Reliability Engineering and System Safety 96 (2011) 403–409

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

# Particle filtering prognostic estimation of the remaining useful life of nonlinear components

Enrico Zio*, Giovanni Peloni

*Politecnico di Milano, Dipartimento di Energia, Via Ponzio 34/3, 20133 Milano, Italy*

ARTICLE INFO

*Article history:*
Received 23 October 2009
Received in revised form
14 August 2010
Accepted 20 August 2010
Available online 31 October 2010

*Keywords:*
Life prognosis
Particle filtering
Monte Carlo estimation
Crack propagation

ABSTRACT

Bayesian estimation techniques are being applied with success in component fault diagnosis and prognosis. Within this framework, this paper proposes a methodology for the estimation of the remaining useful life of components based on particle filtering. The approach employs Monte Carlo simulation of a state dynamic model and a measurement model for estimating the posterior probability density function of the state of a degrading component at future times, in other words for predicting the time evolution of the growing fault or damage state. The approach avoids making the simplifying assumptions of linearity and Gaussian noise typical of Kalman filtering, and provides a robust framework for prognosis by accounting effectively for the uncertainties associated to the estimation. Novel tailored estimators are built for higher accuracy. The proposed approach is applied to a crack fault, with satisfactory results.


## 1. Introduction

Prognosis amounts to the generation of long-term (multi-step) predictions of the evolution in time of a particular signal of interest (fault indicator), with the purpose of estimating the remaining useful life (RUL) of a degrading system [1]. Since it involves the projection in the future of the current system condition using a dynamic model of system state evolution in the absence of future measurements, it is unavoidably affected by large-grain uncertainty [2]. For this reason, the critical state variables used as fault indicators are treated as random variables whose probability distributions need to be estimated, for characterizing the precision of the prediction, e.g. in terms of confidence intervals.

Measurements related to the fault indicator values collected by sensors monitoring provide the starting point for prognosis, based on recursive Bayesian estimation techniques [3]. By this approach, long-term predictions of the fault indicators are generated using dynamic models of state evolution, whose initial conditions are estimated on the basis of incoming measurements. Reasonably, sensor data will be available for a certain observation time window if the incipient failure is detected and isolated at early stages, allowing improvements in the predictions via updates in the model parameter and state estimates; at the end of the observation time window, the predictions on the future evolution of the fault are delivered and a decision is made on which corrective action to take.

Particle filtering (PF) is a Monte Carlo-based computational tool particularly useful for Bayesian-framed prognostics of nonlinear and/or non-Gaussian processes [4–6]. The implementation of sequential importance sampling helps reducing the number of samples required to approximate the future state probability distributions, thus increasing the computational efficiency with respect to other classical Monte Carlo methods [5]. Moreover, it allows information from multiple measurement sources to be fused in a logical manner [2].

Prognosis, however, goes beyond filtering in that it involves future time horizons. Hence, if PF-based algorithms are to be used, it is necessary to devise procedures with the capability of projecting at future times the current particle population, in absence of new observations, adjusting the weights if necessary.

With the purpose of effectively addressing the aforementioned issues, a two-level procedure is presented in this paper, capable of controlling the uncertainty associated with long-term predictions by exploiting the system state model, the estimation of the current state and tailored manipulations of the associated predictions.

The procedure is presented by its application to the estimation of the evolution of a nonlinear fatigue crack growth process, used in the literature to describe typical degradation processes in a number of industrial and structural components [7].

The paper is organized as follows. In Section 2, the Bayesian formulation of the model-based state estimation problem and the basic principles underlying the Monte Carlo sampling methods of estimation are briefly recalled for completeness of the paper. In Section 3, the prognosis procedure is explained in detail and in Section 4 the computational framework is applied to the estimation of the residual useful life (RUL) distributions related to the evolving

* Corresponding author.
*E-mail address:* enrico.zio@polimi.it (E. Zio).

0951-8320/$ - see front matter 
doi:10.1016/j.ress.2010.08.009

degradation process. Finally, some conclusions on the advantages and limitations of the procedure are given in Section 5.

## 2. Particle filtering state estimation

Consider a system whose state at the discrete time step $t_k = k\,\Delta t$ is represented by the vector $\mathbf{x}_k$ whose evolution is described by the state space model

$$\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}) \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \boldsymbol{\upsilon}_k) \tag{2}$$

where

- $\mathbf{f}_k : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\omega} \rightarrow \mathbb{R}^{n_x}$ is the state transition function (possibly nonlinear);
- $\{\boldsymbol{\omega}_k, k \in \mathbb{N}\}$ is an independent identically distributed (i.i.d.) state noise vector sequence of known distribution;
- $\mathbf{h}_k : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\upsilon} \rightarrow \mathbb{R}^{n_z}$ is the measurement function (possibly nonlinear);
- $\{\boldsymbol{\upsilon}_k, k \in \mathbb{N}\}$ is an i.i.d. measurement noise vector sequence of known distribution.

The measurements $\{\mathbf{z}_k, k \in \mathbb{N}\}$ are, thus, assumed to be conditionally independent given the state process $\{\mathbf{x}_k, k \in \mathbb{N}\}$, described by a Markov model of first order.

The Bayesian solution to the problem of estimating the dynamic state $\mathbf{x}_k$, given the measurements $\mathrm{z}_k$ up to time $k$ is sought in terms of the probability density function (pdf) $p(\mathbf{x}_k|\mathbf{z}_{0:k})$. This pdf contains all the information about the state $\mathbf{x}_k$, which is inferred from the measurements $\mathbf{z}_{0:k} = \{\mathbf{z}_m, m = 0,\ldots,k\}$ and the initial distribution of the system state $p(\mathbf{x}_0)$ assumed known.

