Bank marketing decision support system with Bayesian Network

Conceptual design:

Decision-making in business strategies is a challenging task. The management needs wisdom and sensitivity to understand the business environment and customer needs in decision-making. Moreover, brilliant ideas need to be sold and accepted by most of the stakeholders before it benefits the business. Reaching the correct decisions and selling such decisions are complex tasks. Nowadays, with more and more data availability, people are trying to find better ways to assist their decision-making and support decision promotion. A decision support system (DSS) is an information system that supports business decision-making by humans or computers (Wikipedia, 2022). For example, many businesses use the past year's sales as a projection for next year's budget or procurement plan. Such estimation is inaccurate and may lead to short supply when sales boost or budget surge when the procurement amount is lower than the minimum contract quantity. Bayesian Network is a DSS based on probabilistic reasoning, which tends to be more accurate in such cases.

In this project, we try to apply DSS in bank marketing decision-making. The task is to identify and signal which strategy a bank should take in an assumed economic environment and how to allocate resources based on the locational population characteristic.

Tasks: signal marketing and resource allocation strategy given some input economic and client data.

Objectives include:

1. Provide a conceptual description of DSS in enhancing banking marketing and resource allocation strategies by estimating sales behavior based on different economic environments or customer characteristics.
2. Represent the past sales data as a Bayesian network
3. Implement and test the Bayesian network in Python

Constraints:

1. Independence assumption between the economic data and customer traits data
2. Limited by coursework availability of data, our knowledge base may need to be revised in the current market as the data to construct a knowledge base is too old. The economic behavior and population characteristic are developing along with the time
3. Model may not be transferred to some businesses due to limited access to data.

Risks:

1. Using the Bayesian network to predict current marketing behavior may not be accurate as the sales data constructing knowledge base is too old.

Potential users of the example could be bank management. However, the system can easily be transferred to other business fields with its database. Hence, this system could be adopted by most business management.

The system can work independently as a stand-alone system. At the same time, we can easily integrate with other systems. For example, a price index prediction system or a human specialist prediction could provide information about future price indexes, which could be integrated into our system to support more accurate decision makings.

Data and knowledge: in this course work, our Bayesian network DSS requires a background knowledge model that could take in a sales record as a knowledge base. The sales record data should be big enough and specify the economic environment and bank customer characteristics data. By estimating the tendency that a customer subscribes to a term deposit without any prior information, we could define the baseline sales that describe a bank term deposit market size. Depending on different economic data, we can estimate if a market is expanding or not by comparing it with the current market rate. Moreover, depending on the locational population structure, which relates to the types of customers, we can understand if a location requires more resources for market developing and servicing. One could give some input data from both economic and customer (e.g. ([None], ["technician"])) to trigger the function call and activate a signal showing the market "expanding" or more resources allocation, or vice versa.

The system can improve its performance over time by collecting more marketing data on sales in various economic conditions and different types of customers. In addition, the system can be further improved by its integration with other systems, such as the price index estimation system or population distribution system.

Textual description of the background knowledge:

Banks sell various term deposit products. A term deposit, also named a time deposit, is a fixed-term investment (Wikipedia, 2022). When a customer buys a term deposit, the funds will be locked in an account with a financial institution and yield some fixed interest when the term ends. In this course work, we take the open-source data "Bank Marketing Data Set" from https://archive.ics.uci.edu/ml/datasets/Bank+Marketing as our knowledge base. The dataset is related to direct marketing campaigns of a Portuguese banking institution, which contains 20 input features in the time range from May 2008 to November 2010. From this, we can understand the probabilities of people subscribing to a term deposit.

First, we take the probability of people subscribing to a term deposit without any other knowledge as our baseline. Next, we evaluate the factors from two aspects: economic environment and customer characteristics. Assuming these two aspects are independent, we can tell if a market has higher subscriptions given a particular economic climate. When the subscription rate exceeds the current subscription average, we provide a decision signal as an "expanding" market. Else, the "shrinking" market. On the other side, given some customer characteristics, we can understand whether they have a higher probability of subscribing to a term deposit than the average rate. If they are more likely to subscribe to a term deposit, we give a decision signal as "allocate more resources." Or signal as "allocate fewer resources". Else, the sign "no special attention required" is the probability is the same as the average.

Selection of the nodes and links:

Our bank marketing dataset contains 20 features which can be categorized into three types, economic attributes, customer characteristic features, and campaigns related features. This coursework explores how economic factors and customer characteristics could impact management decision-making. Firstly, we analyzed the association between economic factors and subscription status. We found that “emp.var.rate” had a very high correlation(over 0.9) with features “euribor3m” and “nr. employed”. Hence, it is reasonable that “emp.var.rate” could sufficiently represent the other two features. We dropped features “euribor3m” and “nr.employed” as it would lead to little information loss to our model. Moreover, the feature “cons.price.idx” showed little correlation (around 0.05) to product subscription drop. By examing the two features left, “emp.var.rate” and “cons.price.idx” are correlated to each other(over 0.7), while “emp.var.rate” had a stronger correlation with product subscription than the other. We assumed that there is a (causational) connection flowing from “cons.price.idx” to “emp.var.rate” till “y”(product subscription).

Next, we evaluated the customer characteristic data, precisely “age”, “job”, “marital”, “education”, “default”, “housing” and “loan”. By plotting the histogram of “age”, we found that most clients contacted by sales campaign were distributed from age 25 to 60. To understand the association among different client characteristic features, we used a statistical test, the chi-squared test, to examine the distribution of other associated components (Wikipedia, 2022). A chi-squared test in our project assumes two features’ distribution is not associated (following gaussian distribution) and sets the significant values as 5% to reject the assumption. Each part was paired with other features and tested with chi-squared values. Results showed that “housing” and “loan” were not associated with “y” (test value >=5%). Therefore, these two values were removed. We also removed “default” as we are not interested in it. Then, we found some rare categories under different features. These rare categories provide little information to our model and add difficulty to the modeling process as they could lead to zero probability over the variable distributions. Finally, we removed the “unknown” variable in the random variable “job”, “unknown” in “marital”, and “illiterate” in “education”.

To visualize the relationships mentioned above, we plotted pairs of features and found some apparent associations below:

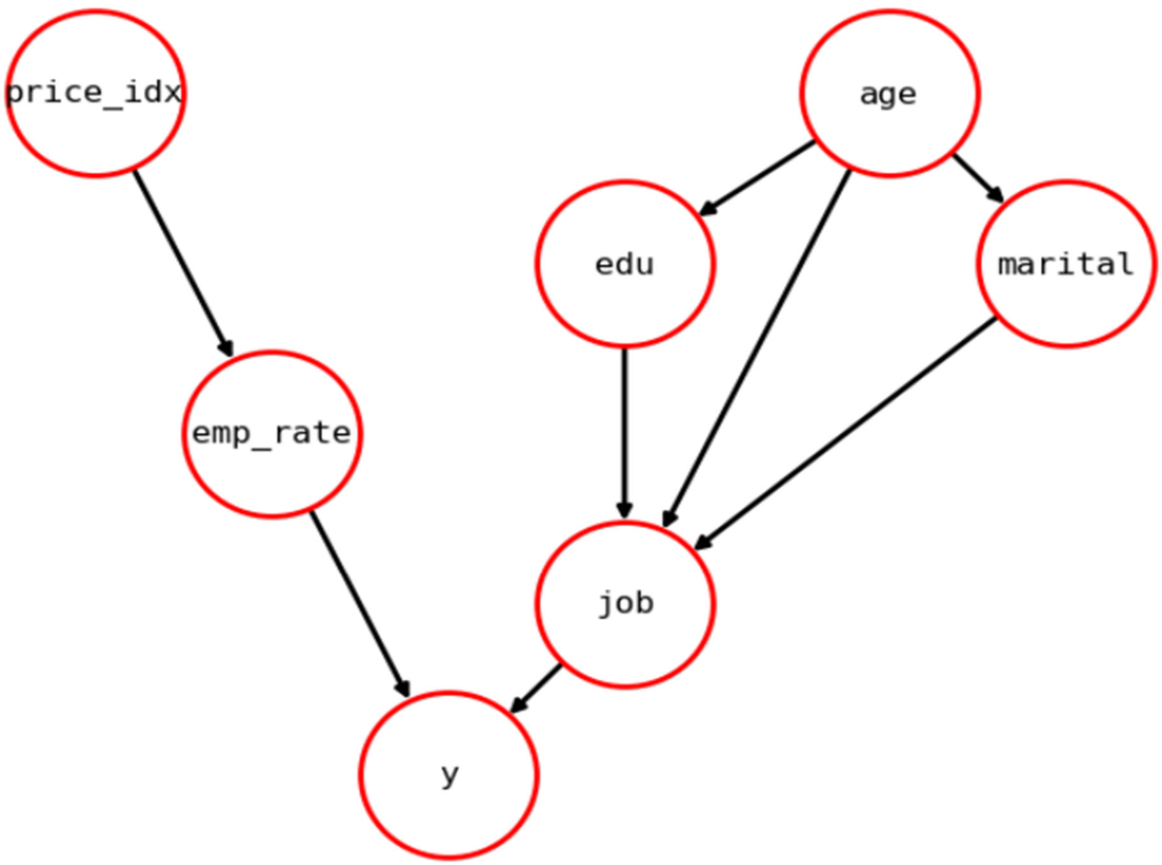
1. Age had an evident influence on random variables “job”, “marital”, and “education”
    a. The marriage and divorce ratio rose when age increased, while the singular ratio decreased.
    b. Older generations (age above 50) tend to have more basic education and less high school education
    c. Most people over 60 were retired, and most students were below 30.
2. Students and retired people are more willing to subscribe to a term than people with other jobs status.
3. Students tend to be single, while the divorce rate rises significantly in retired people.

