THEORETICAL MODELING, ICPSR SUMMER II 2024 HOMEWORK 2

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1 Guessing Two-Thirds of the Average

Assume all players choose x, the target number for each player is $\frac{2}{3}x$. Payoff is:

$$|x - \frac{2}{3}x| = \frac{1}{3}x$$

When x = 0, then it reaches Nash equilibria.

2 Voter Location

$$u_1 = a_1 + \frac{a_2 - a_1}{2} = \frac{a_1 + a_2}{2}$$

$$u_2 = (1 - a_2) + \frac{a_2 - a_1}{2} = 1 - \frac{a_1 + a_2}{2}$$

The only Nash equilibrium is $(\frac{1}{2}, \frac{1}{2})$.

3 Solomon's Wisdom

- **If the object belongs to Player 1:**
  - Player 1 declares "mine".
  - Player 2 then declares:
    - "theirs" → The object is given to Player 1.
    - "mine" → Player 2 gets the object, pays M to King Solomon, and Player 1 pays a small amount to King Solomon.
- **If the object belongs to Player 2:**
  - Player 1 declares "theirs".
  - The object is given to Player 2.

4 Display Costs

4.1

	Hawk	Dove
Hawk	$\frac{v-c}{2}$	v
Dove	0	$\frac{v-d}{2}$

When nearly everyone is a Dove, the expected payoffs are

$$V(D) = w_0 + V(D|D) = w_0 + \frac{v-d}{2}$$

$$V(H) = w_0 + V(H|D) = w_0 + v$$

Since $0 \le d \le v$, meaning $v > \frac{v-d}{2}$, Hawk can always invade a population of Doves.
When nearly everyone is a Hawk, the expected payoffs are

$$V(H) = w_0 + V(H|H) = w_0 + \frac{v-c}{2}$$

$$V(D) = w_0 + V(D|H) = w_0$$

Since $0 \le c \le v$, meaning $\frac{v-c}{2} > 0$, Dove can always invade a population of Hawks.

4.2

An equilibrium exists where V(H) = V(D)

$$w_0 + p\frac{v-c}{2} + (1-p)v = w_0 + (1-p)\frac{v-d}{2}$$

$$p\frac{v-c}{2} + (1-p)\frac{v+d}{2} = 0$$

$$p\frac{-c-d}{2} + \frac{v+d}{2} = 0$$

$$p^* = \frac{v+d}{c+d}$$

4.3

Adding a display cost d leads to a more aggressive frequency of hawks and a more pessimistic but realistic scenarios for doves.