In the so-called *prediction* step, the Chapman–Kolmogorov equation is used to obtain the prior probability distribution of the system state $\mathbf{x}_k$ at time $k$, starting from the probability distribution $p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})$ at time $k-1$ [4,8]:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{z}_{0:k-1}) &= \int p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{z}_{0:k-1}) p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})\, d\mathbf{x}_{k-1} \\ &= \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})\, d\mathbf{x}_{k-1} \end{aligned} \tag{3}$$

in which the transition probability distribution $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ is defined by the system equation (1), with known distribution of the noise vector $\boldsymbol{\omega}_k$, and where the Markovian assumption underpinning the system model (1) has been used.

At time $k$, a new measurement $\mathbf{z}_k$ is collected and used to *update* the prior distribution via Bayes rule, so as to obtain the required posterior distribution of the current state $\mathbf{x}_k$ [4]:

$$p(\mathbf{x}_k|\mathbf{z}_{0:k}) = \frac{p(\mathbf{x}_k|\mathbf{z}_{0:k-1}) p(\mathbf{z}_k|\mathbf{x}_k)}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} \tag{4}$$

where the normalizing constant is

$$p(\mathbf{z}_k|\mathbf{z}_{0:k-1}) = \int p(\mathbf{x}_k|\mathbf{z}_{0:k-1}) p(\mathbf{z}_k|\mathbf{x}_k)\, d\mathbf{x}_k \tag{5}$$

The recurrence relations (3) and (4) give the exact Bayesian solution [4–6]. Unfortunately, except for a few cases, e.g., linear Gaussian state space models (Kalman filter [9]), it is not possible to evaluate analytically these distributions, since they require the evaluation of complex high-dimensional integrals.

One way to overcome this problem is to resort to Monte Carlo sampling or PF methods [4–6,10,11]. Assuming that a set of random samples (particles) $\mathrm{x}_{k-1}^i, i = 1,2,\ldots,N$, of the system state at the time $k-1$ is available as a realization of the posterior probability $p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})$, the predicting step at time $k$ is accomplished by: (i) sampling from the probability distribution of the system noise $\boldsymbol{\omega}_{k-1}$ and (ii) simulating the system dynamics (1) to generate a new set of samples $\mathrm{x}_k^i$ which are realizations of the predicted probability distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k-1})$. In the update step, based on the likelihoods of the observations $\mathbf{z}_k$ collected at time $k$, each sampled particle $\mathrm{x}_{k-1}^i$ is assigned a weight

$$\mathrm{w}_k^i = \frac{p(\mathrm{z}_k|\mathrm{x}_k^i)}{\sum_{j=1}^N p(\mathrm{z}_k|\mathrm{x}_k^j)} \tag{6}$$

An approximation of the posterior distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ can then be obtained from the weighted samples $(\mathrm{x}_k^i, \mathrm{w}_k^i), i = 1,\ldots,N$ [5].

Caution in the implementation of PF is necessary due to the degeneracy problem: as the algorithm evolves in time, the weight variance increases [12] and the importance weight distribution becomes progressively skewed, until (after a few iterations) all but one particle have negligible weights [4–6,13]. As a result, the approximation of the target distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ becomes very poor and significant computational resources are spent trying to update particles with minimum relevance. To avoid the problem, one can proceed to resampling a new swarm of realizations $\mathrm{x}_k^i$ from the approximate posterior distribution, constructed on the weighted samples previously drawn; all particles thereby generated are assigned equal weights, $\mathrm{w}_k^i = 1/N$ [5]; as final step, one has to resample a new swarm of points $\mathrm{x}_k^i$ from the posterior distribution. The prediction, update and resample steps form a single iteration and are recursively applied at each time $k$.

## 3. Particle filtering prognosis

In this section, the computational framework for prognosis by particle filtering is described [2] and estimators of the remaining useful life of a component are proposed.

### *3.1. From the estimation of the current state to an unbiased estimator of the RUL*

The first stage of prognosis regards the generation of a $l$-step ahead, long-term prediction of the state pdf $p(\mathbf{x}_{k+l}|\mathbf{z}_{0:k})$, $l = 1,\ldots,T-k$, where $T$ is the time horizon of interest for the analysis. To this aim, it is necessary to consider that no information is available for estimating the likelihoods of the degrading state following the future paths $\mathbf{x}_{k+1:k+l}$, since future observations $\mathbf{z}_{k+l}$, $l = 1,\ldots,T-k$, have obviously not yet been collected. The only thing that is known is the state dynamic model (1), which allows to say that at the future times $j = k+1,\ldots,k+l$, the state $\mathbf{x}_{j-1}$ will pass into state $\mathrm{x}_j$ with transition pdf $p(\mathbf{x}_j|\mathbf{x}_{j-1})$. It is thus necessary to project the initial condition $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ among all possible future paths weighted by their probability $\prod_{j=k+1}^{k+l} p(\mathrm{x}_j|\mathrm{x}_{j-1})\, d\mathrm{x}_{j-1}$ (Fig. 1).