4. Most people with “basic” education become “blue-collar”, while the majority of “professional.cource” takers are technicians.

With the findings above, we justify connections of random variables by common sense. The random variable “age” should be the first parent node as it is related to time only. Moreover, “age” could be a cause of different “education”, “marriage” and “job” statuses. Furthermore, education and marital status could cause other job options, which may impact investing behaviors.

Associated probabilities were calculated by crosstab the frequency counts of instances related to different variables, presented in the implementation part 1.3, “Calculate CPD”.

Graphical representation was presented in implementation section 1.6.1 “Plot the network,” and attached as below:



Querying Bayesian Network

Question 1: Understanding the market type by sales. What is the subscription rate, given no other information?

Marketing relates to many factors, such as product type, price strategy, and customer behavior. One of the primary factors we need to understand is the type of market by its size. If you sell mineral water, 100% of the population may be your customer. How big could the market be if you sell a bank term contract? It is easy to know the population distribution in different locations. With the Bayesian network, we can infer the market size of a term deposit by estimation from the population. Assume that a city has a population including 100 million adults. There would be around 11 million customers who will subscribe to a term deposit based on our inference from the Bayesian network.

Question 2: What should be the marketing strategy given the assumed economic environment (emp_rate <-2, price_idx >94)?

Our Bayesian network system will give signals based on specific input conditions. If the assumed economic environment is "emp_rate <-2" and "price_idx >94" in the next year, what our marketing strategy shall be? This is a critical strategy impacting overall business development. For instance, if the market expands, we need more human resources to develop the market and support higher business volume. Preparing human resources takes time. Assuming the current market subscription rate is the same as the baseline (around 11% when no prior information is given). If the predicted rate is higher than the current rate of 11%, we provide signals of an "increasing" market. Else, "shrinking" market. In the assumed economic environment, the system will signal management of an "increasing" market as the subscription rate is higher than 11%.

Question 3: What is the resource allocation decision with a locational population distribution high in technicians in the current market?

Once we know there is an uptrend in sales in the assumed economic environment, the next question is how to allocate resources. Cities differ in their population structure, economic component, etc. Which city shall we allocate more resources to? Assuming three cities have similar population sizes, one city's workforce is dominated by technicians, and it has much higher technicians than the sum of the other two cities. Shall we allocate more resources to the city populated with technicians? Or equally split resources among three cities? Based on our Bayesian network, we should allocate more resources to the city dominated by technicians as the subscription rate is higher than average in the assumed economic background.

Total number of words: 1824

References

Wikipedia. (2022) *Chi-squared test* [online]. Available at: https://www.mendeley.com/guides/harvard-citation-guide/ (Accessed: 07 January 2023)

Wikipedia. (2022) *Decision support system* [online]. Available at https://en.wikipedia.org/wiki/Decision_support_system (Accessed: 07 January 2023)

Wikipedia. (2022) *Time deposit* [online]. Available at: https://en.wikipedia.org/wiki/Time_deposit (Accessed: 07 January 2023)

Bayesian Network model

Data

Importing Necessary libraries

In [1]:
```
# import tools and libraries

import pandas as pd
import requests
from zipfile import ZipFile
from io import BytesIO

#tools to analyze categorical variable association
import os as os
from itertools import product
import numpy as np
import scipy.stats as ss

# tools to visulize data
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns

# tools to build bayesian network
from pgmpy.models import BayesianNetwork
from pgmpy.factors.discrete import TabularCPD
import networkx as nx

# tool to make inference from bayesian network
from pgmpy.inference import VariableElimination
```

Exploratory Data Analysis

In [2]:
```
# read data from online data base
r = requests.get("https://archive.ics.uci.edu/ml/machine-learning-databases/00222/bank-additional.zip")
files = ZipFile(BytesIO(r.content))
bank_full=pd.read_csv(files.open("bank-additional/bank-additional-full.csv"), sep=';')

# code refence:
# https://stackoverflow.com/questions/61894641/using-pandas-how-to-read-a-csv-file-inside-a-zip-file-which-you-fetch-using-an
```

In [3]:
```
# quick look at the data
bank_full.head()
```

Out[3]:

	age	job	marital	education	default	housing	loan	contact	month	day_of_week	...	campaign	pdays	previous	poutcome	emp.var.rate	cons.price.idx	cons.conf.idx	euribor3m	nr.employed	y
0	56	housemaid	married	basic.4y	no	no	no	telephone	may	mon	...	1	999	0	nonexistent	1.1	93.994	-36.4	4.857	5191.0	no
1	57	services	married	high.school	unknown	no	no	telephone	may	mon	...	1	999	0	nonexistent	1.1	93.994	-36.4	4.857	5191.0	no
2	37	services	married	high.school	no	yes	no	telephone	may	mon	...	1	999	0	nonexistent	1.1	93.994	-36.4	4.857	5191.0	no
3	40	admin.	married	basic.6y	no	no	no	telephone	may	mon	...	1	999	0	nonexistent	1.1	93.994	-36.4	4.857	5191.0	no
4	56	services	married	high.school	no	no	yes	telephone	may	mon	...	1	999	0	nonexistent	1.1	93.994	-36.4	4.857	5191.0	no

5 rows × 21 columns

In [4]:
```
# data features & information
bank_full.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 41188 entries, 0 to 41187
Data columns (total 21 columns):
 #   Column          Non-Null Count  Dtype
---  ------          --------------  -----
 0   age             41188 non-null  int64
 1   job             41188 non-null  object
 2   marital         41188 non-null  object
 3   education       41188 non-null  object
 4   default         41188 non-null  object
 5   housing         41188 non-null  object
 6   loan            41188 non-null  object
 7   contact         41188 non-null  object
 8   month           41188 non-null  object
 9   day_of_week     41188 non-null  object
 10  duration        41188 non-null  int64
 11  campaign        41188 non-null  int64
 12  pdays           41188 non-null  int64
 13  previous        41188 non-null  int64
 14  poutcome        41188 non-null  object
 15  emp.var.rate    41188 non-null  float64
 16  cons.price.idx  41188 non-null  float64
 17  cons.conf.idx   41188 non-null  float64
 18  euribor3m       41188 non-null  float64
 19  nr.employed     41188 non-null  float64
 20  y               41188 non-null  object
dtypes: float64(5), int64(5), object(11)
memory usage: 6.6+ MB
```

In [5]:
```
# statistics of numerical features
bank_full.describe()
```

Out[5]:

	age	duration	campaign	pdays	previous	emp.var.rate	cons.price.idx	cons.conf.idx	euribor3m	nr.employed
count	41188.00000	41188.000000	41188.000000	41188.000000	41188.000000	41188.000000	41188.000000	41188.000000	41188.000000	41188.000000
mean	40.02406	258.285010	2.567593	962.475454	0.172963	0.081886	93.575664	-40.502600	3.621291	5167.035911
std	10.42125	259.279249	2.770014	186.910907	0.494901	1.570960	0.578840	4.628198	1.734447	72.251528
min	17.00000	0.000000	1.000000	0.000000	0.000000	-3.400000	92.201000	-50.800000	0.634000	4963.600000
25%	32.00000	102.000000	1.000000	999.000000	0.000000	-1.800000	93.075000	-42.700000	1.344000	5099.100000
50%	38.00000	180.000000	2.000000	999.000000	0.000000	1.100000	93.749000	-41.800000	4.857000	5191.000000
75%	47.00000	319.000000	3.000000	999.000000	0.000000	1.400000	93.994000	-36.400000	4.961000	5228.100000
max	98.00000	4918.000000	56.000000	999.000000	7.000000	1.400000	94.767000	-26.900000	5.045000	5228.100000

In [6]:
```
# statistics of categorical features
bank_full.describe(include='object')
```

Out[6]:

	job	marital	education	default	housing	loan	contact	month	day_of_week	poutcome	y
count	41188	41188	41188	41188	41188	41188	41188	41188	41188	41188	41188
unique	12	4	8	3	3	3	2	10	5	3	2
top	admin.	married	university.degree	no	yes	no	cellular	may	thu	nonexistent	no
freq	10422	24928	12168	32588	21576	33950	26144	13769	8623	35563	36548

In [7]:
```
# categories of target variable
bank_full.y.unique()
```

Out[7]:
```
array(['no', 'yes'], dtype=object)
```

In [8]:
```
# check null values in data
bank_full.isna().sum()
```

Out[8]:
```
age               0
job               0
marital           0
education         0
default           0
housing           0
loan              0
contact           0
month             0
day_of_week       0
duration          0
campaign          0
pdays             0
previous          0
poutcome          0
emp.var.rate      0
cons.price.idx    0
cons.conf.idx     0
euribor3m         0
nr.employed       0
y                 0
dtype: int64
```

bank client data

Statistics of bank client characteristic features

In [9]:
```
# what are the bank client feature?
client_feat = bank_full.columns[:7]
print(client_feat)
```

```
Index(['age', 'job', 'marital', 'education', 'default', 'housing', 'loan'], dtype='object')
```

In [10]:
```
# types of features
bank_full[client_feat].info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 41188 entries, 0 to 41187
Data columns (total 7 columns):
 #   Column     Non-Null Count  Dtype 
---  ------     --------------  ----- 
 0   age        41188 non-null  int64 
 1   job        41188 non-null  object
 2   marital    41188 non-null  object
 3   education  41188 non-null  object
 4   default    41188 non-null  object
 5   housing    41188 non-null  object
 6   loan       41188 non-null  object
dtypes: int64(1), object(6)
memory usage: 2.2+ MB
```

