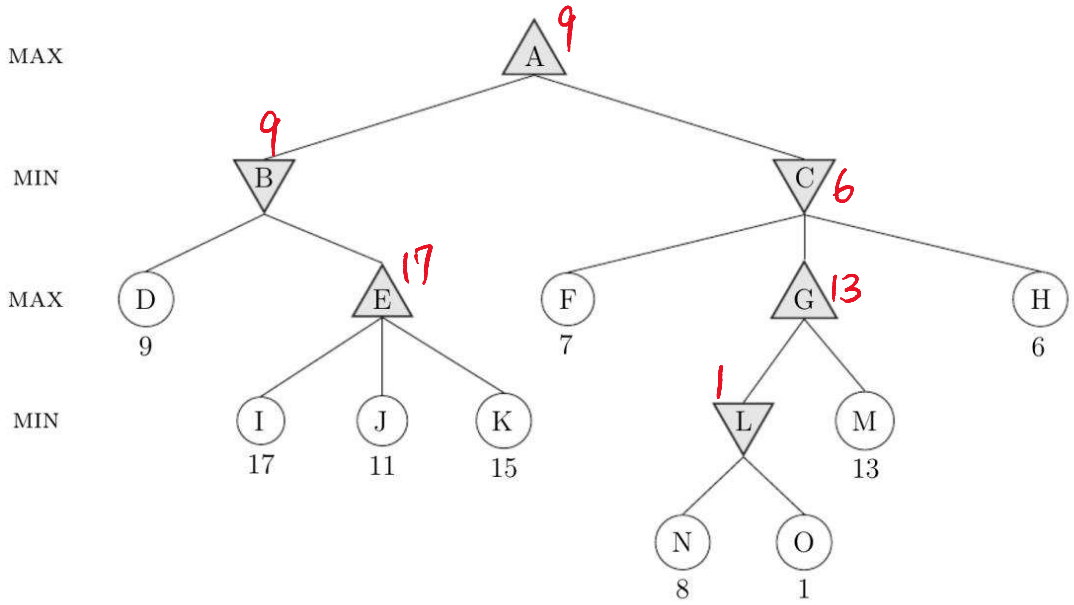
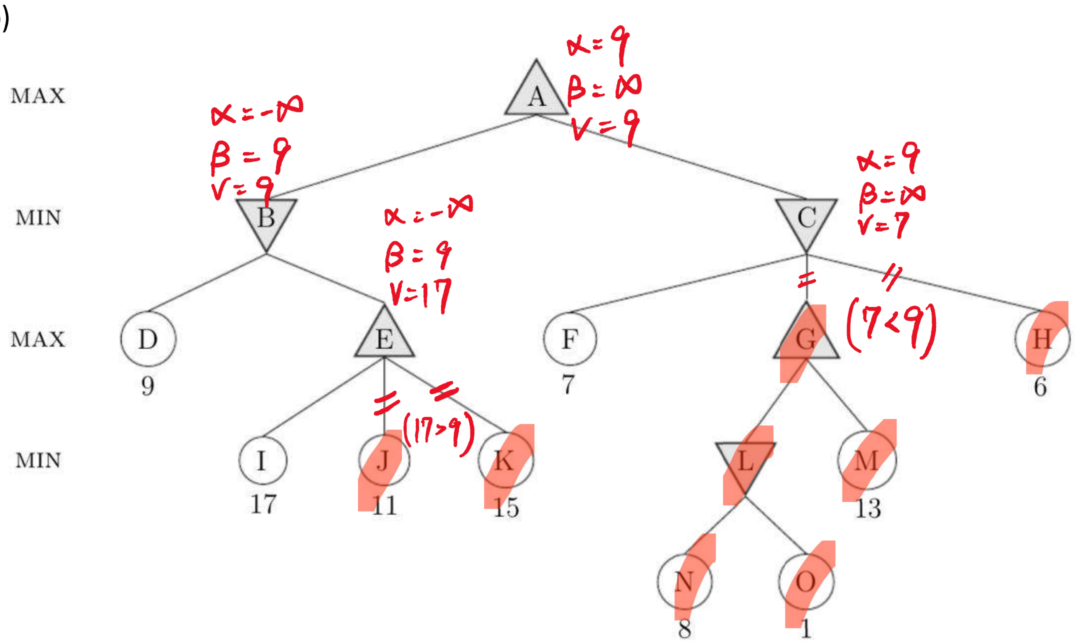
FAI 2023 HW2 Solution