5 Cooperation Invasion

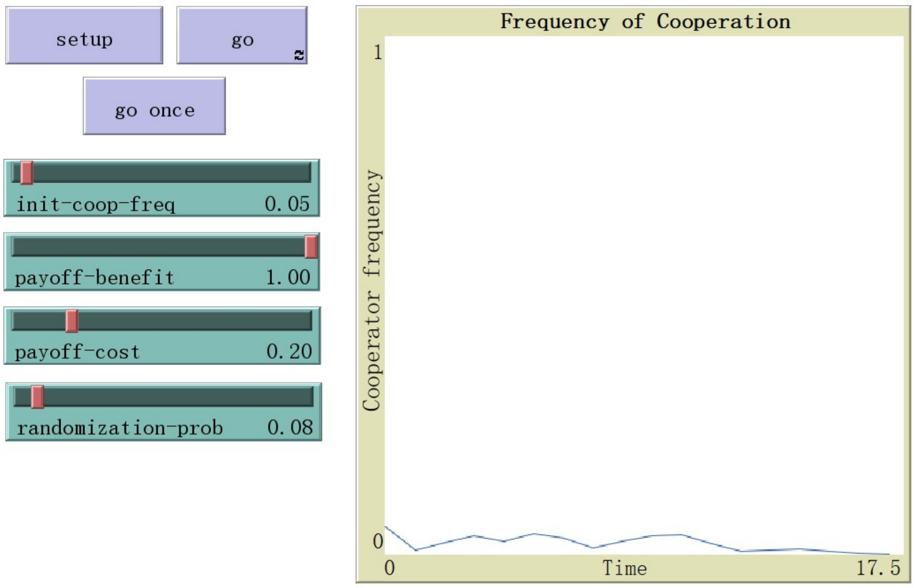

Figure 1: 5% initial cooperators

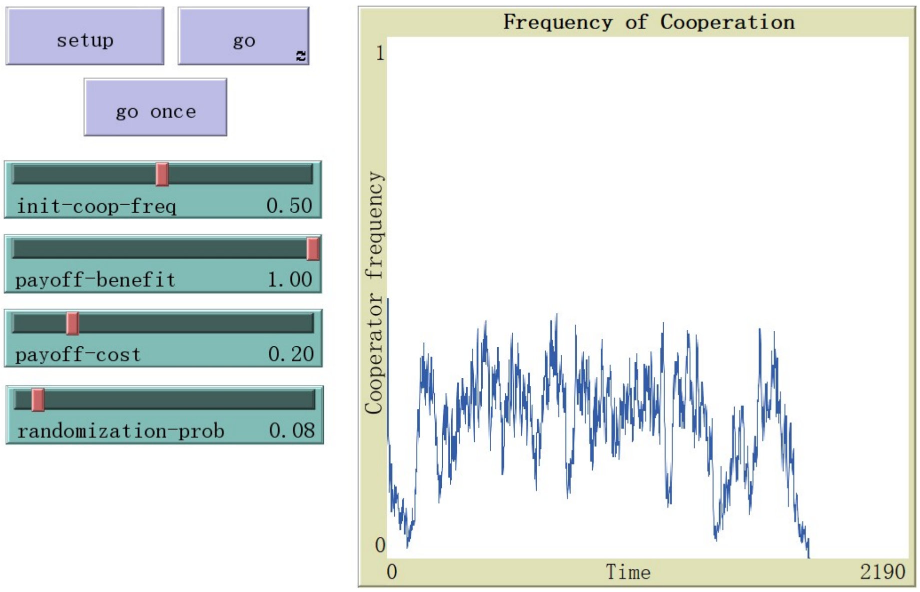

Figure 2: 50% initial cooperators

Starting with 5% cooperators leads to less cooperation compared to starting with 50%. This is because cooperators need clusters to protect themselves from defectors. When cooperators are rare, it's hard to form these clusters. Increased randomization further disrupts cluster formation, making cooperation even more difficult to maintain.

6 Evolutionary Dynamics of TFT

6.1

- When TFT interacts with TFT (probability p):

$$\text{Payoff per round} = b - c$$

$$\text{Total expected payoff} = (b - c) \times \frac{1}{1-w}$$

- When TFT interacts with ALLD (probability $1 - p$):

$$\text{Payoff for the first round} = -c$$

$$\text{Total expected payoff} = -c$$

Therefore,

$$\text{TFT} = p\left((b-c) \times \frac{1}{1-w}\right) + (1-p)(-c)$$

- When ALLD interacts with TFT (probability p):

$$\text{Payoff for the first round} = b$$

- When ALLD interacts with ALLD (probability $1 - p$):

