Information Flow Among NASDAQ Stock Market With Transfer Entropy

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Abstract

The price fluctuation in stock market shares complicated non-linear relationships among different companies. From the perspective of information gain, transfer entropy captures the asymmetrical information flow within the stock market and many studies have been conducted to extract the relationships from a collection of market prices. However, most of them only focus on several companies instead of the whole sector. In this paper, we not only estimate the transfer entropy between different companies but also from the perspective of the whole sector, which contains less noise compared with a single company. Besides, due to the noise introduced at each time step in stock market price, the effectiveness of transfer entropy still remains unexplored. In this paper, we proposed two different experimental settings to validate the effectiveness of transfer entropy and show the information flow between different companies and sectors in the NASDAQ market. We also show the explanations of findings combined with the actual industry market.

1 Introduction

Stefano et al. (2016)[1] emphasized the significance of dynamic complex systems in financial market research, where the evolutionary structure of associated networks exhibits essential dynamic evolution characteristics[2]. Leonidas, George, Celso, Kyuseong, Pengbo, and others[3][4][5][6][7]highlighted the importance of correlation networks in studying financial market indicators, which can be constructed using methods like Pearson correlation, Granger causality, and transfer entropy. Nonetheless, stocks often exhibit nonlinear correlations[8], and when the nonlinear components between two stocks outnumber the linear ones, transfer entropy—useful for estimating nonlinear and causal relationships—attracts widespread interest.

Wiener (1956)[9] suggested a method to determine if a given sequence's prediction relies on another sequence's past information, i.e., for two simultaneously measured signals, if using the past information of the latter is more capable than using information without it predicts the former, then we call the latter a causal relationship of the former, which was formalized by Granger later as Granger causality (Granger, 1969)[10], a robust method widely used in economics for understanding one stock's influence on another. However, it relies on linear models and is unsuitable for nonlinear systems. In contrast, mutual information (MI) methods reveal the information one random variable obtains from another, but MI is symmetric which is deemed as the significant drawback[11]. Furthermore, while correlation analysis is a valuable tool for gauging similarity between variables, causality isn't directly connected to correlation. Thus, Granger causality can only indicate the presence of information flow based on linear relationships, rather than quantifying the information flow.

Shannon (2001)[12] addressed Granger causality's linear model limitations by introducing the concept of entropy in information theory to reduce information uncertainty. To develop a superior mechanism, time lag was incorporated to account for delayed mutual information. Schreiber (2001)[13] formulated transfer entropy (TE) as a nonparametric measure of information transfer between variables based on Shannon entropy, actualizing Wiener's theory. Unlike Granger causality and MI, TE is effective in both linear and nonlinear systems. TE has been applied across numerous scientific fields[14][15] due to its capacity to capture asymmetric interactions and accurately distinguish driving and responding elements within a system[16]. However, TE may produce noise; therefore, Marschinski and Kantz (2002)[17] proposed the effective transfer entropy (ETE) method as a TE extension to obtain more reliable information flow.

In applying the above theories, Okyu Kwon et al.[18] calculated the transfer entropy (TE) of 25 stock indexes from 2000 to 2007, revealing causal relationships between stocks. He et al.[19] analyzed the relationship between nine stocks in the United States, Europe, and China from 1995 to 2015 using the normal transfer entropy (NTE) method. Results indicated that the U.S. led in lagged cases, while China had the most substantial in-trading index influence. During a similar time frame, Mao et al.[20] identified information flow between various stocks from 2011 to 2015 using transfer entropy calculations for high-dimensional sequences; Zhou et al.[21] employed transfer entropy to detect financial distress indicators.[22]

Moreover, utilizing different time series for forecasting offers significant benefits, especially in noisy systems. Combining this technique with transfer entropy theory allows for estimating interactions between multivariate time series within the system. The calculation algorithm of multivariate transfer entropy contains the time-lagged reconstruction theory of phase space and the traditional transfer entropy measurement method[23], where more comprehensive conclusions about the causal relationship between different stock indices can be drawn by combining both methods together.

The analysis of the relationship between stock market shares is an important problem in the field of finance because it can help investors better understand the position of different companies in the market and their relative competitive advantages in the industry. Additionally, this analysis can provide recommendations on stock portfolios, helping investors develop better investment strategies. Tan (2008)[24] pointed out that the relationship between stock market shares is an important market research indicator, which can help investors identify leaders and followers in the market and evaluate their future growth potential. Additionally, analysis of the relationship between stock market shares can provide useful information for portfolio construction. Shubham (2015)[25] proposed a market share-based portfolio construction method and applied it to stock market data.