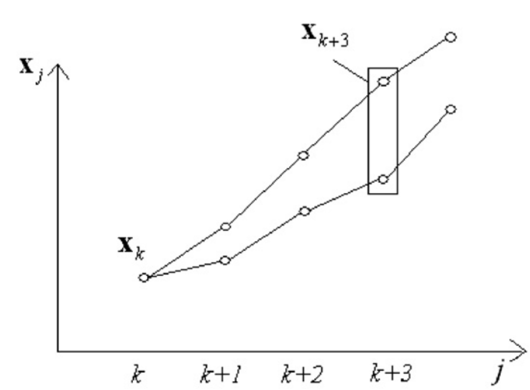


**Fig. 1.** Graphical illustration of two possible future evolutions of the degradation state for $l = 3$ steps ahead, which are integrated in (7), weighed by their probability of occurrence.

Then, by combining the model equation (1) and the current state pdf estimate (4), the $l$-step ahead posterior distribution is obtained:

$$p(\mathbf{x}_{k+l}|\mathbf{z}_{0:k})=\int\cdots\int\prod_{j=k+1}^{k+l}p(\mathbf{x}_j|\mathbf{x}_{j-1})p(\mathbf{x}_k|\mathbf{z}_{0:k})\prod_{j=k}^{k+l-1}d\mathbf{x}_j \tag{7}$$

The evaluation of the integrals in (7) may be difficult and/or require significant computational effort. The PF approach offers an efficient solution for estimating the predictive posterior distribution (7), as follows:

1. At the generic time $k+l-1$, consider the swarm of particles $(\mathrm{x}^i_{k+l-1},\mathrm{w}^i_{k+l-1})$, $i=1,2,\ldots,N$, approximating the pdf $p(\mathbf{x}_{k+l-1}|\mathbf{z}_{0:k})$ (note that at the first step ($l=1$) the particles are those of the PF estimation of the current state $\mathbf{x}_k$ based on all the observations $\mathbf{z}_{0:k}$ collected up to the current time $k$ [11]).
2. With reference to the familiar Monte Carlo pdf approximation [8]:

$$\begin{aligned}p(\mathrm{x}_{k+l}\Big|\mathrm{z}_{0:k})&=\int p(\mathrm{x}_{k+l}|\mathrm{x}_{k+l-1})p(\mathrm{x}_{k+l-1}|\mathrm{z}_{0:k})d\mathrm{x}_{k+l-1}\\&\approx\int\sum_{i=1}^{N}\mathrm{w}^i_{k+l-1}\delta(\mathrm{x}_{k+l-1}-\mathrm{x}^i_{k+l-1})p(\mathrm{x}_{k+l}|\mathrm{x}_{k+l-1})d\mathrm{x}_{k+l-1}\\&=\sum_{i=1}^{N}\mathrm{w}^i_{k+l-1}p(\mathrm{x}_{k+l}\Big|\mathrm{x}^i_{k+l-1})\triangleq\hat{p}(\mathrm{x}_{k+l}|\mathrm{z}_{0:k}),\end{aligned} \tag{8}$$

   where $\delta$ is the Dirac-delta operator, compute $p(\mathrm{x}_{k+l}|\mathrm{x}^i_{k+l-1})$ using the state $\mathrm{x}^i_{k+l-1}$ of every particle and the stochastic process model:

$$p\left(\mathrm{x}_{k+l}|\mathrm{x}^i_{k+l-1}\right)=q_{k+l-1}\left(\mathrm{f}^{-1}_{k+l-1}\left(\mathrm{x}_{k+l},\mathrm{x}^i_{k+l-1}\right)\right)\left|\frac{\partial\mathrm{f}^{-1}_{k+l-1}}{\partial\mathrm{x}_{k+l}}\right| \tag{9}$$

   where $\mathrm{f}^{-1}_{k+l-1}$ is the inverse of the state transition function in (1) and $q_{k+l-1}$ is the pdf of the state noise vector $\boldsymbol{\omega}_{k+l-1}$.
3. To obtain the desired estimate (8), perform an inverse transform re-sampling procedure of the particle population [14,15]. This method obtains samples distributed according to $p(\mathbf{x}_{k+l}|\mathbf{z}_{0:k})$. This is done by first drawing $N$ values of a uniform random variable in the interval [0,1), $u^i\sim U[0,1)$, $i=1,\ldots,N$. The generic $i$th realization $x^i_{k+l}$ of $p(\mathbf{x}_{k+l}|\mathbf{z}_{0:k})$ is then obtained by interpolation of the cumulative state distribution $F(\mathrm{X}_{k+l}\le\mathrm{x}_{k+l})=\int_{-\infty}^{\mathrm{x}_{k+l}}\hat{p}(\mathrm{x}_{k+l}|\mathrm{x}_{1:k+l-1})d\mathrm{x}_{k+l}$, such that $\mathrm{x}^i_{k+l}=F^{-1}(u^i)$ (i.e., $x^i_{k+l}$ is the particle vector such that $F(x^i_{k+l})=u^i$). The weights of the re-sampled particles are kept unchanged, i.e., $\mathrm{w}^i_{k+l}=\mathrm{w}^i_{k+l-1}$.

The procedure is repeated at each time step $k+l$, until the time horizon of interest $T$ is reached.

Once $\hat{p}(\mathrm{x}_{k+l}|\mathrm{z}_{0:k})$, $l=1,\ldots,T-k$, is known, estimates can be obtained of functions of the state $\mathbf{x}_{k+l}$ conditional on the measurements $\mathbf{z}_{0:k}$. In particular, in the case considered in this paper, the interest is on the estimation of $p(RUL|\mathbf{z}_{0:k})$, where $RUL$ is the remaining useful life of a component subject to degradation. In this case, the state $x_k$ (for simplicity but no loss of generality, considered mono-dimensional from now on) represents a fault indicator (e.g., a temperature, a liquid level, a crack depth, etc.), and the RUL is the time remained before its crossing of a pre-defined safety threshold $\lambda$. For each time $k+l$ projected $l$ steps ahead from the current time $k$, the estimate $\hat{p}(RUL\le l|z_{0:k})$ is equal to $\hat{p}(x_{k+l}\ge\lambda|z_{0:k})$. As time proceeds, the estimate is updated on the basis of the new measurements collected $\{\mathbf{z}_j, j=k+1,\ldots,k+l\}$.

