$$
\begin{aligned}
\Upsilon_1 = &\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{M}} \boldsymbol{f}_{m,l}^T \left(\rho_\mathrm{d}\sqrt{\tfrac{\kappa_m\kappa_n}{\bar{\gamma}_{m,\mathrm{u}}\bar{\gamma}_{n,\mathrm{u}}}}\sum_{i\in\mathcal{K}}\sum_{k\in\mathcal{K}}\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{nk,\mathrm{u}}^T\right.\\
&\left.+\tfrac{\rho_\mathrm{d}}{K}\sqrt{(1-\kappa_m)(1-\kappa_n)}\sum_{i\in\mathcal{K}}\sqrt{\tfrac{\eta_{mi}\eta_{ni}}{\gamma_{mi,\mathrm{u}}\gamma_{ni,\mathrm{u}}}}\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{ni,\mathrm{u}}^T+\rho_\mathrm{s}\sqrt{\zeta_m\zeta_n}\boldsymbol{w}_{m,\mathrm{s}}\boldsymbol{w}_{n,\mathrm{s}}^H\right)\boldsymbol{f}_{n,j}^T\\
&+\rho_\mathrm{d}\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{L}}\sum_{m\in\mathcal{M}}\boldsymbol{f}_{m,l}^T\left(\tfrac{\kappa_m}{\bar{\gamma}_{m,\mathrm{u}}}\sum_{i\in\mathcal{K}}\sum_{k\in\mathcal{K}}\psi_{ik}\hat{\boldsymbol{R}}_{mi,\mathrm{u}}\boldsymbol{R}_{mi,\mathrm{u}}^{-1}\boldsymbol{R}_{mk,\mathrm{u}}+\tfrac{(1-\kappa_m)}{K}\sum_{i\in\mathcal{K}}\tfrac{\eta_{mi}}{\gamma_{mi,\mathrm{u}}}\hat{\boldsymbol{R}}_{mi,\mathrm{u}}\right)\boldsymbol{f}_{m,j}^T. \qquad (45)
\end{aligned}
$$

$$
\begin{aligned}
\Upsilon_2 = &\rho_\mathrm{d}\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\tfrac{\kappa_m}{\bar{\gamma}_{m,\mathrm{u}}}\sum_{i\in\mathcal{K}}\sum_{k\in\mathcal{K}}\left(\mathrm{Tr}\left\{\boldsymbol{R}_{ml,\mathrm{r}}\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{mk,\mathrm{u}}^T\right\}+\psi_{ik}\,\mathrm{Tr}\left\{\boldsymbol{R}_{ml,\mathrm{r}}\hat{\boldsymbol{R}}_{mi,\mathrm{u}}\boldsymbol{R}_{mi,\mathrm{u}}^{-1}\boldsymbol{R}_{mk,\mathrm{u}}\right\}\right)\\
&+\rho_\mathrm{d}\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\tfrac{1-\kappa_m}{K}\sum_{i\in\mathcal{K}}\tfrac{\eta_{mi}}{\gamma_{mi,\mathrm{u}}}\mathrm{Tr}\left\{\boldsymbol{R}_{ml,\mathrm{r}}\left(\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^T+\hat{\boldsymbol{R}}_{mi,\mathrm{u}}\right)\right\}+\rho_\mathrm{s}\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\zeta_m\,\mathrm{Tr}\left\{\boldsymbol{R}_{ml,\mathrm{r}}\boldsymbol{w}_{m,\mathrm{s}}\boldsymbol{w}_{n,\mathrm{s}}^H\right\}. \qquad (46)
\end{aligned}
$$

Appendix B
Proof of Proposition 3

The acquisition of the closed-form expression in (19) needs to calculate three expected values. For ease of expression, we denote them as

$$\Upsilon_1 = \mathbb{E}\left\{\left|\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\boldsymbol{f}_{m,l}^T\boldsymbol{x}_m\right|^2\right\}, \qquad (41a)$$

$$\Upsilon_2 = \mathbb{E}\left\{\left|\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\tilde{\boldsymbol{g}}_{ml,\mathrm{r}}^T\boldsymbol{x}_m\right|^2\right\}, \qquad (41b)$$

$$\Upsilon_3 = \mathbb{E}\left\{\left|\sum_{l\in\mathcal{L}}n_{l,\mathrm{s}}\right|^2\right\}. \qquad (41c)$$

