ACTL3162 Task 2

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Ruin Probability

Question 1.

To determine the ruin probability $\psi_{60}(18)$ with N the poisson process with $\lambda = 5$ and Y, the claim size distribution, we first need to calculate the insurer's monthly premium, which can be given by:

$$\pi = (1+\theta)E[N]E[Y].$$

Then under scenario I and II we obtain the following:

$$\pi_I = (1+0.06)(5)\left(\frac{1}{3.0597}\right) = 1.7322,$$
$$\pi_{II} = (1+0.06)(5)\left(\frac{0.67}{2.05}\right) = 1.7322.$$

To find the ruin probability we simulate the surplus function $C(t)$, where if this function falls below 0, then ruin has occurred.

$$C(t) = 18 + \pi t - \sum_{i=1}^{N(t)} Y_i.$$

By first simulating the claim time arrivals with the Poisson process with rate λt for 60 months, we can then generate the claim size at each arrival time, then calculate the surplus function to check whether it has fallen below 0. This process was simulated $1{,}000{,}000$ times to reduce variance and ensure reproducibility of results. Determining the proportion of paths where ruin occurs, we can then determine the ruin probability, an estimation of $\Pr(\tau < 60 | C(0) = 18)$ where τ is the time of ruin $\tau = \inf\{t \geq 0 : C(t) < 0\}$.

With this, we find that under scenario I, $\psi_{60}(18) = 0.00586$ and under scenario II, $\psi_{60}(18) = 0.013862$. Under scenario II, the 5-year survival probability is less than 99%.

Question 2a.

Analysing the gamma distribution given by Y, to determine the ruin probability with reinsurance products, we need to calculate the reinsurer's premium portion and hence net premium in addition to considering the reduced claim amount the insurer faces where ζ is the loading factor.

$$\pi_h = (1+\zeta)\lambda E[h(Y)].$$

For the ruin probability $\psi_{60}(18)$ under reinsurance product A, proportional reinsurance, we need to find the new net premium while considering the reinsurer's premium with the loading factor of 8% and insurer's retention $\alpha = 0.88$ of each claim. For reinsurance A, $h(Y) = \alpha Y$. Then we can find

$$E[h(Y)] = E[(1-\alpha)Y] = (1-\alpha)E[Y].$$

Hence the reinsurer's premium is:

$$\pi_{Ah} = (1+\zeta)(\lambda)(1-a)\left(\frac{0.67}{2.05}\right), (1)$$
$$\pi_{Ah} = (1+0.08)(5)(1-0.88)\left(\frac{0.67}{2.05}\right) = 0.211785.$$

The Cramer-Lundberg process where the claims are now only the amount the insurer retains is:

$$C(t) = 18 + (\pi_{II} - \pi_h)t - \sum_{i=1}^{N(t)} \{Y_i - h(Y_i)\}.$$

Applying a similar method to Question 1 to simulate the ruin probabilities, we find that under reinsurance product A, $\psi_{A60}(18) = 0.007034$.

Similarly, finding the ruin probability under reinsurance product B, with a loading factor of 8% and retentional level $d = 0.881391$, we determine the new net premium where $h(Y) = (Y-d)_+$ for Excess of Loss reinsurance. Finding $E[h(Y)]$ for a gamma distribution, this can be determined to be:

$\frac{\alpha}{\beta}(1 - F_G(d; \alpha + 1, \beta) - d(1 - F_G(d; \alpha, \beta))$ (Jingyu 2016).

Hence the reinsurer's premium is:

$$\pi_{Bh} = (1 + \zeta)\lambda\frac{\alpha}{\beta}(1 - F_G(d; \alpha + 1, \beta) - d(1 - F_G(d; \alpha, \beta)). \text{ (2)}$$

Hence the reinsurer's premium can be determined as 0.211785 in R. Finding the new claim amount that the insurer will face, we check if the claim exceeds the retention level, where if this is exceeded, the insurer pays the retention level, else they pay the claim amount. Simulating for reinsurance B, we find $\psi_{B60}(18) = 0.002065$.

Question 2b.

