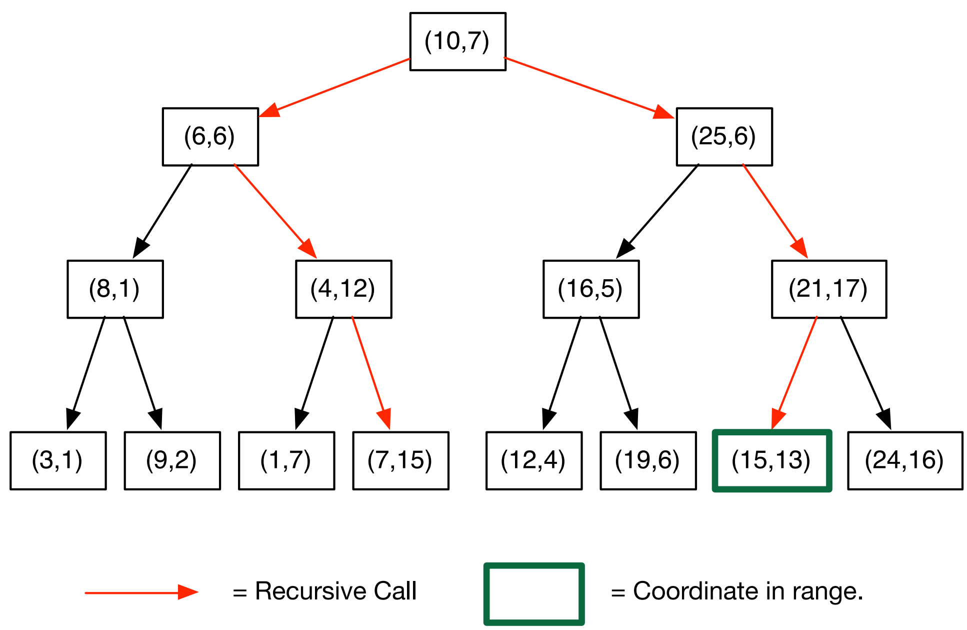
Lecture 18 Exercise Solutions

Mark Eramian

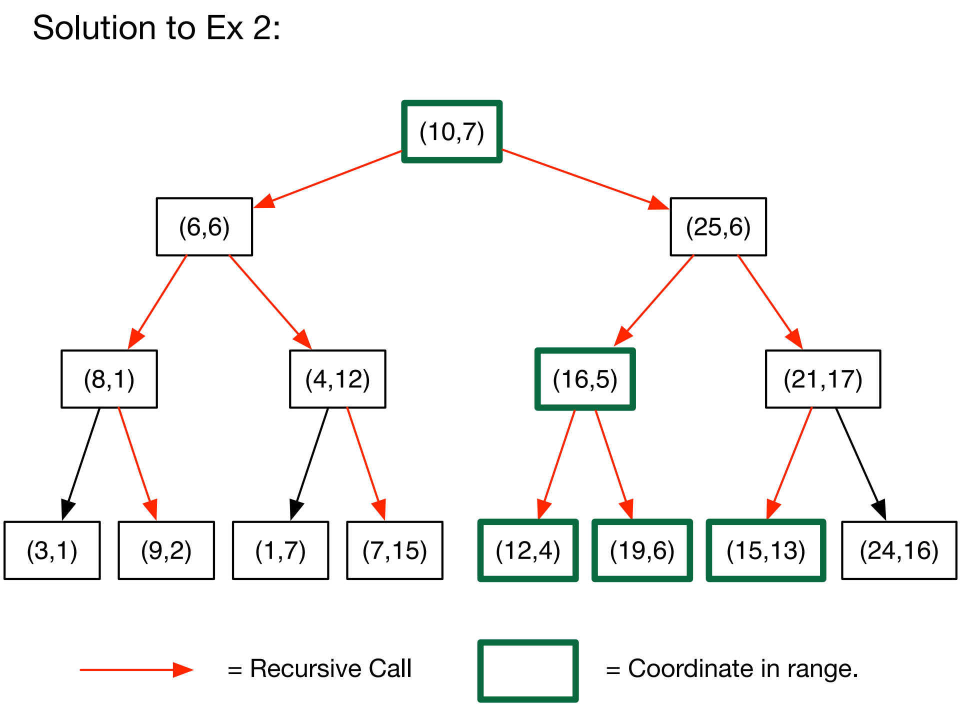
Exercise 1

The red arrows show the recursive calls that are made. All nodes along a red path are visited. The nodes with green outlines show keys that are in-range and are returned.



