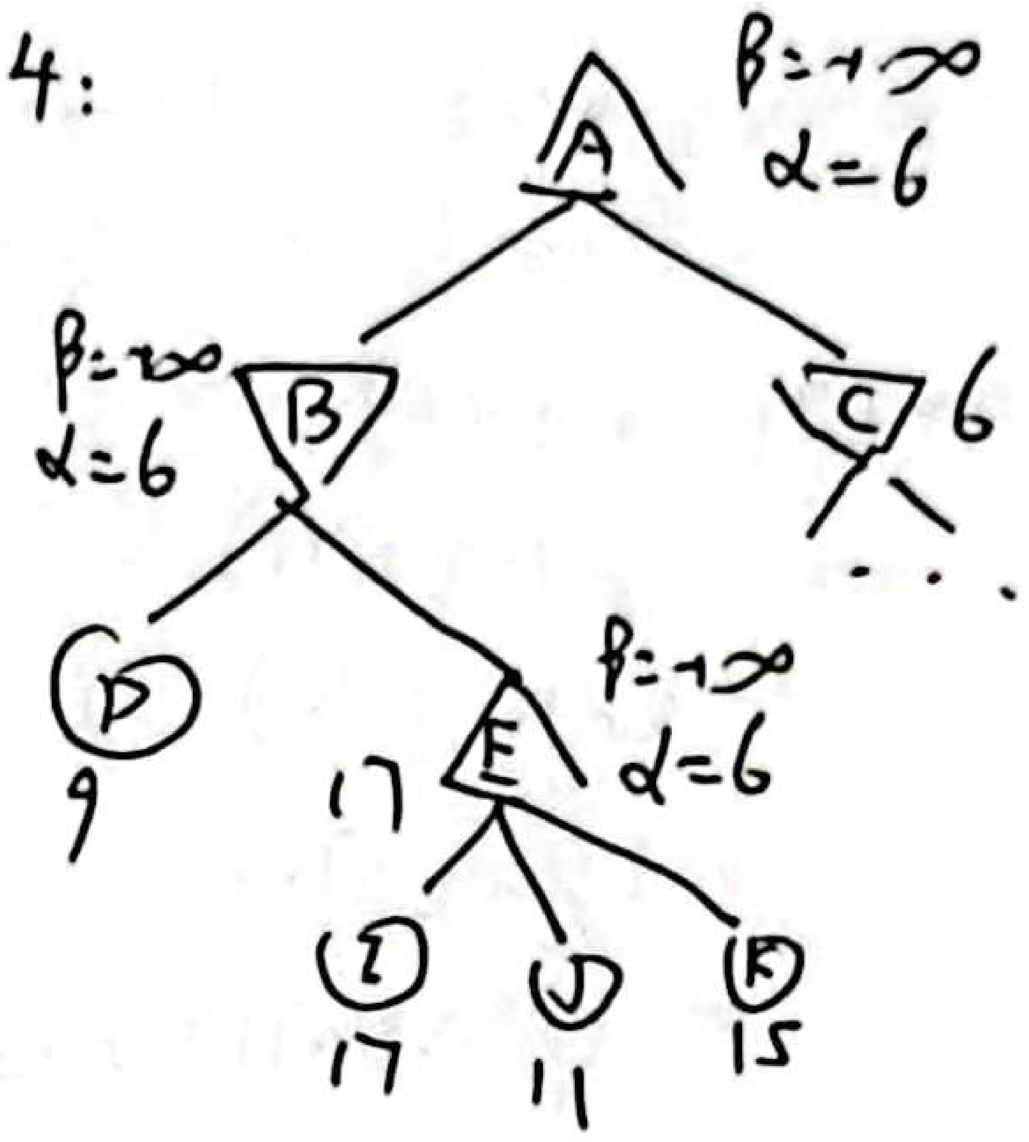
question (c)

Do the same. but with right-to-left α-β pruning. Write the final values for α and β next to every node. and indicate which nodes are not examined due to pruning.

Step 3.



