ICPSR Regression II - Problem Set 3

Ziyuan Gao

August 2024

1 Autocorrelation

1.1

```
R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3_data/ > #1. Begin with OLS routine. Save your residuals.

> cat("Residuals:\n", residuals, "\n")

Residuals:

0.626252 0.5679491 0.3053218 0.533343 -0.05124992 0.142686 -0.1197168 0.3623138 0.6099727

0.7707976 0.7045297 0.2160144 0.1271295 0.06613503 -0.5648776 -0.2603274 -0.07635913 -0.31
61457 -0.5492737 -0.8739607 -0.9608508 -0.8330245 -1.085246 -0.9498431 -0.6955765 -0.20110
63 -0.1636471 -0.6306495 -0.1980108 0.130043 0.09061802 0.001366746 -0.291563 -0.1753669 -0.04851307 0.01520306 -0.4898697 -0.2268086 -0.5019831 -0.1592811 0.3109061 -0.1482657 0.3
337843 0.5879218 0.03374268 0.2099091 0.0531219 0.294991 0.4495031 0.3546462 -0.0386711 0.4606155 0.5944734 0.550828 0.5786432 0.3034045 -0.07538247 -0.2322933 0.07871984 0.4529778
```

Figure 1: Residuals from OLS routine

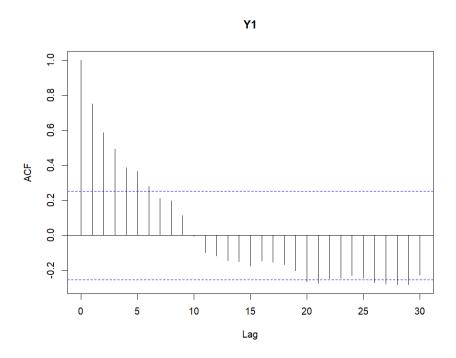


Figure 2: ACF correlogram

Series residuals

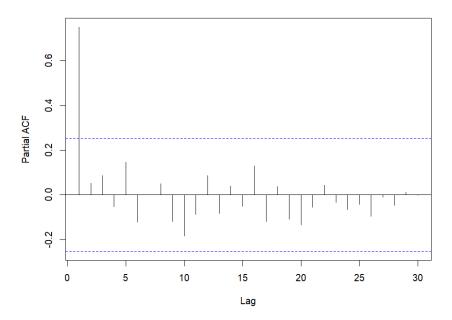


Figure 3: PACF correlogram

1.3

```
Box-Ljung test

data: residuals
X-squared = 104.67, df = 10, p-value < 2.2e-16
```

Figure 4: Box-Ljung test

The null hypothesis of the Box-Ljung test is that there is no autocorrelation up to lag k in the residals. Given the p-value is 2.2e-16, it indicates to reject the null hypothesis and the strong evidence of autocorrelation.

1.4

Figure 5: data transformation in AR1 error process

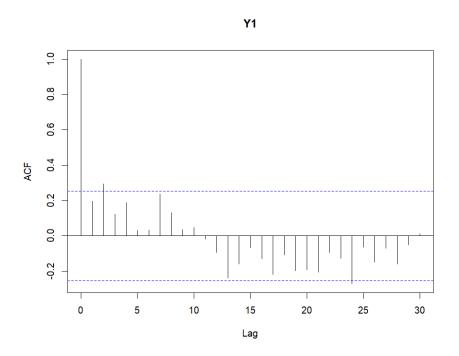


Figure 6: transformed ACF correlogram

Series transformed_residuals

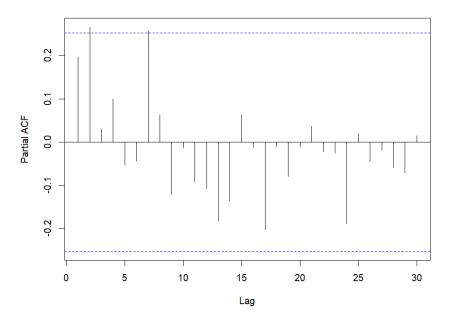


Figure 7: transformed PACF correlogram

```
Box-Ljung test
data: transformed_residuals
X-squared = 16.86, df = 10, p-value = 0.07752
```

Figure 8: transformed Box-Ljung test

From transformed ACF and PACF correlogram, it can be seen that auto correlation has been handled within the significance threshold. The transformed p-value 0.07752 indicates that I failed to reject the null hypothesis, that is there is no autocorrelation after data transformation.