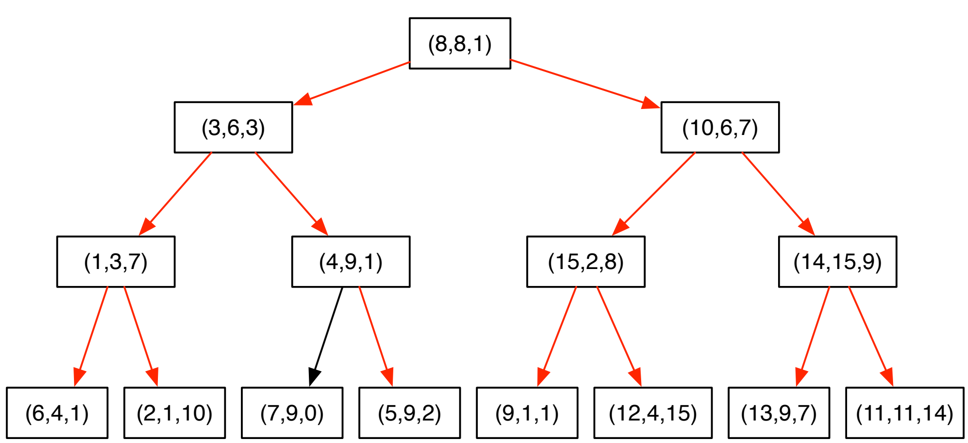
Exercise 2

The red arrows show the recursive calls that are made. All nodes along a red path are visited. The nodes with green outlines show keys that are in-range and are returned.



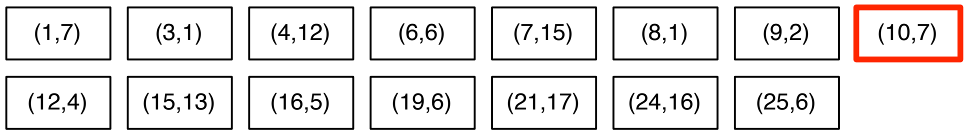
Exercise 3

The red arrows show the recursive calls that are made. All nodes along a red path are visited. The nodes with green outlines show keys that are in-range and are returned. In this case, many nodes were visited, but none of them contained a key for which all three coordinates were in range.

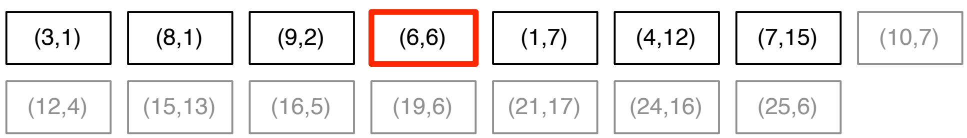


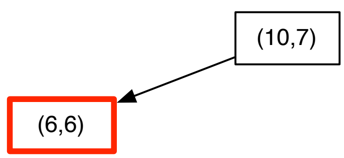
1 Solution to 2-D Tree Build Example

Step 1: Sort the full set of keys on the x-coordinate. The key with the median x-coordinate is (10,7). Make that the root.

