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Sequential seeding for spreading in complex networks: Influence of the network topology

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HIGHLIGHTS

- Sequential seeding strategies are constructed based on network centralities.
- Sequential seeding strategies are examined in both real and synthetic networks.
- Sequential seeding strategies perform better in networks with heterogeneous degrees.
- Average degree and assortativity coefficient also have influence on the performance.

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ABSTRACT

In this paper we investigate the problem of sequential seeding for spreading in complex networks. We focus on the influence of network topology on the performance of seeding strategies. The classic independent cascade model (ICM) is adopted to represent the spreading process. We examine the centrality measures—degree, K-shell, and H-index in several real networks and confirm that degree is a good indicator for spreading efficiency. Scale-free networks with tunable parameters such as power-law exponent, density, and assortativity coefficient are constructed as the testbed of the study. By simulations, we find that the advantage of sequential seeding strategy is large in a degree-heterogeneous network with relatively small average degree and large assortativity coefficient.

1. Introduction

Spreading in complex networks is a very common phenomenon in the real world, ranging from disease spreading among human beings through contact, information and rumor spreading in online social networking websites, to advertising though viral marketing in the society [1,2].

A hot research topic related to spreading in complex networks is how to choose a set of nodes in the network as initial seeds such that the spreading process starting from these seeds can reach the maximum fraction of the population in the network. This is the so-called influence maximization problem [3]. Another similar topic is influential nodes identification problem, i.e., find out a set of nodes which have the highest rank according to some specific measure [4–6]. In fact, if we set the measure as the size of the population reached in the spreading process, the problem of identifying influential nodes is specialized to influence maximization problem. Conversely, choosing influential nodes in the network as seeds to start

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the spreading process is in general a better strategy than randomly choosing seeds among all nodes. Therefore, these two research areas can provide valuable insight and inspiration to each other.

Searching for the optimal seeding strategy is in general an NP-hard problem. Thus, greedy algorithms and other heuristic strategies are used to search for approximate solutions in many applications. Especially, when dealing with a large-scale network, heuristic strategies seem to be the only choice [3,7]. Some classic centrality measures are used in constructing heuristic algorithms, such as degree, betweeness, closeness, PageRank, and K-shell. In a heuristic strategy, a set of nodes with highest centrality measure are selected as the seeds to start the spreading process.

A popular assumption in the existing work is that all of the seeds are activated at the initial time step and then they spread the active state to other nodes through the connections among them. Here we call it single-stage seeding strategy. Most recently, Jankowski et al. propose several sequential seeding strategies, in which only a part of seeds are activated at the initial time step and other seeds are activated in the subsequent steps according to some rules [8,9]. They show that for a fixed number of seeds, sequential seeding strategies can active more nodes than the single-stage seeding strategy. They provide extensive simulations to investigate the influence of many factors, such as centrality measures, seed percentage, and propagation probability, on the performance of sequential seeding strategies. However, the influence of network topology is rarely mentioned in their work. In this paper, we construct scale-free networks and provide a systematic study of the influence of network topological properties (such as network density, degree distribution, and assortativity coefficient) on the performance of sequential seeding strategies.

The paper is organized as follows: In Section 2, we introduce a widely used spreading model—the independent cascade model which captures the spreading dynamic in our work. In Section 3, resorting to real networks, we compare the performance of three sequential seeding strategies, which are constructed based on degree, K-shell, and H-index, respectively. Due to its best performance, we choose degree as the centrality measure in our sequential seeding strategy, and study the influence of network topology in artificial scale-free networks in Section 4. Concluding remarks are given in Section 5.

2. Spreading models

To study the spreading behavior in networks theoretically, we first need to set up the spreading model. In the literature, a variety of spreading models are proposed in different research areas. A review of the models and relations among them can be found in [10]. In our paper, we adopt a commonly used model—the independent cascade model (ICM).