2 Heteroskedasticity

```
R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3_data/ > #1. Begin with OLS routine. Save your residuals.

> cat("Residuals:\n", residuals, "\n")

Residuals:

0.626252 0.5679491 0.3053218 0.533343 -0.05124992 0.142686 -0.1197168 0.3623138 0.6099727

0.7707976 0.7045297 0.2160144 0.1271295 0.06613503 -0.5648776 -0.2603274 -0.07635913 -0.31

61457 -0.5492737 -0.8739607 -0.9608508 -0.8330245 -1.085246 -0.9498431 -0.6955765 -0.20110

63 -0.1636471 -0.6306495 -0.1980108 0.130043 0.09061802 0.001366746 -0.291563 -0.1753669 -

0.04851307 0.01520306 -0.4898697 -0.2268086 -0.5019831 -0.1592811 0.3109061 -0.1482657 0.3

337843 0.5879218 0.03374268 0.2099091 0.0531219 0.294991 0.4495031 0.3546462 -0.0386711 0.

4606155 0.5944734 0.550828 0.5786432 0.3034045 -0.07538247 -0.2322933 0.07871984 0.4529778
```

Figure 9: Residuals from OLS routine

```
> bptest(model_heter)

studentized Breusch-Pagan test

data: model_heter

BP = 13.215, df = 3, p-value = 0.004194
```

Figure 10: Breusch-Pagan Test result

Clearly the p-value (0.004194) from the Breusch-Pagan test is far less than 0.05, I reject the null hypothesis, indicating heteroskedasticity.

```
R 4.4.0 F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw.
> aux_model_X2 <- lm(residuals_sq ~ X2, data = data_heter)</pre>
> summary(aux_mode1_X2)
Call:
lm(formula = residuals_sq ~ X2, data = data_heter)
Residuals:
             1Q Median
   Min
                              3Q
                                     Max
-0.6915 -0.3240 -0.1085 0.3266 1.3576
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.4063
                          0.2617 -1.553 0.125949
              0.3865
                          0.1097
                                  3.525 0.000834 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.434 on 58 degrees of freedom
Multiple R-squared: 0.1764,
                                 Adjusted R-squared: 0.1622
F-statistic: 12.43 on 1 and 58 DF, p-value: 0.0008341
```

Figure 11: glejser test-X2

```
  R 4.4.0 · F;/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3

> aux_model_X3 <- lm(residuals_sq ~ X3, data = data_heter)</pre>
> summary(aux_model_X3)
lm(formula = residuals_sq ~ X3, data = data_heter)
Residuals:
    Min
             1Q Median
                               3Q
-0.5770 -0.3661 -0.1158 0.2424
                                  1.3446
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                                          0.00558 **
(Intercept)
               0.9385
                          0.3260
                                    2.879
                          0.1379
              -0.1911
                                  -1.385 0.17121
X3
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.4705 on 58 degrees of freedom
Multiple R-squared: 0.03204,
                                 Adjusted R-squared: 0.01535
F-statistic: 1.92 on 1 and 58 DF, p-value: 0.1712
```

Figure 12: glejser test-X3

```
R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3
> aux_model_X4 <- lm(residuals_sq ~ X4, data = data_heter)</pre>
> summary(aux_model_X4)
Call:
lm(formula = residuals_sq ~ X4, data = data_heter)
Residuals:
                              3Q
             1Q Median
   Min
-0.5282 -0.3917 -0.1136 0.3131 1.4600
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
                                             0.046 *
                          0.3611
                                    2.039
              0.7362
(Intercept)
             -0.1004
                          0.1480 -0.679
                                             0.500
X4
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.4764 on 58 degrees of freedom
Multiple R-squared: 0.007875, Adjusted R-squared: -0.00923
F-statistic: 0.4604 on 1 and 58 DF, p-value: 0.5001
```

Figure 13: glejser test-X4

I used the Glesjer test to test each predictor individually. Clearly, for X2, a p-value of 0.0008341 strongly indicates heteroskedasticity. While for X3 (p-value 0.1712) and X4 (0.5001) not indicating heteroskedasticity. In conclusion, X2 is the offending variable.

