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Information spreading dynamics in hypernetworks

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HIGHLIGHTS

- Two information diffusion models are proposed by considering both the spreading strategy and network structure.
- Reactive process (RP) strategy and Contact process (CP) strategy present global and local transmission, respectively.
- Numerical simulations are given for different parameters on the models.

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ABSTRACT

Contact pattern and spreading strategy fundamentally influence the spread of information. Current mathematical methods largely assume that contacts between individuals are fixed by networks. In fact, individuals are affected by all his/her neighbors in different social relationships. Here, we develop a mathematical approach to depict the information spreading process in hypernetworks. Each individual is viewed as a node, and each social relationship containing the individual is viewed as a hyperedge. Based on SIS epidemic model, we construct two spreading models. One model is based on global transmission, corresponding to RP strategy. The other is based on local transmission, corresponding to CP strategy. These models can degenerate into complex network models with a special parameter. Thus hypernetwork models extend the traditional models and are more realistic. Further, we discuss the impact of parameters including structure parameters of hypernetwork, spreading rate, recovering rate as well as information seed on the models. Propagation time and density of informed nodes can reveal the overall trend of information dissemination. Comparing these two models, we find out that there is no spreading threshold in RP, while there exists a spreading threshold in CP. The RP strategy induces a broader and faster information spreading process under the same parameters.

1. Introduction

With the advent of Web 2.0, due to the prevalence of diverse social media platforms, such as online social networks, microblogs, and WeChat, information can spread quickly and extensively. Millions of individuals discuss and share their topics ranging from public affairs to personal lives in these social medias, which greatly facilitate people to exchange and disseminate information. As a consequence, understanding the process of information propagation and revealing the spreading characteristics of social medias has important theoretical and application value. Thus it has also become one of the most popular topics. For the research of information diffusion, quite a lot of models have been presented to describe the transmission mechanism. Since it is similar to the diffusion of epidemic spreading, researchers have tried to use classic

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epidemic models to depict the dynamic process. Usually an epidemic is transmitted from an infected individual to a susceptible one through the link between them. Various epidemic models such as SIS [1], SIR [2] and related empirical analysis have been intensively studied [3–7]. Nodes and edges represent individuals and their interactions, respectively, based on which, a complex network can be constructed. In the past years, spreading dynamics on complex networks have attracted an increasing attention from scholars. They have explored approaches to model the information propagation in scale-free network [8], correlated network [9], dynamic network [10], and other different networks, which promoted the development of spreading dynamics. Furthermore, others have studied the information propagation in virtual world [11–13].

All the previous works have contributed to the understanding of spreading dynamics, and revealed that the network structure played a particularly important role in the spreading process. Relationships among individuals in real systems tend to be more complex than those that can be described by simple pairwise relations. Consequently, complex networks based on simple graphs are no longer suitable to represent such systems.

A natural way is to use a generalization known as "networks of networks" [14,15]. Some recent studies on spreading dynamics beyond the single-network framework have been performed under various terms like multilayer networks, interconnected networks, and multiplex networks. Dickinson et al. [16] discussed the SIR model in a two layers network. The inter-layer connections allowed infection to spread between nodes in different layers. Yagan et al. [17] found that the multilayer structure made the propagation process more efficient. Zuzek et al. [18] measured the effect of quarantining infected individuals in multilayer networks. Wu et al. [19] explored the propagation of information and the trust dynamics on a multiplex network. Ma and Liu [20] established a supernetwork model with four subnetworks to depict the public opinion environment. Furthermore, they investigated different strategies of online public opinion intervention [21].

Another possible solution is to utilize hypergraph theory. The hyperedge of a hypergraph can contain arbitrary number of nodes. Thus it provides a useful way to depict interactions among variety nodes. Some scholars have studied the topological properties of hypernetworks. Estrada and Rodríguez-Velázquez [22] extended subgraph centrality and clustering coefficient to hypernetwork, and combined an empirical analysis. Ghoshal et al. [23] proposed a mathematical model for random hypergraphs. Other topological characteristics such as node superdegree, superedge–superedge distance, and superedge overlap were discussed by Ma and Liu [20]. Xiao [24], Kapoor et al. [25] proposed a new multi-criteria measuring method, weighted node degree centrality respectively, to identify the key nodes. Criado et al. [26] defined the concept of hyperstructure and calculated the efficiency of the subway network. Constructing evolving models to depict the evolution of complex systems have also drawn the attention of scholars. Wang et al. [27] and Hu et al. [28] proposed two dynamic models; both based on growth and preferential attachment mechanisms. Guo and Zhu [29] proposed a scale-free model unifying the above models. Furthermore, models combining different preferential attachment mechanisms such as joint degree [30], nonlinear preferential attachment [31], and hyperdegrees with brand effect and competitiveness [32] have also been constructed. Microscopic events including local-world selection [33], connections among old nodes [34] and the influence of aging [35] have also been investigated. Bodó et al. [36] extended the epidemic propagation model to hypergraphs and derived the exact master equations of the propagation process. Other studies in this area are related to community detection, synchronization, cascading failure, collaborative recommendation and so on.

Hypernetwork, based on hypergraph theory, can not only reduce the complexity of network structure, but is also suitable to depict the relationships among multiple nodes. However, there still lack studies focused on spreading pattern in hypernetworks. In fact, designing proper spreading models can promote the understanding of information spreading in social medias. So in this paper we propose two information diffusion models by considering both the spreading strategy and network structure. In fact, information may spread by Reactive Process (RP) strategy or Contact Process (CP) strategy [37,38]. RP means the information published to the whole network, presenting global transmission. In contrary, CP means the information published to the special group, presenting local transmission. Besides different spreading strategies, the topology of network structure is also an important factor affecting the process of information spreading. Here hypernetwork is used to depict the complex social relationships among individuals.

The rest of the paper is organized as follows. Section 2 presents the related concepts of hypernetwork and information propagation process. Section 3 proposes the information propagation model based on RP strategy, and gives the theoretical analysis and simulation results. Section 4 introduces the information propagation model based on CP strategy. Section 5 contains some conclusion remarks.