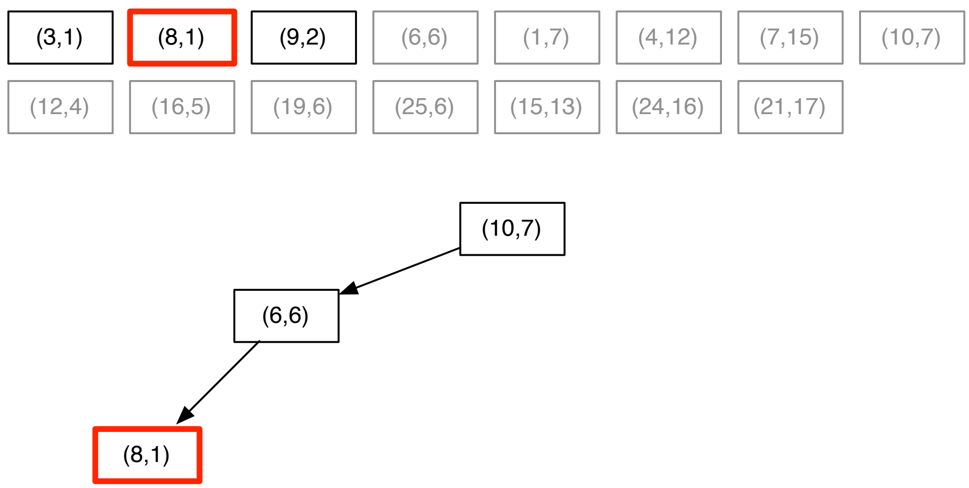


Step 2: We can find the root of the right subtree in a similar fashion. We take the keys from step 1 that were smaller than the median x-coordinate, and sort them by their y-coordinates. From among those keys, the one with the median y-coordinate is the root of the right subtree.

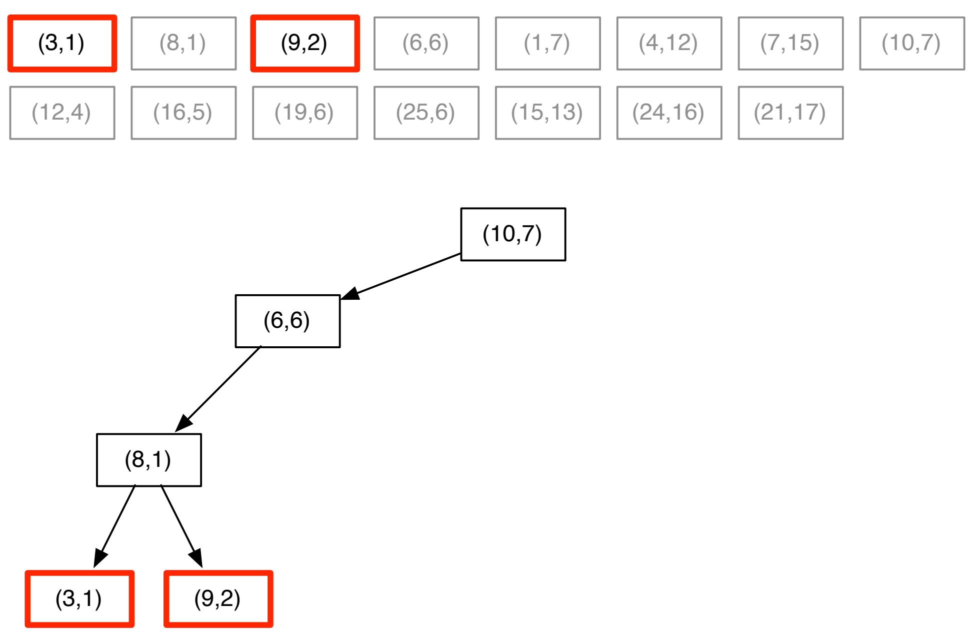




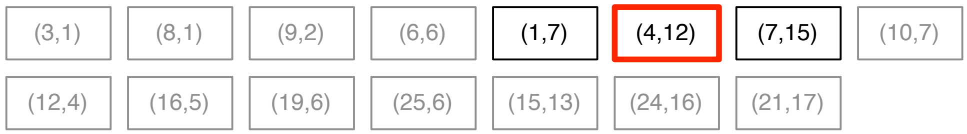
Step 3: From among the set of keys considered in step 2 we recursively consider the keys which are smaller than the median y-coordinate. Since we are about to add nodes on an odd-numbered level of the tree, we sort them by x-coordinate. The median happens to be the key $(8,1)$; this becomes the left child of $(6,6)$.

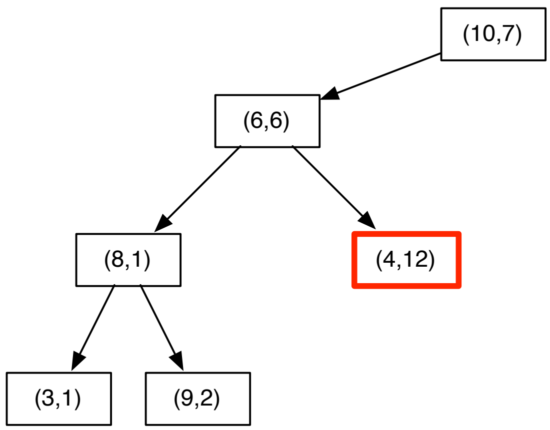


Step 4: Now we recursively consider from among the keys in step 3 those with x-coordinate less than the median. There is only one, so that becomes the left child of the key (8,1). Then we recursively consider from among the keys in step 3 those with x-coordinate greater than the median. Again, there is only one, namely $(9,2)$ so that becomes the right child of $(8,1)$.