Figure 14: transformation and re-estimation code

```
R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2
> # Compare GLS and OLS results
> summary(model_heter) # OLS results
lm(formula = Y4 \sim X2 + X3 + X4, data = data_heter)
Residuals:
     Min
               1Q
                    Median
                                  3Q
                                           Max
-1.39672 -0.48942 0.08721 0.62815 1.03125
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)
              2.5734
                         0.7875
                                  3.268 0.00185 **
                          0.1851 11.375 3.49e-16 ***
X2
              2.1051
X3
              1.3620
                          0.2229
                                  6.110 1.01e-07 ***
                          0.2362 -9.365 4.67e-13 ***
X4
             -2.2116
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.7281 on 56 degrees of freedom
Multiple R-squared: 0.8209,
                               Adjusted R-squared: 0.8113
F-statistic: 85.57 on 3 and 56 DF, p-value: < 2.2e-16
```

Figure 15: OLS summary

```
🕟 R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3_data/ 🕕
> summary(gls_model)
                        # GLS results
Generalized least squares fit by REML
  Model: Y4_transformed ~ X2_transformed + X3_transformed + X4_transformed
  Data: data_heter
      AIC
               BIC
                       logLik
  110.9424 123.0945 -49.47122
Variance function:
Structure: Power of variance covariate
 Formula: ~X2_transformed
 Parameter estimates:
     power
-0.2999196
Coefficients:
                                      t-value p-value
                    Value Std.Error
               -0.3490972 0.4015578 -0.8693571 0.3884
(Intercept)
X2_transformed 0.1597106 0.1788021 0.8932252 0.3756
X3_transformed -0.0617760 0.1813431 -0.3406578 0.7346
X4_transformed 0.0876563 0.1798497 0.4873866 0.6279
Correlation:
               (Intr) X2_trn X3_trn
X2_transformed -0.640
X3_transformed -0.073 -0.287
X4_transformed -0.367 0.009 -0.622
Standardized residuals:
        Min
                                Med
                    Q1
                                              Q3
                                                          Max
-1.56762119 -0.87768709 0.05364205 0.95048461 1.40253507
Residual standard error: 0.6418487
Degrees of freedom: 60 total; 56 residual
```

Figure 16: GLS summary

```
R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3
> summary(aux_model_X2)
lm(formula = residuals_gls ~ X2_transformed, data = data_heter)
Residuals:
                    Median
               1Q
                                  3Q
-0.33667 -0.15515 -0.00459 0.15595 0.39843
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
               0.44548
                           0.11911 3.740 0.000424 ***
(Intercept)
X2_transformed -0.09313
                            0.05995 -1.554 0.125723
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.1912 on 58 degrees of freedom
Multiple R-squared: 0.03995, Adjusted R-squared: 0.0234
F-statistic: 2.414 on 1 and 58 DF, p-value: 0.1257
```

Figure 17: glejser test-X2 transformed

Please noted In data transformation I added 1 to the residuals squared, this is crucial step to ensure the denominator is never zero and always positive, preventing division by very small numbers that could lead to instability in the transformed values.

X2: a p-value of 0.12571 indicates heteroskedasticity has been solved.

Residual Standard Error: The GLS model shows a slightly lower residual standard error (0.6418) compared to the OLS model (0.7281), indicating a better fit after adjusting for heteroscedasticity.

AIC/BIC: The GLS model has lower AIC and BIC values compared to the OLS model. Lower AIC and BIC suggest that the GLS model perform better after accounting for heteroscedasticity. The transformation and the variance modeling applied in GLS appear to have addressed the heteroscedasticity issue,