Step 4:

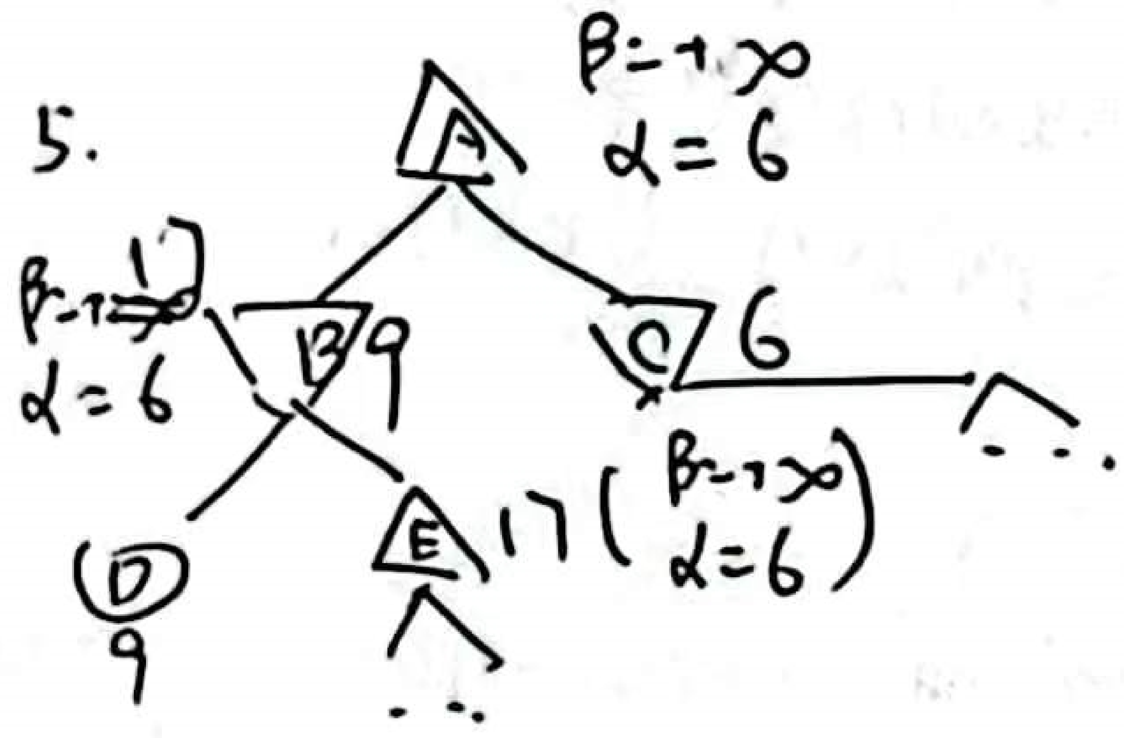


E: max

$17>15>11$

故 $E=17$

Step 5.