In [11]:
```
# stats of "age"
bank_full[client_feat].describe()
```

Out[11]:

	age
count	41188.00000
mean	40.02406
std	10.42125
min	17.00000
25%	32.00000
50%	38.00000
75%	47.00000
max	98.00000

In [12]:
```
# bank client characteristic categorical feature distribution
for feat in client_feat[1:7]:
    print(bank_full[feat].value_counts())
```

```
admin.           10422
blue-collar       9254
technician        6743
services          3969
management        2924
retired           1720
entrepreneur      1456
self-employed     1421
housemaid         1060
unemployed        1014
student            875
unknown            330
Name: job, dtype: int64
married     24928
single      11568
divorced     4612
unknown       80
Name: marital, dtype: int64
university.degree      12168
high.school             9515
basic.9y                6045
professional.course     5243
basic.4y                4176
basic.6y                2292
unknown                 1731
illiterate                18
Name: education, dtype: int64
no         32588
unknown     8597
yes            3
Name: default, dtype: int64
yes        21576
no         18622
unknown      990
Name: housing, dtype: int64
no         33950
yes         6248
unknown      990
Name: loan, dtype: int64
```

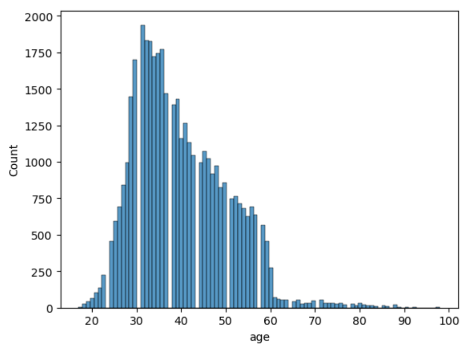
Pre-process bank client features

In [13]:
```
# drop rare categories under different features
rare = bank_full.loc[(bank_full["education"] == "illiterate")
                     | (bank_full["marital"] == "unknown")
                     |(bank_full['job']=='unknown')]
bank_full = bank_full.drop(index=rare.index)
```

In [14]:
```
# combine categories "basic.4y", 'basic.6y', 'basic.9y' as 'basic'
bank_full['education'] = bank_full['education'].replace(["basic.4y", 'basic.6y', 'basic.9y'], "basic")
```

In [15]:
```
# visualize the age feature distribution
sns.histplot( bank_full['age'])
```

Out[15]:
```
<AxesSubplot:xlabel='age', ylabel='Count'>
```



In [16]:
```python
# discretize age feature
bank_full['age'] = bank_full['age'].apply(
                    lambda x: "0.<30"  if x <= 30 else
                    "1.30~50" if 30 < x <= 50 else
                    "2.>50" )
```

Test the associations between different bank client features using Chi-square tests

In [17]:
```python
# features to pair up and be tested by chi2 to see if feature pair are independent from each other
chi2_feat = list(client_feat)
chi2_feat.append('y')
print(chi2_feat)
```

```
['age', 'job', 'marital', 'education', 'default', 'housing', 'loan', 'y']
```

In [18]:
```python
# pairing up different features
group1 = chi2_feat
group2 = chi2_feat
feat_pair = list(product(group1, group2, repeat=1))
# quick look at feature pairs
feat_pair[:5]
```

Out[18]:
```
[('age', 'age'),
 ('age', 'job'),
 ('age', 'marital'),
 ('age', 'education'),
 ('age', 'default')]
```

In [19]:
```python
# calculate chi2 test p-value for each feature pair
result = []
for i in feat_pair:
    if i[0] != i[1]: # exclude a feature pair only containing itself
        result.append((i[0], i[1],
                       list( ss.chi2_contingency(pd.crosstab(bank_full[chi2_feat][i[0]],
                                                              bank_full[chi2_feat][i[1]])))[1]))
```

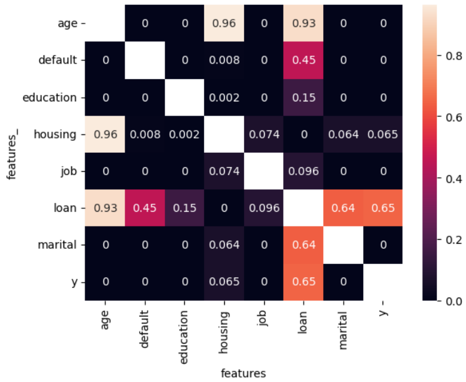
In [20]:
```python
# total number of feature pairs calculated
len(result)
```

Out[20]:
```
56
```

In [21]:
```python
# tabulate the pair-wise results
p_value = pd.DataFrame(result,
                       columns = ['features_', 'features', 'p']).pivot(
                       index='features_', columns='features', values='p').round(decimals=3)
```

In [22]:
```python
# visualize Chi2 test result (p-values)
sns.heatmap(p_value, annot=True)
print('There is association between 2 variables if p<0.05. y is not associated with (housing, loan),\
other variables are correlated with each other.')
```

```
There is association between 2 variables if p<0.05. y is not associated with (housing, loan),other variables are correlated with each other.
```



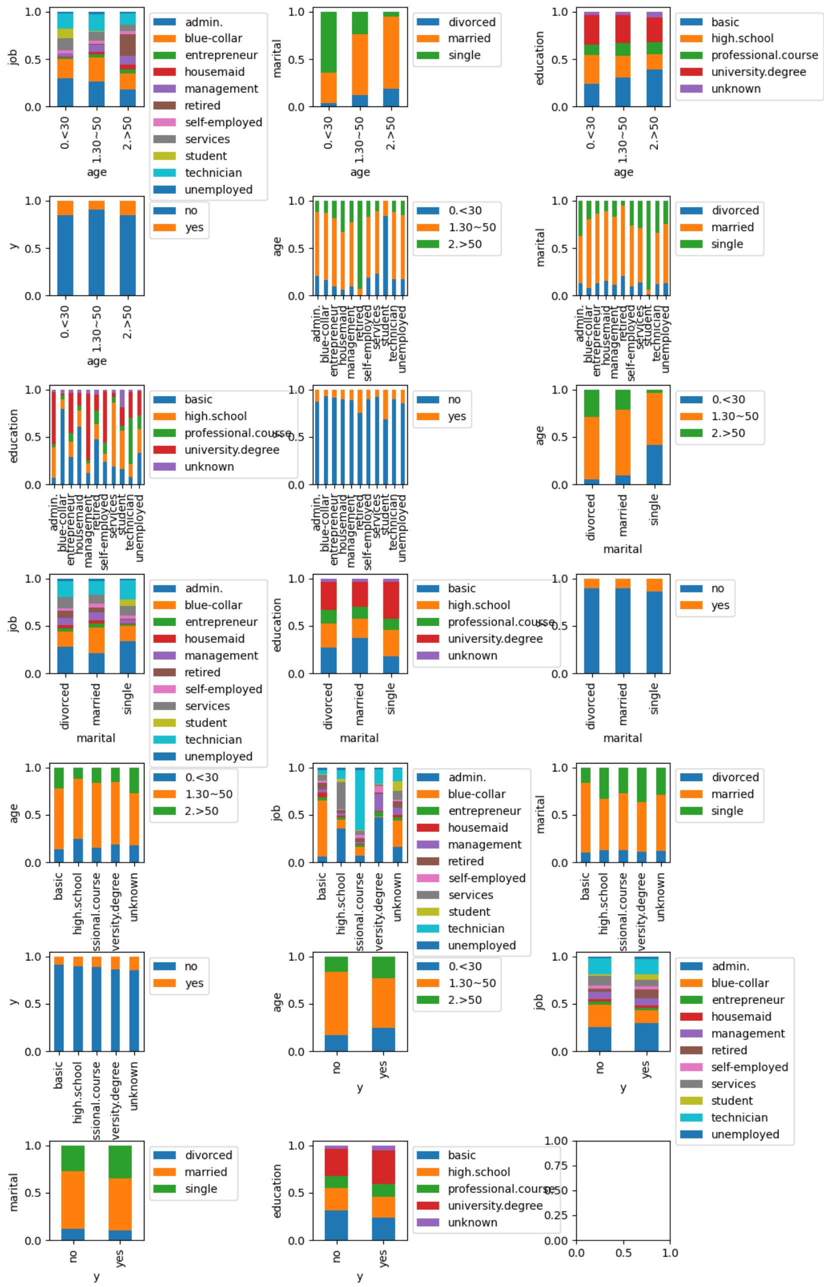
"y" is associated with age, default, education, job, and marital—no association with housing or loan. We are not interested in "default". Hence, the selected features would be "age", "job", "marital" and "education".