2. Theoretical backgrounds

2.1. The concept of hypergraph and hypernetwork

The concept of hypergraph was first proposed by Berge [39,40]. In a graph, an edge relates only two nodes, but a hyperedge in hypergraph can connect arbitrary number of nodes. Estrada and Rodríguez-Velázquez [22] identified that complex systems described by hypergraphs could be regarded as hypernetworks. The mathematical definition of hypergraph is given as follows. Let $V = \{v_1, v_2, \ldots, v_n\}$ be a finite set, and $E_i = \{E_1, E_2, \ldots, E_m\}$ be a family of subsets of V($|E_i|$ denotes the cardinality of set E_i). The pair $H = (V, E)$ is called a hypergraph. And $H = (V, E)$ is a k -uniform hypergraph if $|E_i| = k$. $H = (V, E)$ degenerates into a graph if $|E_i| = 2$. Thus graph can be considered as a special case of hypergraph.

The mathematical definition of the hypernetwork [29] is given as follows. Suppose $\Omega = \{(V, E)|(V, E)$ is a finite hypergraph$\}$ and G is a map from $T = [0, +\infty)$ into Ω; for any given $t \geq 0$, $G(t) = (V(t), E(t))$ is a finite hypergraph.

A hypernetwork $\{G(t), t \in T\}$ is a collection of hypergraphs evolving with time t. The hyperdegree of node i, k_i, is defined as the number of connected hyperedges of that node. In the example shown in Fig. 1, at time $t = 1$, $k_1 = 1$, $k_4 = 3$ and $k_5 = 2$. The hypernetwork studied here is assumed to be k-uniform for simplicity, and it can be expanded into non-uniform following the method provided in Ref. [32].

2.2. The evolving model of hypernetwork

In order to explore the dynamics of information spreading in social networks, we first generate some hypernetworks for the following study. Wang et al. [27] proposed an evolving hypernetwork model satisfied the following steps.

(i) The hypernetwork starts with a small number of nodes and a hyperedge containing these initial nodes.

(ii) At each time step, m_1 new nodes are added to the hypernetwork. These newly added nodes and an existing node is selected to form a new hyperedge, totally m new hyperedges are constructed with no repetition. The probability that an existing node being selected is proportional to the hyperdegree of that node.

$$W(k_i) = \frac{k_i}{\sum_j k_j} \tag{1}$$

where k_i is the hyperdegree of node i .

After a few time steps, a hypernetwork is constructed, and its hyperdegree distribution is $P(k) = \frac{m_1+1}{m}(\frac{m}{k})^{m_1+2}$.

2.3. Information spreading process

In this paper, Susceptible–Informed–Susceptible (SIS) model is used to illustrate the information spreading process in social networks. Each individual can either be informed or uninformed. The informed refer to spreading individuals, who have known the information and have the ability to continue to spread it to others. We define them I-state for short. The uninformed refer to those who are unaware of the information, or those who do not care about the information and are unwilling to spread it. We call them S-state for short. Information is transmitted among individuals through different social relationships. The uninformed may accept the information with a certain probability if they connected to their informed neighbors. As a result, they become new spreaders. At the same time, the informed individual may recover to uninformed again with a certain probability.

3. Information spreading model with RP strategy in hypernetworks

3.1. Model description

In hypernetworks, a node represents an individual, and an hyperedge represents the unit of social relationships such as household, workplace, meeting or virtual community. The hyperdegree of an individual refers to the number of social communities he/she belonged to. WeChat, as a popular social media platform, provide an innovative way to communicate through information exchange. Information is spread through official account and personal moments. In RP strategy, the official account or individual publics a piece of information on his/her moment, resulting in all his/her friends get the news.

According to the above mechanism, the evolution of dynamic spreading model runs as follows:

(i) Initialization. A hypernetwork is constructed. The propagation process starts with a single spreader. The informed is in I-state while all the others are in S-state.

(ii) At each time step, a node in I-state tries to transmit information to all of its neighbors in S-state with probability β. On the other hand, it may recover to S-state with probability γ.

(iii) During the evolution of time, the density of nodes in I-state is calculated.

(iv) steps (ii) and (iii) are repeated until the system reaches to its steady state.

The schematic representation is shown in Fig. 1. Green and red represents S-state and I-state, respectively. At the initial time step $t = 1$, a node (node 7) is randomly selected to be I-state, and all the others are in S-state. At time step $t = 2$, since node 7 is encircled by hyperedges e2 and e4, we suppose nodes 4, 5, 11, 12 are informed into I-state. At time step $t = 3$, the nodes in I-state (nodes 4, 5, 7, 11, 12) infect their neighbors in S-state with probability β. Since nodes 4, 5, 7, 11, 12 are encircled by hyperedges e1, e2, e3, e4 and e5, we suppose nodes 1, 6, 8, 13 and 14 are informed into I-state, nodes 4, 5 and 11 recover to S-state.

3.2. Theoretical analysis

In this section we will present an analytical framework for the model based on mean-field theory. The dynamical mean-field reaction rate equation can be written as

$$\partial_t \rho_k(t) = -\gamma \rho_k(t) + \beta k m_1 [1 - \rho_k(t)] \Theta(\rho(t)) \tag{2}$$

where $\rho_k(t)$ is the relative density of informed nodes with given hyperdegree k. The first term in the right-hand side considers the probability that informed nodes with hyperdegree k recover to S-state. The second term in the right-hand side considers

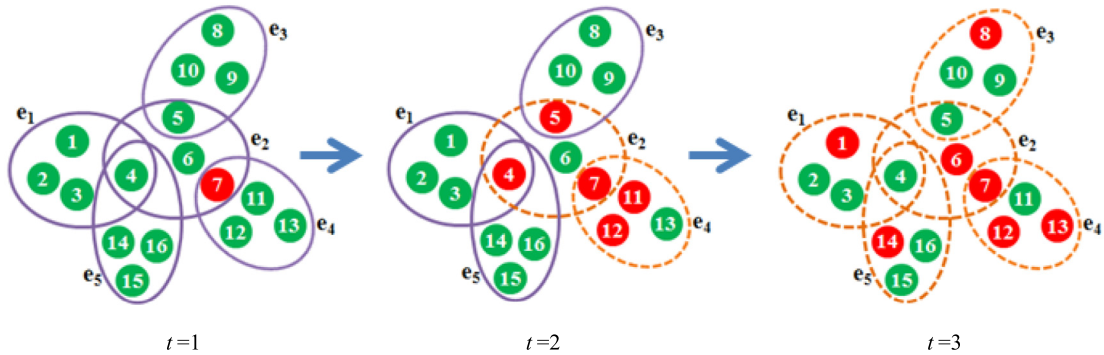


Fig. 1. Schematic illustration of the spreading process based on RP strategy.