3 Multicollinearity

```
156
    dataset_Q3 <- read.csv("Singular.csv")</pre>
    head(dataset_Q3)
    y <- as.matrix(dataset_Q3['y'])</pre>
    x <- as.matrix(dataset_Q3[, c('X2','X3','X4','X5','X6','X7')])</pre>
161
162
    model_full \leftarrow lm(y \sim X2 + X3 + X4 + X5 + X6 + X7, data = dataset_Q3)
    summary(model_full)
166
167
    vif_value <- vif(model_full)</pre>
    vif_value
    #Option 1 discard independent variables of multicollinearity
    model_discard <- lm(y \sim X3 + X4 + X5, data = dataset_Q3)
    summary(model_discard)
    vif_value_discard <- vif(model_discard)</pre>
    vif_value_discard
    anova(model_discard, model_full)
    AIC(model_discard, model_full)
    BIC(model_discard, model_full)
    pca <- prcomp(dataset_Q3[, c('X2', 'X3', 'X4', 'X5', 'X6', 'X7')], scale. = TRUE)</pre>
    pca_scores <- pca$x
    model_pca <- lm(y ~ pca_scores[, 1:3], data = data.frame(y = dataset_Q3$y, pca_scores))</pre>
    summary(model_pca)
188
    anova(model_pca, model_full)
    AIC(model_pca, model_full)
    BIC(model_pca, model_full)
```

Figure 18: Q3 code

Workflow

1. Fit the model.

2. Diagnose multicollinearity using Variance Inflation Factor (VIF). It can be seen there is strong evidence of multicollinearity issue in variable X2, X6, X7 as they have high vif value.

Figure 19: VIF

- 3. To deal with collinear explanatory variables, I tried two options:
 - Option 1: Discard independent variables exhibiting multicollinearity.
 - Option 2: Apply Principal Component Analysis (PCA).
- 4. After transforming the data, I compared the results with the original model using ANOVA, AIC, and BIC to determine the best model with the best estimates.

Option 1: Discard independent variables exhibiting multicollinearity.

From ANOVA test, The null hypothesis for the F-test is that the new model improve the model fit. However, the extremely small p-value 2.2e-16 leads us to reject this null hypothesis, indicating that discarding high multicollineared variables X1 X3 X4 performs worse on the model fit. Also, from AIC and BIC, the new model discarding high multicollineared variables has both a much higher AIC (860.5856) and a higher BIC (870.1458) compared to the original model with AIC (758.2161) and BIC(773.5123), indicating that the new model performed much worse then the original one. So discarding variables is not good operation.

```
vif_value_discard
      x3
 003330 1.003071 1.001454
> anova(model_discard, model_full)
Analysis of Variance Table
Model 1: y \sim X3 + X4 + X5
Model 2: y
           \sim X2 + X3 + X4 + X5 + X6 + X7
              RSS Df Sum of Sq
  Res.Df
                   3 63356199 110.88 < 2.2e-16 ***
2
      43
         8190352
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> AIC(model_discard, model_full)
              df
                       AIC
               5 860.5856
model_discard
model_full
               8 758.2161
> BIC(model_discard, model_full)
model_discard
                 870.1458
               5
model_full
               8
                 773.5123
```

Figure 20: ANOVA AIC BIC for op1

Option 2: Apply Principal Component Analysis (PCA)

From ANOVA test, The null hypothesis for the F-test is that the new model improves the model fit. After PCA, the p-value 0.3457 fail to reject this null hypothesis, indicating that PCA improve the model fit. From AIC and BIC, the new model after PCA has both a lower AIC (757.2622) and a lower

BIC (770.6464) compared to the original model with AIC (758.2161) and BIC(773.5123), indicating it performs better then the original model. Therefore, we pick PCA for data transformation leading to a good fit.

```
> anova(model_pca, model_full)
Analysis of Variance Table
Model 1: y \sim pca_scores[, 1:5]
Model 2: y \sim X2 + X3 + X4 + X5 + X6 + X7
             RSS Df Sum of Sq
  Res.Df
                                    F Pr(>F)
      44 8363524
      43 8190352
                  1
                        173172 0.9092 0.3457
2
 AIC(model_pca, model_full)
           df
                    AIC
            7 757.2622
model_pca
model_full 8 758.2161
> BIC(model_pca, model_full)
           df
                    BIC
            7 770.6464
model_pca
model_full 8 773.5123
```

Figure 21: ANOVA AIC BIC for op2

4 Model Specification

4.1

I build two model. Model 1 is Life expectancy - People/TV. Model 2 is Life expectancy - People/TV + People/ physician. Clearly Model 1 is nested in Model 2.