Visualized association between selected feature pairs

In [23]:
```python
# pair up selected features including y
pairs = list(product(["age", "job", "marital", "education", "y"], ["age", "job", "marital", "education", "y"], repeat=1))
# delete the pairs where feature pair with itself
for pair in pairs:
    if pair[0] == pair[1]:
        pairs.remove(pair)
```

In [24]:
```python
# visualize paired features distribution

# set a subplot grid
fig, axs = plt.subplots(nrows=7, ncols=3, figsize=(10, 20))
# set heights and widths between axes
plt.subplots_adjust(hspace=0.9, wspace=1.8)
# plot feature pairs on axes
for pair, ax in zip(pairs, axs.ravel()):
    # exclude pairs which feature paired with itself
    pd.crosstab(bank_full[pair[0]], bank_full[pair[1]], normalize='index').plot(kind='bar',stacked=True, ax=ax)
    ax.legend(bbox_to_anchor=(1, 1))
    ax.set_ylabel(pair[1])
plt.show()
```



There are some significant associations as follow:

1. Age had an evident influence on random variables "job", "marital", and "education"
   a. The marriage and divorce ratio rose when age increased, while the singular ratio decreased.
   b. Older generations (age above 50) tend to have more basic education and less high school education
   c. Most people over 60 were retired, and most students were below 30.
1. Students and retired people are more willing to subscribe to a term than people with other jobs status.
1. Students tend to be single, while the divorce rate rises significantly in retired people.
1. Most people with "basic" education become "blue-collar", while the majority of "professional.cource" takers are technicians.

social and economic context attributes

In [25]:
```
# name an object "SE" to represent social and economic features and "y"
SE = bank_full.iloc[:,15:].replace({'y':{'yes':1, 'no':0}})
```

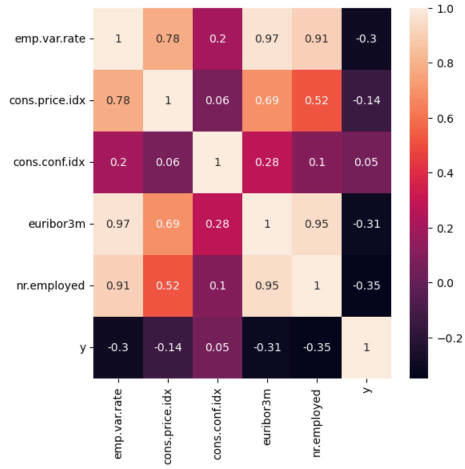
In [26]:
```
SE.corr().round(decimals=2)
```

Out[26]:

	emp.var.rate	cons.price.idx	cons.conf.idx	euribor3m	nr.employed	y
emp.var.rate	1.00	0.78	0.20	0.97	0.91	-0.30
cons.price.idx	0.78	1.00	0.06	0.69	0.52	-0.14
cons.conf.idx	0.20	0.06	1.00	0.28	0.10	0.05
euribor3m	0.97	0.69	0.28	1.00	0.95	-0.31
nr.employed	0.91	0.52	0.10	0.95	1.00	-0.35
y	-0.30	-0.14	0.05	-0.31	-0.35	1.00

In [27]:
```
# visualize correlations between variables
plt.figure(figsize=(6,6))
sns.heatmap(SE.corr().round(decimals=2), annot=True)
```

Out[27]:
```
<AxesSubplot:>
```



It seems 'emp.var.rate' could informativly represent 'euribor3m' and 'nr.employed', while 'cons.conf.idx' is not informative to 'y'

In [28]:
```
# stats of "SE"
SE[['emp.var.rate', 'cons.price.idx']].describe()
```

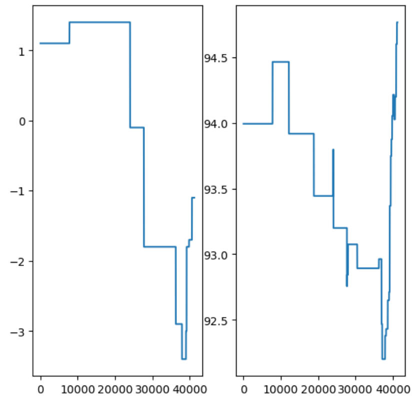
Out[28]:

	emp.var.rate	cons.price.idx
count	40769.000000	40769.000000
mean	0.080610	93.574918
std	1.570076	0.578516
min	-3.400000	92.201000
25%	-1.800000	93.075000
50%	1.100000	93.749000
75%	1.400000	93.994000
max	1.400000	94.767000

Discretize continous social and economic features

In [29]:
```
fig = plt.figure(figsize=(6,6))
j=1

for i in ['emp.var.rate', 'cons.price.idx']:
    plt.subplot(1,2,j)
    SE[i].plot()
    j = j+1
plt.show()
```



In [30]:
```
SE['emp.var.rate'] = SE['emp.var.rate'].apply(lambda x:
                                              ">0" if x >= 0 else
                                              "-2~0" if 0 > x >=-2 else
                                              "<-2"
                                              )
```

In [31]:
```
SE['cons.price.idx'] = SE['cons.price.idx'].apply(lambda x:
                                                  ">94" if x >= 94 else
                                                  "93~94" if 94 > x >= 93 else
                                                  "<93"
                                                  )
```

Choose variables and instantiate bayesian network model with edges

In [32]:
```
# create new data frame with the target data
df = pd.concat([SE[['emp.var.rate', 'cons.price.idx']],
                bank_full[['age', 'education','marital', 'job', 'y']]
               ], axis=1)
```

In [33]:
```
# quick look at target data
df.head()
```

Out[33]:

	emp.var.rate	cons.price.idx	age	education	marital	job	y
0	>0	93~94	2.>50	basic	married	housemaid	no
1	>0	93~94	2.>50	high.school	married	services	no
2	>0	93~94	1.30~50	high.school	married	services	no
3	>0	93~94	1.30~50	basic	married	admin.	no
4	>0	93~94	2.>50	high.school	married	services	no

Calculate CPD values (visualize in two decimal place)

P('cons.price.idx')

In [34]:
```
pd.crosstab(df['cons.price.idx'], 'Empty', margins = False, normalize = 'columns').apply(lambda x: round(x,2))
```

Out[34]:

col_0	Empty
cons.price.idx	
93~94	0.64
<93	0.22
>94	0.14

P('age')

In [35]:
```
pd.crosstab(df['age'], 'Empty', margins = False, normalize = 'columns').apply(lambda x: round(x,2))
```

Out[35]:

col_0	Empty
age	
0.<30	0.18
1.30~50	0.65
2.>50	0.17

P('emp.var.rate'|'cons.price.idx')

In [36]:
```
pd.crosstab(df['emp.var.rate'],df['cons.price.idx'], margins = True, normalize = 'columns').apply(lambda x: round(x,2))
```

Out[36]:

cons.price.idx	93~94	<93	>94	All
emp.var.rate				
-2~0	0.26	0.68	0.24	0.35
<-2	0.00	0.32	0.00	0.07
>0	0.74	0.00	0.76	0.58

P('education'|'age')

In [37]:
```
pd.crosstab(df['education'],df['age'], margins = True, normalize = 'columns').apply(lambda x: round(x,2))
```

Out[37]:

age	0.<30	1.30~50	2.>50	All
education				
basic	0.23	0.30	0.39	0.30
high.school	0.31	0.23	0.17	0.23
professional.course	0.11	0.13	0.12	0.13
university.degree	0.31	0.30	0.26	0.30
unknown	0.04	0.03	0.06	0.04

P('marital'|'age')

In [38]:
```
pd.crosstab(df['marital'],df['age'], margins = True, normalize = 'columns').apply(lambda x: round(x,2))
```

Out[38]:

age	0.<30	1.30~50	2.>50	All
marital				
divorced	0.03	0.12	0.19	0.11
married	0.32	0.64	0.76	0.61
single	0.64	0.24	0.05	0.28

P('job'|'education', 'marital')

In [39]:
```
pd.crosstab(df['job'],[df['education'],df['marital']], margins = True, normalize = 'columns').apply(lambda x: round(x,2))
```

Out[39]:

education	basic			high.school			professional.course			university.degree			unknown			All
marital	divorced	married	single	divorced	married	single	divorced	married	single	divorced	married	single	divorced	married	single	
job																
admin.	0.08	0.05	0.09	0.37	0.33	0.37	0.07	0.07	0.07	0.50	0.43	0.53	0.14	0.16	0.16	0.26
blue-collar	0.45	0.61	0.60	0.06	0.09	0.11	0.05	0.11	0.06	0.00	0.00	0.01	0.32	0.32	0.18	0.23
entrepreneur	0.02	0.04	0.02	0.04	0.03	0.01	0.02	0.03	0.01	0.06	0.06	0.03	0.05	0.04	0.01	0.04
housemaid	0.08	0.06	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.05	0.03	0.01	0.03
management	0.02	0.03	0.02	0.03	0.04	0.01	0.02	0.02	0.01	0.18	0.22	0.09	0.07	0.10	0.03	0.07
retired	0.14	0.07	0.01	0.04	0.04	0.01	0.08	0.06	0.01	0.04	0.03	0.01	0.05	0.09	0.01	0.04
self-employed	0.02	0.03	0.02	0.01	0.01	0.01	0.04	0.04	0.02	0.05	0.07	0.06	0.02	0.02	0.02	0.03
services	0.10	0.05	0.08	0.29	0.30	0.25	0.04	0.04	0.05	0.01	0.01	0.02	0.09	0.09	0.09	0.10
student	0.00	0.00	0.06	0.00	0.00	0.11	0.00	0.00	0.03	0.00	0.00	0.03	0.02	0.00	0.35	0.02
technician	0.05	0.04	0.05	0.10	0.09	0.09	0.63	0.60	0.72	0.11	0.13	0.19	0.18	0.13	0.12	0.17
unemployed	0.03	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.02

P('y'|'emp.var.rate', 'job')

In [40]:
```
pd.crosstab(df['y'], [df['emp.var.rate'],df['job']], margins = True, normalize = 'columns').apply(lambda x: round(x,2)).T
```