There are many variants of ICM in the literature. Here we adopt the conceptually simplest version that is studied in [3,11]. The spreading process unfolds in discrete steps as follows:

(1) At the initial time step t_0, a set of nodes are activated, which are regarded as the seeds to trigger the spreading process.
(2) At any time step $t > t_0$, each of the active nodes, say v, has a change to activate each of the inactive nodes, say w, which is directly connected with it. It succeeds with a probability p called spreading rate, and whether or not it succeeds, it could not make further attempts to activate w in the subsequent time steps.
(3) The process runs until no more nodes can be possibly activated. The size of the population activated represents the coverage of the spreading process.

Based on this ICM spreading model, next we will study how the seeding strategies (sequential and single-stage) affect the final population coverage of the spreading process.

3. Sequential seeding in real networks

3.1. Single-stage seeding and sequential seeding

We compare two kinds of seeding strategies in this paper: single-stage seeding and sequential seeding. After ranking the nodes descendingly according to a given centrality measure, the two strategies are as follows:

Single-Stage Seeding: Select the top k nodes as the seeds and activate them all at the initial time step.

Sequential Seeding: Only select the top $k/2$ nodes as the first half number of seeds and activate them at the initial time step. Once the spreading process ends, select top $k/2$ nodes which are not activated yet as the other half number of the seeds and activate them.

The comparison of the two strategies is first conducted on several real networks from SNAP Datasets [12]. Among all the networks in SNAP datasets, we mainly choose social networks, where the spreading phenomenon is usually observed. And due to the restrict of our computational capacity, we have to choose medium size networks with less than 100 thousand nodes. We choose seven networks from SNAP and provide the summary of them in Table 1, where the first column shows the names of the networks, the second column contains the types of the networks (directed or undirected), the subsequent two columns show the numbers of nodes and edges, respectively, and the last column has brief descriptions of the networks and their corresponding references.

Table 1
Basic information of seven real networks.

Networks	Type	Nodes	Edges	Description
Email	Undirected	36 692	183 831	Email communication network from Enron [13]
Bitcoin	Directed	5 881	35 592	Bitcoin OTC web of trust network [14]
Epinions	Directed	75,879	508,837	Who-trusts-whom network of Epinions.com [15]
Facebook	Undirected	4,039	88,234	Social circles from Facebook [16]
P2P	Directed	8,846	31,839	Gnutella peer to peer network from August 5 2002 [17]
Slashdot	Directed	77,360	905,468	Slashdot social network from November 2008 [13]
Wiki	Directed	7,115	103,689	Wikipedia who-votes-on-whom network [18]

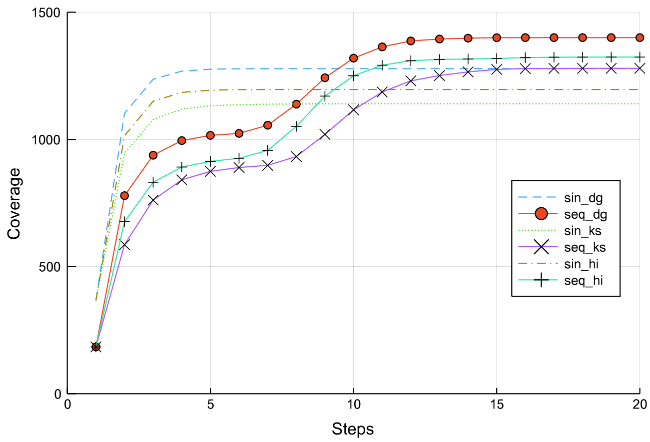

Fig. 1. Sequential seeding vs. single-stage seeding in the email network.

3.2. Centrality measures to construct sequential seeding strategies

Many centrality measures can be used to construct seeding strategies, among which degree might be the simplest and most intuitive one: a celebrity who attracts considerable attention of other people (i.e., has many connections in the social network) usually has more influence on information dissemination in the society. However, it is shown in [19] that K-shell is a better indicator if only one seed is allowed. K-shell also has its own drawbacks. The first is that calculating K-shell requires global topological information, which is relatively hard especially in a large-scale network. In addition, K-shell is not a distinguishable index in the sense that a large portion of nodes in a network may have the same K-shell value.