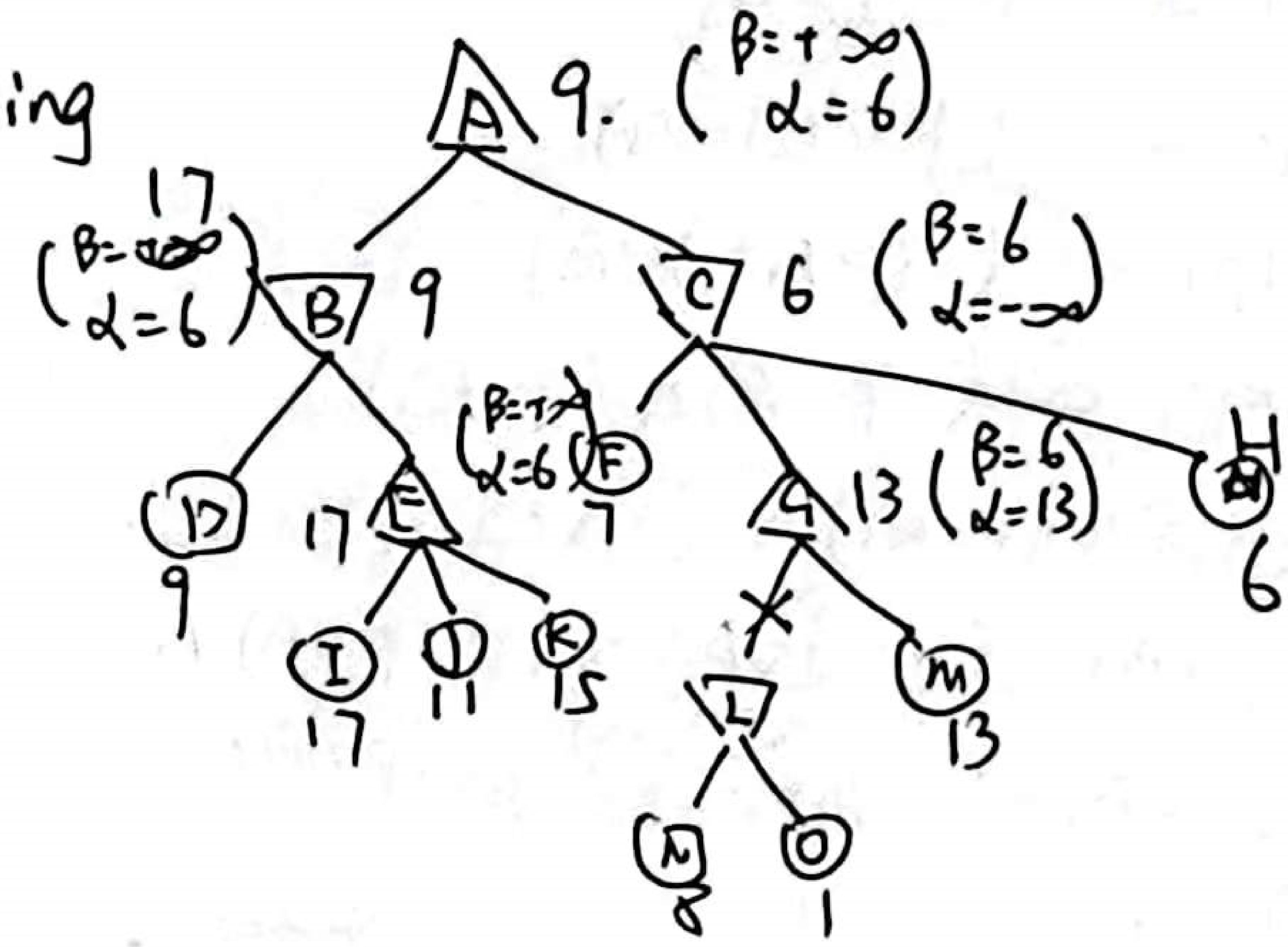


B: min $9<17$

故 $B=9$

A: max $9>6$. 故 $A=9$.

Resulting



not examined node:

L. N. O

Problem. 2

(a) - Rewrite the following propositional formulae into Conjunctive Normal Form

1. $((((p\to q)\to \neg p)\to \neg q)\to \neg r)\to r$

$\Rightarrow (((\neg p\lor q)\to \neg p)\to \neg q)\to \neg r)\to r$

$\Rightarrow ((\neg(\neg p\lor q)\lor \neg p)\to \neg q)\to \neg r)\to r$

$\Rightarrow ((\neg(\neg(\neg p\lor q)\lor \neg p)\lor \neg q)\to \neg r)\to r$

$\Rightarrow (\neg(\neg(\neg(\neg p\lor q)\lor \neg p)\lor \neg q)\lor \neg r)\to r$

$\Rightarrow (\neg(\neg((p\land \neg q)\land p)\lor \neg q)\lor \neg r)\to r$

$\Rightarrow (\neg((\neg p\lor q)\lor \neg p)\land q)\lor \neg r)\to r$

$\Rightarrow (((p\land \neg q)\land p)\lor \neg q)\land \neg r)\to r$

$\Rightarrow (((p\land \neg q)\lor \neg q)\lor \neg r)\to r$

$\Rightarrow (\neg(\neg((p\land \neg q)\lor \neg p)\lor \neg q)\lor \neg r)\to r$

$\Rightarrow (\neg(\neg(\cancel{p\lor\neg p}\ \neg q\lor \neg p)\lor \neg q)\lor \neg r)\to r$