the probability that nodes with hyperdegree k is in S-state$[1-\rho_k(t)]$ and get informed through connected informed nodes. The probability of this term is proportional to the spreading rate β, hyperdegree k, the number of neighbors in one hyperedge m_1, and the probability $\Theta(\rho(t))$ that any given hyperedge link to an informed node. Here we do not consider the connectivity correlations. So the probability that a hyperedge encircles an informed node is only a function of the total density of informed nodes. The total density of informed nodes can be obtained by $\rho(t)=\sum_k P(k)\rho_k(t)$. In the steady state, ρ is a function of spreading rate β and recovering rate γ. Thus, the probability Θ becomes an implicit function of β and γ. The system reaches its stationarity which can be described as $\partial_t \rho_k(t)=0$. Thus we obtain

$$\rho_k=\frac{\beta k m_1 \Theta(\beta,\gamma)}{\gamma+\beta k m_1 \Theta(\beta,\gamma)}. \tag{3}$$

Eq. (3) shows that nodes with more neighbors will have higher probability to be informed. In the computation of $\Theta(\beta,\gamma)$, the heterogeneity must be taken into account. Indeed, the probability that a hyperedge encircles a node with hyperdegree s is proportional to $sP(s)$, yielding,

$$\Theta(\beta,\gamma)=\sum_k \frac{kP(k)\rho_k}{\sum_s sP(s)}. \tag{4}$$

Substituting Eq. (3) into Eq. (4), leading to

$$\Theta(\beta,\gamma)=\frac{1}{\langle k\rangle}\int_m^{\infty} k\cdot P(k)\cdot\frac{\beta k m_1 \Theta(\beta,\gamma)}{\gamma+\beta k m_1 \Theta(\beta,\gamma)}dk. \tag{5}$$

From Ref. [27], we know that the hyperdegree distribution of the constructed hypernetwork is $P(k)=\frac{(m_1+1)}{m}(\frac{m}{k})^{m_1+2}$, yielding

$$\langle k\rangle=\int_m^{\infty} kP(k)dk=(m_1+1)m^{m_1+1}\int_m^{\infty}\frac{1}{k^{m_1+1}}dk=\frac{(m_1+1)m}{m_1}. \tag{6}$$

Substituting the hyperdegree distribution of $P(k)$ and Eq. (6) into Eq. (5), leading to

$$\Theta(\beta,\gamma)=\frac{m_1}{(m_1+1)m}\int_m^{\infty} k\cdot\frac{(m_1+1)}{m}(\frac{m}{k})^{m_1+2}\cdot\frac{\beta k m_1 \Theta(\beta,\gamma)}{\gamma+\beta k m_1 \Theta(\beta,\gamma)}dk. \tag{7}$$

Reducing Eq. (7), we have

$$\frac{1}{\beta m_1^2 m^{m_1}}=\int_m^{\infty}\frac{1}{k^{m_1}}\cdot\frac{1}{\gamma+\beta k m_1 \Theta(\beta,\gamma)}dk. \tag{8}$$

The right hand side of Eq. (8) can be reduced as follows.

$$\int_m^{\infty}\frac{1}{k^{m_1}}\cdot\frac{1}{\gamma+\beta k m_1\Theta(\beta,\gamma)}dk=\int_m^{\infty}\frac{1}{\gamma}(\frac{1}{k^{m_1}}-\frac{\beta m_1\Theta(\beta,\gamma)\cdot\frac{1}{k^{m_1-1}}}{\gamma+\beta k m_1\Theta(\beta,\gamma)})dk$$

$$=\frac{1}{\gamma(m_1-1)}\cdot\frac{1}{m^{m_1-1}}-\frac{\beta m_1\Theta(\beta,\gamma)}{\gamma^2(m_1-2)}\cdot\frac{1}{m^{m_1-2}}+\cdots+\frac{(-\beta m_1\Theta(\beta,\gamma))^{m_1-2}}{\gamma^{m_1-1}}\cdot\frac{1}{m}$$
$$+\frac{(-\beta m_1\Theta(\beta,\gamma))^{m_1-1}}{\gamma^{m_1}}\cdot\ln(\frac{\gamma}{\beta m m_1\Theta(\beta,\gamma)}+1).$$

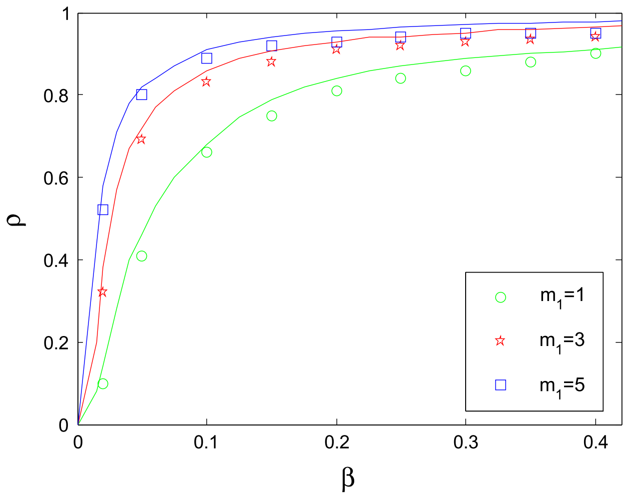


Fig. 2. Densities of ρ obtained by theoretical analysis and simulations in the stable state.

Substituting Eq. (3) into $\rho = \sum_k P(k)\rho_k$, leading to

$$\begin{aligned}\rho &= \int_m^\infty \frac{(m_1+1)}{m}\left(\frac{m}{k}\right)^{m_1+2} \cdot \frac{\beta k m_1 \Theta(\beta,\gamma)}{\gamma + \beta k m_1 \Theta(\beta,\gamma)} dk \\ &= (m_1+1) m^{m_1+1} \beta m_1 \Theta(\beta,\gamma) \int_m^\infty \frac{1}{k^{m_1+1}} \cdot \frac{1}{\gamma + \beta k m_1 \Theta(\beta,\gamma)} dk \\ &= (m_1+1) m^{m_1+1} \beta m_1 \Theta(\beta,\gamma) \left(\int_m^\infty \frac{1}{\gamma} \cdot \frac{1}{k^{m_1+1}} dk - \frac{\beta m_1 \Theta(\beta,\gamma)}{\gamma} \int_m^\infty \frac{1}{k^{m_1}} \cdot \frac{1}{\gamma + \beta k m_1 \Theta(\beta,\gamma)} dk \right).\end{aligned} \tag{9}$$

Substituting Eq. (8) into Eq. (9), leading to

$$\rho = \frac{(m_1+1) m \beta \Theta(\beta,\gamma)(1-\Theta(\beta,\gamma))}{\gamma}. \tag{10}$$

From Eq. (10), in the steady state, ρ is only a function independent of t. Once the spreading rate, recovering rate and the structure parameters of hypernetwork are given, ρ can be calculated.