### 3.2. Alternative biased estimators for prognostics

The estimator above described is unbiased; on the other hand, there are cases when a biased estimator can be more efficient, e.g. when large grain uncertainty is involved which possibly introduces outliers [16]. Indeed, the biased estimator $\hat{\theta}$ of a generic parameter $\theta$ may have a smaller variance than an unbiased estimator, such to reduce the mean squared error ($MSE$) in accordance with [3]:

$$MSE(\hat{\theta})=Var(\hat{\theta})+(E(\hat{\theta})-\theta)^2 \tag{10}$$

The proposed biased estimator takes the following general form:

$$\tilde{p}(RUL\le l|z_{0:k+l})=\frac{\sum_{r=k}^{k+l-1}\hat{v}_r(z_{r+1:k+l})\hat{p}(RUL\le l|z_{0:r})}{\sum_{r=k}^{k+l-1}\hat{v}_r(z_{r+1:k+l})} \tag{11}$$

where $\hat{v}_r(z_{r+1:k+l})$ are called *credibility weights*, properly defined to measure the credibility of the estimates of $\hat{p}(RUL\le l|z_{0:r})$ made at time steps $r=k,\ldots,k+l-1$ prior to $t_{0:k+l}$. The credibility of an estimate at the generic instant $k\le r<k+l$ is measured on the basis of the accuracy in the estimation of the future states $\mathbf{x}_{r+1:k+l}$, with respect to the measurements actually collected $\mathbf{z}_{r+1:k+l}$.

Obviously, (11) is equal to $\hat{p}(RUL\le l|z_{0:k+l})$ only if $\hat{v}_r(z_{r+1:k+l})=\delta_{r,k+l}\ \forall r\in[k,k+l-1]$, i.e., if the estimation of the state $\mathbf{x}_{r+1:k+l}$ is perfectly accurate. Still, a proper definition of these weights could allow obtaining an improved estimator in the sense of a reduced MSE (10). The idea is to choose the credibility weights so to give smaller relevance to the contributions of the estimates $\hat{p}(RUL\le l|z_{0:r})$ in (11) which are associated to less accurate state estimates at previous times. If the latest available measurements are good realizations of the measurement process (i.e., if they are not outliers), the PF algorithm described in the previous Section will lead to a good approximation $\hat{p}(RUL\le l|z_{0:r})$. For example, projecting from $k=45$ to $k+l=63$ and 81, in the left side of Fig. 2, the true state (indicated by a dashed thick horizontal segment) is included in the interval between the thirtieth and the seventieth percentiles of $\hat{p}(x_{k+l}|z_{0:r})$ (indicated by the solid thick horizontal segments). On the contrary, if the last available measurements are outliers, the PF algorithm will lead to a bad approximation $\hat{p}(RUL\le l|z_{0:r})$. This latter situation is shown in the right side of

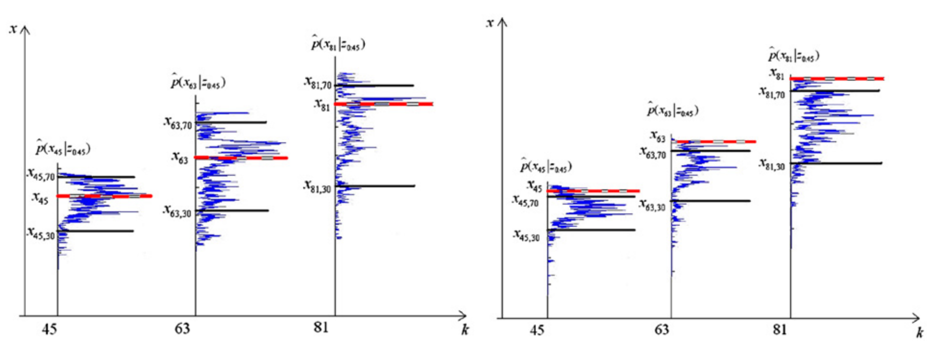


**Fig. 2.** Pdfs of particles on the basis of good measurements up to $k=45$ leading to a good estimation of $p(RUL|z_{0:45})$ (left side). Pdfs of particles on the basis of outliers in the measurement process up to $k=45$ leading to a poor estimation of $p(RUL|z_{0:45})$ (right side).

Fig. 2, where the true state is not included in the interval between the thirtieth and the seventieth percentiles of $\hat{p}(x_{k+l}|z_{0:r})$.