Focusing on Υ_1, it can be expanded as

$$
\begin{aligned}
\Upsilon_1 &= \mathbb{E}\left\{\left|\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\boldsymbol{f}_{m,l}^T\boldsymbol{x}_m\right|^2\right\}\\
&=\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{M}}\mathbb{E}\left\{\boldsymbol{f}_{m,l}^T\boldsymbol{x}_m\boldsymbol{x}_n^H\boldsymbol{f}_{n,j}^T\right\}. \qquad (42)
\end{aligned}
$$

The key component in (42) is $\mathbb{E}\left\{\boldsymbol{x}_m\boldsymbol{x}_n^H\right\}$, which can be categorized into two distinct cases: $m=n$ and $m\neq n$, each calculated separately. For $m=n$, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\boldsymbol{x}_m\boldsymbol{x}_n^H\right\} = \rho_\mathrm{d}\kappa_m\mathbb{E}\left\{\boldsymbol{w}_{m,\mathrm{c}}\boldsymbol{w}_{m,\mathrm{c}}^H\right\}+\rho_\mathrm{s}\zeta_m\boldsymbol{w}_{m,\mathrm{s}}\boldsymbol{w}_{m,\mathrm{s}}^H\\
&\qquad+\tfrac{\rho_\mathrm{d}(1-\kappa_m)}{K}\sum\nolimits_{i\in\mathcal{K}}\eta_{mi}\mathbb{E}\left\{\boldsymbol{w}_{mi,\mathrm{p}}\boldsymbol{w}_{mi,\mathrm{p}}^H\right\}\\
&=\tfrac{\rho_\mathrm{d}\kappa_m}{\bar{\gamma}_{m,\mathrm{u}}}\left(\sum_{i\in\mathcal{K}}\sum_{k\in\mathcal{K}}\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{mk,\mathrm{u}}^T+\sum_{i\in\mathcal{K}}\sum_{k\in\mathcal{K}}\psi_{ik}\hat{\boldsymbol{R}}_{mi,\mathrm{u}}\boldsymbol{R}_{mi,\mathrm{u}}^{-1}\boldsymbol{R}_{mk,\mathrm{u}}\right)\\
&+\tfrac{\rho_\mathrm{d}(1-\kappa_m)}{K}\sum_{i\in\mathcal{K}}\tfrac{\eta_{mi}}{\gamma_{mi,\mathrm{u}}}\left(\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^T+\hat{\boldsymbol{R}}_{mi,\mathrm{u}}\right)+\rho_\mathrm{s}\zeta_m\boldsymbol{w}_{m,\mathrm{s}}\boldsymbol{w}_{m,\mathrm{s}}^H. \qquad (43)
\end{aligned}
$$

While for $m\neq n$, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\boldsymbol{x}_m\boldsymbol{x}_n^H\right\}\\
&=\rho_\mathrm{d}\sqrt{\tfrac{\kappa_m\kappa_n}{\bar{\gamma}_{m,\mathrm{u}}\bar{\gamma}_{n,\mathrm{u}}}}\sum_{i\in\mathcal{K}}\sum_{k\in\mathcal{K}}\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{nk,\mathrm{u}}^T+\rho_\mathrm{s}\sqrt{\zeta_m\zeta_n}\boldsymbol{w}_{m,\mathrm{s}}\boldsymbol{w}_{n,\mathrm{s}}^H\\
&+\tfrac{\rho_\mathrm{d}}{K}\sqrt{(1-\kappa_m)(1-\kappa_n)}\sum_{i\in\mathcal{K}}\sqrt{\tfrac{\eta_{mi}\eta_{ni}}{\gamma_{mi,\mathrm{u}}\gamma_{ni,\mathrm{u}}}}\bar{\boldsymbol{g}}_{mi,\mathrm{u}}^*\bar{\boldsymbol{g}}_{ni,\mathrm{u}}^T. \qquad (44)
\end{aligned}
$$

Combining the results in (43) and (44) leads to (45). Concerning the term Υ_2, since $\tilde{\boldsymbol{g}}_{ml,\mathrm{r}}$ exhibits no correlation with $\boldsymbol{x}_m$ and has zero mean, we can employ an analytical approach analogous to that used in deriving Υ_1. For conciseness, we present only the final expression, given as (46). Both (45) and (46) are shown at the bottom of the page.

At last, Υ_3 is expressed as

$$\Upsilon_3 = \sum_{l\in\mathcal{L}}\mathbb{E}\left\{\left|n_{l,\mathrm{s}}\right|^2\right\} = LN_0. \qquad (47)$$

It is easy to get (20) after substituting (45)–(47) into (19). Thus, Proposition 3 is proved.