Problem 1

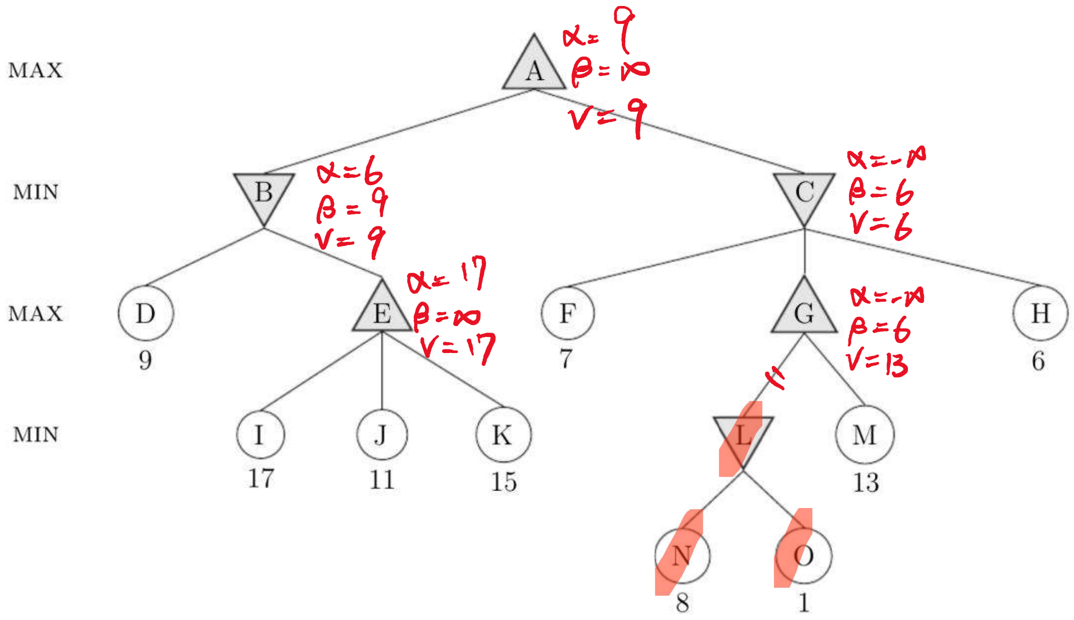
a)



b)



c)



Problem 2

(a.1)

$((((p \to q) \to \neg p) \to \neg q) \to \neg r) \to r$

$((((\neg p \lor q) \to \neg p) \to \neg q) \to \neg r) \to r$ (by implication elimination)

$(((\neg (\neg p \lor q) \lor \neg p) \to \neg q) \to \neg r) \to r$

$((((p \land \neg q) \lor \neg p) \to \neg q) \to \neg r) \to r$

$(((p \land q) \lor \neg q) \to \neg r) \to r$

$((p \lor \neg q) \to \neg r) \to r$

$(\neg (p \lor \neg q) \lor \neg r) \to r$

$\neg (\neg (p \lor \neg q) \lor \neg r) \lor r$

$((p \lor \neg q) \land r) \lor r$

r

(a.2)

$(p \to (q \to r)) \to ((p \to \neg r) \to (p \to \neg q))$

$(\neg p \lor (\neg q \lor r)) \to ((\neg p \lor \neg r) \to (\neg p \lor \neg q))$

$(\neg p \lor (\neg q \lor r)) \to ((p \land r) \lor (\neg p \lor \neg q))$

$\neg (r \lor \neg p \lor \neg q) \lor (r \lor \neg p \lor \neg q)$

1

(b)

$((A \land q) \to \neg p) \to ((p \to \neg q) \to A)$

$(\neg (A \land q) \lor \neg p) \to ((\neg p \lor \neg q) \to A)$

$(\neg (A \wedge q) \vee \neg p) \rightarrow ((p \wedge q) \vee A)$

$((A \wedge q) \wedge p) \vee ((p \wedge q) \vee A)$

A being True can make this formula always be true.

(c)

$(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$

or

$(\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r)$

(d)

$(r \rightarrow A) \equiv (r \rightarrow (p \wedge q))$ and $(A \rightarrow r) \equiv (\neg(p \vee q) \rightarrow r)$

When r is true, $A = (p \wedge q)$ makes the formulae equivalent.

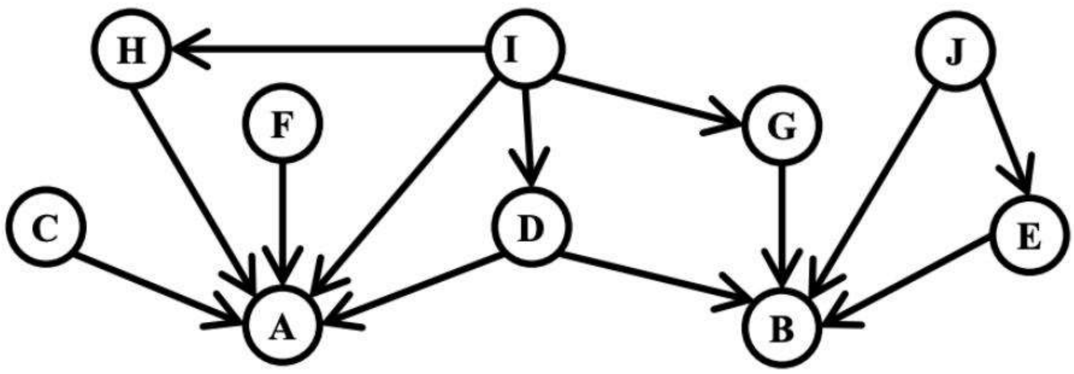
When r is false $A = \neg(p \vee q)$ makes the formulae equivalent.

Therefore, we let

$A = (r \wedge (p \wedge q)) \vee (\neg r \wedge \neg(p \vee q))$, which makes it be $(p \wedge q)$ when r is true and $\neg(p \vee q)$ when r is false.

Problem 3

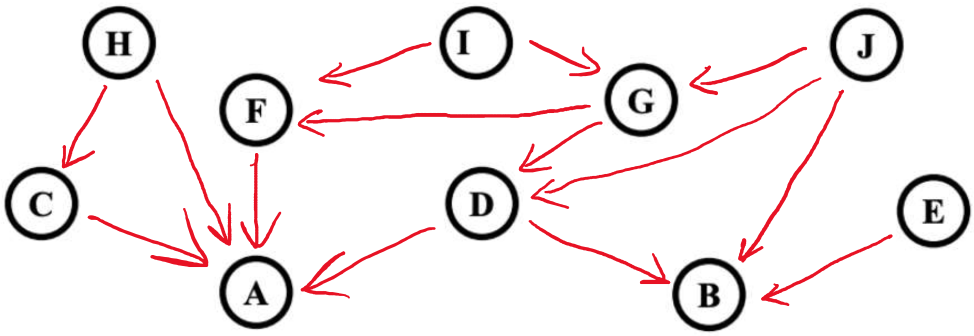
(a)



P(A|C,H,F,I,D) P(C) P(H|I) P(F) P(D|I) P(I) P(B|D,G,J,E) P(G|I) P(J) P(E|J)

(b)

$$P(A|C,D,F,H)P(B|D,E,J)P(C|H)P(D|G,J)P(E)P(F|G,I)P(G|I,J)P(H)P(I)P(J)$$



(c)

P(C = f) P(R = f | C = f) P(S = t | C = f) P(W = t | R = f, S = t)

Problem 4

(a) A, C, D, G, I, J

(b) L