

# Lecture 19 Exercise Solutions

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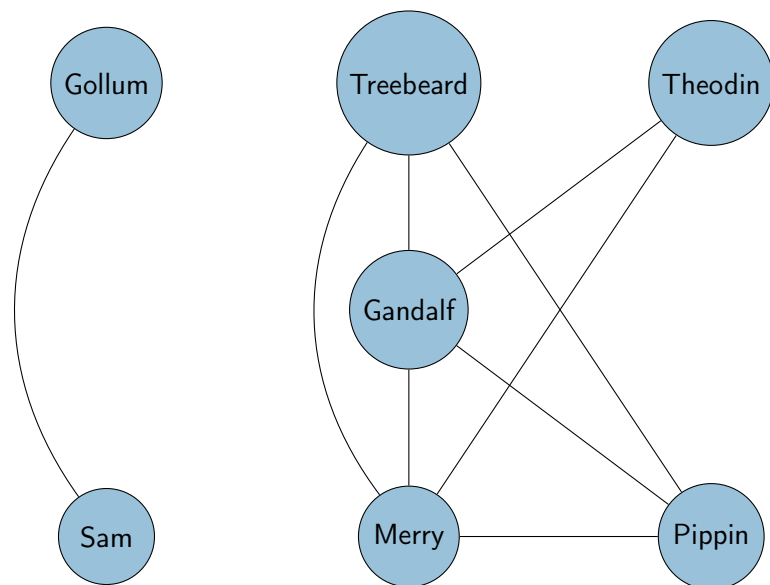
## Exercise 1

- a)  $V = \{\text{CMPT 111, Calculus, CMPT 115, MATH 110, CMPT 270, CMPT 260, CMPT 214, CMPT 280, CMPT 215}\}$   
 $E = \{$   
     $(\text{CMPT 111, CMPT 115}),$   
     $(\text{Calculus, CMPT 270}),$   
     $(\text{CMPT 115, CMPT 260}),$   
     $(\text{CMPT 115, CMPT 270}),$   
     $(\text{CMPT 115, CMPT 214}),$   
     $(\text{MATH 110, CMPT 260}),$   
     $(\text{MATH 110, CMPT 214}),$   
     $(\text{CMPT 270, CMPT 280}),$   
     $(\text{CMPT 214, CMPT 215}),$   
     $\}$
- b) The indegree of CMPT 214 is 2, the indegree of CMPT 280 is 1.
- c) The outdegree of CMPT 115 is 3. The outdegree of MATH 110 is 2.

## Exercise 2

- a) The degree of Gandalf is 6. The degree of Gollum is 2.
- b) Examples of walks from Treebeard to Gollum:  
Treebeard, Gandalf, Theodin, Elrond, Gandalf, Merry, Pippin, Gandalf, Frodo, Gollum.  
Treebeard, Gandalf, Frodo, Sam, Gollum  
Treebeard, Pippin, Merry, Gandalf, Elrond, Sam, Frodo, Sam, Gollum.  
Any sequence of connected nodes that starts with Treebeard and ends with Gollum.
- c) Trails may not repeat edges. Every trail is also a walk, but not all walks are trails. Examples of trails from Theodin to Sam:  
Theodin, Merry, Gandalf, Theodin, Elrond, Gandalf, Frodo, Sam  
Theodin, Elrond, Gandalf, Frodo, Sam  
Note: The first two walks in part b) are trails, but the third is not (it requires two traversals of the [Frodo, Sam] edge).

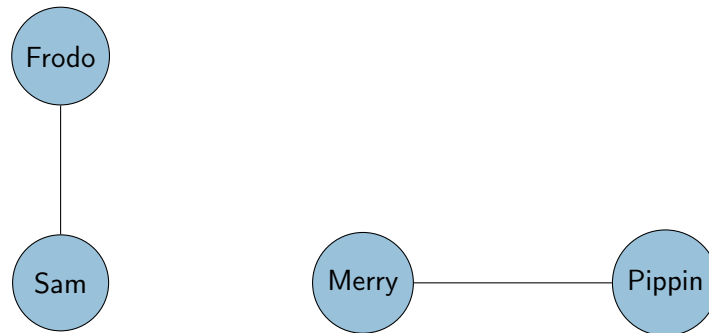
- d) A path may not visit a vertex more than once. Therefore a path also cannot traverse the same edge more than once, because that implies visiting a vertex more than once. Every path is therefore a trail, but not every trail is a path. Some paths from Merry to Frodo are:  
Merry, Gandalf, Frodo  
Merry, Theodin, Elrond, Gandalf, Frodo  
Merry, Pippin, Treebeard, Gandalf, Elrond, Frodo
- e) The shortest path from Gollum to Pippin is:  
Gollum, Frodo, Gandalf, Pippin (length 3).
- f) A Hamiltonian path in the graph:  
Gollum, Frodo, Sam, Elrond, Theodin, Gandalf, Treebeard, Merry, Pippin.
- g) A circuit that starts and ends at Gandalf: Gandalf, Frodo, Sam, Gollum, Frodo, Elrond, Gandalf.
- h) A cycle that starts and ends at Gandalf: Gandalf, Frodo, Gollum, Sam, Elrond, Theodin, Gandalf.
- i) Since there is a path from every node to every other node, the entire graph is a single connected component.
- j) Here's the subgraph:



So it's clear that the graph would have two connected components, one consisting of Same and Gollum, the other consisting of the remaining nodes.

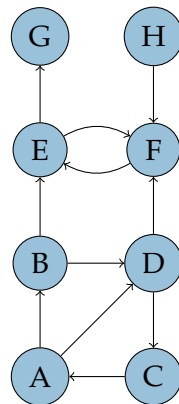
### Exercise 3

The subgraph that includes only the Hobbit nodes is:



It has two connected components.

### Exercise 4



$\{A, B, C, D\}$  is a strongly connected component because there is a path from each node in this set to every other node. No superset of nodes has this property. The other connected components are  $\{E, F\}$ ,  $\{G\}$ , and  $\{H\}$ .  $G$  and  $H$  are their own components because every node has a self-loop (not drawn). If we hadn't made the assumptions about self-loops, then this graph then  $G$  and  $H$  would not belong to any strongly connected component. Notice that each subgraph that is a strongly connected component contains a subset of edges that form a Hamiltonian Cycle of that subgraph!