Out[40]:

emp.var.rate	job	no	yes
-2~0	admin.	0.80	0.20
	blue-collar	0.90	0.10
	entrepreneur	0.90	0.10
	housemaid	0.76	0.24
	management	0.86	0.14
	retired	0.62	0.38
	self-employed	0.85	0.15
	services	0.89	0.11
	student	0.64	0.36
	technician	0.82	0.18
	unemployed	0.78	0.22
<-2	admin.	0.60	0.40
	blue-collar	0.66	0.34
	entrepreneur	0.66	0.34
	housemaid	0.60	0.40
	management	0.62	0.38
	retired	0.59	0.41
	self-employed	0.69	0.31
	services	0.61	0.39
	student	0.58	0.42
	technician	0.60	0.40
	unemployed	0.63	0.37
>0	admin.	0.95	0.05
	blue-collar	0.95	0.05
	entrepreneur	0.95	0.05
	housemaid	0.97	0.03
	management	0.95	0.05
	retired	0.94	0.06
	self-employed	0.95	0.05
	services	0.95	0.05
	student	0.94	0.06
	technician	0.96	0.04
	unemployed	0.96	0.04
All		0.89	0.11

Define the network

In [41]:
```
# model with list of edges
model = BayesianNetwork([ ('price_idx', 'emp_rate'), ('age', 'edu'),
                          ('age', 'marital'), ('age', 'job'),('marital', 'job'),
                          ('edu', 'job'), ('job', 'y'), ('emp_rate', 'y')])
```

In [42]:
```
# check the leave node
model.get_leaves()
```

Out[42]:
```
['y']
```

In [43]:
```
# children nodes of 'age'
model.get_children('age')
```

Out[43]:
```
['edu', 'marital', 'job']
```

Build Conditional Probability Cards (CPDs)

CPD of price_idx

In [44]:
```
cpd_price_idx = TabularCPD(
    variable = 'price_idx',
    variable_card=3,
    values=[[0.64],[0.22],[0.14]],
    state_names={
        'price_idx':['93~94', '<93', '>94']
    }
)
```

CPD of emp_rate

In [45]:
```
cpd_emp_rate = TabularCPD(
    variable = 'emp_rate',
    variable_card=3,
    evidence = ['price_idx'],
    evidence_card=[3],
    values=[
       [0.25835564, 0.67909275, 0.2440049 ],
       [0.        , 0.32090725, 0.        ],
       [0.74164436, 0.        , 0.7559951 ]

    ],
    state_names={
        'emp_rate': ['-2~0', '<-2', '>0'],
        'price_idx':['93~94', '<93', '>94']
    }
)
```

CPD of age

In [46]:
```
cpd_age = TabularCPD(
    variable = 'age',
    variable_card=3,
    values=[
        [0.17996517],
       [0.64723197],
       [0.17280286]],
    state_names={
        'age': ['0.<30', '1.30~50', '2.>50']
    }
)
```

CPD of education

In [47]:
```
cpd_edu = TabularCPD(
    variable = 'edu',
    variable_card=5,
    evidence=['age'],
    evidence_card=[3],
    values=[
       [0.23320158, 0.3015879 , 0.38594748],
       [0.31007224, 0.22776367, 0.16735273],
       [0.1087638 , 0.13468754, 0.12391767],
       [0.30898187, 0.30291431, 0.26061036],
       [0.03898051, 0.03304658, 0.06217175]
    ],
    state_names={
        'edu': ['basic', 'high.school', 'professional.course', 'university.degree','unknown'],
        'age': ['0.<30', '1.30~50', '2.>50']
    }
)
```

CPD of marital

In [48]:
```python
cpd_marital = TabularCPD(
    variable = 'marital',
    variable_card=3,
    evidence=['age'],
    evidence_card=[3],
    values=[
       [0.03107537, 0.11535984, 0.18807665],
       [0.32479215, 0.64232387, 0.758978  ],
       [0.64413248, 0.24231629, 0.05294535]
    ],
    state_names={
        'marital': ['divorced', 'married', 'single'],
        'age': ['0.<30', '1.30~50', '2.>50']
    }
)
```

CPD of job

In [49]:
```python
cpd_job = TabularCPD(
    variable = 'job',
    variable_card=11,
    evidence=['edu', 'marital','age'],
    evidence_card=[5,3,3],
    values=[
        [0.13461538, 0.08224543, 0.0626506 , 0.042362  , 0.04868852,
        0.05553027, 0.07386364, 0.09157509, 0.13084112, 0.30337079,
        0.40423032, 0.27309237, 0.38073394, 0.34135546, 0.272203  ,
        0.32049608, 0.42487047, 0.36507937, 0.15384615, 0.06302521,
        0.09677419, 0.0648855 , 0.06759802, 0.06006006, 0.09215686,
        0.06519208, 0.05769231, 0.52830189, 0.55386417, 0.3962704 ,
        0.48963317, 0.45544111, 0.30132709, 0.52110901, 0.54208754,
        0.53174603, 0.125     , 0.03092784, 0.27272727, 0.32786885,
        0.15118397, 0.13095238, 0.09677419, 0.2079646 , 0.36      ],
       [0.46153846, 0.54308094, 0.29156627, 0.75353017, 0.67016393,
        0.39827037, 0.59886364, 0.62728938, 0.3271028 , 0.14606742,
        0.05757932, 0.03614458, 0.09785933, 0.09488243, 0.05767013,
        0.0926893 , 0.13212435, 0.01587302, 0.        , 0.06722689,
        0.01935484, 0.1221374 , 0.11762055, 0.06306306, 0.05294118,
        0.06519208, 0.        , 0.        , 0.00234192, 0.        ,
        0.00797448, 0.00492611, 0.00312256, 0.01071204, 0.01646091,
        0.        , 0.5       , 0.36082474, 0.25974026, 0.27868852,
        0.42622951, 0.16666667, 0.13824885, 0.2079646 , 0.36      ],
       [0.07692308, 0.02741514, 0.00963855, 0.03594352, 0.04180328,
        0.03186163, 0.00568182, 0.02289377, 0.05607477, 0.01123596,
        0.03642773, 0.04417671, 0.01987768, 0.03651452, 0.01730104,
        0.01044386, 0.00906736, 0.        , 0.        , 0.02521008,
        0.01935484, 0.05343511, 0.03064443, 0.02702703, 0.00588235,
        0.01979045, 0.        , 0.03773585, 0.05386417, 0.07925408,
        0.03668262, 0.06739812, 0.07025761, 0.01764335, 0.02880658,
        0.05555556, 0.        , 0.05154639, 0.06493506, 0.01639344,
        0.04918033, 0.04166667, 0.        , 0.01327434, 0.08      ],
       [0.        , 0.07049608, 0.1060241 , 0.01797176, 0.05016393,
        0.08784706, 0.00454545, 0.02197802, 0.04672897, 0.        ,
        0.02115159, 0.03614458, 0.02599388, 0.02130014, 0.02652826,
        0.00718016, 0.0123057 , 0.        , 0.        , 0.01260504,
        0.03225806, 0.00381679, 0.00901307, 0.01951952, 0.        ,
        0.0104773 , 0.09615385, 0.        , 0.0117096 , 0.01398601,
        0.01116427, 0.01298701, 0.0156128 , 0.00252048, 0.01122334,
        0.01587302, 0.        , 0.01030928, 0.1038961 , 0.        ,
        0.01821494, 0.05059524, 0.00460829, 0.02212389, 0.        ],
       [0.05769231, 0.02349869, 0.00963855, 0.01283697, 0.02885246,
        0.04369595, 0.01363636, 0.02197802, 0.07476636, 0.02247191,
        0.02820212, 0.04016064, 0.00917431, 0.04923928, 0.03690888,
        0.00652742, 0.02266839, 0.        , 0.        , 0.01260504,
        0.0516129 , 0.        , 0.01847679, 0.03153153, 0.00196078,
        0.0104773 , 0.05769231, 0.01886792, 0.16861827, 0.23076923,
        0.12759171, 0.22503359, 0.27010148, 0.0863264 , 0.09165731,
        0.03174603, 0.        , 0.10309278, 0.02597403, 0.01639344,
        0.11657559, 0.09821429, 0.01382488, 0.03982301, 0.04      ],
       [0.        , 0.01305483, 0.40481928, 0.        , 0.00819672,
        0.25216204, 0.00340909, 0.01373626, 0.12149533, 0.        ,
        0.00117509, 0.20080321, 0.        , 0.0033195 , 0.22145329,
        0.00065274, 0.00259067, 0.25396825, 0.        , 0.00840336,
        0.30322581, 0.        , 0.00315457, 0.25525526, 0.00196078,
        0.        , 0.23076923, 0.        , 0.00234192, 0.12820513,
        0.        , 0.00067174, 0.15612802, 0.        , 0.00112233,
        0.16666667, 0.        , 0.        , 0.11688312, 0.        ,
        0.00728597, 0.24107143, 0.        , 0.        , 0.16      ],
       [0.        , 0.02872063, 0.00963855, 0.01797176, 0.03377049,
        0.02503414, 0.00681818, 0.02106227, 0.05607477, 0.        ,
        0.01057579, 0.01204819, 0.00917431, 0.01604426, 0.00922722,
        0.00522193, 0.01489637, 0.04761905, 0.        , 0.03991597,
        0.02580645, 0.07633588, 0.03109509, 0.03303303, 0.01960784,
        0.02444703, 0.01923077, 0.11320755, 0.05854801, 0.02797203,
        0.08931419, 0.0575459 , 0.08821233, 0.08128544, 0.04938272,
        0.07142857, 0.        , 0.03092784, 0.01298701, 0.01639344,
        0.01821494, 0.01785714, 0.01843318, 0.01769912, 0.        ],
       [0.23076923, 0.11227154, 0.05060241, 0.05776637, 0.0547541 ,
        0.03413746, 0.06931818, 0.09615385, 0.06542056, 0.38202247,
        0.29259694, 0.26506024, 0.34556575, 0.31147994, 0.23298731,
        0.25391645, 0.23834197, 0.28571429, 0.03846154, 0.04621849,
        0.02580645, 0.07251908, 0.04326273, 0.01501502, 0.06470588,
        0.03725262, 0.        , 0.        , 0.01522248, 0.01631702,
        0.02392344, 0.01074787, 0.0078064 , 0.02772527, 0.01346801,
        0.        , 0.        , 0.13402062, 0.05194805, 0.24590164,
        0.08743169, 0.07440476, 0.09677419, 0.10176991, 0.        ],
       [0.        , 0.00130548, 0.        , 0.        , 0.00032787,
        0.        , 0.14431818, 0.00641026, 0.        , 0.        ,
        0.        , 0.        , 0.00611621, 0.00276625, 0.        ,
        0.21475196, 0.00906736, 0.        , 0.        , 0.00210084,
        0.        , 0.        , 0.00270392, 0.        , 0.06666667,
        0.00232829, 0.        , 0.        , 0.00468384, 0.        ,
        0.02392344, 0.00044783, 0.        , 0.06616257, 0.01646091,
        0.        , 0.125     , 0.02061856, 0.        , 0.        ,
        0.00364299, 0.        , 0.55299539, 0.18584071, 0.        ],
       [0.03846154, 0.05483029, 0.03614458, 0.0436457 , 0.03721311,
        0.04642695, 0.05909091, 0.04487179, 0.04672897, 0.08988764,
        0.11633373, 0.05220884, 0.05963303, 0.09598893, 0.09803922,
        0.06657963, 0.11334197, 0.03174603, 0.80769231, 0.69117647,
        0.4       , 0.57251908, 0.6408292 , 0.47447447, 0.68235294,
        0.74738068, 0.53846154, 0.26415094, 0.11358314, 0.09090909,
        0.16427432, 0.14151366, 0.07259953, 0.16446125, 0.20538721,
        0.12698413, 0.25      , 0.24742268, 0.09090909, 0.08196721,
        0.10928962, 0.16369048, 0.07373272, 0.18584071, 0.        ],
       [0.        , 0.04308094, 0.01927711, 0.01797176, 0.02606557,
        0.02503414, 0.02045455, 0.03205128, 0.07476636, 0.04494382,
        0.03172738, 0.04016064, 0.04587156, 0.02710927, 0.02768166,
        0.02154047, 0.02072539, 0.        , 0.        , 0.03151261,
        0.02580645, 0.03435115, 0.03560162, 0.02102102, 0.01176471,
        0.01746217, 0.        , 0.03773585, 0.01522248, 0.01631702,
        0.02551834, 0.02328706, 0.01483216, 0.02205419, 0.02394314,
        0.        , 0.        , 0.01030928, 0.        , 0.01639344,
        0.01275046, 0.01488095, 0.00460829, 0.01769912, 0.        ]
       ],
    state_names={
        'job': ['admin.', 'blue-collar', 'entrepreneur', 'housemaid', 'management',
                'retired', 'self-employed', 'services', 'student', 'technician','unemployed'],
        'edu': ['basic', 'high.school', 'professional.course', 'university.degree','unknown'],
        'marital': ['divorced', 'married', 'single'],
        'age': ['0.<30', '1.30~50', '2.>50']
    }

)
```