Three weighting procedures are here defined, to be applied at each temporal step $\tau$ within $r \le \tau < k+l$:

(a) *Interval weights:* To determine the consistency between the estimate $\hat{x}_\tau$ conditional on $\mathbf{z}_{0:r}$ and that conditional on $\mathbf{z}_{0:\tau}$, one may define the distance:

$$d(\hat{x}_\tau, z_{k+1:\tau}) = \begin{cases} \dfrac{1}{k+l-(r+1)} & \text{if } x_{\tau,30} \le E_{p(x_\tau|z_{0:r})}(X_\tau) \le x_{\tau,70} \\ 0 & \text{otherwise} \end{cases}$$

where $x_{\tau,30}$ and $x_{\tau,70}$ are the thirtieth and the seventieth percentiles of the state variable $X_\tau$ conditional on the measurements $\mathbf{z}_{0:\tau}$, respectively. The credibility weight $\hat{v}_r(z_{r+1:k+l})$ is thus equal to $\sum_{\tau=r+1}^{k+l} d(\hat{x}_\tau, z_{k+1:\tau})$ and the estimate $\hat{p}(RUL \le l|z_{0:r})$ will be the more credible the more $\hat{v}_r(z_{r+1:k+l})$ is close to 1 (Fig. 3).

(b) *Likelihood weights*: In this case the estimates $\hat{x}_\tau$, conditional on the measurements $\mathbf{z}_{0:r}$ and on the measurements $z_{r+1:\tau}$, are said to be close if the likelihood $l(z_\tau|\hat{x}_\tau)$ is high to have a measurement equal to $z_\tau$ when the state of the system $x_\tau$ at $\tau$ is taken equal to the estimate $\hat{x}_\tau$ (Fig. 4). Operatively, one needs to compute

$$\hat{v}_r(z_{r+1:\tau}) = \left( \frac{1}{N} \prod_{\tau=r+1}^{k+l} l(z_\tau|\hat{x}_\tau) \right)^{1/(k+l-(r+1))}$$

where

$$l(z_\tau|\hat{x}_\tau) = p(\hat{x}_\tau|z_\tau) = q_\tau(h_\tau^{-1}(\hat{x}_\tau, z_\tau)) \left| \frac{\partial h_\tau^{-1}}{\partial \hat{x}_\tau} \right|$$

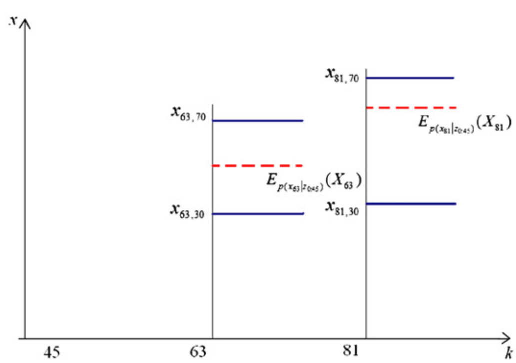


**Fig. 3.** In this example $E_{p(x_{63}|z_{0:45})(X_{63})}$ and $E_{p(x_{81}|z_{0:45})(X_{81})}$, indicated by dashed lines, are included between the thirtieth and seventieth percentiles of $p(x_{63}|z_{0:63})$ and $p(x_{81}|z_{0:81})$, indicated by solid thick horizontal segments.

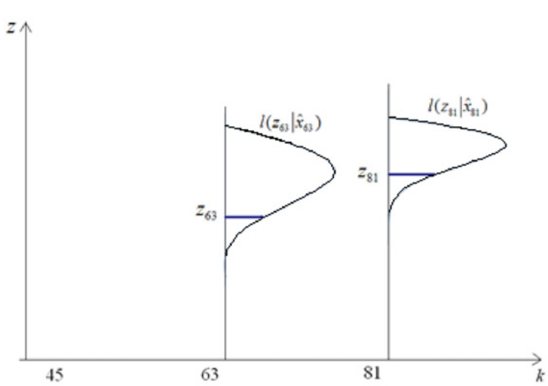


**Fig. 4.** The likelihoods $l(z_{63}|\hat{x}_{63})$ and $l(z_{81}|\hat{x}_{81})$ are built on the basis of $\hat{x}_{63}$ and $\hat{x}_{81}$, conditional on the measurements $z_{0:45}$.

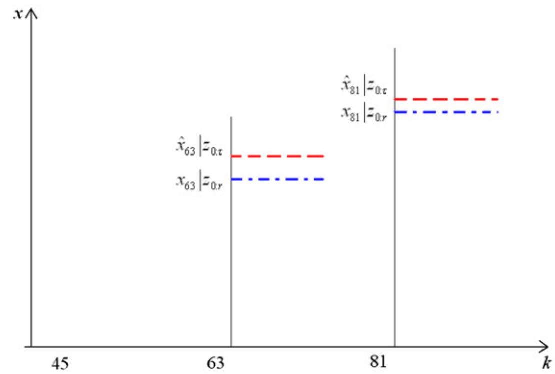


**Fig. 5.** The normalized squared distance between $\hat{x}_{63}|z_{0:\tau}$ and $\hat{x}_{63}|z_{0:r}$ are compared.

where $q_\tau$ is the pdf of the state measurement vector and $h_\tau^{-1}$ is the inverse of the measurement function in (2).

(c) *RMSE weights:* In this case, the normalized squared distance is computed between the state estimates $\hat{x}_\tau|z_{0:\tau}$ and $\hat{x}_\tau|z_{0:r}$, conditional on the measurements $z_{0:\tau}$ and $z_{0:r}$, respectively (Fig. 5):

$$v_r(z_{r+1:k+l}) = \sqrt{\frac{1}{1/(k+l-(r+1)) \sum_{\tau=r+1}^{k+l} ((\hat{x}_\tau|z_{0:\tau} - \hat{x}_\tau|z_{0:r})^2)/\hat{x}_\tau|z_{0:r}}}$$

It is worth noticing that whereas the interval and RMSE weights depend on all the measurements $z_{0:\tau}$, the likelihood weights depend only on the last measurement $z_\tau$, thus partially removing the outliers problem as it will be shown in the application.

