2026 Spring p-adic Analysis(Reading Course)

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Abstract

The p-adic theory is a very important part in the modern number theory, which presents new ways of completing $\mathbb{Q}$. As an idea, it's natural for us to try to do calculus on p-adic fields, which is quite hard because of its weird topology. In this course, we will learn the p-adic version of calculus and properties of these special functions: p-adic L-functions, especially for ζ. We'll also learn something about Fontaine's rings as prerequisites for the p-adic Hodge theory by following P. Colmez's lecture notes in Tsinghua.

- **Language**: Mainly English.
- **Schedule**: The course will last for 17 weeks with 3 class periods per week.
- **Content**:

1. **Chapter 1**—Modular Forms
   (a) Basic definitions
   (b) Things when $\Gamma = \mathrm{SL}_2(\mathbb{Z})$
   (c) The algebra of all modular forms
   (d) Hecke operators
   (e) Petersson inner product
2. **Chapter 2**—L-Functions Related to Modular Forms
   (a) L-functions of modular forms
   (b) Higher level modular forms
   (c) Somethings for special L-values: global case
3. **Chapter 3**—The p-adic L-Functions
   (a) Riemann's ζ on negative integers
   (b) Continuity, Banach spaces and measures
   (c) The p-adic ζ
   (d) Smoothness and locally analytic functions
   (e) About distributions
   (f) Somethings for special L-values: p-adic case

4. **Chapter 4**—Cohomological Properties for (φ, Γ)-Modules
   (a) Fontaine's rings
   (b) The (φ, Γ)-modules: definitions
   (c) Galois cohomology
   (d) The complex $C_{\varphi,\gamma}(K, V)$
   (e) Euler-Poincaré formula
   (f) Tate's duality and residues
5. **Chapter 5**—(φ, Γ)-Modules for Iwasawa Theory
   (a) Iwasawa modules $H^i_{\mathrm{Iw}}(K, V)$
   (b) Description of H^i_{Iw} in terms of $D(V)$
   (c) Structure of $H^i_{\mathrm{Iw}}(K, V)$
   (d) Back to p-adic analysis: $D(\mathbb{Z}_p)^{\psi=1}$
   (e) Kummer theory
   (f) Coleman's power series
   (g) An explicit reciprocity law
6. **Chapter 6**—(φ, Γ)-Modules and p-adic L-Functions
   (a) Tate-Sen's theory
   (b) Overconvergent (φ, Γ)-modules
   (c) Tales about Bloch-Kato and Coates-Wiles

References

[1] P. Colmez, *Fontaine's rings and p-adic L-functions.* https://webusers.imj-prg.fr/ pierre.colmez/tsinghua.pdf

[2] P. Colmez, *Notes du nombres p-adiques et fonctions L.* https://webusers.imj-prg.fr/ pierre.colmez/M2.html

[3] N. Koblitz, *p-adic numbers, p-adic analysis, and zeta-functions.* Second edition Grad. Texts in Math., 58 Springer-Verlag, New York, 1984. xii+150 pp. ISBN:0-387-96017-1

Remark [1] *will be our main textbook.*