To deal with these problems, H-index is introduced as a trade-off between degree and K-shell in [20,21]. This index is originally proposed to measure the citation impact of a scholar, which is defined as the maximum value h such that the scholar publishes at least h papers and each paper has at least h citations [22]. Similarly, the H-index of a node is defined as the maximum value h such that the node has at least h neighbors of degree no less than h. It is shown that in the situation of single seed, H-index outperforms both degree and K-shell in many real networks.

Even though degree is not the best indicator for a seed in single-seed cases, it is shown in [19] that if multiple seeds are used, degree has a better performance than K-shell in many networks. The priority of degree, K-shell, and H-index is also studied in [20] and the result is that, in multiple-seed situation, degree has the best performance among the three centrality measures. In this section, we check the performance of degree, K-shell, and H-index in sequential seeding situations, and then we only choose the one with the best performance to construct our seeding strategy when study the influence of network topology.

3.3. Simulations on real networks

In all of the following simulations, we set the number of seeds as 1% of the number of nodes in the network, and the probability of activation by a contact as 0.01. Due to the randomness in the spreading process, we repeat the simulation 1000 times and obtain the average.

We first provide a detailed description of the simulation on the Email network. The result is shown in Fig. 1, where "sin" represents single-stage seeding and "seq" represents sequential seeding. Combined with three centrality measures: "dg" for degree, "ks" for K-shell, and "hi" for H-index, we have six seeding strategies in total.

From Fig. 1 we have the following observations:

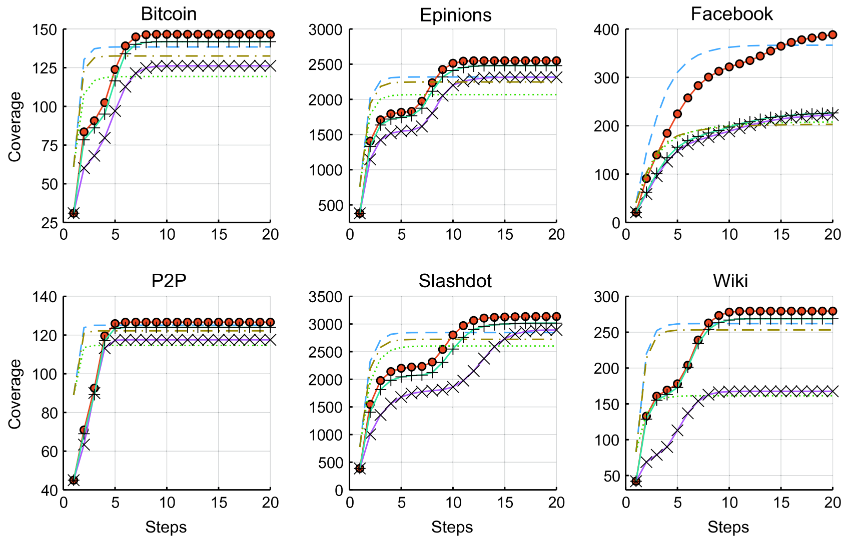

Fig. 2. Sequential seeding vs. single-stage seeding in six real network.

(1) For a given centrality measure, the single-stage seeding generally has a rapid spreading in the early stage compared with its sequential seeding counterpart, but the latter obtains a larger coverage in the network eventually. Thus, there is a trade-off between speed and coverage.
(2) The priority of the three measures—degree, K-shell, and H-index, is consistent in both single-stage and sequential strategies: degree is the best, H-index is the second, and K-shell is the last.

Then we run simulations on other six networks, and the result is shown in Fig. 2. Note that the legend in Fig. 2 is the same as in Fig. 1. We can see that the trend observed in the Email network also occurs in all the six networks. Due to the random selection of networks, the trend might be quite common in other real networks.