However, due to the high complexity of modeling dependent time series with additive noise, the effectiveness of transfer entropy still remains unexplored. Most studies only conducted the transfer entropy and its variations without considering the effectiveness of the results given by transfer entropy. To solve the issue, we validate the effectiveness by measuring the distribution and strength of transfer entropy through experiments with synthetic data. However, few studies describe the relationship between two stock market shares through the power of transfer entropy over different time horizons while removing noise from the time series. Therefore, we consider transfer entropy (Yao and Li, 2020)[26] arising from conditionally relevant information, which quantifies the reduction of uncertainty and makes it easier to model statistical causality between variables in natural phenomena.

In this paper, based on the method of transfer entropy, we analyze the relationship between significant stocks in the NASDAQ market and the related industries, then compare and explore the results. First, we selected the daily data of the top 20 stocks from 2013 to 2022 according to the size of each stock's market share. Second, we use the daily data of the sum of the stock closing points of each sector for each day from 2013 to 2022. In the choice of method, we use the original transfer entropy method to analyze the above data.

Section 2 illustrates the basic conception of information theory and the definition of transfer entropy approach. Section 3 simply introduces the synthetic data and real-world data used in our estimation together with pre-processing step. Section 4 demonstrates the estimating results, which are followed by limitations and conclusion in Section 5

2 Methodology

This section introduces the methodology of transfer entropy. We first introduce basic conceptions of Shannon entropy and the measurement of mutual information. Then based on the mutual information, the definition of transfer entropy will be given. Lastly, we articulate our implementation of transfer entropy on the time series of the stock market.

2.1 Preliminary

Shannon first proposed information theory in 1948[27], which is used to measure the information of communication. After that, information theory has been widely used in computer science, economics, bio-informatics, etc. The definition of Shannon entropy is given by

$$\begin{aligned} H(X) &= H(p_1, p_2, ..., p_K) \\ &= -\sum_k p_k \cdot \log P(x_k) \end{aligned} \tag{1}$$

In general, this equation measures the uncertainty of the information. For example, the uncertainty of an event that definitely happens or not happens is the lowest. In that case, the probability p of that event will be 1 or 0, which makes $p \cdot \log p = 0$. On the contrary, when the uncertainty of an event reaches the maximum, indicating $p = 0.5$ mathematically, the equation $p \cdot \log p$ reaches its maximum.

To quantify the amount of information from a random variable after observing the other variable, mutual information, which is also known as information gain, will be used. According to the definition of mutual entropy

$$I(X;Y) = H(X) - H(X|Y) \tag{2}$$

Mutual information can be quantified by the reduction of the uncertainty of one random variable X given the other random variable Y. Then we can articulate the calculation based on the chain rule of entropy

$$H(X,Y) = H(X) + H(Y|X) = H(Y) + H(X|Y) \tag{3}$$

where $H(X,Y)$, $H(X|Y)$, and $H(Y|X)$ refer to the joint entropy of two random variables (X,Y), conditional entropy of X given Y, and conditional entropy of Y given X respectively.

Applying formula 3 to formula 2, we have

$$\begin{aligned} I(X;Y) &= H(X) + H(Y) - H(X,Y) \\ &= \sum_x p(x)\ \log \frac{1}{p(x)} + \sum_y p(y)\ \log \frac{1}{p(y)} + \sum_{x,y} p(x,y)\ \log \frac{1}{p(x,y)} \\ &= \sum_{x \in X} \sum_{y \in Y} p(x,y)\ \log \frac{p(x,y)}{p(x)p(y)} \end{aligned} \tag{4}$$

If we take the equation from the view of Kullback-Leibler divergence, we have

$$I(X;Y) = D_{KL}(p(x,y)|p(x) \otimes p(y)) \tag{5}$$

where D_{KL} indicates the measurement of Kullback-Leibler divergence and $\otimes$ indicates cartesian product. From this perspective, the inherent dependence is measured with the expression of joint distribution and marginal distribution.

2.2 Transfer Entropy

Transfer entropy[13], a measurement of asymmetric interaction between different variables, can be used to quantify the degree of the driving factors from the response factors effectively. Given two variables X, Y, assuming Y is influenced by the former k state of inherent Y and l state of X, we have the definition of transfer entropy

$$\begin{aligned} T_{X \to Y} &= I(y_{n+1}; x_n^l | y_n^k) \\ &= H(y_{n+1}|y_n^k) - H(y_{n+1}|y_n^k, x_n^l) \\ &= (-\sum p(y_{n+1}, y_n^k, x_n^l) \log(p(y_{n+1}|y_n^k))) - (-\sum p(y_{n+1}, y_n^k, x_n^l) \log(p(y_{n+1}|y_n^k, x_n^l))) \\ &= \sum p(y_{n+1}, y_n^k, x_n^l) \log(\frac{p(y_{n+1}|y_n^k, x_n^l)}{p(y_{n+1}|y_n^k)}) \end{aligned} \tag{6}$$

