Theoretical modeling for social behavior

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1 Model 1: Vaccine Efficacy in the SIS Model

This research analyzes the SIS model incorporating vaccination with efficacy e, where a fraction V of the population is vaccinated. The objective is to determine the vaccination threshold V^* required for herd immunity, accounting for imperfect vaccine efficacy.

1.1 Model Overview

The rate of new infections is governed by the following equation:

$$\Delta I = \tau I \left(1 - \frac{I}{N}\right) ((1 - V) + V(1 - e)) - \gamma I$$

where:

- ΔI is the change in the number of infected individuals,
- τ is the transmission rate per interaction,
- I is the current number of infected individuals,
- N is the total population size,
- V is the proportion of the population vaccinated,
- e is the vaccine efficacy, i.e., the probability that the vaccine prevents infection,
- γ is the recovery rate of infected individuals.

At the start of an outbreak, when the number of infected individuals is small ($I \approx 0$), the equation simplifies to:

$$\Delta I = \tau I ((1 - Ve)) - \gamma I$$

The infection will not spread if:

$$R_0 (1 - Ve) \leq 1$$

where $R_0 = \frac{\tau}{\gamma}$ is the basic reproduction number. This leads to the threshold vaccination rate for herd immunity:

$$V^* = \frac{1}{e}\left(1 - \frac{1}{R_0}\right)$$

1.2 Threshold Behavior

A plot of V^* as a function of efficacy $e \in [0, 1]$, using $\tau = 0.1$ and $\gamma = 0.05$, demonstrates the relationship between herd immunity and vaccine efficacy. As e increases, the required vaccination proportion decreases, illustrating that higher vaccine efficacy reduces the need for widespread vaccination to achieve herd immunity.

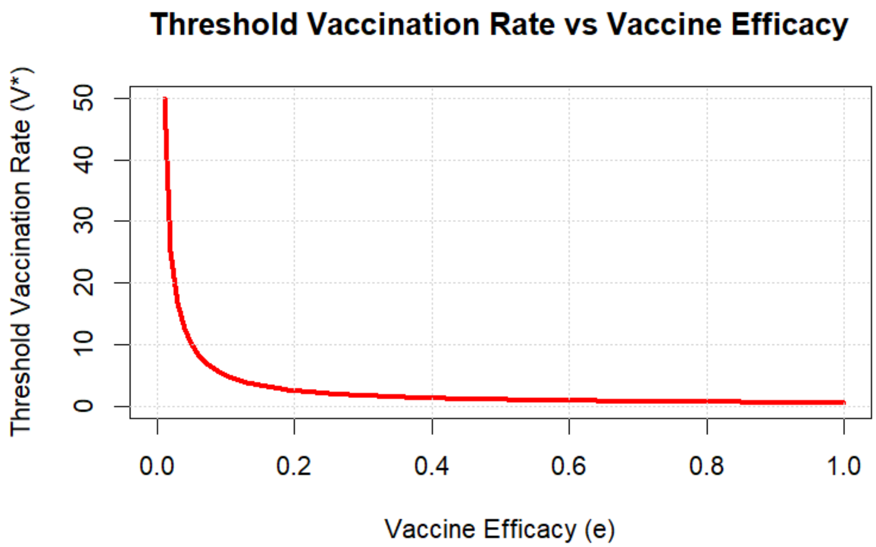

Figure 1: Threshold Vaccination Rate vs Vaccine Efficacy

1.3 Conclusion

The derived threshold $V^* = \frac{1}{e}\left(1 - \frac{1}{R_0}\right)$ shows that vaccine efficacy e significantly impacts the proportion of the population that must be vaccinated to achieve herd immunity. Lower efficacy e requires higher vaccination coverage V, while near-perfect efficacy allows for a lower vaccination rate to achieve herd immunity.

2 Model 2: The Hawk-Dove Game with Display Costs

The Hawk-Dove model is a fundamental concept in evolutionary game theory that describes the conflict between two strategies in the context of resource competition. It explores how individuals can adopt aggressive (Hawk) or passive (Dove) behaviors when competing for a shared resource.

This research analyzes the modified Hawk-Dove game that incorporates display costs incurred by Doves during interactions. The game dynamics change significantly, leading to different equilibria and behavioral outcomes.

2.1 Payoff Matrix

The modified payoff matrix is defined as follows, where:

- v: Value of the resource contested.
- c: Cost incurred by a Hawk in a contest.
- d: Display cost incurred by Doves when they interact with other Doves.

	Hawk	Dove
Hawk	$\frac{v-c}{2}$	v
Dove	0	$\frac{v-d}{2}$

2.2 Payoff Analysis

- **When most are Doves:**

$$V(D) = w_0 + V(D|D) = w_0 + \frac{v-d}{2} \quad \text{(Expected payoff for a Dove)}$$
$$V(H) = w_0 + V(H|D) = w_0 + v \quad \text{(Expected payoff for a Hawk)}$$

  Since $v > \frac{v-d}{2}$, Hawks can invade a population of Doves.