$\Rightarrow (\neg((q\land p)\lor \neg q)\lor \neg r)\to r$

$\Rightarrow (\neg(p\lor \neg q)\lor \neg r)\to r$

$\Rightarrow ((\neg p\land q)\lor \neg r)\to r$

$\Rightarrow \cancel{(\neg p\lor \neg r)}$

$(\neg((\neg p\lor \neg r)\land(q\lor \neg r)))\lor r$

$\Rightarrow (p\land r)\lor(\neg q\land r)\lor r$

$(\neg(\neg p\land q)\land r)\lor r$

$\Rightarrow ((p\lor \neg q)\land r)\lor r$

$\Rightarrow r\land(p\lor \neg q\lor r)$

(5) 2. $(p\to(q\to r))\to((p\to\neg r)\to(p\to\neg q))$

$\Rightarrow (p\to(\neg q\vee r))\to((\neg p\vee\neg r)\to(\neg p\vee\neg q))$

$\Rightarrow (\neg p\vee\neg q\vee r)\to((p\wedge r)\vee(\neg p\vee\neg q))$

$\Rightarrow (p\wedge q\wedge\neg r)\vee((p\wedge r)\vee(\neg p\vee\neg q))$

$\Rightarrow (p\wedge q\wedge\neg r)\vee(\cancel{p\wedge r}\,(p\vee\neg p\vee\neg q)\wedge(r\vee\neg p\vee\neg q))$

$\Rightarrow (p\wedge q\wedge\neg r)\vee(\neg q\wedge(\neg p\vee\neg q\vee r))$

$\Rightarrow ((p\wedge q\wedge\neg r)\vee\neg q)\wedge((p\wedge q\wedge\neg r)\vee(\neg p\vee\neg q\vee r))$

$\Rightarrow ((p\wedge q\wedge\neg r)\vee\neg q)\wedge(\neg(\neg p\vee\neg q\vee r)\vee(\neg p\vee\neg q\vee r))$

$\Rightarrow ((p\wedge q\wedge\neg r)\vee\neg q$

$\Rightarrow (p\vee\neg q)\wedge(\neg r\vee\neg q)$

(b) - Construct a formula A such that the formula $((A\wedge q)\to\neg p)\to((p\to\neg q)\to A)$ is always true.

$((A\wedge q)\to\neg p)\to((p\to\neg q)\to A)$

$\Rightarrow (\neg(A\wedge q)\vee\neg p)\to((\neg p\vee\neg q)\to A)$

$\Rightarrow (A\vee\neg q\vee\neg p)\to((p\wedge q)\vee A)$

$\Rightarrow (\neg A\wedge q\wedge p)\vee((p\vee A)\wedge(q\vee A))$

$\Rightarrow ((\neg A\wedge q\wedge p)\vee(p\vee A))\wedge((\neg A\wedge q\wedge p)\vee(q\vee A))$

$\Rightarrow (\neg A\vee p\vee A)\wedge(q\vee p\vee A)\wedge(p\vee p\vee A)\wedge(\neg A\vee q\vee A)\wedge$

$\Rightarrow (q\vee p\vee A)\wedge((p\wedge q)\vee A)$ $(p\vee A)\wedge(q\vee A)$ $(q\vee q\vee A)\wedge(p\vee q\vee A)$

$(q\vee p)\vee A=1$ $\quad p\vee A=1$ $\quad p\vee q=1$

$A=\neg(q\vee p)\vee\neg p\vee\neg q$

$=(\neg q\wedge\neg p)\vee\neg p\vee\neg q$

$=(\neg q\vee\neg p)\,A$

Construct a formula with variables p. q. r that is true if and only if at least two of the variables are true

(c) $(p\wedge q)\vee(p\wedge r)\vee(r\wedge q)$

(d) - Construct a formula A such that $(r\to A)\equiv(r\to(p\wedge q))$ and $(A\to r)\equiv(\neg(p\vee q)\to r)$

$(r\to A)\equiv(r\to(p\wedge q))$

$\neg r\vee A\equiv(\neg r\vee(p\wedge q))$

$(A\to r)\equiv(\neg(p\vee q)\to r)$

$\neg A\vee r\equiv((p\vee q)\vee r)$

$A\wedge\neg r\equiv\neg r\wedge\neg p\wedge\neg q$

$(\neg r\vee A)\wedge(A\wedge\neg r)=(\neg r\vee(p\wedge q))\wedge\neg r\wedge\neg p\wedge\neg q$