## 4. Particle filtering prognostics of the RUL of a mechanical component subject to fatigue crack growth

The system considered is a mechanical component subject to fatigue. The following state space model describes the dynamic evolution of the mono-dimensional crack depth $x$ as a function of the load cycles $N$ [7]:

$$\frac{dx}{dN} = C(\Delta K)^n \tag{12}$$

where $C$ and $n$ are constants related to the material properties, to be estimated from experimental data, and $\Delta K$ is the stress intensity amplitude roughly proportional to the square root of $x$:

$$\Delta K = \beta\sqrt{x} \tag{13}$$

In (13), the parameter $\beta$ is again a constant to be estimated from experimental data. The intrinsic stochasticity of the process may be inserted in the model modifying (12) as follows [7]:

$$\frac{dx}{dN} = e^{\omega} C(\Delta K)^n \tag{14}$$

where $\omega \sim N(0, \sigma_\omega^2)$ is a white Gaussian noise.

For $\Delta N$ sufficiently small, the state space model (14) can be discretized to give:

$$x_k = x_{k-1} + e^{\omega_k} C(\Delta K)^n \Delta N \tag{15}$$

which represents a nonlinear Markov process with independent, non-stationary degradation increments.

The degradation state $\boldsymbol{x}_k$ is generally not directly measurable. In the case of non-destructive ultrasonic inspections, a logit model for the observation $z_k$ can be introduced [17]:

$$\ln \frac{z_k}{d - z_k} = \beta_0 + \beta_1 \ln \frac{x_k}{d - x_k} + v_k \tag{16}$$

where $d$ is the component material thickness, $\beta_0 \in (-\infty, +\infty)$ and $\beta_1 > 0$ are parameters to be estimated from experimental data and $\mathbf{v}$ is a white Gaussian noise such that $v \sim N(0,\sigma_v^2)$. Introducing the following standard transformations:

$$y_k = \ln \frac{z_k}{d - z_k} \tag{17}$$

$$\mu_k = \beta_0 + \beta_1 \ln \frac{x_k}{d - x_k} \tag{18}$$

then $Y_k \sim N(\mu_k, \sigma_v^2)$ is a Gaussian random variable. The likelihood $p(z_k|x_k)$ is thus

$$p(z_k|x_k) = \frac{1}{\sqrt{2\pi}\sigma_v} e^{-(1/2)(\ln(z_k/(d-z_k)) - \mu_k)/\sigma_v)^2} \frac{d}{z_k(d-z_k)} \tag{19}$$

Without loss of generality, in the numerical example that follows the variables and parameters appearing in the above equations are taken in arbitrary units of measurement. In particular, the numerical values of the parameters in the state equation (15) are $C=0.005$, $n=1.3$, $\beta=1$, whereas those in the measurement equation (16) are $\beta_0=0.06$, $\beta_1=1.25$ [17]. The process and measurement noise variances are $\sigma_\omega^2 = 2.89$, $\sigma_v^2 = 0.22$, respectively [17]. The cycle time step is $\Delta N = 1$. The component is assumed to fail when the crack depth $x > \lambda = 40$ in arbitrary units.

### 4.1. Results

The PF state estimation of Section 2 has been performed on the basis of $N=1000$ particles. Fig. 6 shows the results: the solid line corresponds to the reference crack depth evolution considered as the real evolution occurring, the dashed line is the median of the posterior distribution estimated with the $N=1000$ particles, whereas the dot-dashed line corresponds to the fifth percentile.

The particles $x_k^i$, $i=1,\ldots,1000$ are then projected in the future to estimate $\hat{p}(x_{k+l}|z_{0:k})$ according to the prognosis procedure of Section 3.1.

Updating the estimated pdf on the basis of the measurements collected every 20 temporal steps, one obtains the RUL median estimate of Fig. 7 and corresponding 90% confidence interval. The diagonal dashed line represents the real RUL of the component and the solid line corresponds to the median value of $\hat{p}(RUL|z_{0:k})$.

At two updating times, the confidence intervals do not include the true RUL value because the realizations of the measurement process, which the estimates of these intervals are based on, are outliers, which then perturb the estimates.

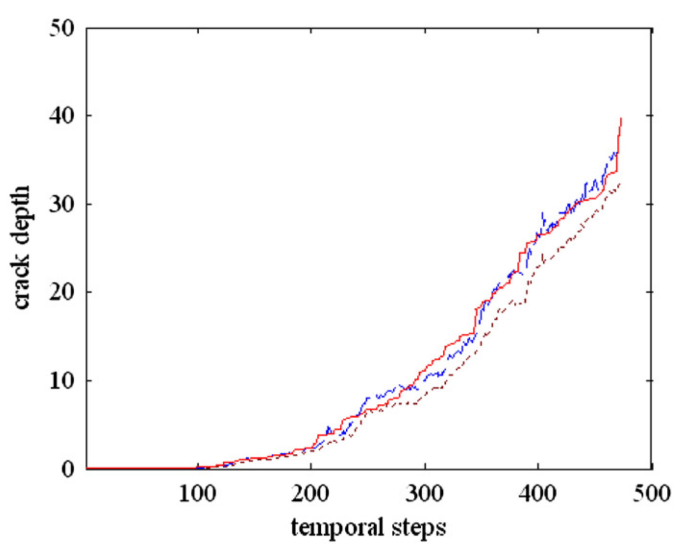


**Fig. 6.** Crack depth evolution: real (solid line), median of the estimated posterior distribution (dashed line), fifth percentile of the estimated posterior distribution.