3.3. Simulation results and analysis

To test the model, we compare its theoretical results to stochastic simulations in hypernetworks. The hypernetworks are generated according to the algorithms introduced in Section 2. Usually social members have relatively stable relations to communicate, so we assume the hypernetwork structure is static. Initially, one node is chosen as spreading seed. The simulation continues until the system reaches to its steady state. To eliminate random effects, each simulation result is obtained by averaging over 100 independent runs.

(1) The simulation results of final densities of informed in the steady state

The effective spreading rate $\lambda = \beta/\gamma$ only affects the speed of information diffusion. Without loss of generality, we let $m_1 = 1, 3, 5$, $m = 1$, and the recovering rate is set as $\gamma = 0.05$ in the simulations. In Fig. 2, the curves are theoretical results calculated by Eq. (10), and the discrete points are the densities of informed nodes ρ in the steady state. As it shows, the theoretical analysis is in good agreement with simulations. Information will be spread to the whole hypernetwork even with a small value of effective spreading rate, thus there is no spreading threshold in RP strategy.

(2) Effects of hypernetwork scale

To illustrate the spreading process, we focus on the time evolution of the densities of informed nodes. Fig. 3 shows a comparison for different hypernetwork scales. The other parameters are setting as follows, $m_1 = 3$, $\beta = 0.3$, $\gamma = 0.05$. Note that these three curves almost overlap with each other. It can be obtained that the hypernetwork scale has little impact on the spreading process. So the following simulations are performed under the scale of $N = 1000$.

(3) Effects of spreading rate and recovering rate

First, we want to find out the diversity behavior of information diffusion with different spreading rate β. The parameters are setting as follows, $\gamma = 0.05$, $m_1 = 1, 3, 5$. It can be observed from Fig. 4 that for different hypernetworks, the spreading behavior are similar. Information spread more rapidly for a larger value of β, resulting in the network evolves

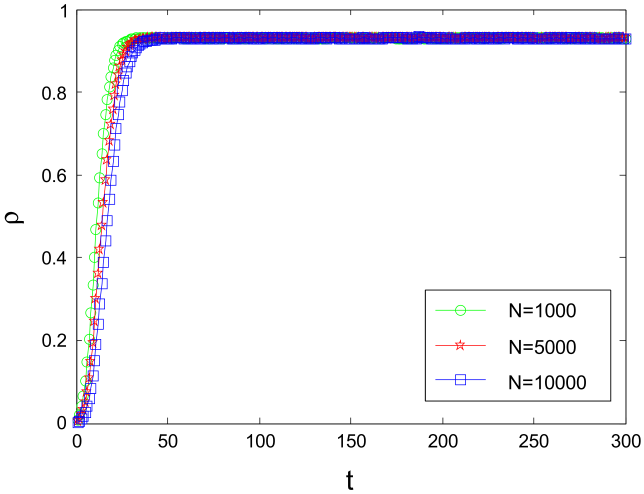
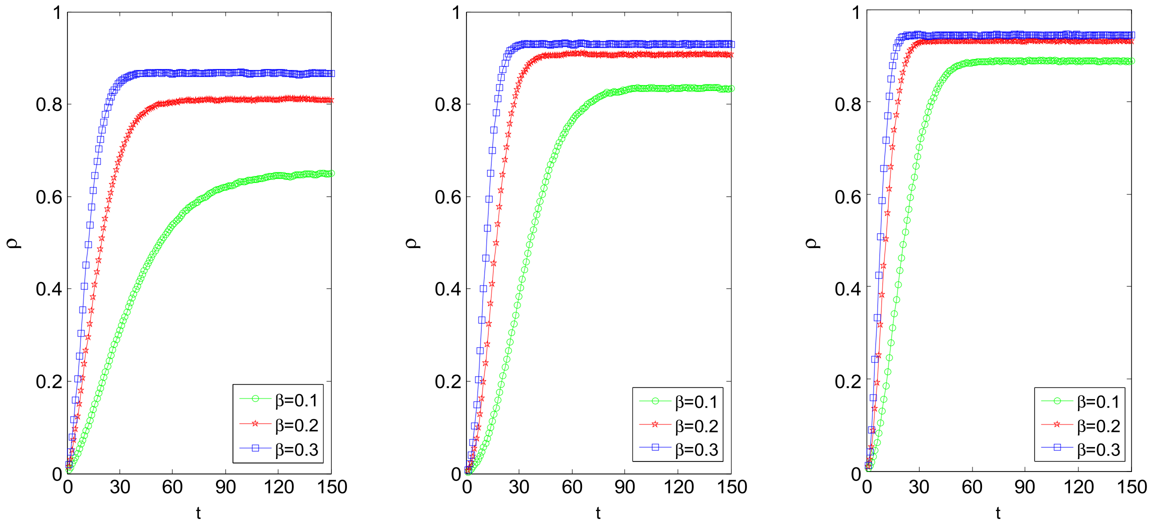


Fig. 3. The simulation results with different hypernetwork scales.



(a) $m_1 = 1$. (b) $m_1 = 3$. (c) $m_1 = 5$.

Fig. 4. The evolution of informed with different spreading rate.

to the stable state in a shorter time step. Increasing β will exaggerate the information diffusion efficiency and capacity. Once the information breaks out in the network, it will spread quickly to the whole network.