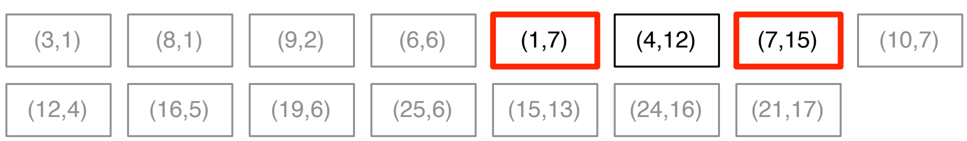


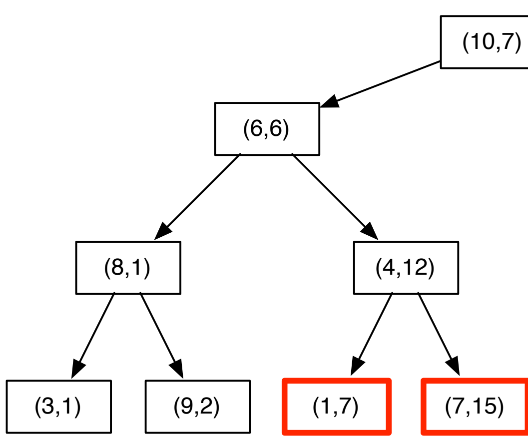
Step 5: Now we are finished processing the left subtree of $(6,6)$, so we recursively consider those nodes from step 2 that were larger than the y-coordinate median. Again we are at an odd-level, so we sort on the x-coordinate and find the median x-coordinate which is $(4,12)$. Thus, $(4,12)$ becomes the right child of $(6,6)$.



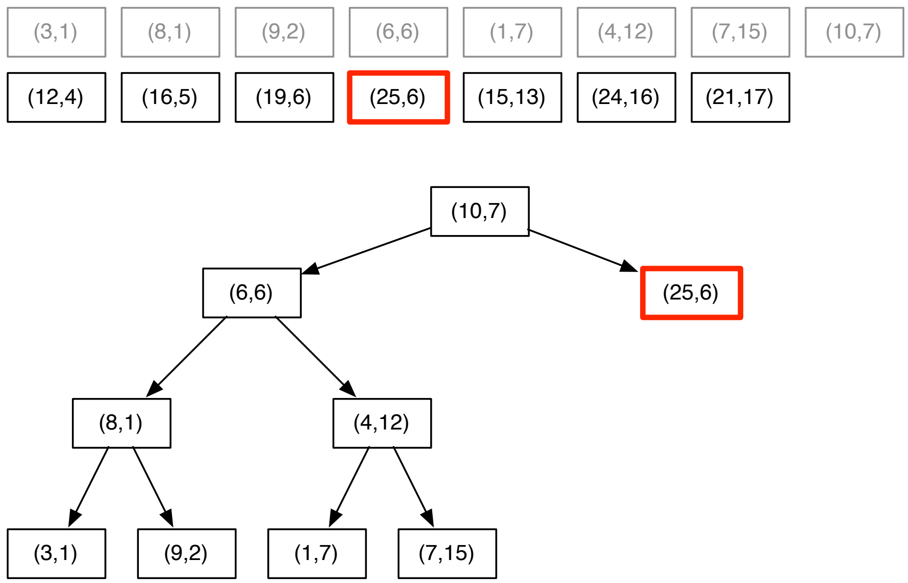


Step 6: Next we recursively consider from among the keys in step 5 those which had an x-coordinate less than the median. There is only one, $(1,7)$, which becomes the left child of $(4,12)$. Similarly, when we recursively consider from among those keys in step 5 which had an x-coordinate larger than the median, we find there is only one, $(7,15)$, so that key becomes the right child of $(4,12)$.