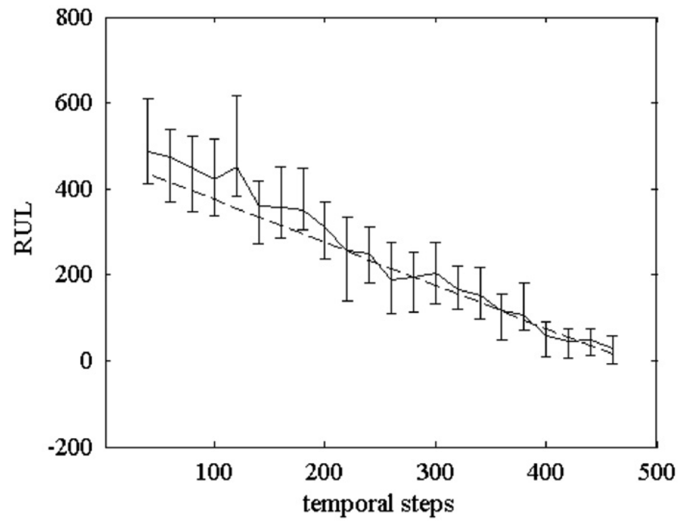


**Fig. 7.** RUL real value (dashed line), median (solid line) and 90% confidence interval estimates (bars).

The left side of Figs. 8–10 show the estimates of the RUL median and 19th confidence interval computed by (11) with the three credibility weights-methods presented in Section 3.2. The right side of the Figures shows the corresponding values of the credibility weights. As expected, all three methods seem to offer a more precise, although biased, estimator of $p(RUL|z_{0:k})$, as shown by the smoother behavior of the median and 19th confidence interval estimates. Also as expected, the credibility weights show a growing trend due to the fact that, in principle, the last estimates are more accurate. The fluctuations in the weights values highlight the presence of some outliers in the realizations of the measurement process $z$. In any case, the 19th confidence intervals estimated by the three methods almost always include the true RUL, except for the method using likelihood credibility weights at the temporal step 120. The smaller robustness of the likelihood method is due to the fact that in this case $p(z_r|\hat{x}_r)$depends only on the last measured $z_r$ and not on $z_{0:r}$, so that no smoothing occurs.

## 5. Conclusions

Failure prognostics is the 'holy grail' of RAMS analysis [17]. The possibility of predicting the remaining life of the components, systems and structures of an industrial plant constitute an incredibly attractive opportunity for reducing maintenance costs and enhancing productivity, while not compromising safety.

In this work, a Particle Filtering-based prognostics framework has been introduced for the precise estimation of the component remaining useful life. The simulation method is capable of handling nonlinear dynamics and of dealing with non-Gaussian noises at no further computational or design expenses. As such, it represents a valuable prognostic tool which can drive effective condition-based maintenance strategies for improving the availability, safety and cost effectiveness of plant operation. Original estimators have been introduced to improve the precision of the method; these have been shown effective in a prognostic case study regarding the fatigue crack dynamic evolution of a mechanical component.

Future research is needed to verify how the Particle Filtering estimators introduced can cope with the challenges and open issues typical of many other prognostic methods, some of which are briefly recalled in the following: (i) state estimations and predictions must be accompanied by a measure of the associated error,

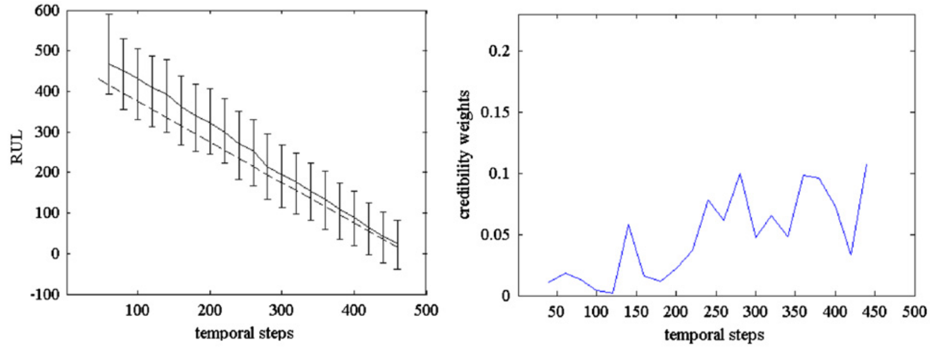


**Fig. 8.** RUL real value (dashed line), median (solid line) and 90% confidence interval estimates (bars) (left) and credibility interval weights (right).

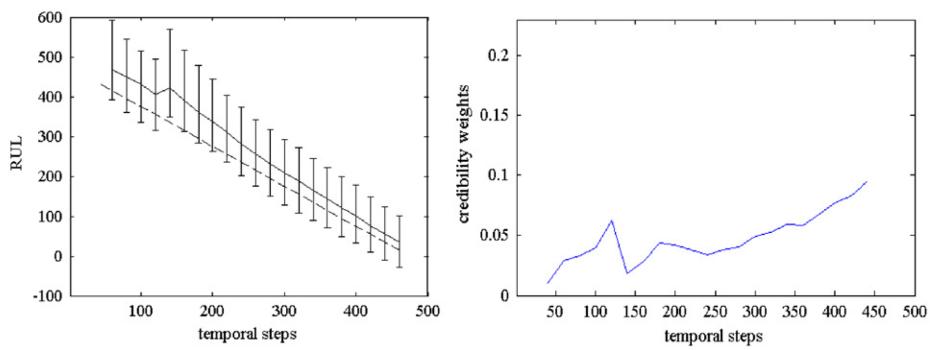


**Fig. 9.** RUL real value (dashed line), median (solid line) and 90% confidence interval estimates (bars) (left) and credibility likelihood weights (right).