$$\text{Payoff per round} = 0$$

Therefore,

$$\text{ALLD} = p \cdot b$$

6.2

When,

$$\text{ALLD} = \text{TFT}$$

$$p\left((b-c) \times \frac{1}{1-w}\right) + (1-p)(-c) = p \cdot b$$

$$p^* = \frac{(1-w)c}{w(b-c)}$$

6.3

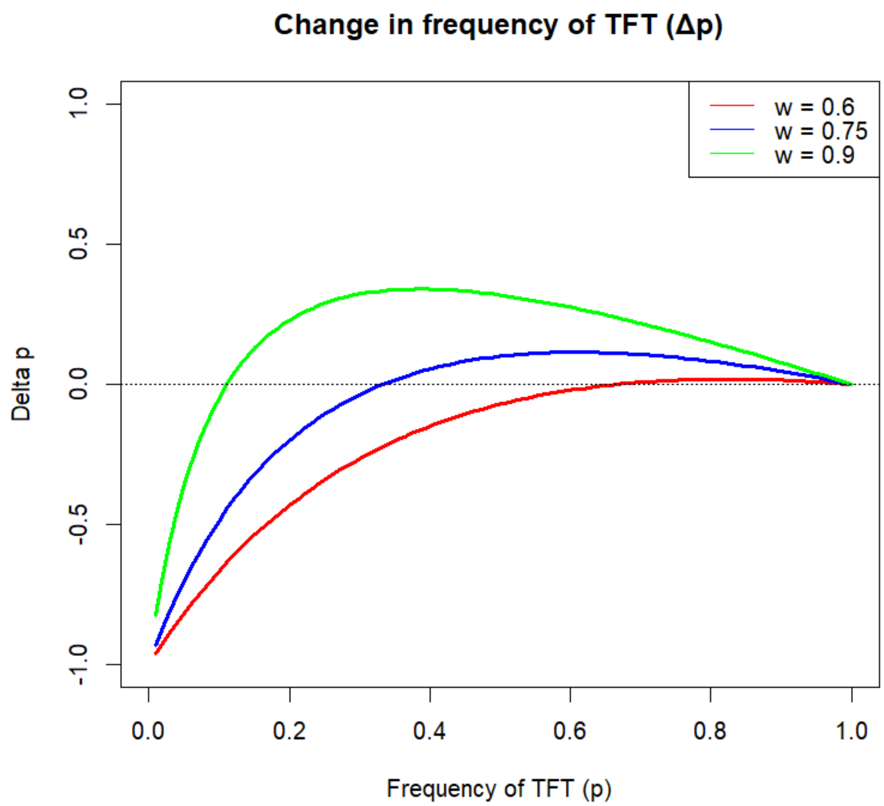

Figure 3: Change in frequency of TFT

As w increases, the basin of attraction for ALLD decreases.

7 Simple the bESSt

7.1

When Norm 1 is common ($p \approx 1$):

The expected payoff for a Norm 1 player:

$$V_1 \approx 1 + \delta + g$$

The expected payoff for a rare Norm 2 player:

$$V_2 \approx 1 + g$$

Since $\delta > 0$:

$$1 + \delta + g > 1 + g$$

Thus, $V_1 > V_2$, indicating that Norm 2 cannot invade when Norm 1 is common.

When Norm 2 is common ($p \approx 0$):

The expected payoff for a Norm 2 player:

$$V_2 \approx 1$$

The expected payoff for a rare Norm 1 player:

$$V_1 \approx 1 - h$$

Since $h > 0$:

$$1 > 1 - h$$

Thus, $V_2 > V_1$, indicating that Norm 1 cannot invade when Norm 2 is common.

7.2

To find the minimum threshold frequency p^* where the payoffs are equal, $V_1 = V_2$:

$$p(1 + \delta + g) + (1 - p)(1 - h) = p(1 + g) + (1 - p)(1)$$

$$p^* = \frac{h}{\delta + h}$$

8 Structured Coordination

For Structured Population: Norm 1 is more likely to increase in frequency due to the formation of supportive clusters, even when starting below the threshold. For Well-Mixed Population: Norm 1 struggles to increase in frequency when starting below the threshold because agents do not benefit from localized support. I set the random-seed to 123 for both clusters and the following screenshots show the comparison.

8.1

```
to evolve
  ask turtles[
  let best-neighbor max-one-of (turtles-on neighbors4) [payoff]
  if ([payoff] of best-neighbor) > payoff ;;if their payoff is better than mine, imitate them
    [set norm1? [norm1?] of best-neighbor]
  ]
end
```