An intuitive interpretation of the advantage of sequential seeding strategy is that it has a better usage of the natural diffusion process. Note that some of the seeds might be wasted in the sense that they can be activated by other nodes even if they are not initially activated as seeds. Statistically, the less seeds being wasted, the more nodes we can activate. In this respect, the sequential seeding does better than the single-stage seeding. Inspired by this intuition, the best sequential seeding strategy is to choose only one seed, wait until the spreading process terminates, and choose another seed, while the time consumed in this case is also the greatest.

4. Influence of the network topology

To systematically investigate the influence of network topology on the performance of sequential seeding strategies, we next construct scale-free networks with tunable parameters as the controlled test platform [23]. We are mainly interested in three topological parameters: *degree distribution* which is represented by power-law exponent of the network, *average degree* which reflects the network density, and *assortativity coefficient* which represents how nodes are connected with other nodes of similar degrees. We use Python and its package *igraph* to construct the required networks. There exist many network-related functions in *igraph* such that we can simply call them to build a network with given degree distribution and average degree. However, *igraph* does not have a function to produce a network with a specific assortativity coefficient. To deal with this problem, we adopt Xalve–Brunet–Sokolov algorithm to generate a required network [24].

In the following simulations, the basic values of parameters are:

- Number of nodes: 10^4
- Number of edges: 10^5
- Power-law exponent: 3
- Assortativity coefficient: 0.

That means when we investigate the influence of a parameter, the other parameters are fixed at these basic values. And we only use degree to construct seeding strategies because of its good performance.

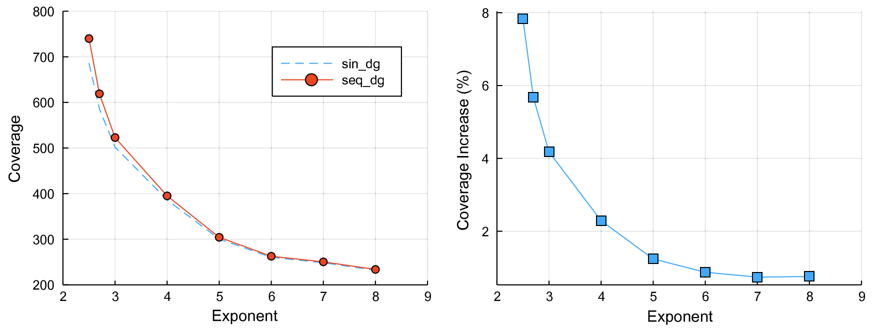

Fig. 3. Sequential seeding vs. single-stage seeding as power-law exponent varies. (Left) Coverage as the spreading process ends; (Right) Coverage increase under the sequential seeding strategy compared with its single-stage counterpart.

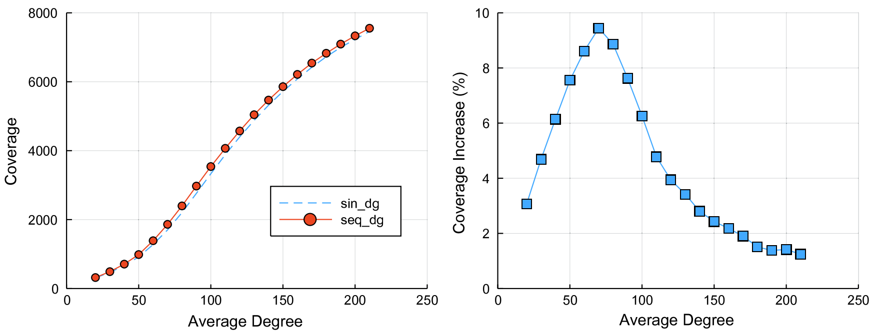

Fig. 4. Sequential seeding vs. single-stage seeding as the average degree varies. (Left) Coverage as the spreading process ends; (Right) Coverage increase under the sequential seeding strategy.