Since the most important day that influences the next day is the day before the next day, we assume $k = l = 1$,

$$\begin{aligned} T_{X \to Y} &= \sum p(y_{n+1}, y_n, x_n) \log(\frac{p(y_{n+1}|y_n, x_n)}{p(y_{n+1}|y_n)}) \\ &= \sum p(y_{n+1}, y_n, x_n) \log(\frac{p(y_{n+1}, y_n, x_n)p(y)}{p(y_{n+1}, y_n)p(x_n, y_n)}) \end{aligned} \tag{7}$$

The advantage of transfer entropy compared with mutual entropy is the asymmetric information flow, which distinguishes the driving random variable and response variable. Compared with Granger causality or dynamic modeling, transfer entropy does not require a pre-defined model, making it more flexible and adaptable. Besides, transfer entropy can capture non-linear relationships, making it powerful to analyze complex systems.

2.3 Time Series Analysis with Transfer Entropy

After introducing the concept of transfer entropy, we still need to apply it the real-world time series data. Because different processing steps generate various results, we introduce the detailed procedures there to ensure the reproducibility of our experiment.

Transfer entropy can be applied to both discrete and continuous cases. However, in reality, time series data, e.g. stock market data, is always discrete. Therefore, to get the marginal probability or joint probability, we need to catalog each data point. In that case, how to decide the bin width and bin number for each variable is the first step. There we use the Freedman-Diaconis rule to get the optimal width for random variables X and Y

$$W(x) = 2\frac{IQR(x)}{\sqrt[3]{n_x}},\ W(y) = 2\frac{IQR(y)}{\sqrt[3]{n_y}} \tag{8}$$

where $IQR(x), IQR(y)$ refers to the interquartile range of the random variable X, Y respectively, and n_x, n_y refers to the sample number of the random variable X, Y respectively. After getting the optimal bin width, we derive the bin number

$$N(x) = \frac{max\{x\} - min\{x\}}{W(x)},\ N(y) = \frac{max\{y\} - min\{y\}}{W(y)} \tag{9}$$

that is, we get the bin number $N(x), N(y)$ by taking the whole range of the random variable divided by bin width. Then, for each bin $[N_{x1}, N_{x2}, ..., N_{xk}; N_{y1}, N_{y2}, ..., N_{yk}]$, we catalogue every data point in the given time series $[x_1, x_2, ..., x_{nx}; y_1, y_2, ..., y_{ny}]$ following the rules

$$x_i \in N_{k_i}\ s.t. \begin{cases} x \leq (k_i - 1)W(x) \\ x < k_i W(x) \end{cases} \tag{10}$$

$$y_i \in N_{k_j}\ s.t. \begin{cases} y \leq (k_j - 1)W(y) \\ y < k_j W(y) \end{cases} \tag{11}$$

Then we can get the probability distribution of each variables

$$P(x \in N_{ki}) = \sum_{i=1}^{xk} I(x_i \in N_{ki}) \tag{12}$$

$$P(y \in N_{kj}) = \sum_{j=1}^{yk} I(y_j \in N_{kj}) \tag{13}$$

where $I(event) = 1$ when the event in brackets happens, otherwise $I(event) = 0$. As for joint probability, we have

$$P(x \in N_{ki}, y \in N_{kj}) = \sum_{i=1}^{xk}\sum_{j=1}^{yk} I(x_i \in N_{ki}, y_j \in N_{kj}) \tag{14}$$

Based on these probability distributions, we can calculate transfer entropy $T_{X\to Y}$ and $T_{Y\to X}$ with formula 7.

3 Data Preparation

Both synthetic data and real-world data are used in our experiments. Synthetic data with explicit dependency is adapted to validate the effectiveness of transfer entropy. Real-world data is used to analyze the underlying dependency among companies or sectors. In the following subsections, the details of the data preparation are introduced.