To avoid that ultimate ruin is certain, we must ensure that the insurer's net premium exceeds their expected aggregate claims per unit. To find the range of α we need to satisfy the net profit condition $E[X_1] > 0$ where $X_1 = \pi_1 - S_1$. Solving the following inequality for reinsurance product A:

$$\pi_{II} - \pi_{Ah} > \lambda E[aY].$$

We find that $\alpha > \frac{\zeta-\theta}{\zeta}$ and since α is bounded, the range of values for $\alpha \,\epsilon (0.25,1]$.

For reinsurance product B, we must satisfy the following condition:

$$\pi_{II} - \pi_{Bh} > \lambda E[Y - h(Y)].$$

$E[Y - h(Y)]$ is the equivalent of finding $E[\min(Y, d)] = \int_0^d P(Y > x)dx$. Using this integral and the result for reinsurance B's premium in Question 2a (2) in R, we find the following condition:

$$d > 0.105478.$$

Hence, we find that the range of values for the retention level $d \,\epsilon\, (0.105478, \infty)$.

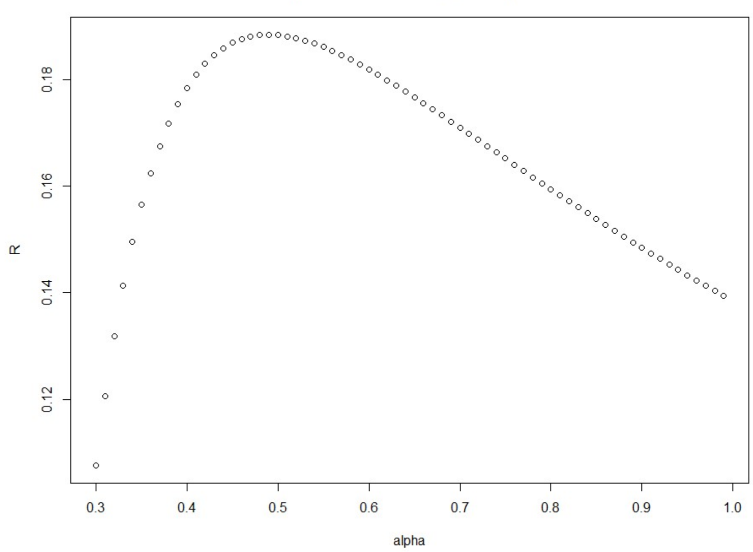
Question 2c.

To find the insurer's retained proportion a_m that maximises the adjustment coefficient, we solve the equation:

$$\lambda\left(E\left[e^{r(Y-h(Y))}\right] - 1\right) = (\pi_{II} - \pi_{Ah})r.$$

For a gamma distribution $E\left[e^{r(Y-h(Y))}\right] = (1 - \frac{a_m r}{\beta})^{-a}$. Using the premium equation obtained in Question 2a (1), by moving all terms to one side and solving for the root using R's "uniroot" function with *Figure 1.1*, we find that the a value that maximises the adjustment coefficient is 0.49 and hence $R_h = 0.188$.

Figure 1.1: Maximising R for alpha



To find the insurer's limit d for reinsurance product B that will maximise the adjustment coefficient, we employ the following function:

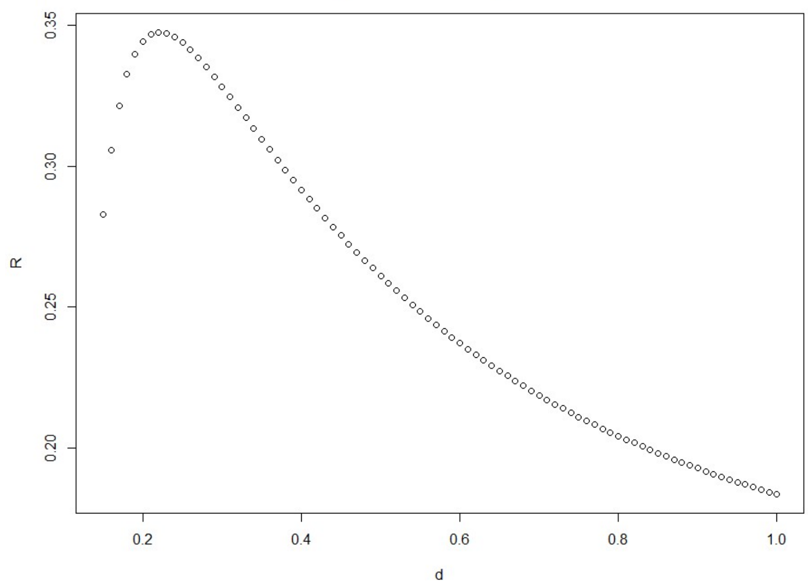
$$\lambda\left(E\left[e^{r(Y-h(Y))}\right]-1\right)=(\pi_{II}-\pi_{Bh})r.$$