CPD of y

In [50]:
```python
cpd_y = TabularCPD(
    variable = 'y',
    variable_card=2,
    evidence=['emp_rate','job'],
    evidence_card=[3,11],
    values=[
        [0.80300083, 0.90314465, 0.89643463, 0.76339286, 0.86166667,
        0.6179966 , 0.84554455, 0.8913649 , 0.64285714, 0.81559588,
        0.78393352, 0.60107527, 0.65882353, 0.6557377 , 0.60493827,
        0.61904762, 0.5920398 , 0.68867925, 0.60655738, 0.5840708 ,
        0.59850374, 0.6328125 , 0.95406601, 0.95426726, 0.95006242,
        0.97203728, 0.95033557, 0.94174757, 0.95386534, 0.95051975,
        0.93604651, 0.95548823, 0.96346154],
       [0.19699917, 0.09685535, 0.10356537, 0.23660714, 0.13833333,
        0.3820034 , 0.15445545, 0.1086351 , 0.35714286, 0.18440412,
        0.21606648, 0.39892473, 0.34117647, 0.3442623 , 0.39506173,
        0.38095238, 0.4079602 , 0.31132075, 0.39344262, 0.4159292 ,
        0.40149626, 0.3671875 , 0.04593399, 0.04573274, 0.04993758,
        0.02796272, 0.04966443, 0.05825243, 0.04613466, 0.04948025,
        0.06395349, 0.04451177, 0.03653846]
        ],
```

```
    state_names={
        'y': ['no', 'yes' ],
        'emp_rate': ['-2~0', '<-2', '>0'],
        'job': ['admin.', 'blue-collar', 'entrepreneur', 'housemaid', 'management',
               'retired', 'self-employed', 'services', 'student', 'technician','unemployed'],
    }
)
```

In [ ]:

Associate the CPDs to the network

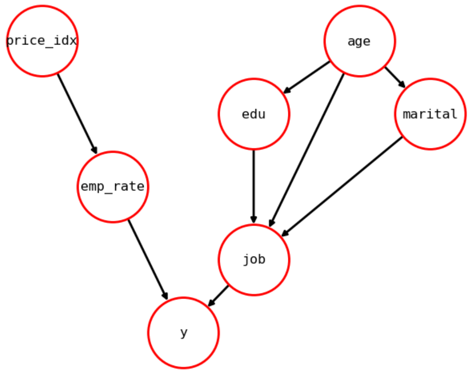
In [51]:
```
model.add_cpds(cpd_price_idx, cpd_emp_rate, cpd_age, cpd_edu, cpd_marital, cpd_job, cpd_y)
```

Plot the network

In [52]:
```
pos = {'price_idx': (0, 0), 'emp_rate': (1, -2),
       'y': (2, -4), 'job': (3, -3), 'edu': (3, -1),
       'marital':(5.5,-1), 'age': (4.5,0)}
options = {
    "with_labels": True,
    "font_size": 12,
    "node_size": 4000,
    "node_color": 'white',
    "node_shape": 'o',
    "edgecolors": "red",
    "linewidths": 2,
    "width": 2,
    "font_family": 'monospace' # 'fantasy' 'monospace'
}

nx.draw(model, pos, **options)

# Set margins for the axes so that nodes aren't clipped
ax = plt.gca()
ax.margins(0.10)
plt.axis("off")
plt.show()
```



Understanding the model

In [53]:
```
# check the network structure and CPDs,
# verify if the CPDs are correctly defined and sum to 1.
model.check_model()
```

Out[53]:
```
True
```

In [54]:
```
# list all cpds
model.get_cpds()
```

Out[54]:
```
[<TabularCPD representing P(price_idx:3) at 0x2465f9e3d00>,
 <TabularCPD representing P(emp_rate:3 | price_idx:3) at 0x2465fa007c0>,
 <TabularCPD representing P(age:3) at 0x2465f9f2c10>,
 <TabularCPD representing P(edu:5 | age:3) at 0x2465fa00550>,
 <TabularCPD representing P(marital:3 | age:3) at 0x2465fa002e0>,
 <TabularCPD representing P(job:11 | edu:5, marital:3, age:3) at 0x2465fa005e0>,
 <TabularCPD representing P(y:2 | emp_rate:3, job:11) at 0x2465fbd74c0>]
```

In [55]:
```
# check cpd of 'y'
print(model.get_cpds('y'))
```

```
+----------+----------------+------------------+-----+---------------+-----------------+-----------------+
| emp_rate | emp_rate(-2~0) | emp_rate(-2~0)   | ... | emp_rate(>0)  | emp_rate(>0)    | emp_rate(>0)    |
+----------+----------------+------------------+-----+---------------+-----------------+-----------------+
| job      | job(admin.)    | job(blue-collar) | ... | job(student)  | job(technician) | job(unemployed) |
+----------+----------------+------------------+-----+---------------+-----------------+-----------------+
| y(no)    | 0.80300083     | 0.90314465       | ... | 0.93604651    | 0.95548823      | 0.96346154      |
+----------+----------------+------------------+-----+---------------+-----------------+-----------------+
| y(yes)   | 0.19699917     | 0.09685535       | ... | 0.06395349    | 0.04451177      | 0.03653846      |
+----------+----------------+------------------+-----+---------------+-----------------+-----------------+
```