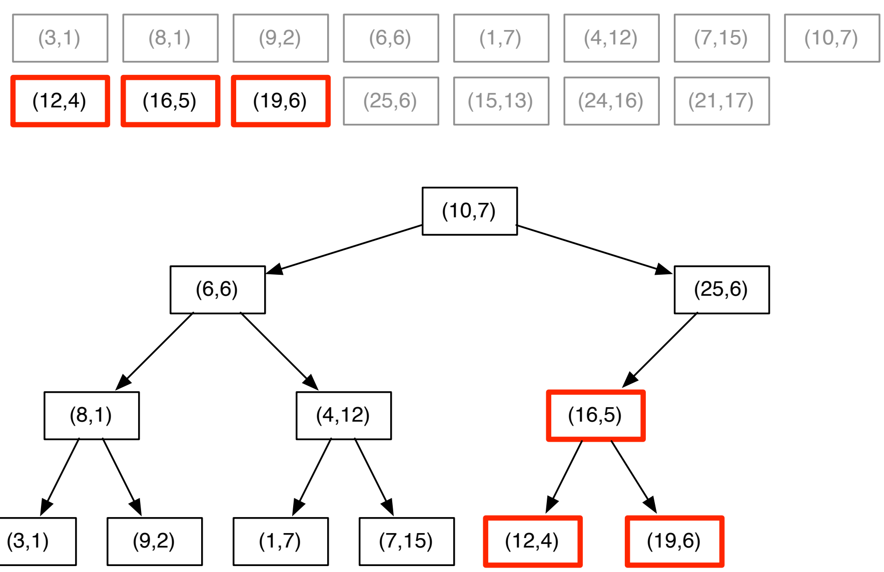




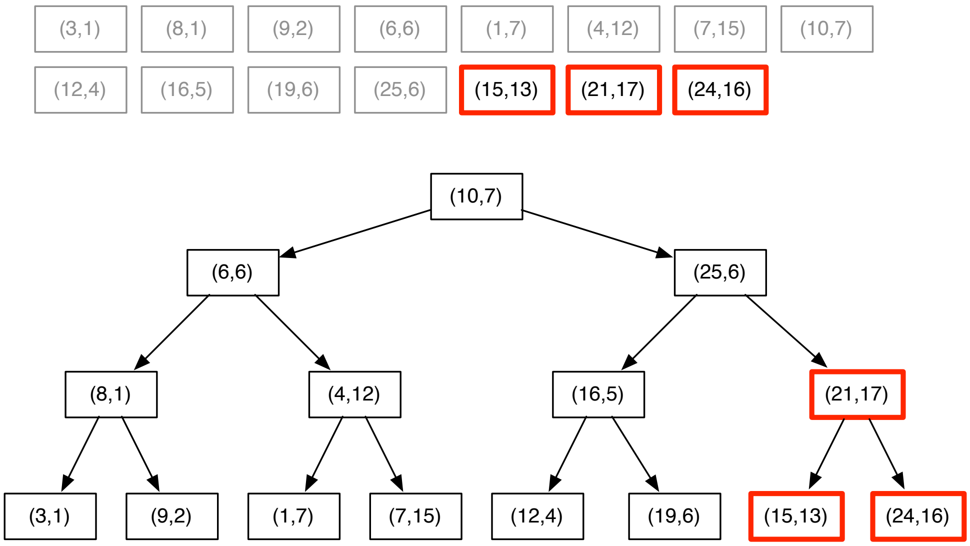
Step 7: We are now done with the keys from Step 1 that were smaller than the median x-coordinate; the recursion unwinds until we are back at the root level. Now we recursively consider the keys from step 1 that were larger than the median x-coordinate. These get sorted by y-coordinate (because we are on level 2, an even level) and we find that the key with the median y-coordiante is $(25, 6)$. So $(25, 6)$ becomes the right child of the root.



Step 8: Now we must recursively consider the keys from Step 7 whose y-coordinates are smaller than the median x-coordinate. From among these, the key with median x-coordinate (x because we're on an odd numbered tree level) is $(16, 5)$, so it becomes the left child of $(25, 6)$. There is only one coordinate with x coordinate respectively less than and greater than the median, namely $(12, 4)$, and $(19, 6)$, so recursively processing those singleton sets of keys result in them becoming the left and right children of $(16, 5)$ respectively.



Step 9: Finally we must process the keys from Step 7 that had y-coordinates larger than the median. The key with median x-coordinate from among these is $(21, 17)$, so it becomes the right child of $(25, 6)$. If we recursively process the keys that have smaller and larger x-coordinates, respectively, we find that in each case there is only one, so they become the left and right children of $(21, 17)$, respectively.



And that matches the tree from Exercise 1 in the slides!