4.1. Influence of the power-law exponent

The parameter of power-law exponent captures the extent of degree heterogeneity in the network. Generally speaking, a small exponent corresponds to a network containing a small number of nodes with extremely large degrees and a large number of nodes with very small degrees. Conversely, a large exponent corresponds to a network where nodes have similar degrees.

The performance of single-stage seeding and sequential seeding in networks with different power-law exponents is shown in Fig. 3, where the exponent varies from 2.5 to 8. The coverages of both strategies become smaller as the exponent increases. This phenomenon is consistent with the existing result that heterogeneous degrees can promote information spread [25]. It is also shown that the difference between two seeding strategies is smaller when the degrees become homogeneous. The underlying reason is apparent. As the exponent increases, the scale-free network resembles a random network which has nodes with similar degrees, and thus, either of the seeding strategies which are based on degree could not work well.

4.2. Influence of the network density

As the number of nodes in the network is fixed at 10^4, the network density is positively correlated with the average degree of the nodes. In the simulation, the number of edges has the values from 10^5 to 1.05×10^6 with an interval of 0.5×10^5. That is, the average degree has its values from 20 to 210 with an interval of 10. Other parameters adopt the basic values as mentioned before.

The result is shown in Fig. 4, which also contains two subfigures: the coverage as a function of average degree is shown on the left and the coverage increase under sequential seeding is shown on the right. It is obvious that the coverage becomes larger as the network density increases. It is consistent with the intuition that a network with a great number of contacts among individuals is more likely to spread information or rumors to a large area.

The coverage increase of sequential seeding compared with single-stage seeding is not a monotonic function of the average degree. Note that the study based on several real networks in [8] suggests that the advantage of sequential seeding might be larger in sparse networks. Here through systematic simulations on a parameter-tunable platform, we show that it is not always the case.

Table 2
Coverage comparison between sequential seeding (seq) and single-stage seeding (sin).

Average degree	20	70	200
Seq	318	1861	7552
Sin	309	1701	7460
Seq–Sin	9	160	92
(Seq–Sin)/Sin	2.9%	9.4%	1.2%

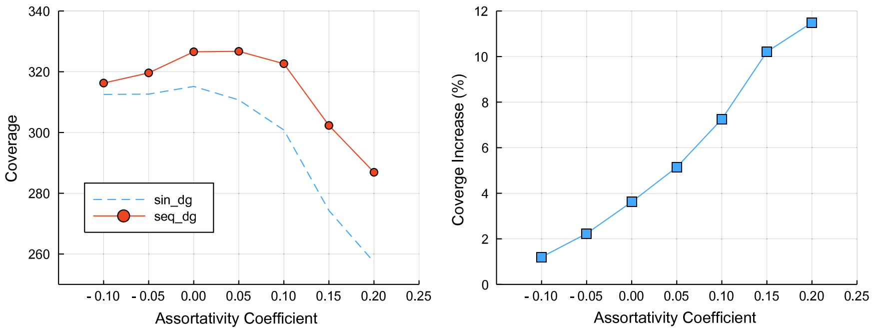

Fig. 5. Sequential seeding vs. single-stage seeding as the assortativity coefficient varies. (Left) Coverage as the spreading process ends; (Right) Coverage increase under the sequential seeding strategy.

To understand the non-monotonic curve in Fig. 4, we detail the data at some key points on the curve as shown in Table 2.

(1) As the average degree is relatively lower, the spreading is confined into a small region (about 3% of the total population) under both seeding strategies, and the absolute coverage difference is also very small.
(2) For a very large average degree, both of the seeding strategies can result in large coverages (about 75% of the total population). The absolute coverage difference is larger than that under low average degree, but the relative difference is quite small due to the large denominator.
(3) The advantage of sequential seeding is more apparent under medium average degree when the spreading covers about 20% population and also leaves some room for improvement under the sequential seeding strategy. In this case, the sequential seeding can improve the infected population by almost 10%.