Where the following MGF of the insurer's loss is:

$$E\left[e^{r(Y-h(Y))}\right]=\int_0^d e^{rx}f_Y(x)dx+\int_d^\infty e^{rd}f_Y(x)dx.$$

Similarly, using the premium equation obtained in Question 2a (2), moving all terms to one side and solving in R to calculate the integrals, plot *Figure 1.2* and hence d, it is found that $d=0.221$ and corresponding $R_h=0.348$.

Figure 1.2: Maximising R for d



Hence the upper bound for the probability of ruin for the reinsurance product A is $e^{-R_h c_0}=0.03364$ and reinsurance product B is 0.0019172.

Question 3.

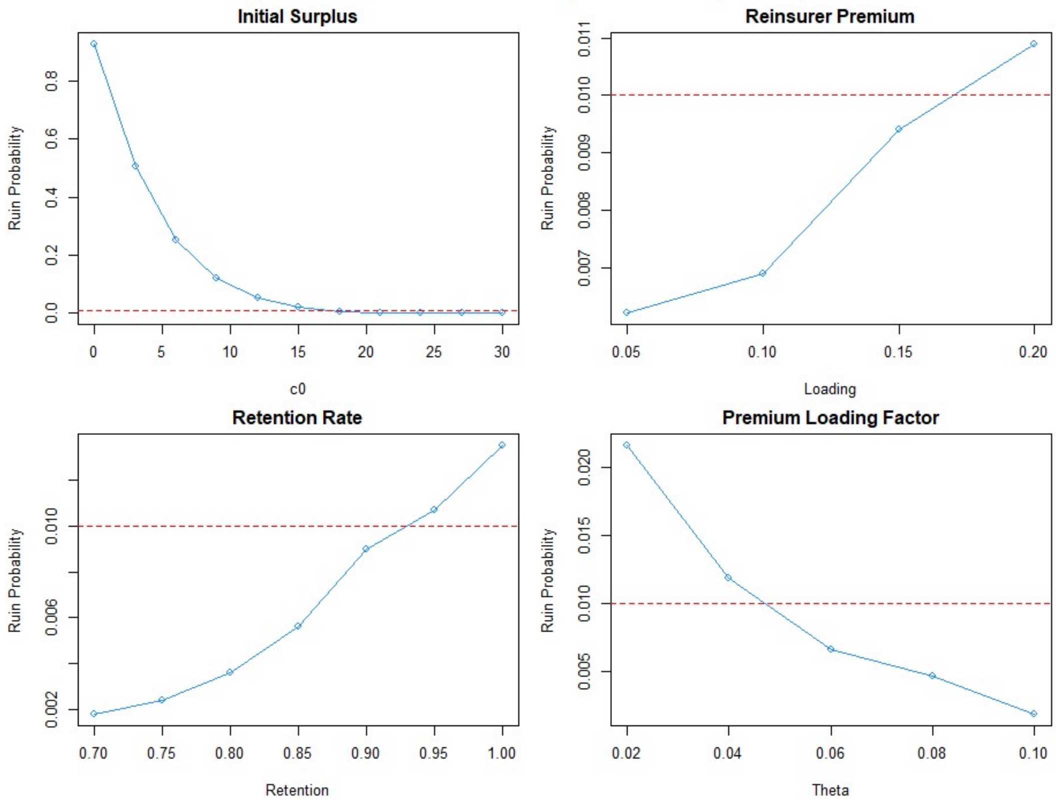
Reinsurance products transfer a portion of risk from the insurer to the reinsurer, reducing their overall claims faced, however also reduces their premium income. Under reinsurance product A, proportional reinsurance, the insurer shares the burden of claims which can reduce the impact of extreme claim amounts in catastrophic events. This protects the insurer and overall reduces their ruin probability as previously determined, where under reinsurance A, the ruin probability is lower than that without. However, by purchasing reinsurance, the insurer makes a tradeoff with profitability. By calculating the change in net profit condition when purchasing reinsurance in R:

