ACTL3162 General Insurance Techniques
ACTL5106 Insurance Risk Models
Assignment, 2024 T3

1 Learning outcomes

This assignment is designed to develop the course learning outcomes (CLO1-CLO4) as outlined in the course syllabus. It also evaluate programe learning outcomes in the areas of "Business Knowledge", "Problem solving", and "Business Communication". You are expected to demonstrate your ability to analyse an actuarial problem, apply appropriate relevant theories and logical reasoning to interpret the problem, and formulate solutions and conclusions. The clarity and effectiveness of your communication will also be assessed.

2 TASK 2 [15 marks]

Insurer's surplus process:

The surplus of the direct insurer at time t (measured in months) can be described as

$$C(t) = c_0 + \pi t - \sum_{i=1}^{N(t)} Y_i, \quad t \geq 0$$

where c_0 is the initial surplus at time 0, Y_i is the i-th claim amount and π is the constant rate of premium income paid continuously, and $N(t)$ is a number of claims up to time t.

Ruin probability:

The ruin probability is crucial for assessing the financial stability and solvency of insurance companies, particularly in the context of extreme events. In a regulatory context, it is essential to determine the appropriate level of capital required to mitigate the risk of ruin. For example, the probability of ruin within one year should be no more than 0.005 (1 in 200 years event).

- Let $\psi_t(c_0)$ denote the probability that ruin occurs within t periods of time, given an initial surplus c_0, i.e. $\Pr(\min_{s \leq t} C(s) < 0 | C(0) = c_0)$.
- For efficient use of capital, the minimum capital is determined to ensure a certain level of solvency. Specifically, it is often required that the 1-year survival probability is at least 99.5%, and the 5-year survival probability is at least 99%, i.e. $\psi_{12}(c_0) \leq 0.005$ and $\psi_{60}(c_0) \leq 0.01$.

Claim arrival process and claim size distributions:

Based on recent experience, claim arrives according to a Poisson Process with parameter $\lambda = 5$ *per month*. The individual claim sizes are well-described by the following distributions under two scenarios:

- Scenario I: Exponential distribution with rate $c = 3.0597$;
- Scenario II: Gamma distribution with shape $\gamma = 0.67$ and rate $c = 2.05$.

Other information:

- The insurer's premium is paid continuously at a constant rate π and is calculated so that the relative security loading θ is 6%.
- The initial surplus is $c_0 = 18$.

Reinsurance products:

The insurer plans to purchase one of the following reinsurance options:

(A) **Proportional reinsurance** from another reinsurance company, with the direct insurer retaining $\alpha = 0.88$ of each claim. The reinsurance company changes a premium loading factor of 8% (ξ).

(B) **Excess of loss (EoL) reinsurance** with a retention level $d = 0.881391$. The reinsurance company charges a premium loading factor of 8% (ξ).

Questions:

1. **Estimation of ruin probability**:

   Under two scenarios, estimate the ruin probability $\psi_{60}(c_0)$ and check whether the 5-year survival probability less than 99%.

2. **Analysis of reinsurance products**:

   For scenario in which $\psi_{60}(c_0) > 1\%$, consider reinsurance products (A) and (B) to improve financial stability (i.e. reduce the ruin probability). Perform the following analysis.

   (a) With an initial surplus $c_0 = 18$, find the approximated ruin probabilities within 5 years.

   (b) To avoid that ultimate ruin is certain, the insurer's net of reinsurance premium income per unit time must be larger than the expected aggregate claims per unit time. Find the range of α in (A) and d in (B), respectively.

   (c) Numerically determine the direct insurer's retained proportion $\alpha \in [0, 1]$ in (A) and limit d in (B) that will maximise the adjustment coefficient for the direct insurer. Then calculate the upper bound for the probability of ruin with this choice of α in (A) and limit d in (B).

3. **Comparison of reinsurance strategies**:

   Explain how the above reinsurance products (A) and (B) can enhance an insurer's stability and discuss the trade-off between stability and profitability of the insurance company.

4. **Sensitivity analysis of ruin probability**:

   For Senario 2 with reinsurance product (A), conduct a sensitivity analysis on the ruin probability $\psi_{60}(c_0)$ with respect to key parameters such as the initial surplus c_0 and premium loading factors (θ, ξ). For instance, consider increasing or decreasing each parameter by a certain percentage and analyse how these changes impact the ruin probability. Based on your findings, recommend adjustments to the insurer's reinsurance strategy or premium rates.

3 Required document

You are asked to provide a **report and R code**. There will be **TWO** submission boxes (one business report and one R code in Moodle.

- The report should provide results for all of the above tasks in word or pdf format. You do not need to provide a table of contents in your report. and should think of a clear and effective structure for your responses. There is no specific formatting requirement; however, you should ensure that the report is professional in the business context.
  - The main body of the report should be **no more than 4 pages** (i.e. maximum 4).
  - Cover pages, appendices and reference lists are not counted towards the page limit.
  - No page limit for the appendix.
  - You need to provide a reference list if any references are used in your report.

- You should **not**
  - Include a chunk of programming codes in the main body of your report
  - Have figures or tables that are not referred to or analysed in the main body of your report
  - Include materials that are not highly relevant in the main body of your report
- Intermediate steps for questions involving any form of derivation are required. Your comments and conclusions should be well justified and charts should be used to support your conclusions where applicable.
- You are **strongly recommended to use the software R for programming**, although the use of other software will also be accepted. Some sample R codes for fitting are available on the course website which may be of use. In addition, feel free find packages online to perform your computations (but always check that your answer is sensible!).
- When making a comment or conclusion based on R outputs (or other software outputs), you should include the relevant outputs in the main body of your report. You should make sure that the marker can read and understand your arguments and statements without referring to the separate R code file.
- Your R codes (or codes of other software) should be included in the separate file. The marker will choose a portion of the reports to check the code. He/she will need to copy the code, run it and check whether it is correct, implementable and consistent with the output presented in your answer. **Students will risk failing the assignment if the code cannot be run or the output provided in the answer is not consistent with the output generated by the code.**

4 Assessment criteria

Please see the file, “Rubric".