Figure 4: modified code

8.2

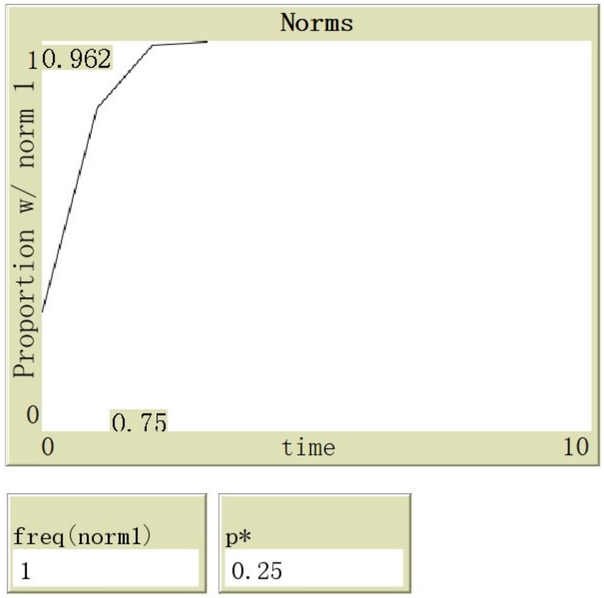

Figure 5: Frequency change in terms of norm1 - 4 neighbours

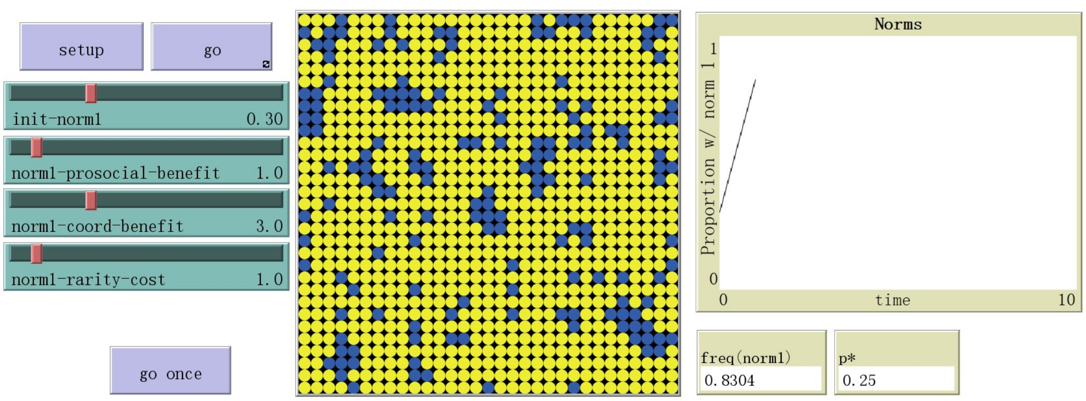

Figure 6: 0.25-4 neighbours-step1

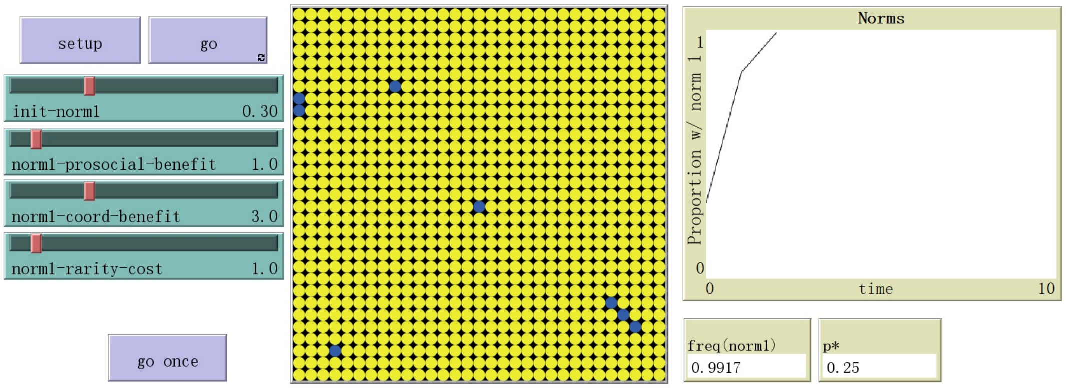