$(\neg r\wedge A)\vee(A\wedge\neg r)=(\neg r\wedge\neg p\wedge\neg q)\vee\neg r$

~~A: (p∧q)∨(¬p∧¬q)~~

if $r=1$
 $A=p\wedge q$
else if $r=0$
 $A=\neg(p\vee q)$

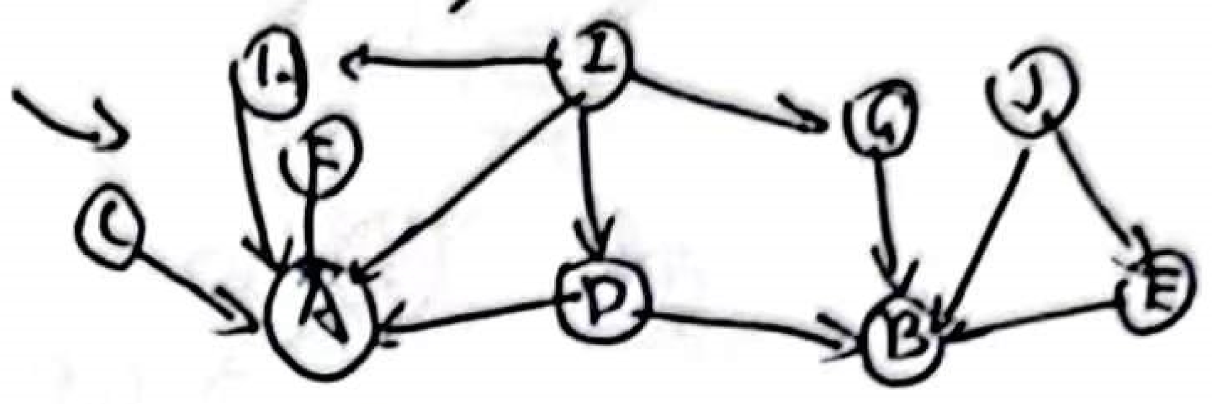
$\Rightarrow A=(r\wedge p\wedge q)\vee(\neg r\wedge\neg(p\vee q))$

(a) Write down the factored joint probability distribution according to the following Bayesian Network.

$P(A \mid C, D, F, H, I)$ $P(B \mid D, E, G, J)$

$P(C)$ $P(D \mid I)$ $P(E \mid J)$ $P(F)$

$P(G \mid I)$ $P(H \mid I)$ $P(I)$ $P(J)$



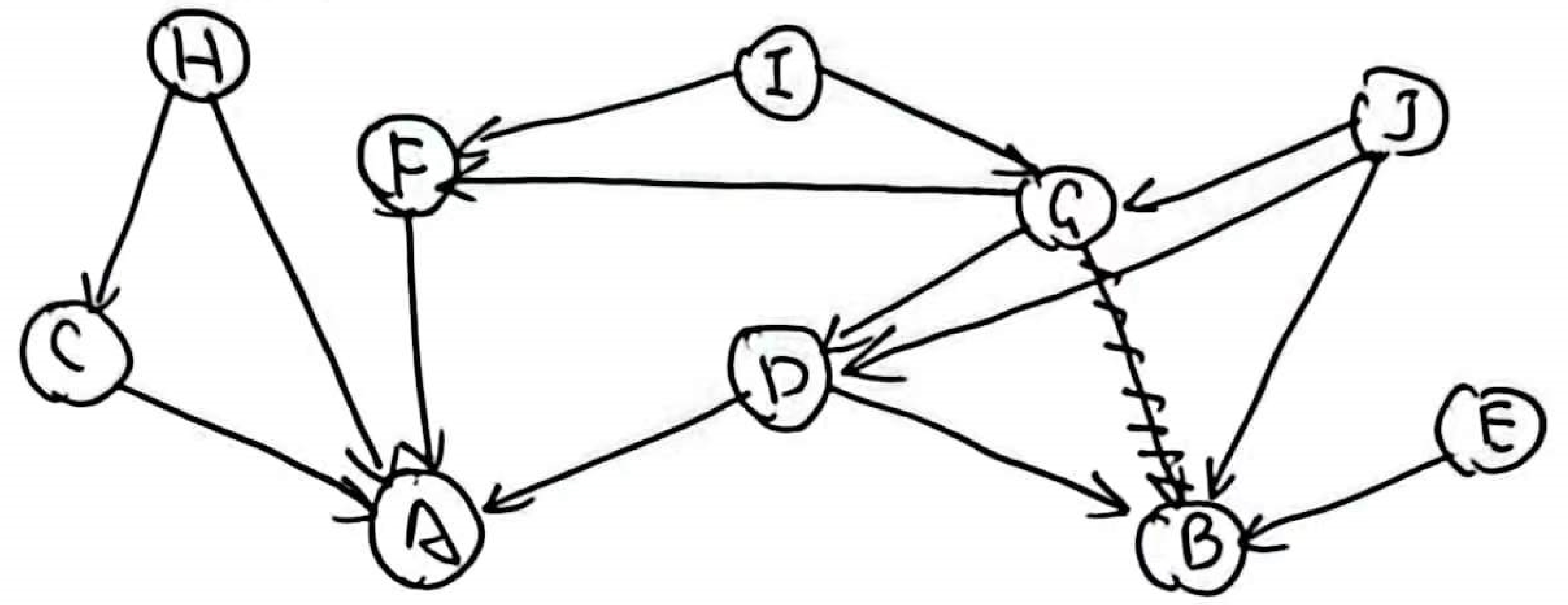
(b) Draw the Bayesian Network that corresponds to this conditional probability