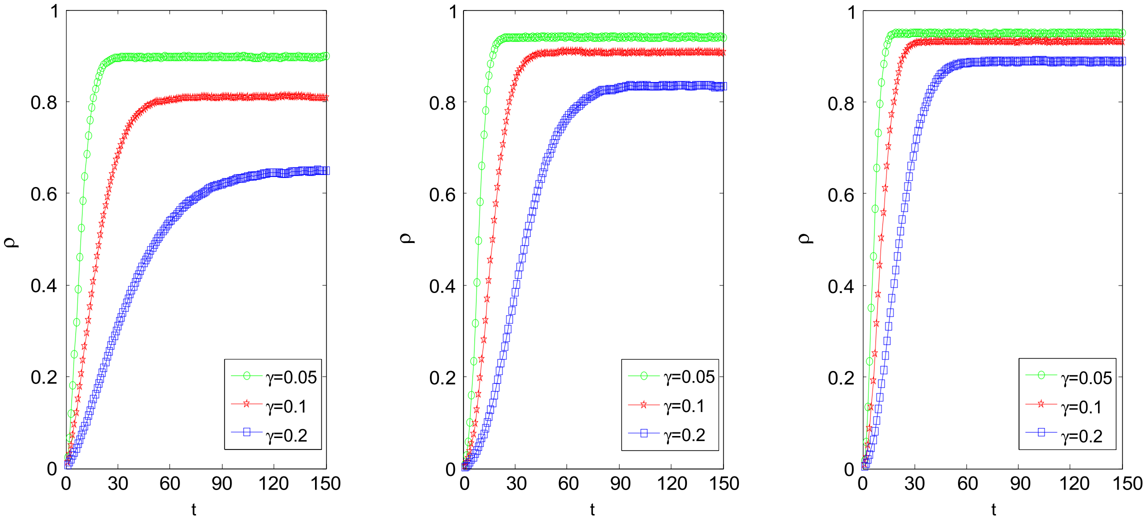
We next consider the effect of recovering rate γ. The parameters are setting as follows, $\beta = 0.4$, $m_1 = 1, 3, 5$. It can be observed from Fig. 5 that information spread more slowly for a larger value of γ, resulting in the network evolves to the stable state in a longer time step. Furthermore, the proportion of informed in the stable state is lower with the increasing of γ. Recovering rate is on behalf of information resistance capacity, which will reduce the information diffusion efficiency.

(4) Effects of structure parameters

Besides, we also want to examine the effect of structure parameters m_1 and m. The parameters are setting as follows, $\gamma = 0.05$, $\beta = 0.05$. A large value of m_1 means nodes have more neighbors in their social networks, and a large value of m means nodes participate in many social networks, both resulting in more chance for individuals to spread information. From Fig. 6(a), it can be seen that the information spreading enhancement with the increasing of m can be very significant when m_1 is small. From Fig. 6(c), it can be seen that the proportion of informed increase sharply at the first several time steps when m_1 is big.

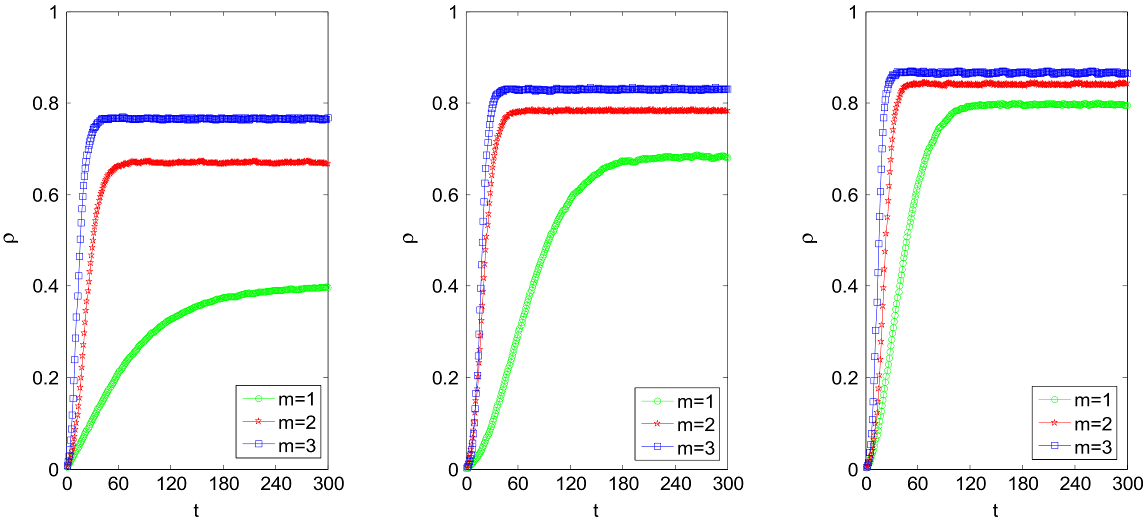
(5) Effects of spreading seed

Now let us focus on the impact of spreading seed. For the sake of comparison, we fix the value of parameters $m_1 = 3$, $\gamma = 0.05$, and we select node with largest and smallest hyperdegree as the initial informed seed, respectively. Fig. 7 exhibits that



(a) $m_1 = 1$. (b) $m_1 = 3$. (c) $m_1 = 5$.

Fig. 5. The evolution of informed with different recovering rate.



(a) $m_1 = 1$. (b) $m_1 = 3$. (c) $m_1 = 5$.

Fig. 6. The evolution of informed with different structure parameters.

both the diffusion speed and the proportion of informed are larger in the first case. On the contrary, it requires a certain time to spread the information in the second case. However, both curves converge to the same value in the stable state. This is mainly due to the high connectivity in social networks. Nodes with large hyperdegree correspond to individuals with great influence, which is an important feature of social network. This is in accord with reality, as the news published by celebrities can spread rapidly in network [41].

4. Information spreading model with CP strategy in hypernetworks

4.1. Model description

The RP strategy describes the characteristics of information to be spread throughout the whole network, which does not consider the influence of local structure. In fact, sometimes individuals may spread information according to its content. Take WeChat as an example, an individual may divide his/her friends into different groups representing different social relations.

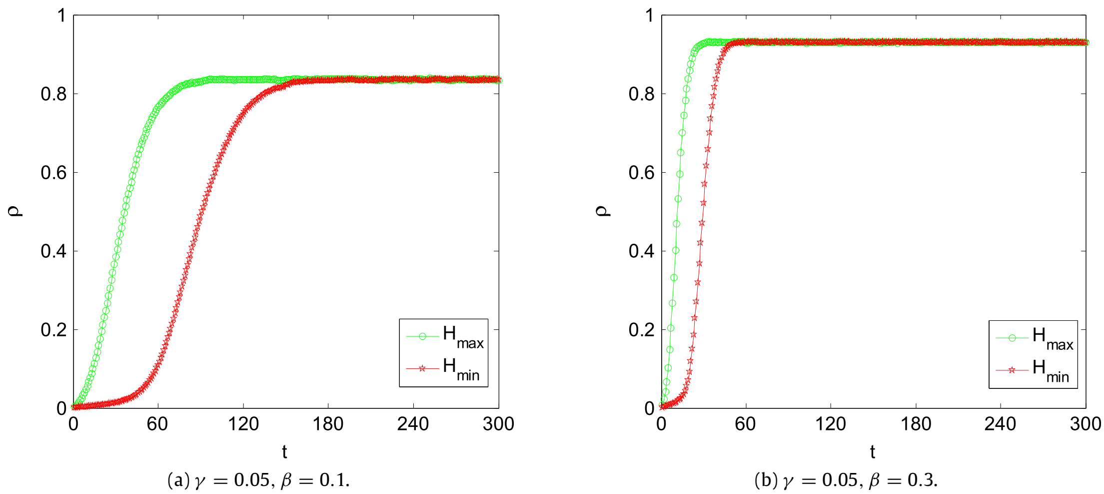


Fig. 7. The simulation results with different initial spreading seed.