- **When most are Hawks:**

$$V(H) = w_0 + V(H|H) = w_0 + \frac{v-c}{2} \quad \text{(Expected payoff for a Hawk)}$$
$$V(D) = w_0 + V(D|H) = w_0 \quad \text{(Expected payoff for a Dove)}$$

  Given that $\frac{v-c}{2} > 0$, Doves can invade a population of Hawks.

2.3 Mixed Equilibrium

A mixed equilibrium occurs when the expected payoffs of Hawks and Doves are equal:

$$V(H) = V(D)$$

Substituting the expected payoffs leads to:

$$w_0 + p\frac{v-c}{2} + (1-p)v = w_0 + (1-p)\frac{v-d}{2}$$

where:

- p: Frequency of Hawks in the population at equilibrium.

  Simplifying this equation results in:

$$p\frac{v-c}{2} + (1-p)\frac{v+d}{2} = 0$$

  This implies:

$$p^* = \frac{v+d}{c+d} \quad \text{(Frequency of Hawks in the mixed equilibrium)}$$

2.4 Effect of Display Costs

The introduction of display costs alters the behavioral dynamics:

A higher display cost d increases the frequency of Hawks in the population, leading to less cooperation. The mixed equilibrium frequency p^* demonstrates that even a small cost can have substantial effects on population dynamics.

In conclusion, display costs fundamentally shift the equilibrium dynamics of the Hawk-Dove game, resulting in a greater prevalence of aggressive strategies and highlighting the challenges of sustaining cooperation in populations with low initial cooperation levels.

3 Model 3: Dynamic Cooperation Model with TFT and ALLD Strategies

In the evolutionary iterated Prisoner's Dilemma, strategies such as Tit-for-Tat (TFT) and Always Defect (ALLD) play a critical role in shaping the dynamics of cooperation. Understanding these strategies provides insight into the conditions under which cooperation can thrive or fail within a population.

3.1 Strategy Definitions

3.1.1 Tit-for-Tat (TFT)

Tit-for-Tat (TFT) is characterized by its simple yet effective approach to fostering cooperation. Key features of TFT include:

- **Cooperative Initial Move**: TFT begins interactions by cooperating with the opponent.
- **Reciprocal Behavior**: In subsequent rounds, TFT mimics the previous action of its opponent—continuing to cooperate if the opponent cooperates and defecting in response to defection.
- **Forgiveness**: After a defection, TFT will return to cooperation if the opponent also cooperates again.

3.1.2 Always Defect (ALLD)

In contrast, Always Defect (ALLD) represents a more aggressive strategy:

- **Consistent Defection**: ALLD always defects, irrespective of the opponent's actions.
- **Maximization of Immediate Payoff**: This strategy seeks to maximize its own payoff without regard for the long-term impact on cooperation.
- **Lack of Reciprocity**: ALLD does not adapt to the opponent's behavior, leading to a cycle of mutual defection if faced with a cooperative strategy.

3.2 Mathematical Formulation

3.2.1 Variable Definitions

- p: Frequency of TFT in the population.
- b: Payoff received by a player when cooperating with another cooperative player.
- c: Cost incurred by a player when defecting.
- w: Weighting factor that reflects the probability of interacting with a TFT player.

3.2.2 Payoff Calculations

For TFT:

- When TFT interacts with TFT (probability p):

$$\text{Payoff per round} = b - c$$

$$\text{Total expected payoff} = (b - c) \times \frac{1}{1 - w}$$

- When TFT interacts with ALLD (probability $1 - p$):

$$\text{Payoff for the first round} = -c$$

$$\text{Total expected payoff} = -c$$

Combining these results, we derive the expected payoff for TFT:

$$\text{TFT} = p\left((b - c) \times \frac{1}{1 - w}\right) + (1 - p)(-c)$$

For ALLD:

- When ALLD interacts with TFT (probability p):

$$\text{Payoff for the first round} = b$$

- When ALLD interacts with ALLD (probability $1 - p$):

$$\text{Payoff per round} = 0$$

Thus, the expected payoff for ALLD is:

$$\text{ALLD} = p \cdot b$$

3.2.3 Equilibrium Condition

To find the threshold frequency where both strategies have equal fitness, we set:

$$\text{ALLD} = \text{TFT}$$

This leads to:

$$p^* = \frac{(1 - w)c}{w(b - c)}$$

3.3 Implications of Evolutionary Dynamics

The interaction between TFT and ALLD yields important insights into the evolutionary stability of these strategies. In well-mixed populations, both TFT and ALLD can be evolutionarily stable strategies (ESS) against each other. There exists a threshold frequency of TFT (p^*), below which TFT's prevalence will decline and above which it will thrive.