$P(A \mid C, D, F, H)$ $P(B \mid D, E, J)$

$P(C \mid H)$ $P(D \mid G, J)$ $P(E)$

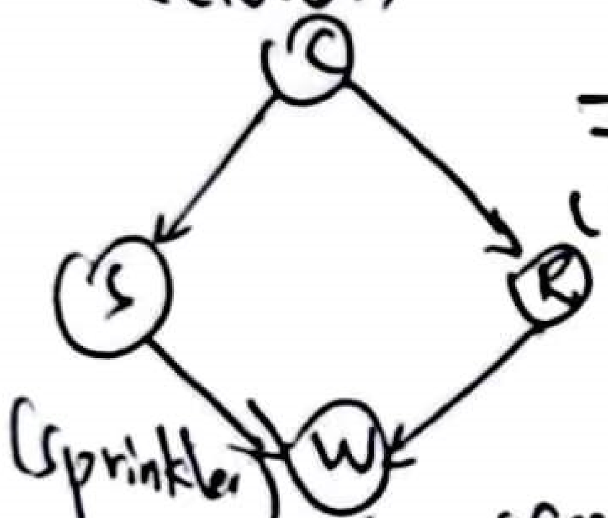
$P(F \mid G, I)$ $P(G \mid I, J)$

$P(H)$ $P(I)$ $P(J)$



(c) Below is the Bayesian network for the WetGrass problem. Write down an expression that will evaluate to $P(C=f \wedge R=f \wedge S=t \wedge W=t)$.

(You don't need to carry out the multiplication to produce a single number (probability))



$= P(C=f) \cdot P(R=f \mid C=f) \cdot P(S=t \mid C=f) \cdot P(W=t \mid S=t, R=f)$