From ANOVA test, The null hypothesis for the F-test is that the nested model/Model 1 fits better then Model 2. The p-value 0.043 reject this null hypothesis, indicating that Model 2 performs better.

From AIC and BIC, Model 2 has both a lower AIC (260.1059) and a lower BIC (266.8614) compared to Model 1 (262.5792) and BIC(267.6459), indicating it performs better then the nested model. Therefore, we pick Model 2.

Figure 22: comparision of two nested models code

```
R 4.4.0 F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3_data/
> anova(model1, model2)
Analysis of Variance Table
Model 1: V2 ~ V3
Model 2: V2 ~ V3 + V4
  Res.Df
            RSS Df Sum of Sq
                                     F Pr(>F)
      38 1430.1
2
      37 1278.8
                 1
                        151.31 4.3781 0.04332 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> AIC(model1, model2)
       df
                ATC
model1 3 262.5792
model2 4 260.1059
> BIC(model1, model2)
       df
model1 3 267.6459
model2 4 266.8614
```

Figure 23: ANOVA AIC BIC

4.2

I build two model. Model A is y - X3 + X4. Model B is y - X3 + X5. Model C is y - X5 + X6 + X7. Clearly Model B should be "overlapping non-nested" with Model A, and Model C should be "strictly non-nested" with respect to Model A.

From AIC and BIC, Model C has both the lowest AIC (848.0703) and the lowest BIC (857.6305) compared to Model A and Model B, indicating it performs best as it achieves a better trade-off between fit and complexity.

Figure 24: comparision of three non-nested models code

```
> print(c(AIC_A = aicA, AIC_B = aicB, AIC_C = aicC))
    AIC_A     AIC_B     AIC_C
862.8973 875.4680 848.0703
> print(c(BIC_A = bicA, BIC_B = bicB, BIC_C = bicC))
    BIC_A     BIC_B     BIC_C
870.5454 883.1161 857.6305
```

Figure 25: AIC BIC

5 Logistic Regression and GLM

```
239 dataset_Q5 <- read.csv("eo_survey.csv")</pre>
240 head(dataset_Q5)
     dataset_Q5$party_new <- as.numeric(factor(dataset_Q5$party,</pre>
                                                    "No Party, Independent, Decline to state")))
     dataset_Q5$party <- dataset_Q5$party_new</pre>
     dataset_Q5 <- dataset_Q5 %>% select(-party_new)
     head(dataset_Q5)
     # build the logit model
model_logit <- glm(const ~ approve + ideo + party + inc, data = dataset_Q5, family = binomial)</pre>
     summary(model_logit)
256 predicted_probability <- predict(model_logit, type = "response")[1:2]</pre>
     predicted_probability
260 - calculate_pred_probability <- function(approve,ideo,party,inc){
       X_beta <- -0.07649+(0.70263*approve)+(-0.15691*ideo)+(-0.24840*party)+(0.03409*inc)
pred_probability <- exp(X_beta)/(1+exp(X_beta))</pre>
        return(pred_probability)
264 - 1
     pred_probability_first <- calculate_pred_probability(1,6,2,7)</pre>
     pred_probability_first
     pred_probability_second <- calculate_pred_probability(3,2,1,11)</pre>
     pred_probability_second
```

Figure 26: Q5 code

```
R 4.4.0 F;/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3_data/
Call:
glm(formula = const ~ approve + ideo + party + inc, family = binomial,
    data = dataset_Q5)
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
                                                0.8864
(Intercept)
             -0.07649
                           0.53533
                                      -0.143
                                       7.794
                                             6.48e-15
approve
               0.70263
                           0.09015
                                                0.0116
ideo
              -0.15691
                           0.06215
                                      -2.525
party
              -0.24840
                           0.11672
                                      -2.128
                                                0.0333
               0.03409
inc
                           0.02823
                                       1.208
                                                0.2272
```

Figure 27: sign and significance of the variables

Obama approval (approve): The coefficient for Obama approval is positive and highly significant (estimate = 0.70263, p = 6.48e-15). This suggests that as respondents' approval of Obama increases, they are significantly more likely to believe that executive orders are constitutional.

Ideology (ideo): The ideology coefficient is negative and significant (estimate = -0.15691, p = 0.0116). This indicates that as respondents' political ideology becomes more conservative, the likelihood of believing that executive orders are constitutional decreases.