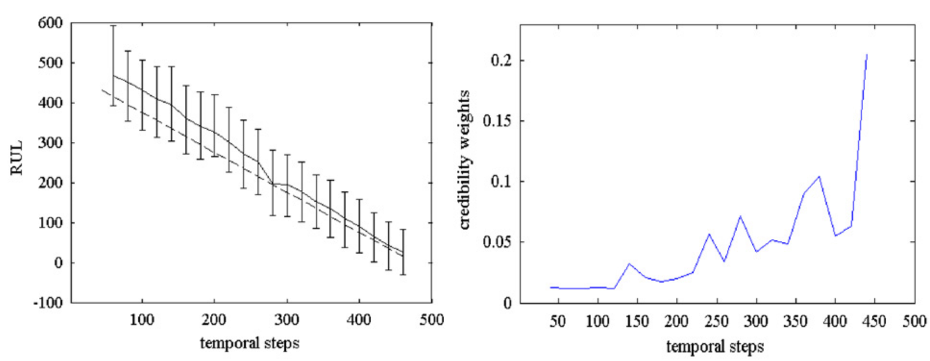


**Fig. 10.** RUL real value (dashed line), median (solid line) and 90% confidence interval estimates (bars) (left) and credibility RMSE weights (right).

accounting for the incomplete and imprecise information available on the process; this measure is fundamental for projecting the confidence on the predictions and the related decisions on the actions to take on the component to effectively and reliably control its function; (ii) processes in general change in time, as do the functioning of component due to changes in external environments, structural changes, changes in the input, retrofitting, etc.; prognostic methods in general must be able to accommodate these changes and perform equally well in the different working conditions experienced; (iii) for timely prognostics, the amount of modeling required, and the storage and computational burden associated must be properly gauged to the application; (iv) the treatment of multiple faults may represent a significant computational challenge as the more combinations are possible, the more powerful and informative set of measurements we need for discriminating among the different occurrences and avoiding shielding effects, i.e., conditions for which the failures of certain components render impossible the retrieval of information on the state of other components.

## Acknowledgments

The authors are thankful to the two anonymous reviewers for their comments and suggestions.

## References

[1] Coble JB, Hines W. Prognostic algorithm categorization with PHM challenge application. International conference on prognostics and health management (PHM08), Marriott Tech Center, Denver, CO, USA, 6–9 October 2008.
[2] Orchard ME, Vachtsevanos G. A particle filtering framework for failure prognosis. In: Proceedings of the WTC2005, world tribology congress III, Washington, DC, USA, 2005.
[3] Berger JO. Statistical decision theory and Bayesian analysis. second ed. New York: Springer Verlag; 1985.
[4] Arulampalam MS, Maskell S, Gordon N, Clapp T. A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. IEEE Transactions on Signal Processing 2002;50(2):174–88.
[5] Doucet A, de Freitas JFG, Gordon NJ. An introduction to sequential Monte Carlo methods. In: Doucet A, de Freitas JFG, Gordon NJ, editors. Sequential Monte Carlo in practice. New York: Springer-Verlag; 2001.

[6] Doucet A, Godsill S, Andreu C. On sequential Monte Carlo sampling methods for Bayesian filtering. Statistics and Computing 2000;10:197–208.
[7] Myotyri E, Pulkkinen U, Simola K. Application of stochastic filtering for lifetime prediction. Reliability Engineering and System Safety 2006;91:200–8.
[8] Li P, Kadirkamanathan V. Particle filtering based likelihood ratio approach to fault diagnosis in nonlinear stochastic systems. IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews 2001;31(3):337–43.
[9] Kalman RE. A new approach to linear filtering and prediction problems. Journal of Basic Engineering 1960;1(82):35–45.
[10] Pulkkinen U. A stochastic model for wear prediction through condition monitoring. In: Holmberg K, Folkeson A, editors. Operational reliability and systematic maintenance. London, New York: Elsevier; 1991. p. 223–43.
[11] Seong S-H, Park H-Y, Kim D-H, Suh Y-S, Hur S, Koo I-S, et al. Development of fast running simulation methodology using neural networks for load follow operation. Nuclear Science and Engineering 2002;vol. 141:66–77.
[12] Doucet A. On sequential Monte Carlo methods for Bayesian filtering. Technical Report, Engineering Department, University of Cambridge, UK, 1998.
[13] Andrieu C, Doucet A, Punskaya E. Sequential Monte Carlo methods for optimal filtering. In: Doucet A, de Freitas N, Gordon N, editors. Sequential Monte Carlo methods in practice. New York: Springer-Verlag; 2001.
[14] Kalos MH, Whitlock PA. Monte Carlo methods, vol. I: Basics, vol. I. New York: John Wiley; 1986.
[15] Marseguerra M, Zio E. Basics of the Monte Carlo method with application to system reliability. LiLoLe-Verlag GmbH (Publ. Co. Ltd.); 2002.
[16] Lehmann EL. Classical estimation. In: Kotz S, Read CB, Balakrishnan N, Vidakovic B, editors. Encyclopedia of statistical sciences. John Wiley & Sons, Inc.; 2006.
[17] Cadini F, Zio E, Avram D. Monte Carlo-based filtering for fatigue crack growth estimation. Probabilistic Engineering Mechanics 2009;24(3):367–73.