4.3. Influence of the assortativity coefficient

The definition of assortativity coefficient is originally proposed by M. Newman in [26], which is the Pearson correlation coefficient between the degrees found at the two ends of the same edge. The assortativity coefficient captures the extent that a node is connected with another node with similar degrees. More precisely, in an assortative network (the coefficient is greater than zero), the hubs tend to connect to other hubs and the small-degree nodes tend to connect to other small-degree nodes; while in a disassortative network (the coefficient is less than zero), the hubs avoid each other, connecting to small-degree nodes.

Note that even though the possible range of assortativity coefficient is $[-1, 1]$, a scale-free network with a fixed degree distribution might have a much smaller range [27]. According to the result from [28] the theoretically lower bound of the assortativity coefficient in our work (10^4 nodes, 10^5 edges, power-law exponent equals 3) is -0.17. Using the algorithm proposed in [24] and after tens of millions of rewiring operations, we obtain the assortativity coefficient as -0.13. We conduct simulations in the range from -0.1 to 0.2. The simulations in these networks are shown in Fig. 5.

It is shown that the coverage of active nodes decreases dramatically as the assortativity coefficient becomes larger. The underlying reason may be that as the assortativity coefficient increases, the hubs have more connections with one another and avoid the nodes with small degrees, which implies that the seeds chosen according to degree tend to be clustered closed to one another. Thus the spreading process starting from these seeds can hardly reach remote small-degree nodes. In such an assortative network, the advantage of sequential seeding strategy is much apparent compared with its single-stage counterpart, because the first seeding at the initial time step can activate many hubs due to the close connections among them, and thus those small-degree nodes have opportunities to be chosen as seeds in the second seeding stage.

5. Conclusion

We have studied the influence of network topology on the performance of sequential seeding strategies. We adopt a widely used spreading model—ICM model to capture the spreading process. We have examined three centrality measures:

degree, K-shell, and H-index, and found that degree is a very suitable indicator for seeds, in both single-stage and sequential cases. The topological properties of networks such as power-law exponent, network density, and assortativity coefficient are considered in the investigation of a class of parameter-tunable scale-free networks.

Note that we focus our attention on the influence of network topology. Other spreading dynamic related parameters such as the spreading rate p and the number of seeds are not concerned. We simply set these parameters to be some fixed values. In fact, all the parameters in the spreading model affect the performance of the proposed seeding strategy. For example, we have tested our results for p in the range from 0.01 to 0.1 and found that the advantage of sequential seeding strategy is not apparent as p increases to 0.1, since a large population has been infected under the relatively large spreading rate by the single-stage seeding strategy, leaving little improvement space for the sequential seeding strategy. To deal with the problem of parameter sensitivity, Liu et al. provide an enlightening solution [29]. In addition to the topological information, the authors get down to the underlying spreading dynamics and propose a dynamics-sensitive (DS) centrality, which takes into account the spreading rate. It is shown that this DS centrality performs much better than other well-known indices for a very broad range of spreading rate.

For the future work on the sequential seeding strategy, a promising research direction is to consider the sequential seeding strategy for competitive spreading in the networks. In the noncompetitive market, the sequential seeding strategy can obtain a larger population coverage at the cost of slow speed. In a highly competitive circumstance, the speed of spreading is crucial for the competitors to occupy the market. Therefore, the problem that whether the sequential seeding still outperforms the single-stage seeding deserves further studies.

In addition, more realistic modeling of the spreading process is also an important research direction. For example, besides peer-to-peer interaction in networks, the out-of-social-network influence is also considered in [30], and the results highlight the necessity of considering diverse information channels in studying the spreading on real social networks. Moreover, information spreading on dynamic social networks should also be investigated. For example, a link rewiring strategy based on the Fermi function is introduced in [31] and the result indicates that the information would spread faster and broader in such a dynamic social network.

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