$= (1-0.5) \times (1-0.2) \times (0.1) \times (0.9)$

$= 0.5 \times 0.8 \times 0.1 \times 0.9 = 0.036$

P(C)
0.5

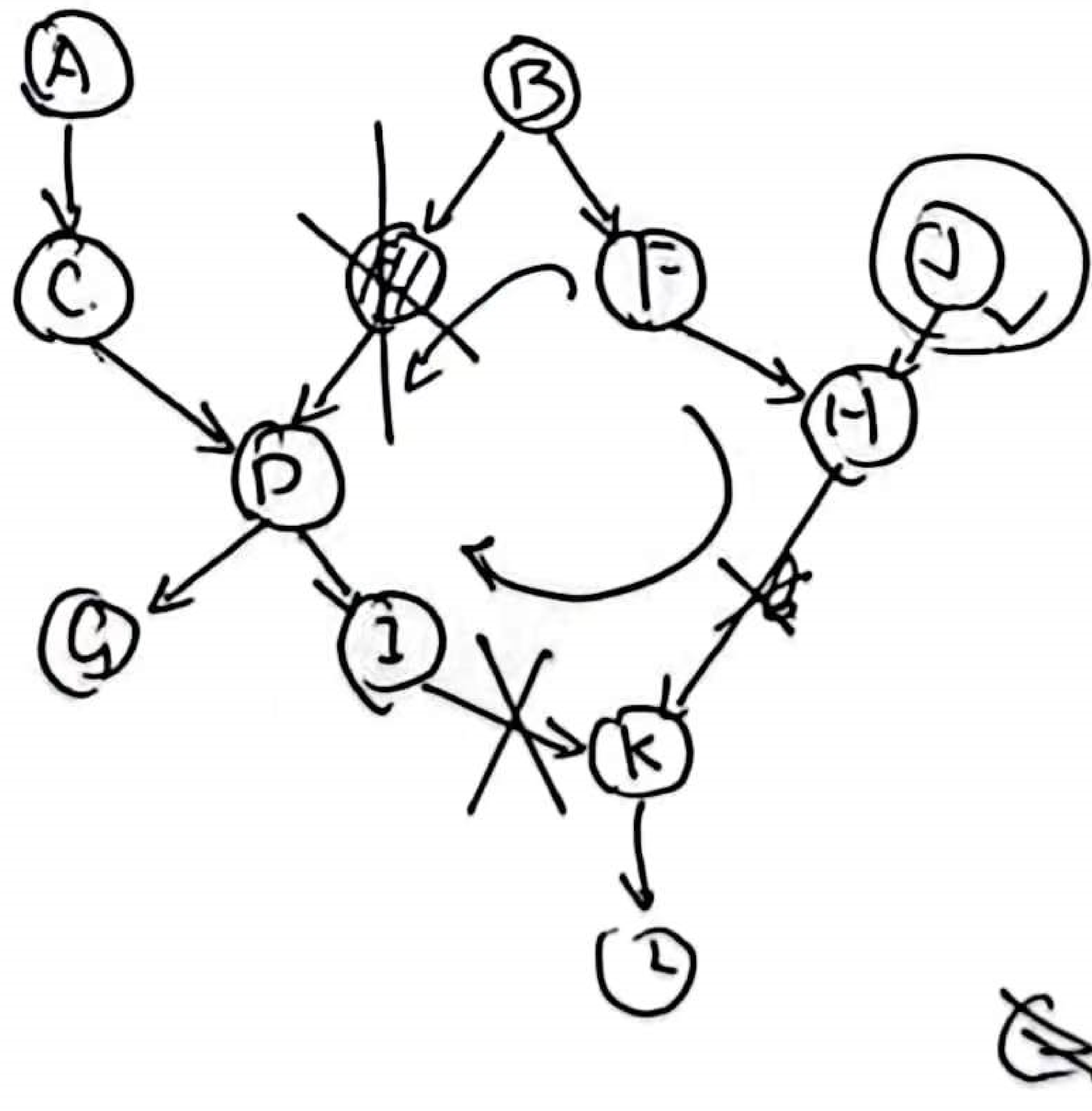
C	P(S)
t	0.1
f	0.5

C	P(R)
t	0.8
f	0.2

S	R	P(W)
t	t	0.99
t	f	0.90
f	t	0.90
f	f	0.00

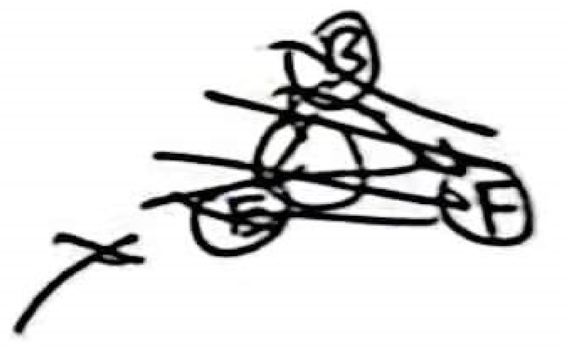
Problem 4 According to the following Bayesian Network

(a) List all the variables that are d-separated from F given E.



given E.