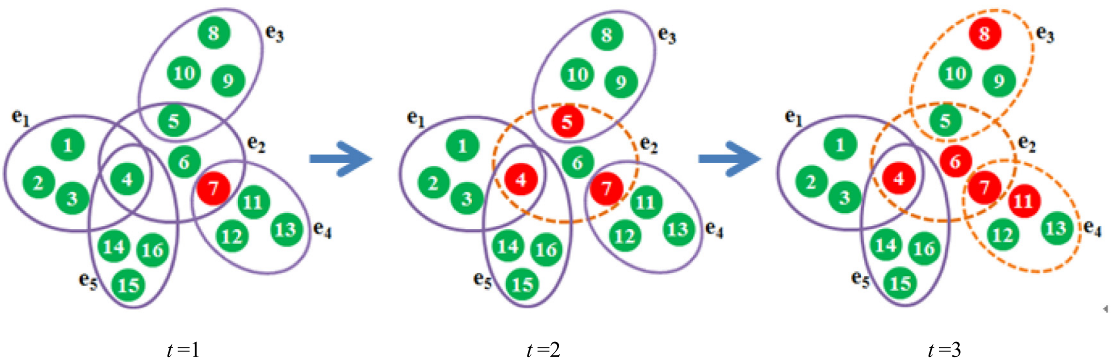


Fig. 8. Schematic illustration of the spreading process based on CP strategy.

Sometimes he/she may share information to his/her workplace group. Or he/she may share privacy information only to close friends group. Thus we use CP strategy to depict such situations. Once the individual spreads a piece of information just to a social relation group, resulting in all his/her neighbors in the particular group get and spread the information with certain probability.

According to the above mechanism, the evolution of dynamic spreading model runs as follows:

(i) Initialization is the same as set in RP strategy.

(ii) At each time step, a node in I-state randomly choose one of its connecting hyperedges, and tries to transmit information to nodes in S-state in this particular hyperedge with probability β. On the other hand, it may recover to S-state with probability γ.

(iii) and (iv) are the same as set in RP strategy.

The schematic representation is shown in Fig. 8. At the initial time step $t = 1$, a node (node 7) is randomly selected to be I-state, and all the others are in S-state. At time step $t = 2$, a hyperedge (e2) encircling node 7 is randomly selected. We suppose nodes 4, 5, 7 in e2 are informed into I-state. At time step $t = 3$, a hyperedge encircling nodes in I-state (nodes 4, 5, 7) is randomly selected. We suppose e2, e3 and e4 are selected, and nodes 6, 8, 11 are informed into I-state, node 5 recovers to S-state.

4.2. Simulation results and analysis

In order to explore how different strategies affect the process of information diffusion, the hypernetworks are kept the same with that of RP.

(1) The simulation results of final densities of informed in the steady state

As Fig. 9 shows, different from RP strategy, the CP strategy emerges a continuous transition at a critical spreading threshold value λ_c(λ is the effective spreading rate, and $\lambda = \beta/\gamma$). λ_c separates an absorbing phase from an active one [42–46].

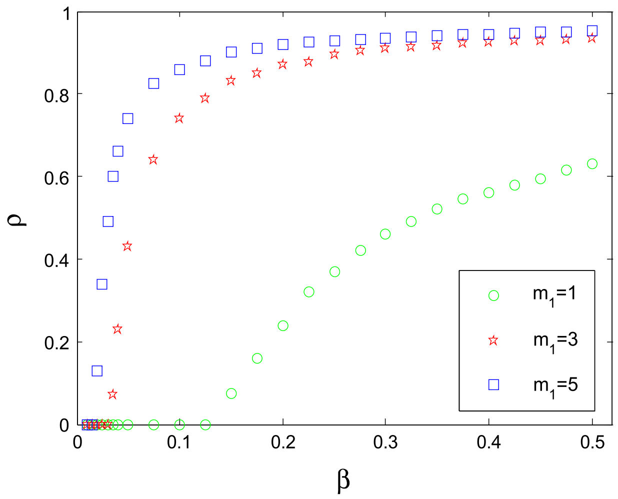


Fig. 9. Densities of ρ obtained by simulations in the stable state.

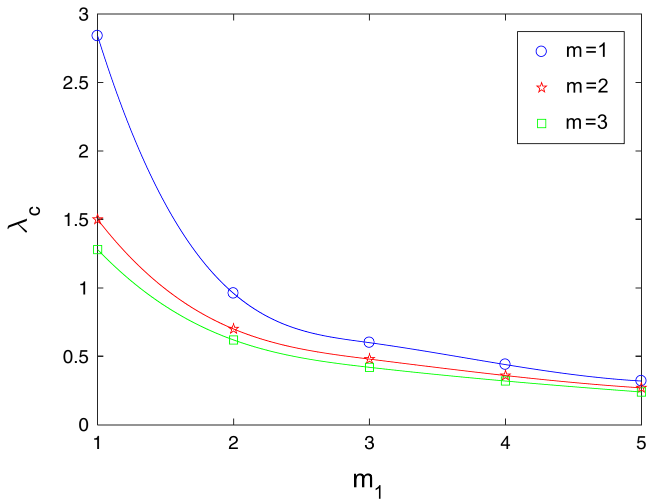


Fig. 10. The spreading threshold value with different structure parameters.