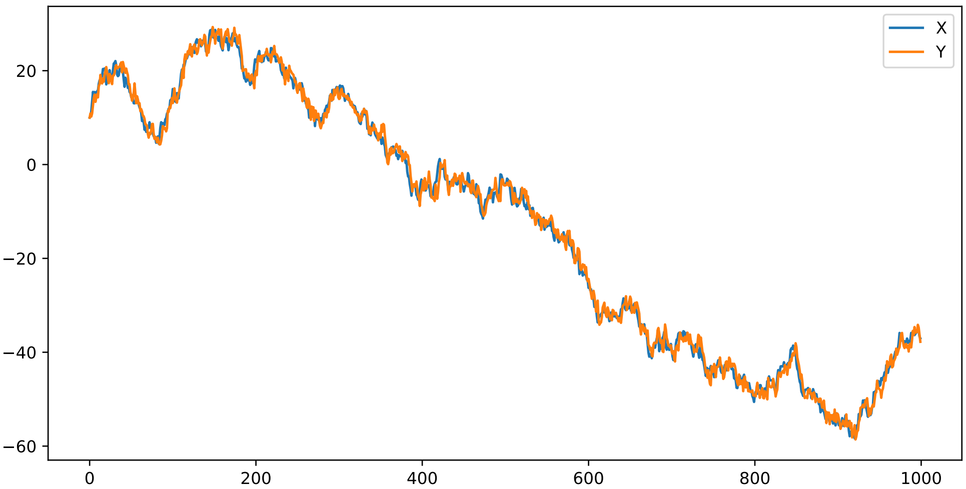

Figure 1: Two random variables generated by our model with parameter setting $R = 0.5, seed = 0$.

3.1 Synthetic Data

Even if the definition of transfer entropy shows the dependency of two random variables inherently, how to measure the effectiveness of transfer entropy still remains a problem. Here we design two dependent variables to justify the effectiveness of transfer entropy in extracting the information flow.

$$\begin{cases} x_t & = x_{t-1} + \epsilon_x \\ y_t & = r x_{t-1} + (r-1) y_{t-1} + \epsilon_y \end{cases} \tag{15}$$

Equation 15 shows our model. At time step t, random variables x_t generated based on its previous state x_{t-1} and y_t partially depends on its previous state y_{t-1} and the previous state of x_{t-1}. Noise ϵ_x, ϵ_y are conformed to normal distribution $\Phi(0, 1)$. r refers to the dependent factor which controls the degree of information flow. Figure 1 shows 1000 randomly generated time series by this model.

Unlike real-world data where the dependency is much more complex and implicit, the benefit of synthetic data is the pre-defined dependency. With the designed dependency, we can validate the effectiveness through two different approaches: 1) Measure the ratio of $T_{X\rightarrow Y}$ and $T_{Y\rightarrow X}$ to show asymmetric information flow direction, 2) Measure the change of $T_{X\rightarrow Y}$ or $T_{Y\rightarrow X}$ after increasing the dependent factor r.

Instead of computing the time series data, we pre-process the data with asset return, which has good statistical properties. The nature of heteroscedasticity and multi-collinearity in the data is weakened through logarithms, benefiting calculations without changing the relationship. We adapt log-return as our asset return

$$x_t^{'} = ln(x_t) - ln(x_{t-1}) \tag{16}$$

3.2 Real-world Data

The Nasdaq stock market is one of the most active stock markets in the US. In our experiment, we collect the stock market prices of the 3457 largest companies by market capitalization from 2015 to 2022. Our analysis is conducted in two settings: 1) transfer entropy among top companies; 2) transfer entropy among different sectors.

Our assumption is that the companies with higher market capitalization typically show stronger dependency on each other. As shown in table 1, we select top-20 companies based on their market capitalization to analyze stock price dependency. Among the top-20 companies, Apple, Microsoft, and Google are the most influential tech giants. Stock price fluctuation of these companies is believed to

Symbol	Name	Market Cap	Last Sale	Volume	Sector
AAPL	Apple	2.91e+12	$167	27079416	Technology
MSFT	Microsoft	2.15e+12	$288	9980710	Technology
GOOG	Google	1.35e+12	$105	8245032	Technology
AMZN	Amazon	1.06e+12	$104	33613987	Consumer Discretionary
TSLA	Tesla	5.78e+11	$182	67732723	Consumer Discretionary
JNJ	Johnson &Johnson	5.05e+11	$161	3474933	healthcare
V	Visa	4.89e+11	$232	1490079	Consumer Discretionary
XOM	Exxon Mobil	4.74e+11	$116	5434005	Energy
UNH	UnitedHealth	4.55e+11	$487	2757728	healthcare
JPM	JP Morgan	4.12e+11	$140	4658484	Finance
WMT	Walmart	4.05e+11	$150	1945661	Consumer Discretionary
NVO	Novo Nordisk	3.71e+11	$165	1003523	healthcare
PG	Procter &Gamble	3.56e+11	$150	1600091	Consumer Discretionary
MA	Mastercard	3.56e+11	$373	714218	Consumer Discretionary
LLY	Eli Lilly	3.53e+11	$370	722264	healthcare
HD	Home Depot	3.01e+11	$297	1317095	Consumer Discretionary
ABBV	AbbVie	2.83e+11	$160	1495254	healthcare
AVGO	Broadcom	2.64e+11	$634	683710	Technology
PEP	PepsiCo	2.54e+11	$184	822248	Consumer Staples
ASML	ASML Holding	2.47e+11	$626	1485385	Technology

Table 1: Selected top-20 companies with respect to market capitalization, data information collected in 22th April, 2023

strongly influence other companies. From the perspective of the sector, almost all top-20 companies belong to the technology, consumer discretionary, or healthcare sector, showing the complex relationship of stock price fluctuation.

To further discover the stock price relationship among different sectors, we sum up the stock prices in the same sector. Then, each sector is treated as an identity to analysing the dependency or information flow among different sectors. As shown in table2, the total market capitalization of different sectors differs greatly.

4 Estimation and Results

This section introduces the estimation and results included in this paper. The Effectiveness of transfer entropy is estimated on synthetic data. Then, on the real-world data, we estimate the transfer entropy among top-20 companies and different sectors to draw conclusions.

Sector	Ave stock price	Sector	Ave stock price
Industrials	1.45e+04	Miscellaneous	3.81e+02
Consumer Discretionary	3.03e+09	Telecommunications	6.84e+02
Finance	2.24e+04	Utilities	3.31e+03
Technology	1.45e+04	Consumer Staples	3.16e+03
Real Estate	6.09e+03	Energy	3.58e+03
healthcare	5.72e+04	Basic Materials	5.90e+02

Table 2: Total average market captialization by each sector, data information collected in 22th April, 2023

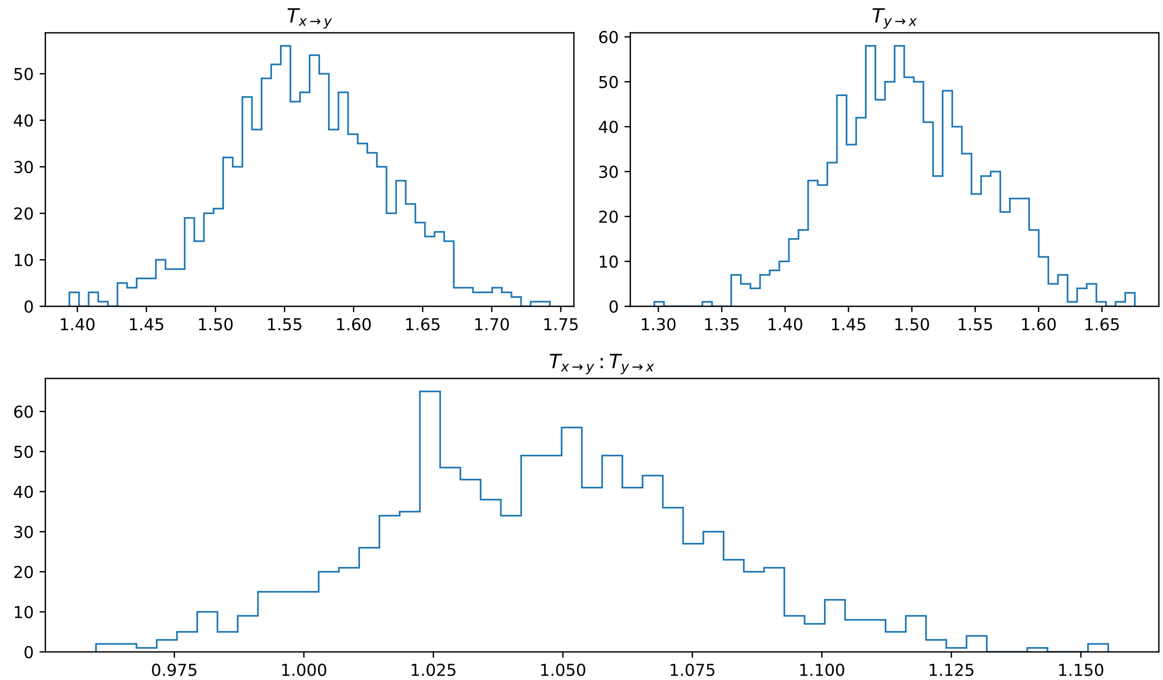

Figure 2: Histogram of transfer entropy of two random variables in our synthetic model.

4.1 Effectiveness of Transfer Entropy

Following the definition 15 of our model, we validate the effectiveness of transfer entropy by checking the distribution of $T_{x\to y}$, $T_{y\to x}$, and $\frac{T_{x\to y}}{T_{y\to x}}$. The dependency factor r is set to 0.5, initial conditions of random variables are $x_0 = 10$ and $y_0 = 10$. Because of the uncertainty of noise in each experiment, we generate the data 1000 times with different random seeds, which removes random effects. As shown in figure 2, histograms of transfer entropy are given. It is clear that $T_{x\to y}$ and $T_{y\to x}$ conform to normal distribution. The difference is that the mean of $T_{x\to y}$ is slightly higher than $T_{y\to x}$, indicating the information flow direction from variable x to variable y. Then, in terms of the ratio, we can see that $\frac{T_{x\to y}}{T_{y\to x}}$ is more likely to be larger than 1, showing the influence from x to y.

To further discover the effectiveness of transfer entropy, we change the dependency factor r and observe the response of $T_{x\to y}$, $T_{y\to x}$, and $\frac{T_{x\to y}}{T_{y\to x}}$. The dependency factor r is set to $0.1, 0.2, ..., 1.0$ As for $T_{x\to y}$, no evident response is observed in this case. On the contrary, a significant continuous decrease of $T_{x\to y}$ can be observed. Therefore, the ratio $\frac{T_{x\to y}}{T_{y\to x}}$ goes up as $T_{x\to y}$ decreases. It is worth mentioning that $\frac{T_{x\to y}}{T_{y\to x}}$ is always above 1 and it increases linearly as the growth of dependency factor r.

From the two synthetic settings above, two essential findings are concluded: First, transfer entropy is shown to conform to a normal distribution with additive Gaussian noise, but the direction information flow can still be observed by the ratio of transfer entropy of two variables. Second, the ratio of transfer entropy reflects the strength of dependency, in which the transfer entropy of the dependent variable is influenced by the strength of dependency with a negative correlation while the transfer entropy of the independent variable is not affected by it.

4.2 Transfer Entropy of Top-20 Companies

As shown in figure 4, the transfer entropy matrix of top-20 companies is given. Darker red refers to stronger information flow. One notable observation is the stock market price of top-3 companies(Apple, Microsoft, and Google) influence each other. This could be caused by the industry trend of the technology sector. The positive trends or news of technology impact these tech companies together. Besides, because the stock market prices are influenced by investor sentiment, if investors see positive developments in Microsoft, they might as be positive about Apple and Google to reduce risks or increase

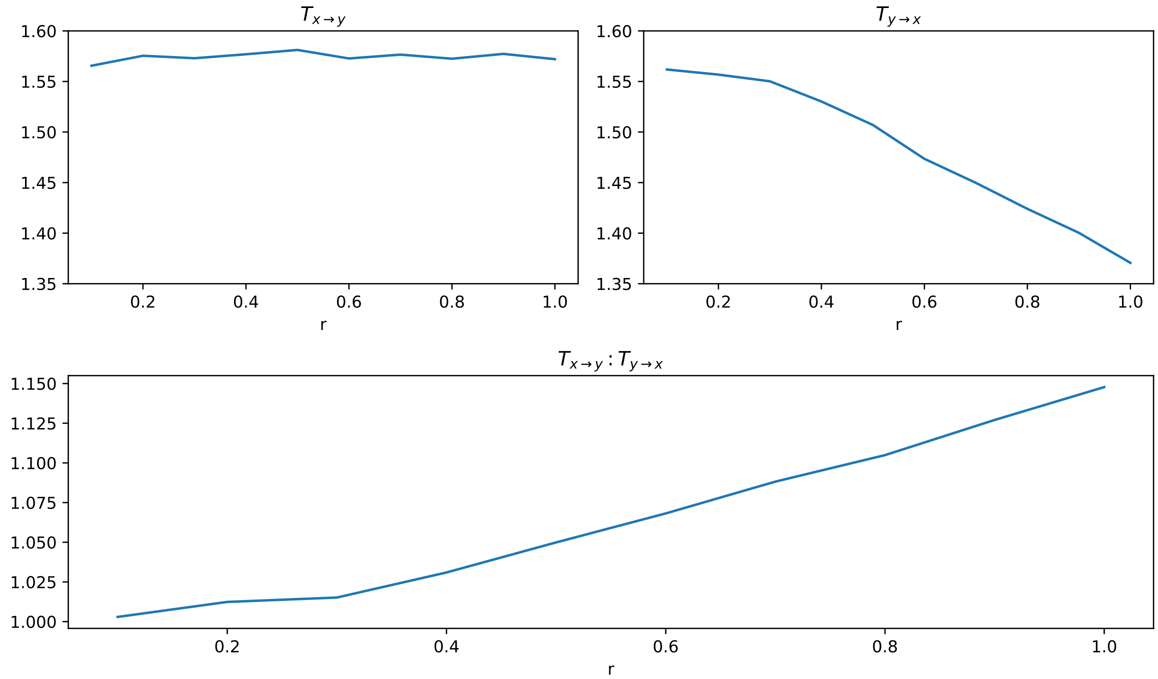

Figure 3: The influence of dependency factor r on transfer entropy.

profits. Moreover, the competition among these technology companies leads to positive information flow. For instance, when Microsoft release its Microsoft365 integrated with ChatGPT, Google, and Apple also announce their new products in big natural language processing models, attracting the interest of investors.

The other notable observation is that Visa and MasterCard show the strongest transfer entropy in top-20 companies. The potential reason could be that these two companies are highly similar with respect to their functionality. In the global payments industry, Visa and MasterCard are both major players. The development of e-payments has positive impact on both companies. For example, in 2019, to explore joint development of the digial payments industry, MasterCard announced that it signed a memorandum of understanding with China Central Bank, impacting Visa's stock prices because it shows the potential growth in the world's largest payment market.

4.3 Transfer Entropy of different sectors

We analyze the information flow among different sectors of stock market prices, and log-return is included in the pre-processing step. As shown in figure 6, some sectors show strong information flow to other sectors like technology, industrials, and finance while the consumer discretionary and healthcare sector shows even no information flow to other sectors. Intuitively, consumer discretionary and healthcare sectors are less interconnected with other sectors in the economy. Then, consumer discretionary sector is related to non-essential consumer goods like leisure, retail, and entertainment, which is influenced by consumer sentiment instead of overall economic trends. As a result, the information flow of consumer discretionary is less since fewer relationships among consumer discretionary with other sectors. Similarly, healthcare sector contains less information flow, because the demand for healthcare products and services will be influenced by laws changes, drug approvals, or experimental results, rather than economic trends in other sectors.

According to our observation, it is worth noting that the industrials sector influences the finance and technology sectors. Industrial companies often need large capital expenditures or loans to set up infrastructures, leading to a high demand for financing. In terms of supply chain and logistics, industrial companies provide raw materials, components, or finished products to high-tech companies. Disruptions in the supply chain impact the companies in the supply chain greatly, resulting in the fluctuation in stock prices.

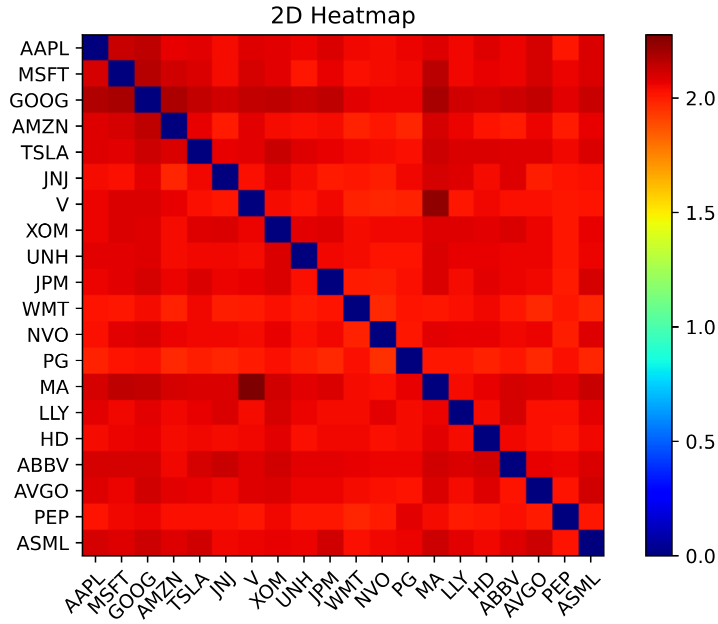

Figure 4: Transfer entropy matrix on log-return among top-20 companies in Nasdaq stock market. The diagonal is set to 0 because transfer entropy is estimated between two different variables.

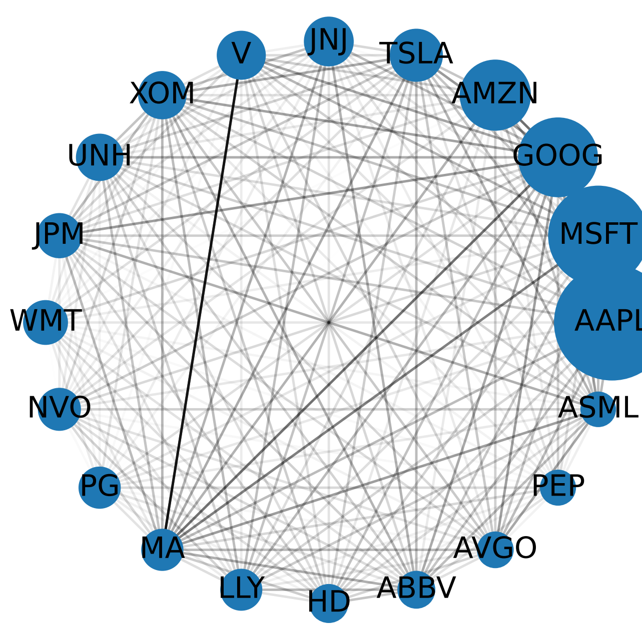

Figure 5: Information flow network of on log-return among top-20 companies in Nasdaq stock market. The size of nodes represents the total stock market capitalization. The transparency of edges represents the strength of information flow.

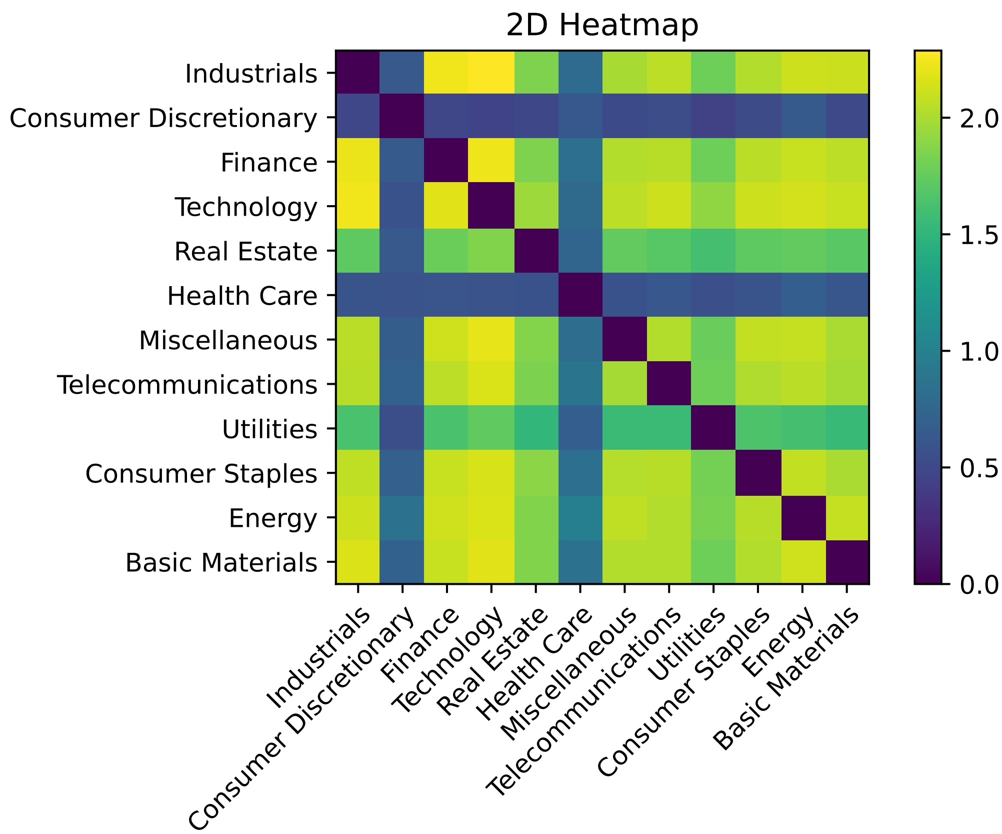

Figure 6: Transfer entropy matrix on log-return among different sectors in Nasdaq stock market. The diagonal is set to 0 because transfer entropy is estimated between two different variables.

5 Conclusion

In this paper, we introduce the concept of transfer entropy with its detailed estimation in time series data. We use synthetic data to validate the effectiveness of transfer entropy and real-world data to analyze the information flow among top companies or different sectors. We find that the transfer entropy is conform to a normal distribution with additive noise and the ratio of transfer entropy reflects the strength of dependency. In the real-world data, we conclude that high-tech companies or payment companies have strong relationships within their sectors in terms of log return. Furthermore, in the estimation of different sectors, we conclude that the consumer discretionary and healthcare sector is much more stable than other sectors.

However, there still remain some issues with this work. Although this paper shows the approach to validate the effectiveness of transfer entropy from the experiments, there is still a lack of mathematical proof in current research, because additive noise introduced each time poses a great influence on the next state with aggregation. Furthermore, how to define effectiveness still remains unexplored. In terms of application, how to select the length of two time series and delay period still remain unexplored. For the future work, we hope to solve the above-mentioned issues to extract information flow among stock market prices precisely.

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