In [56]:
```
# check independencies
model.get_independencies()
```

Out[56]:
```
(price_idx ⊥ age, marital, edu, job)
(price_idx ⊥ age, job, edu | marital)
(price_idx ⊥ age, marital, job | edu)
(price_idx ⊥ marital, edu, age, job, y | emp_rate)
(price_idx ⊥ marital, job, edu | age)
(price_idx ⊥ age, marital, edu | job)
(price_idx ⊥ age, job | marital, edu)
(price_idx ⊥ age, job, edu, y | marital, emp_rate)
(price_idx ⊥ job, edu | age, marital)
(price_idx ⊥ age, edu | marital, job)
(price_idx ⊥ age, marital, job, y | edu, emp_rate)
(price_idx ⊥ marital, job | age, edu)
(price_idx ⊥ age, marital | edu, job)
(price_idx ⊥ job, marital, edu, y | age, emp_rate)
(price_idx ⊥ age, marital, edu, y | job, emp_rate)
(price_idx ⊥ age, marital, edu, job | y, emp_rate)
(price_idx ⊥ marital, edu | age, job)
(price_idx ⊥ age, marital, edu | job, y)
(price_idx ⊥ age, job, y | marital, edu, emp_rate)
(price_idx ⊥ job | age, marital, edu)
(price_idx ⊥ age | marital, edu, job)
(price_idx ⊥ job, edu, y | age, marital, emp_rate)
(price_idx ⊥ age, edu, y | marital, job, emp_rate)
(price_idx ⊥ age, edu, job | y, marital, emp_rate)
(price_idx ⊥ edu | age, marital, job)
(price_idx ⊥ age, edu | marital, job, y)
(price_idx ⊥ marital, job, y | age, edu, emp_rate)
(price_idx ⊥ age, marital, y | job, edu, emp_rate)
(price_idx ⊥ age, marital, job | y, edu, emp_rate)
(price_idx ⊥ marital | age, edu, job)
(price_idx ⊥ age, marital | y, edu, job)
(price_idx ⊥ marital, edu, y | age, job, emp_rate)
(price_idx ⊥ marital, edu, job | age, y, emp_rate)
(price_idx ⊥ age, marital, edu | y, job, emp_rate)
(price_idx ⊥ marital, edu | age, job, y)
(price_idx ⊥ job, y | age, marital, edu, emp_rate)
(price_idx ⊥ age, y | job, marital, edu, emp_rate)
(price_idx ⊥ age, job | y, marital, edu, emp_rate)
(price_idx ⊥ age | y, marital, edu, job)
(price_idx ⊥ edu, y | age, marital, job, emp_rate)
(price_idx ⊥ edu, job | y, age, marital, emp_rate)
(price_idx ⊥ age, edu | y, marital, job, emp_rate)
(price_idx ⊥ edu | age, marital, job, y)
(price_idx ⊥ marital, y | age, job, edu, emp_rate)
(price_idx ⊥ marital, job | age, y, edu, emp_rate)
(price_idx ⊥ age, marital | job, y, edu, emp_rate)
(price_idx ⊥ marital | age, y, edu, job)
(price_idx ⊥ marital, edu | age, y, job, emp_rate)
(price_idx ⊥ y | marital, job, edu, emp_rate, age)
(price_idx ⊥ job | marital, y, edu, emp_rate, age)
(price_idx ⊥ age | marital, job, y, edu, emp_rate)
(price_idx ⊥ edu | marital, job, y, emp_rate, age)
(price_idx ⊥ marital | job, y, edu, emp_rate, age)
(marital ⊥ price_idx, emp_rate)
(marital ⊥ emp_rate | price_idx)
(marital ⊥ price_idx, emp_rate | edu)
(marital ⊥ price_idx | emp_rate)
(marital ⊥ price_idx, edu, emp_rate | age)
(marital ⊥ price_idx, emp_rate, y | job)
(marital ⊥ emp_rate | price_idx, edu)
(marital ⊥ edu, emp_rate | price_idx, age)
(marital ⊥ emp_rate, y | price_idx, job)
(marital ⊥ price_idx | edu, emp_rate)
(marital ⊥ price_idx, emp_rate | age, edu)
(marital ⊥ price_idx, emp_rate, y | edu, job)
(marital ⊥ price_idx, edu | age, emp_rate)
(marital ⊥ price_idx, y | job, emp_rate)
(marital ⊥ price_idx | y, emp_rate)
(marital ⊥ price_idx, emp_rate, y | age, job)
(marital ⊥ price_idx, emp_rate | job, y)
(marital ⊥ emp_rate | price_idx, age, edu)
(marital ⊥ emp_rate, y | price_idx, edu, job)
(marital ⊥ edu | price_idx, age, emp_rate)
(marital ⊥ y | price_idx, job, emp_rate)
(marital ⊥ emp_rate, y | price_idx, age, job)
(marital ⊥ emp_rate | price_idx, job, y)
(marital ⊥ price_idx | age, edu, emp_rate)
(marital ⊥ price_idx, y | job, edu, emp_rate)
(marital ⊥ price_idx | y, edu, emp_rate)
(marital ⊥ price_idx, emp_rate, y | age, edu, job)
(marital ⊥ price_idx, emp_rate | y, edu, job)
(marital ⊥ price_idx, y | age, job, emp_rate)
(marital ⊥ price_idx | age, y, emp_rate)
(marital ⊥ price_idx | y, job, emp_rate)
(marital ⊥ price_idx, emp_rate | age, job, y)
(marital ⊥ y | price_idx, job, edu, emp_rate)
(marital ⊥ emp_rate, y | price_idx, age, edu, job)
(marital ⊥ emp_rate | price_idx, y, edu, job)
(marital ⊥ y | price_idx, age, job, emp_rate)
(marital ⊥ emp_rate | price_idx, age, job, y)
(marital ⊥ price_idx, y | age, job, edu, emp_rate)
(marital ⊥ price_idx | age, y, edu, emp_rate)
(marital ⊥ price_idx | job, y, edu, emp_rate)
(marital ⊥ price_idx, emp_rate | age, y, edu, job)
(marital ⊥ price_idx | age, y, job, emp_rate)
(marital ⊥ y | job, price_idx, edu, emp_rate, age)
(marital ⊥ emp_rate | job, y, price_idx, edu, age)
(marital ⊥ price_idx | job, y, edu, emp_rate, age)
(edu ⊥ price_idx, emp_rate)
(edu ⊥ emp_rate | price_idx)
(edu ⊥ price_idx, emp_rate | marital)
(edu ⊥ price_idx | emp_rate)
(edu ⊥ price_idx, marital, emp_rate | age)
(edu ⊥ price_idx, emp_rate, y | job)
(edu ⊥ emp_rate | price_idx, marital)
(edu ⊥ marital, emp_rate | price_idx, age)
(edu ⊥ emp_rate, y | price_idx, job)
(edu ⊥ price_idx | marital, emp_rate)
(edu ⊥ price_idx, emp_rate | age, marital)
(edu ⊥ price_idx, emp_rate, y | marital, job)
(edu ⊥ price_idx, marital | age, emp_rate)
(edu ⊥ price_idx, y | job, emp_rate)
(edu ⊥ price_idx | y, emp_rate)
(edu ⊥ price_idx, emp_rate, y | age, job)
(edu ⊥ price_idx, emp_rate | job, y)
(edu ⊥ emp_rate | price_idx, age, marital)
(edu ⊥ emp_rate, y | price_idx, marital, job)
(edu ⊥ marital | price_idx, age, emp_rate)
(edu ⊥ y | price_idx, job, emp_rate)
(edu ⊥ emp_rate, y | price_idx, age, job)
(edu ⊥ emp_rate | price_idx, job, y)
(edu ⊥ price_idx | age, marital, emp_rate)
(edu ⊥ price_idx, y | marital, job, emp_rate)
(edu ⊥ price_idx | y, marital, emp_rate)
(edu ⊥ price_idx, emp_rate, y | age, marital, job)
(edu ⊥ price_idx, emp_rate | marital, job, y)
(edu ⊥ price_idx, y | age, job, emp_rate)
(edu ⊥ price_idx | age, y, emp_rate)
(edu ⊥ price_idx | y, job, emp_rate)
(edu ⊥ price_idx, emp_rate | age, job, y)
(edu ⊥ y | price_idx, marital, job, emp_rate)
(edu ⊥ emp_rate, y | price_idx, age, marital, job)
(edu ⊥ emp_rate | price_idx, marital, job, y)
(edu ⊥ y | price_idx, age, job, emp_rate)
(edu ⊥ emp_rate | price_idx, age, job, y)
(edu ⊥ price_idx, y | age, marital, job, emp_rate)
(edu ⊥ price_idx | y, age, marital, emp_rate)
(edu ⊥ price_idx | y, marital, job, emp_rate)
(edu ⊥ price_idx, emp_rate | age, marital, job, y)
(edu ⊥ price_idx | age, y, job, emp_rate)
(edu ⊥ y | marital, job, price_idx, emp_rate, age)
(edu ⊥ emp_rate | marital, job, y, price_idx, age)
(edu ⊥ price_idx | marital, job, y, emp_rate, age)
(emp_rate ⊥ age, marital, edu, job)
(emp_rate ⊥ age, marital, job, edu | price_idx)
(emp_rate ⊥ age, job, edu | marital)
(emp_rate ⊥ age, marital, job | edu)
(emp_rate ⊥ marital, job, edu | age)
(emp_rate ⊥ age, marital, edu | job)
(emp_rate ⊥ age, edu, job | price_idx, marital)
(emp_rate ⊥ age, marital, job | price_idx, edu)
(emp_rate ⊥ marital, edu, job | price_idx, age)
(emp_rate ⊥ age, marital, edu | price_idx, job)
(emp_rate ⊥ age, job | marital, edu)
(emp_rate ⊥ edu, job | age, marital)
(emp_rate ⊥ age, edu | marital, job)
(emp_rate ⊥ marital, job | age, edu)
(emp_rate ⊥ age, marital | edu, job)
(emp_rate ⊥ marital, edu | age, job)
(emp_rate ⊥ age, marital, edu | job, y)
```

```
(emp_rate ⊥ age, job | price_idx, marital, edu)
(emp_rate ⊥ edu, job | price_idx, age, marital)
(emp_rate ⊥ age, edu | price_idx, marital, job)
(emp_rate ⊥ marital, job | price_idx, age, edu)
(emp_rate ⊥ age, marital | price_idx, edu, job)
(emp_rate ⊥ marital, edu | price_idx, age, job)
(emp_rate ⊥ age, marital, edu | price_idx, job, y)
(emp_rate ⊥ job | age, marital, edu)
(emp_rate ⊥ age | marital, edu, job)
(emp_rate ⊥ edu | age, marital, job)
(emp_rate ⊥ age, edu | marital, job, y)
(emp_rate ⊥ marital | age, edu, job)
(emp_rate ⊥ age, marital | y, edu, job)
(emp_rate ⊥ marital, edu | age, job, y)
(emp_rate ⊥ job | price_idx, age, marital, edu)
(emp_rate ⊥ age | price_idx, marital, edu, job)
(emp_rate ⊥ edu | price_idx, age, marital, job)
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(emp_rate ⊥ age, marital | price_idx, y, edu, job)
(emp_rate ⊥ marital, edu | price_idx, age, job, y)
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(emp_rate ⊥ marital | age, y, edu, job)
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(emp_rate ⊥ edu | marital, job, y, price_idx, age)
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(age ⊥ price_idx, emp_rate)
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(age ⊥ price_idx | emp_rate)
(age ⊥ price_idx, emp_rate, y | job)
(age ⊥ emp_rate | price_idx, marital)
(age ⊥ emp_rate | price_idx, edu)
(age ⊥ emp_rate, y | price_idx, job)
(age ⊥ price_idx, emp_rate | marital, edu)
(age ⊥ price_idx | marital, emp_rate)
(age ⊥ price_idx, emp_rate, y | marital, job)
(age ⊥ price_idx | edu, emp_rate)
(age ⊥ price_idx, emp_rate, y | edu, job)
(age ⊥ price_idx, y | job, emp_rate)
(age ⊥ price_idx | y, emp_rate)
(age ⊥ price_idx, emp_rate | job, y)
(age ⊥ emp_rate | price_idx, marital, edu)
(age ⊥ emp_rate, y | price_idx, marital, job)
(age ⊥ emp_rate, y | price_idx, edu, job)
(age ⊥ y | price_idx, job, emp_rate)
(age ⊥ emp_rate | price_idx, job, y)
(age ⊥ price_idx | marital, edu, emp_rate)
(age ⊥ price_idx, emp_rate, y | marital, edu, job)
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(age ⊥ price_idx, y | job, edu, emp_rate)
(age ⊥ price_idx | y, edu, emp_rate)
(age ⊥ price_idx, emp_rate | y, edu, job)
(age ⊥ price_idx | y, job, emp_rate)
(age ⊥ emp_rate, y | price_idx, marital, edu, job)
(age ⊥ y | price_idx, marital, job, emp_rate)
(age ⊥ emp_rate | price_idx, marital, job, y)
(age ⊥ y | price_idx, job, edu, emp_rate)
(age ⊥ emp_rate | price_idx, y, edu, job)
(age ⊥ price_idx, y | job, marital, edu, emp_rate)
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(age ⊥ price_idx, emp_rate | y, marital, edu, job)
(age ⊥ price_idx | y, marital, job, emp_rate)
(age ⊥ price_idx | job, y, edu, emp_rate)
(age ⊥ y | marital, job, price_idx, edu, emp_rate)
(age ⊥ emp_rate | marital, job, y, price_idx, edu)
(age ⊥ price_idx | marital, job, y, edu, emp_rate)
(job ⊥ price_idx, emp_rate)
(job ⊥ emp_rate | price_idx)
(job ⊥ price_idx, emp_rate | marital)
(job ⊥ price_idx, emp_rate | edu)
(job ⊥ price_idx | emp_rate)
(job ⊥ price_idx, emp_rate | age)
(job ⊥ emp_rate | price_idx, marital)
(job ⊥ emp_rate | price_idx, edu)
(job ⊥ emp_rate | price_idx, age)
(job ⊥ price_idx, emp_rate | marital, edu)
(job ⊥ price_idx | marital, emp_rate)
(job ⊥ price_idx, emp_rate | age, marital)
(job ⊥ price_idx | edu, emp_rate)
(job ⊥ price_idx, emp_rate | age, edu)
(job ⊥ price_idx | age, emp_rate)
(job ⊥ price_idx | y, emp_rate)
(job ⊥ emp_rate | price_idx, marital, edu)
(job ⊥ emp_rate | price_idx, age, marital)
(job ⊥ emp_rate | price_idx, age, edu)
(job ⊥ price_idx | marital, edu, emp_rate)
(job ⊥ price_idx, emp_rate | age, marital, edu)
(job ⊥ price_idx | age, marital, emp_rate)
(job ⊥ price_idx | y, marital, emp_rate)
(job ⊥ price_idx | age, edu, emp_rate)
(job ⊥ price_idx | y, edu, emp_rate)
(job ⊥ price_idx | age, y, emp_rate)
(job ⊥ emp_rate | price_idx, age, marital, edu)
(job ⊥ price_idx | age, marital, edu, emp_rate)
(job ⊥ price_idx | y, marital, edu, emp_rate)
(job ⊥ price_idx | y, age, marital, emp_rate)
(job ⊥ price_idx | age, y, edu, emp_rate)
(job ⊥ price_idx | marital, y, edu, emp_rate, age)
(y ⊥ price_idx | emp_rate)
(y ⊥ age, marital, edu | job)
(y ⊥ age, marital, edu | price_idx, job)
(y ⊥ price_idx | marital, emp_rate)
(y ⊥ age, edu | marital, job)
(y ⊥ price_idx | edu, emp_rate)
(y ⊥ age, marital | edu, job)
(y ⊥ price_idx | age, emp_rate)
(y ⊥ price_idx, age, marital, edu | job, emp_rate)
(y ⊥ marital, edu | age, job)
(y ⊥ age, edu | price_idx, marital, job)
(y ⊥ age, marital | price_idx, edu, job)
(y ⊥ age, marital, edu | price_idx, job, emp_rate)
(y ⊥ marital, edu | price_idx, age, job)
(y ⊥ price_idx | marital, edu, emp_rate)
(y ⊥ age | marital, edu, job)
(y ⊥ price_idx | age, marital, emp_rate)
(y ⊥ price_idx, age, edu | marital, job, emp_rate)
(y ⊥ edu | age, marital, job)
(y ⊥ price_idx | age, edu, emp_rate)
(y ⊥ price_idx, age, marital | job, edu, emp_rate)
(y ⊥ marital | age, edu, job)
(y ⊥ price_idx, marital, edu | age, job, emp_rate)
(y ⊥ age | price_idx, marital, edu, job)
(y ⊥ age, edu | price_idx, marital, job, emp_rate)
(y ⊥ edu | price_idx, age, marital, job)
(y ⊥ age, marital | price_idx, job, edu, emp_rate)
(y ⊥ marital | price_idx, age, edu, job)
(y ⊥ marital, edu | price_idx, age, job, emp_rate)
(y ⊥ price_idx | age, marital, edu, emp_rate)
(y ⊥ price_idx, age | job, marital, edu, emp_rate)
(y ⊥ price_idx, edu | age, marital, job, emp_rate)
(y ⊥ price_idx, marital | age, job, edu, emp_rate)
(y ⊥ age | marital, job, price_idx, edu, emp_rate)
(y ⊥ edu | marital, job, price_idx, emp_rate, age)
(y ⊥ marital | job, price_idx, edu, emp_rate, age)
(y ⊥ price_idx | marital, job, edu, emp_rate, age)
```