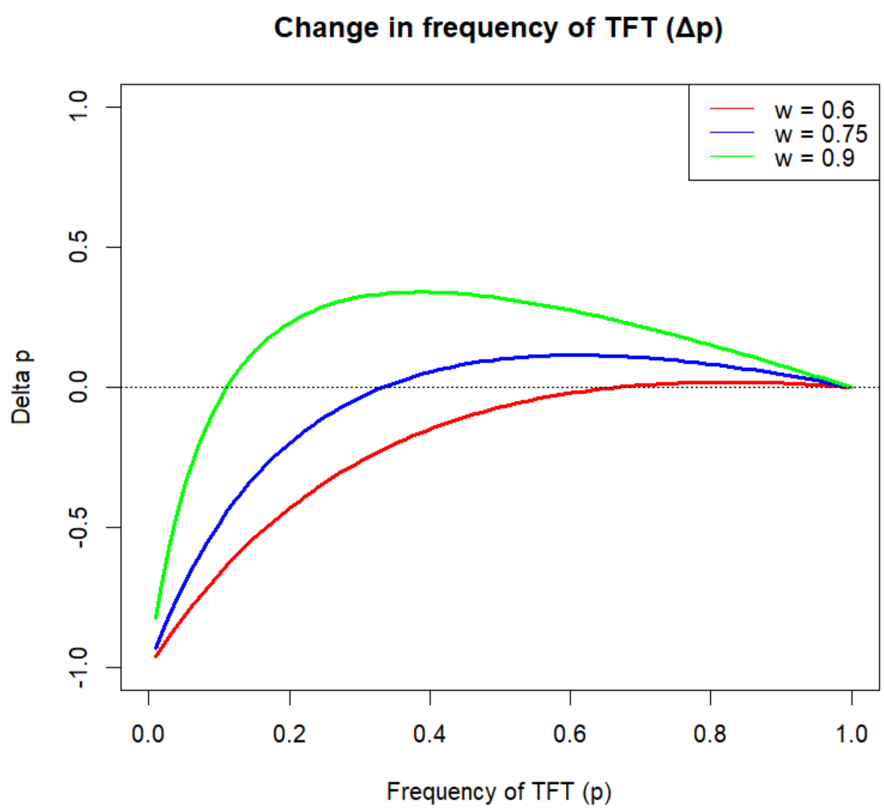

Figure 2: Change in frequency of TFT

As w increases, the basin of attraction for ALLD decreases.

3.4 Conclusion

The analysis demonstrates that TFT can effectively resist invasion by ALLD under certain conditions. As the frequency of TFT increases, the population shifts towards greater cooperation, while the dynamics change as different parameters, such as the cost of defection and baseline payoff, are adjusted. Understanding these dynamics is crucial for comprehending how cooperation can evolve and stabilize in various social and biological contexts.

4 Model 4: Expected Payoff to Social Learnering Model

This research explores the expected payoff to social learners within a population. The aim is to mathematically derive the expected payoff based on various influencing factors, such as baseline fitness, the benefits of learning, and the probabilities of behavior adaptation over generations. By examining these relationships, insights into the evolution of social learning strategies in response to changing environments are provided.

4.1 Expected Payoff Formula

The expected payoff for social learners can be represented mathematically as follows:

$$\tilde{w}_S = w + b$$

where:

- w: The inherent fitness level of an individual, representing its ability to survive and reproduce in its environment.
- b: The additional fitness gain an individual receives by learning from others.

4.2 Adaptive Behavior Over Generations

To factor in the adaptive behavior stemming from generations of learning, the equation is extended as follows:

$$\tilde{w}_S = w + b\sum_{\tau=1}^{\infty} p^{\tau-1}(1-p)(1-u)^{\tau}$$

where:

- p: The likelihood that a behavior stems from individual learning efforts.
- u: The likelihood that the environment changes, affecting the adaptability of learned behaviors.

This summation represents the cumulative benefits of adaptive traits acquired through learning over time, accounting for the probabilities of individual learning p and environmental stability $(1-u)$.

4.3 Incorporating Adaptive Behavior

To further refine the expected payoff formula, adjustments are made for the likelihood that behaviors remain adaptive in the current environment:

$$\tilde{w}_S = w + b(1-p)(1-u)$$

4.4 Deriving K - The Expected Value

The expected value K captures the anticipated outcome of behaviors over generations and is expressed as follows:

$$K = \sum_{\tau=0}^{\infty} p^{\tau}(1-u)^{\tau}$$

This expression considers the persistence of learned behaviors and their impact on individual fitness.