$$1-\frac{\pi_{II}-\pi_{h-\lambda E[Y-h(Y)]}}{\pi_{II-\lambda E[Y]}}.$$

We find that by transferring 12% of losses to the reinsurer, this will decrease the firm's expected profit by approximately 16%. In return for increased stability, the insurer must sacrifice a greater proportion of their profits. For reinsurance B, excess of loss reinsurance, there is greater protection against larger losses, as the insurer will only pay up to a certain retention level amount. However, for smaller claims, the insurer bares the full loss amount. Thus, in the presence of larger claims such as catastrophic events, the insurer will experience greater stability, as only paying up to the retention level will reduce the ruin probability. Furthermore, by eliminating heavy tailed extreme values, the loss faced by the insurer will reduce in variance as all loss amounts are now bounded, whereas in reinsurance product A this was not the case. Calculating the profitability-stability tradeoff, we find that by setting the retention level at 0.881391 for reinsurance, the insurer will also face an expected decrease in expected profit by approximately 16%. Reinsurance B may provide greater stability due to its ability to bound losses faced by the insurer while decreasing the expected profit by the same amount as reinsurance A. Thus, purchasing reinsurance can increase the insurer's stability by sharing risk and bounding losses at a certain amount, however as a trade-off this will reduce the firm's profits.

Question 4.

Figure 1.3: Ruin Probability Sensitivity Analysis



The ruin probability derived from Question 2a under reinsurance product A is 0.007034. Since this probability is above the required 5-year survival probability of 99%, we conduct a sensitivity analysis to either reduce the insurer's required initial surplus or increase their profitability while maintaining adequate stability. Analysing *Figure 1.3,* reducing the initial surplus of 18 we find that this value can be reduced to approximately 15 before exceeding the required survival probability. This indicates that the insurer can further reduce their initial surplus while still maintaining stability, where redirecting these funds to other policies may be more profitable. Increasing the insurer's security loading rate reduces the ruin probability, indicating that by increasing their net premium income, the insurer will be less likely to face ruin as they will have more funds to withstand greater losses. However, by increasing premium rates, the insurer may face reduced clients and other potential knock-on effects as a result from decreasing affordability of their insurance product. Increasing the reinsurer's premium results in a greater chance of ruin, indicating that when the reinsurer's premium rises, thereby reducing the insurer's net premium, the reduced overall income will increase the chance of ruin. By changing the retention rate α the proportion of claims that the insurer will face will increase as the they now bare greater risk of the losses. This is expected as by taking on a greater proportion of claims, while the reinsurer's premium decreases, the tradeoff results in greater risk faced by the insurer despite the increase net profit.

Thus, since the ruin probability is within the required 5-year required survival probability with some leeway, the insurer can look to either increase their net profit income or reduce their initial surplus investment. The firm could allocate fewer funds for their initial surplus, which could be directed to other insurance initiatives for greater profit as a minor decrease in the initial surplus will still ensure stability. Furthermore, should the firm wish to drive up profits, could look to increase their own premium loading factor, however, should ensure that this would not result in having to change their insurance product or cause a loss of clients. Similarly, since under reinsurance A, the ruin probability is under the required value, the insurer should look for other proportional reinsurance products that have a lower premium loading, which will increase their premium income while ensuring stability. Changing the retention rate is more complicated as while the firm will face greater claims, which reduces their profitability and increases ruin probability, the premium paid to reinsurers are also reduced, hence, finding an optimal retention rate which balances an increase in profit and increased claims is vital. Therefore, the insurer has room to seek increased profits by reducing their initial surplus, increasing their premium loading factor or finding a lower reinsurance premium loading factor.

Reference List

She, Jingyu 2016, *Problem 4.1 a) Solution: Expected excess loss in Gamma distribution,* accessed 24 October 2024, https://www.youtube.com/watch?v=SMcqZEl5n00&ab_channel=JingyuShe