~~F ⫫ B | E.~~



inactive Triples

$B \to E \to D$ $B \perp\!\!\!\perp D \mid E$

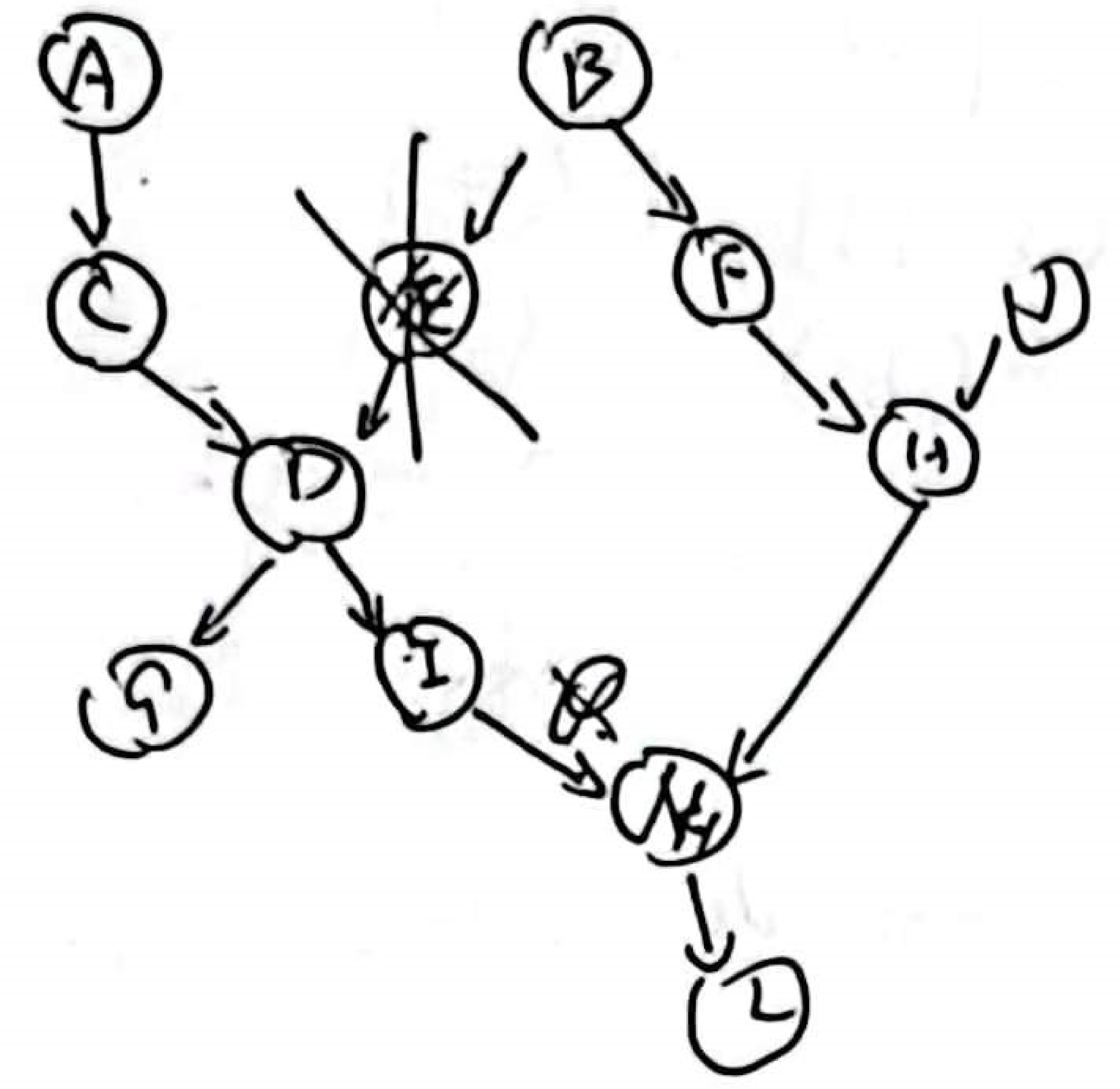
$F \to H \leftarrow J$ $F \perp\!\!\!\perp J \mid E$

$I \to K \leftarrow H$ $I \perp\!\!\!\perp H \mid E$

~~G~~ are d-separated

A. C. D G H J from F giv. E

(b). List all the variables that are d-separated forom F give E and K



inactive triple.

$B \rightarrow E \rightarrow D$ $\quad B \perp\!\!\!\perp D \mid E, K$

$F \rightarrow H \leftarrow J$ $\quad F \perp\!\!\!\perp J \mid E, K$

$H \rightarrow K \rightarrow L$ $\quad H \perp\!\!\!\perp L \mid E, K$

J, L are d-separated from F given E and K