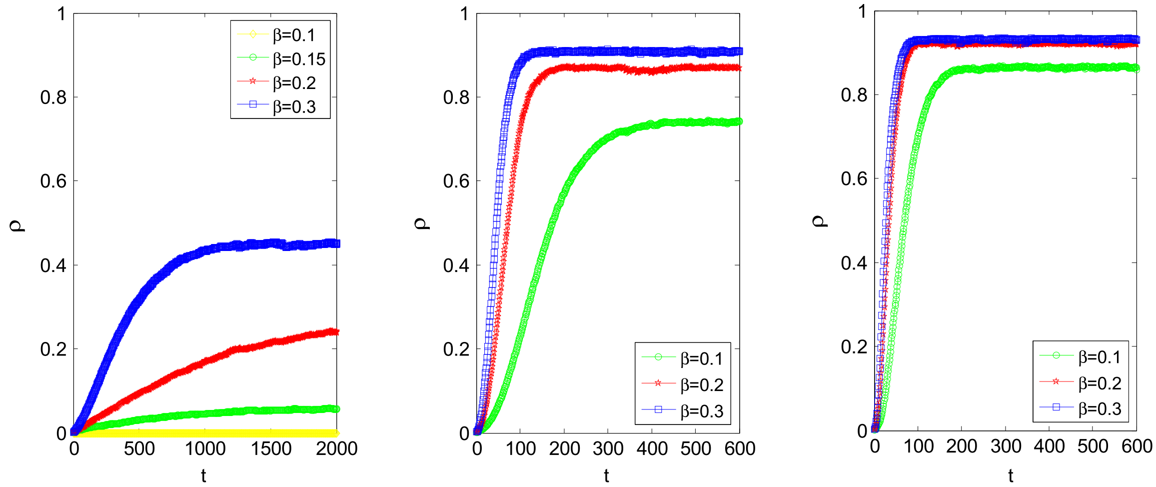
For $\lambda > \lambda_c$, the proportion of informed will tend to be positive constant. Otherwise, the spreading disappears and the proportion of informed goes to zero. We present the numerical simulation results of λ_c in different hypernetworks in Fig. 10. With the increase of structure parameter m and m_1, the value of λ_c decrease in a non-linear form. Especially when $m = 1$, λ_c decrease fastest with the increase of m_1. In addition, when $m_1 = 5$, λ_c tend to be approximately equal with the increase of m. Thus it can be concluded that λ_c is related to the topological effect of hypernetworks. However, the exact functional relationship between λ_c and structure parameters requires further discussion in future studies.

(2) Effects of spreading rate and recovering rate

Figs. 11–12 show the time evolutions of information diffusion under different β and γ. In Fig. 11 the parameters are setting as follows, $\gamma = 0.05$, $m_1 = 1, 3, 5$. In Fig. 12 the parameters are setting as follows, $\beta = 0.4$, $m_1 = 1, 3, 5$. The spreading process is similar to RP strategy. However, one noticeable difference is that in the case of $m_1 = 1$, information cannot be transmitted under a small value of spreading rate or a large value of recovering rate.

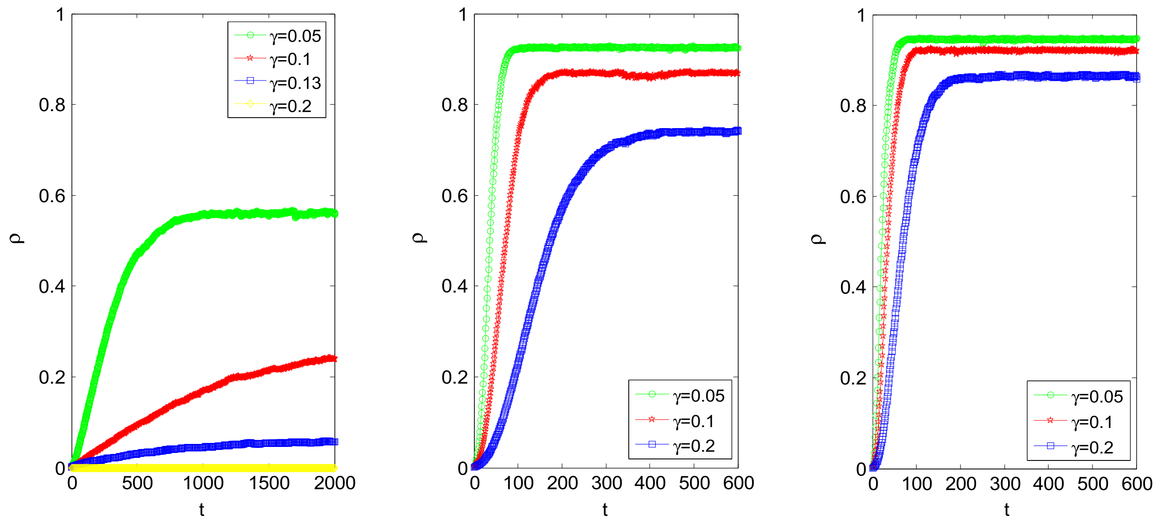
(3) Effects of structure parameters

Now we discuss the effect of structure parameters m_1 and m. In Fig. 13(b) and (c), the parameters are setting as follows, $\gamma = 0.05$, $\beta = 0.05$. From Fig. 13(b), it can be seen that when m is increased from 1 to 2, the spreading enhancement is significant. In contrary, the proportion of informed is not obviously increased when m is increased from 2 to 3. Furthermore, from Fig. 13(c), the spreading increases very little when m is increased from 1 to 3. The increase of m results in individuals participating in more social networks. However, since only one social network is randomly selected in CP strategy, the spreading ability is constrained. In Fig. 13(a), the parameters are setting as follows, $\gamma = 0.05$, $\beta = 0.1$. It is interesting



(a) $m_1 = 1$. (b) $m_1 = 3$. (c) $m_1 = 5$.

Fig. 11. The evolution of informed with different spreading rate.



(a) $m_1 = 1$. (b) $m_1 = 3$. (c) $m_1 = 5$.

Fig. 12. The evolution of informed with different recovering rate.