In [57]:
```python
# all reachable nodes of 'y' from the graph
model.active_trail_nodes("y")
```

Out[57]:
```
{'y': {'age', 'edu', 'emp_rate', 'job', 'marital', 'price_idx', 'y'}}
```

In [58]:
```python
# all reachable nodes of 'edu' from the graph
model.active_trail_nodes("edu")
```

Out[58]:
```
{'edu': {'age', 'edu', 'job', 'marital', 'y'}}
```

Make inference

What is the subscription rate, given no other information?

What is the probability of customer subscribe a term deposit given no other information?

In [59]:
```
infer = VariableElimination(model)
```

In [60]:
```
y_distribution = infer.query(variables=['y'])
```

In [61]:
```
print(y_distribution)
```

```
+--------+----------+
| y      |   phi(y) |
+========+==========+
| y(no)  |   0.8878 |
+--------+----------+
| y(yes) |   0.1122 |
+--------+----------+
```

What should be the marketing strategy given the assumed economic environment (emp_rate <-2, price_idx >94)?

What is the bank term deposit subscription rate given (emp_rate <-2, price_idx >94) ?

In [62]:
```
y_given_0_94 = infer.query(
    variables = ['y'],
    evidence = {'price_idx':'>94', 'emp_rate':'<-2'}
)
print(y_given_0_94)
```

```
+--------+----------+
| y      |   phi(y) |
+========+==========+
| y(no)  |   0.6206 |
+--------+----------+
| y(yes) |   0.3794 |
+--------+----------+
```

Assuming the current market subscription rate is the same as the baseline (around 11% when no prior information is given). If the predicted rate is higher than the current rate of 11%, we provide signals of an "increasing" market. Else, "shrinking" market. In the assumed economic environment, the system will signal management of an "increasing" market as the subscription rate is much higher than 11%.

What is the resource allocation decision with a locational population distribution high in technicians in the current market?

In [63]:
```
y_given_technician = infer.query(
    variables = ['y'],
    evidence = { 'job': "technician",'price_idx':'>94', 'emp_rate':'<-2'}
)
print(y_given_technician)
```

```
+--------+----------+
| y      |   phi(y) |
+========+==========+
| y(no)  |   0.5985 |
+--------+----------+
| y(yes) |   0.4015 |
+--------+----------+
```

Assuming three cities have similar population sizes, one city's workforce is dominated by technicians, and it has much higher technicians than the sum of the other two cities. Shall we allocate more resources to the city populated with technicians? Or equally split resources among three cities? Based on our Bayesian network, we should allocate more resources to the city dominated by technicians as the subscription rate(around 40%) is higher than average(around 38%) in the assumed economic background.

In [ ]: