Statistical Models

Selected Topic from STAT 221/231

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University of Waterloo

Choosing a Statistical Models

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Purposes of using statistical models: To study and explain the uncertainty and variability in the inferences made to the population when only a small proportion of the units from the population is reachable.

How to choose a statistical model: Depends on the problem of interest

Choosing a Statistical Model

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We assume that

$$X \sim f(x;\theta) \, .$$

$f(x;\theta)$ is indexed by the parameter θ, which could be a scalar or a vector.

Choosing a Statistical Model

Tossing a fair coin once. The possible outcomes are $\{\text{Head}, \text{Tail}\}$.
Want to know the **chance of getting a head**.

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Model the outcome by a random variable X, a sensible model choice would be the **Bernoulli distribution** indexed by $\theta = \mathbb{P}(X = \text{Head})$, where $\mathbb{P}(X = \text{Head})$ is the probability of getting a head:

$$X \sim Bernoulli(\theta)$$

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In this case, the true value of θ is known ($\frac{1}{2}$) because it is a fair coin.

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The process of selecting a value for θ based on the information (the observed data) we have is called "**estimating**" the value of θ or "**fitting**" the model.

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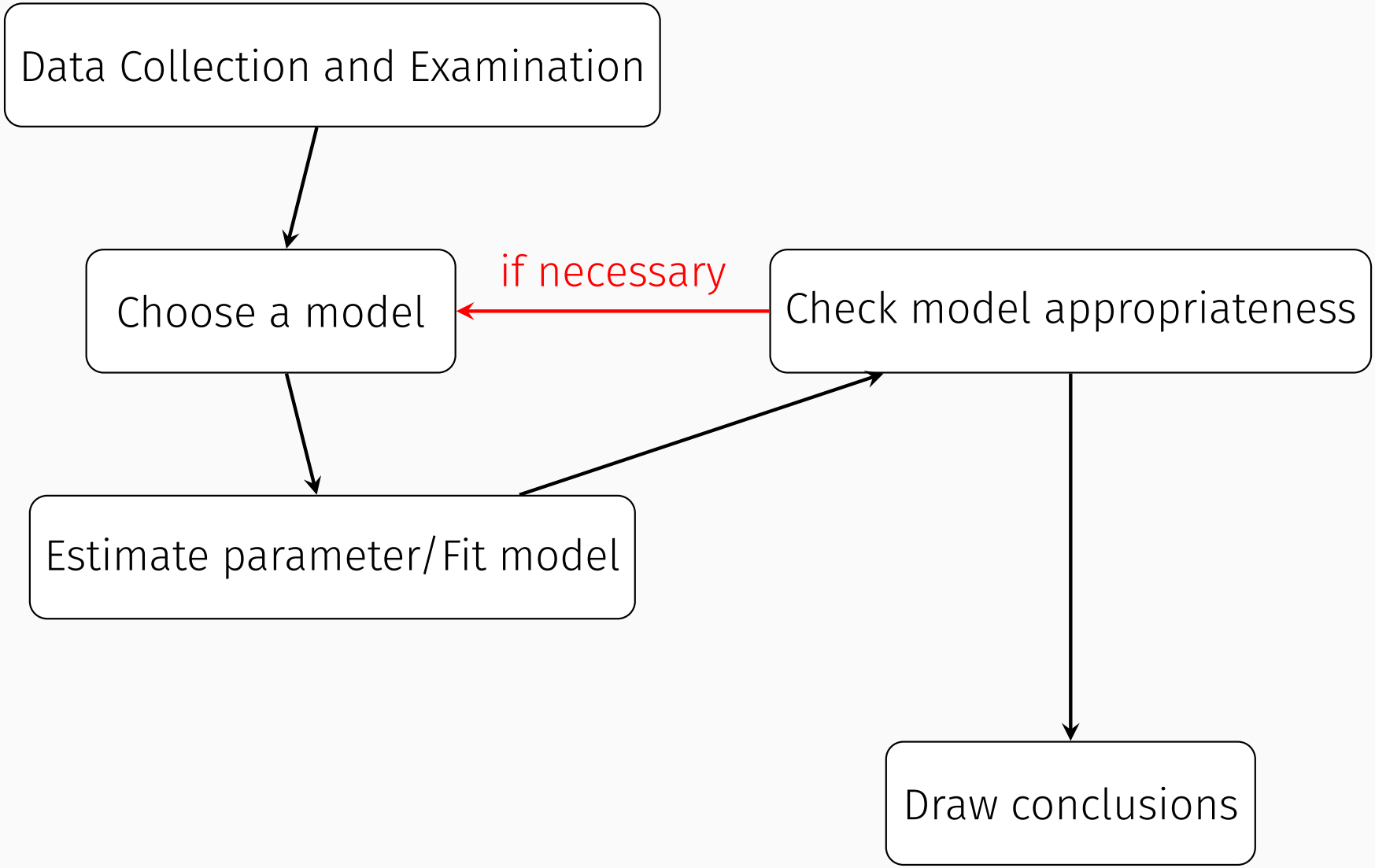

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