4.5 Recurrence Relation

A recurrence relation for K can be developed:

$$K = p(1-u)[K+1]$$

Rearranging this equation leads to:

$$K[1-p(1-u)] = 1 \quad \Rightarrow \quad K = \frac{1}{1-p(1-u)}$$

4.6 Final Payoff Calculation

The expected payoff formula for social learners, incorporating the derived K, is given by:

$$\tilde{w}_S = w + \frac{b(1-p)(1-u)}{1-p(1-u)}$$

This equation illustrates the combined effects of baseline fitness w, learning benefits b, and adaptive behaviors on social learners' payoffs.

4.7 Equilibrium Condition

To establish relationships between social and individual learning strategies, the expected payoffs can be set equal:

$$\tilde{w}_S = w + b(1-c)$$

where:

- c: The cost associated with learning through individual efforts.

By rearranging, relationships between the proportions of different learner types can be explored:

$$b(1-c)(1-p(1-u)) = b(1-s)(1-p)(1-u)$$

where:

- s: The proportion of agents in the population that engage in consensus learning strategies.

4.8 Final Formulation

Ultimately, the probability of adopting certain behaviors can be expressed as follows:

$$p = \frac{(1-u)(c-s) - (1-c)u}{(1-u)(c-s)}$$

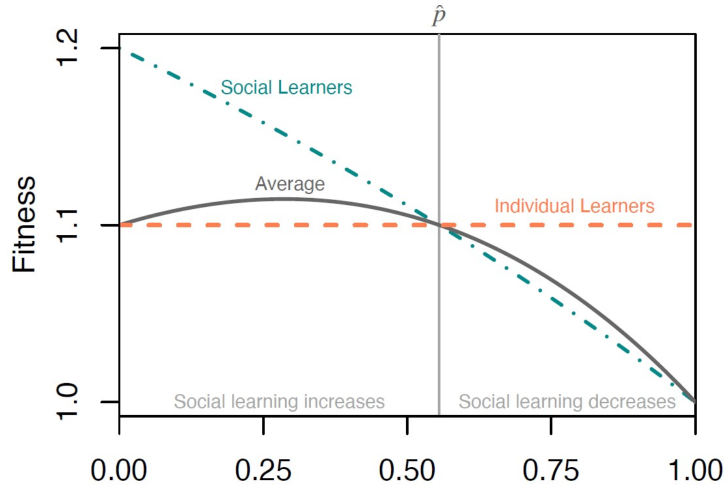

Figure 3: frequency of social learners

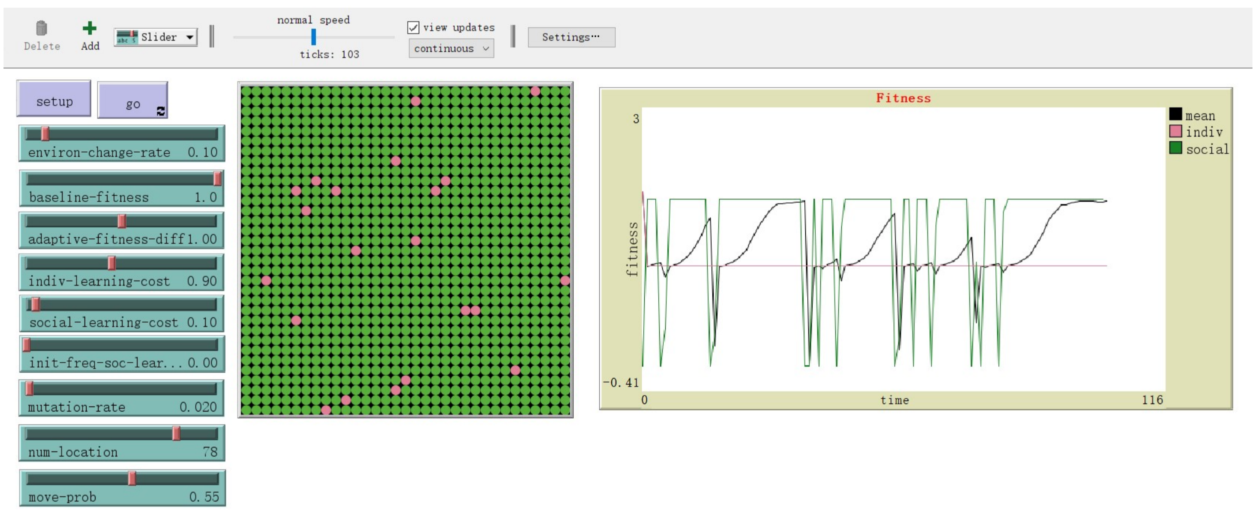

Figure 4: social-learning simulation

4.9 Conclusion

This analysis provides a comprehensive mathematical framework for understanding the expected payoff to social learners in evolving populations. By delineating the relationships between key variables, insights are gained into how learning strategies adapt over time and the factors that influence their success. This understanding is crucial for exploring social dynamics in various contexts, including ecology, economics, and human behavior.