Party (party): The coefficient is negative and significant (estimate = -0.24840, p = 0.0333). This means that respondents identifying as Republican or Independent are significantly less likely to believe that executive orders are constitutional compared to Democrats.

Income (inc): The income coefficient is positive but not statistically significant (estimate = 0.03409, p = 0.2272). This indicates that income does not have a significant effect on the belief in the constitutionality .

```
> predicted_probability
1 2
0.3604285 0.8634394
```

Figure 28: predicted probability for the first two observations

```
260 - calculate_pred_probability <- function(approve,ideo,party,inc){
          X_beta <- -0.07649+(0.70263*approve)+(-0.15691*ideo)+(-0.24840*party)+(0.03409*inc)
          pred_probability <- exp(X_beta)/(1+exp(X_beta))</pre>
          return(pred_probability)
       pred_probability_first <- calculate_pred_probability(1,6,2,7)</pre>
       pred_probability_first
  271
       pred_probability_second <- calculate_pred_probability(3,2,1,11)</pre>
 272
       pred_probability_second
249:17
       (Untitled) #
                                                                                                       X Copil
        Terminal
                  Background Jobs
🕟 R 4.4.0 · F:/ICPSRworkshop/02-Regression Analysis II Linear Models/Assignments/Problem Set 3/reg2_hw3_data/ 🖈
 pred_probability_first
[1] 0.3604319
 pred_probability_second
[1] 0.8634411
```

Figure 29: hand-calculate for the predicted probablity

Question 5 Bonus

I tried my best but the plot looks very weird... I attached my code and plots below and do appreciate if I could have any feedback on it!

```
278
     library(ggplot2)
      library(tidyverse)
      library(dplyr)
      dataset_Q5$pred_prob <- predict(model_logit, dataset_Q5, type = "response")</pre>
283
      dataset_Q5_plot <- dataset_Q5 %>%
        mutate(predlow = plogis(const - (1.96 * pred_prob)),
                pred = plogis(const),
                predhigh = plogis(const + (1.96 * pred_prob)))
      dataset_Q5_plot
     dataset_Q5_plot %>%
        ggplot(aes(x = ideo, y = pred_prob)) +
        geom_ribbon(aes(ymin = predlow,
296
                           ymax = predhigh,
                           fill = const, alpha = 0.3) +
298
        geom_line(aes(colour = const), size = 1) +
299
        labs(y = "Predicted Probability",
              x = "Ideology",
              title = "Predicted Probability by Ideology",
              subtitle = "Holding other variables constant",
              color = "Constitutionality",
              fill = "Constitutionality") +
        scale\_color\_manual(values = c("\frac{\mathref{#3368d8}}{\mathref{#3368d8}}", "\frac{\mathref{#33a2d8}}{\mathref{#33a2d8}}"), \\ labels = c("Unconstitutional", "Constitutional")) +
        scale\_fill\_manual(values = c("#3368d8", "#33a2d8"), \\ labels = c("Unconstitutional", "Constitutional")) +
        guides(color = guide_legend(reverse = TRUE)) +
        guides(fill = guide_legend(reverse = TRUE)) +
311
        theme_minimal()
312
```

Figure 30: Buggy code. I do appreciate if I could have any feedback on it!

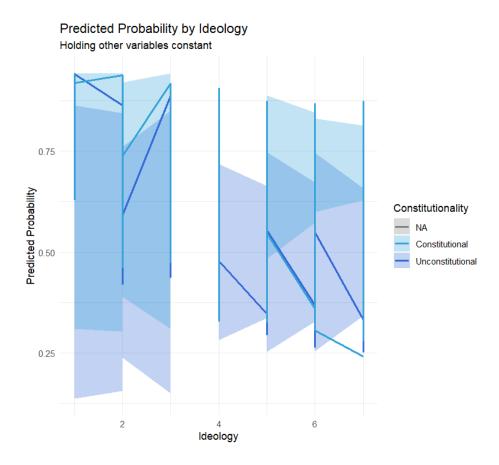


Figure 31: Buggy plot. I tried my best but the plot looks very weird... I do appreciate if I could have any feedback on it!