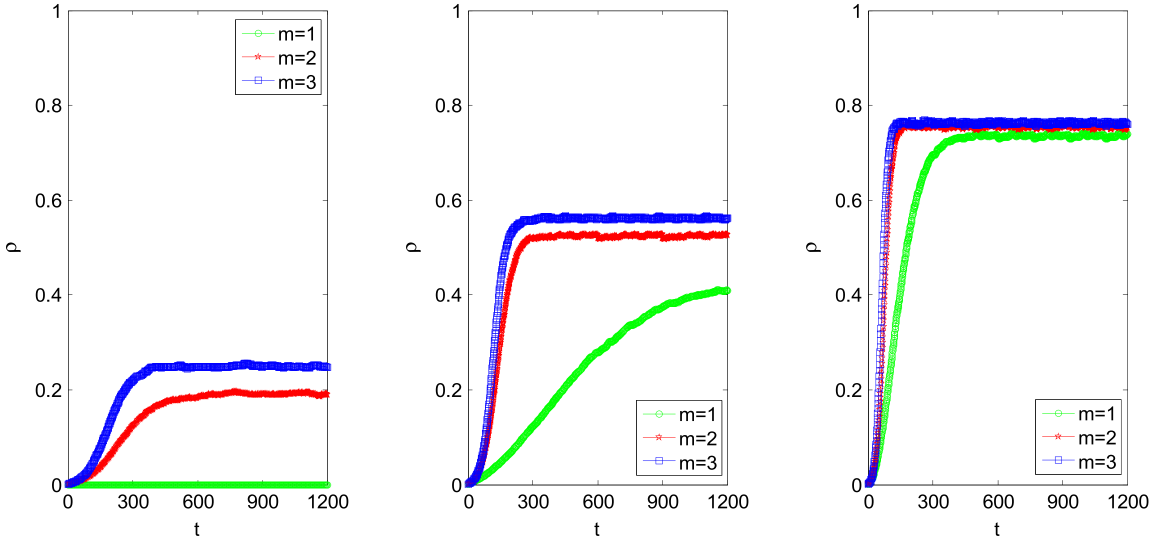
to find that the system reaches the absorbing state when $m = 1$. With the increase of m, the system becomes to a stationary active state. It can be concluded that m has strengthened information spreading to some extent.

(4) CP strategy compared with RP strategy

Finally, we compare the diversity characteristics of information diffusion between two different strategies. The parameters are setting as follows, $m_1 = 3$, $\gamma = 0.05$, $\beta = 0.1$. By comparing the curves in Figs. 2 and 9, we find out that there is no spreading threshold in RP, while there exists a spreading threshold in CP. The RP strategy induces a broader and faster information spreading process under the same parameters, which is in conformity with the reality. In Fig. 14, gray line represents simulation results in one single simulation run, while red line represents the average results over 100 independent runs. Since a hyperedge is randomly selected to spread information at each time step, it can be seen that the gray curves fluctuate bigger in CP strategy. In contrast, the curves are relatively stable in RP strategy.

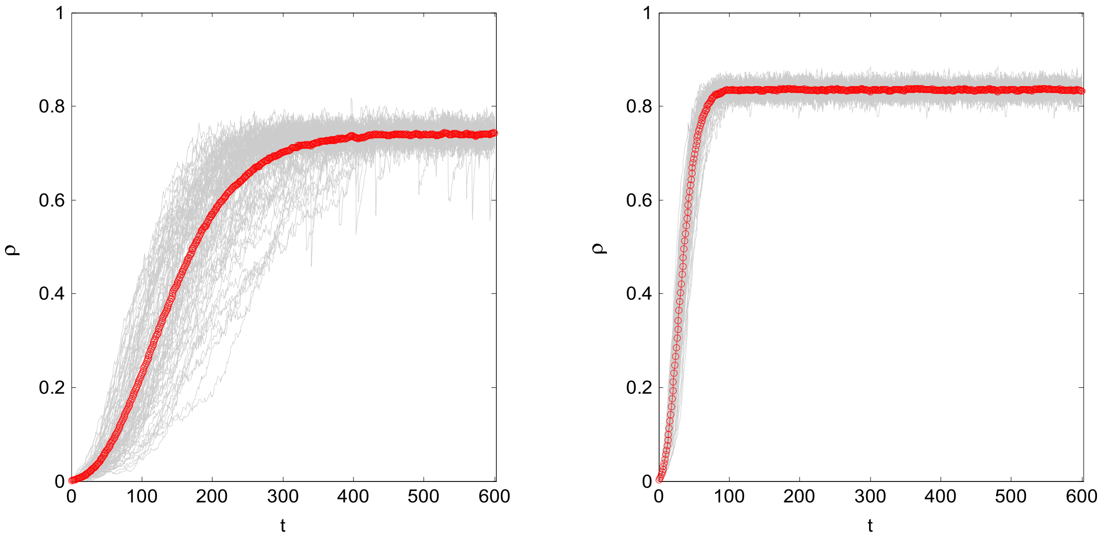
5. Conclusions and discussion

Information can spread more extensively and rapidly through online social networks. Thus it is important to capture the dynamical process of information spreading in those networks. In this paper, we focus on models of information propagation



(a) $m_1 = 1$. (b) $m_1 = 3$. (c) $m_1 = 5$.

Fig. 13. The evolution of informed with different structure parameters.



(a) CP. (b) RP.

Fig. 14. The simulation results in two different strategies.

in social networks which are presented by hypernetworks. Two models are introduced. One is based on RP strategy, the other is based on CP strategy. We simulate the specific spreading process in hypernetworks with different parameters, and try to explore how they impact the spreading process. Obviously, information spread much faster in hypernetworks than in complex networks. Individuals with more neighbors have a high ability to spread information, thus resulting in a greater impact. Both the spreading rate and recovering rate affect the diffusion speed and results in the steady state. The selection of initial spreading seed only affects the early evolution process. Comparing these two models, the obviously difference is whether there exists a spreading threshold. The absence of phase transition in PR model, and the presence of phase transition in CP model have been found. The results also uncover macro characteristics of the process of information diffusion among individuals.

The models proposed in the paper offer a starting point for analyzing spreading process in hypernetworks. To supplement the current study, several future research directions are proposed as follows.

(1) It should be noted that epidemic models with spatial diffusion have been considered [47–49]. And patch invasion has been obtained [49]. Discussing the influence of spatial pattern on information spreading will be interesting and valuable. This would be a potentially valuable area for further investigation.

(2) Although theoretical models have been proposed here, the corresponding empirical studies are needed to validate the current research.

(3) The hypernetwork structure is based on a k-uniform hypergraph. In reality, the number of individuals in each social network is generally different. Whether non-uniform hypernetwork demonstrates unique spreading characteristics, need to be further discussed.

(4) The spreading process discussed focus on static hypernetworks. However, the structure and contacts among individuals are changing according to their frequently social activities. So it should be straightforward to extend the study to dynamic hypernetworks.

(5) For CP strategy, the exact theoretical solution of the model and the expression of critical spreading threshold are badly needed. As reported in Ref. [50], the model cannot be accounted for by mean-field techniques. So theoretical framework based on Markov chains at the individual level will be attempted in the future study.

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