Figure 7: 0.25-4 neighbours-step2

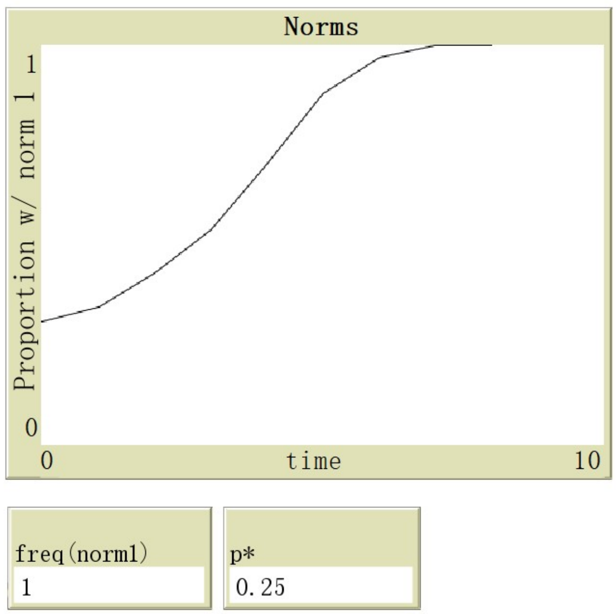

Figure 8: Frequency change in terms of norm1 - mixed

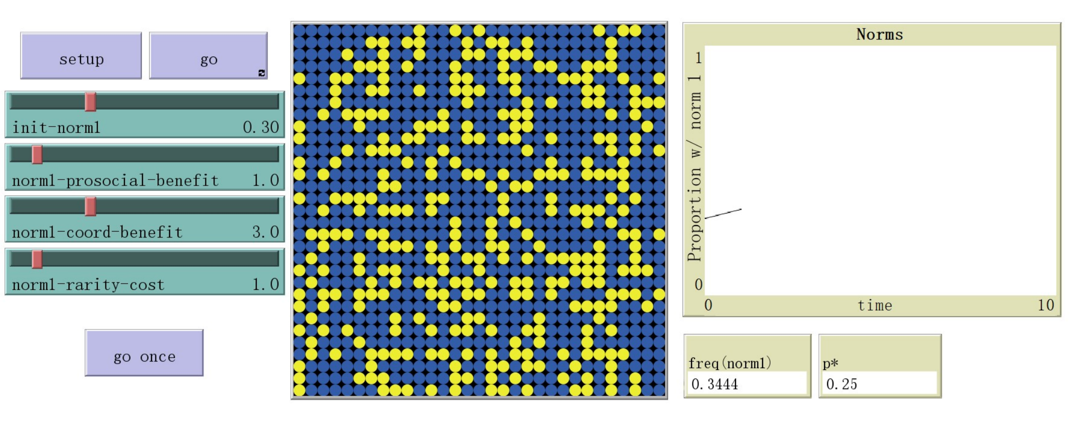

Figure 9: 0.25-mix-step1

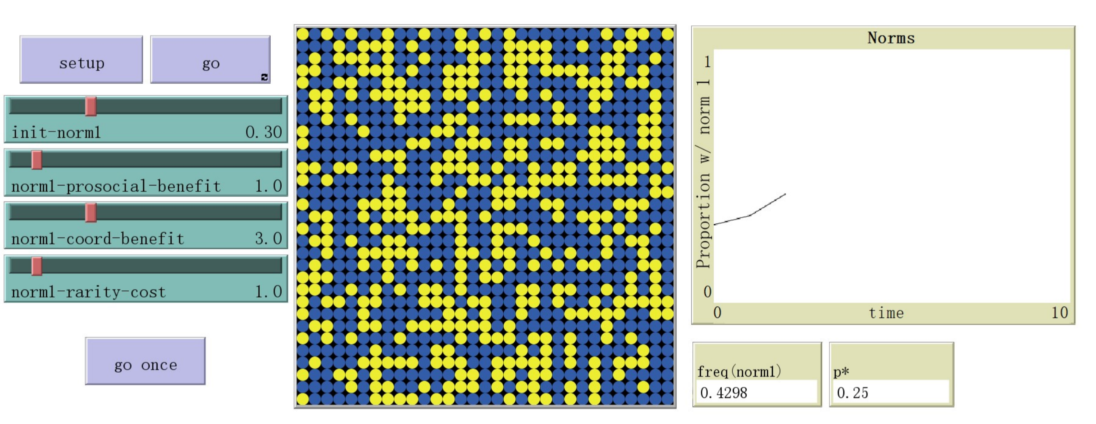

Figure 10: 0.25-mix-step2