National University of Computer and Emerging Sciences

FAST School of Computing **Spring 2020** **Islamabad Campus**

CS-302: Design and Analysis of Algorithms

Thursday 25th of June

Course Instructors

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Remote Final Exam:
Attempt Time: 3 Hours
Submission (on LMS and email) Time: 15 min
Total Marks: 140

Instructions:

1. The final exam will be attempted offline in the student's own handwriting (in a readable way).
2. The students will use A4 size blank white sheets to attempt the exam (portrait format unless a diagram or table requires landscape). Each sheet of the A4 size paper **MUST** have the Roll Number, Name, the course code, name of the course and Signature of the student at the top of **EACH** sheet.
3. Students will use cam-scanner, MS lens, or an equivalent application to scan and convert their hand-written answer sheets into a **SINGLE** pdf file (keeping the correct order of pages and question numbers), which they will submit on LMS and **MUST also** email to the email address (of the concerned course/lab instructor) which will be provided. They will be given 15 minutes (after the 3 hours attempt time) for this purpose. All students must use the standard file name format (Full course code - Roll number e.g. CS-302-18i-0123). Submissions after 30 minutes may not be accepted. **Try to submit soon after 3 hours of attempt time and do not wait for 15 minutes to be elapsed.**
4. For proven cheating/ plagiarism, a student will get an F grade even if the student had opted for S/U grade, and the case will be referred to DDC (Department's Disciplinary Committee). Instructors will conduct vivas of randomly selected students or in case of doubt (significantly different attempt as compared to past performance in the course or matching attempt with other students). Plagiarism includes sharing an attempt to other students (copy providing). Students who are not able to satisfactorily answer instructor's questions (based on the exam as well as slightly lateral but related concepts) during viva will also be considered as plagiarism cases.
5. Students should carry a clean scanning that is free from any marks/stains etc.
6. Attempt all of the questions. Read the question carefully, understand the question, and then attempt it.
7. **Solutions without description or working will not carry any credit.**

	Q-1	Q-2	Q-3	Q-4	Q-5	Q-6	Q-7	Q-8	Total
Marks Obtained									
Total Marks	**16**	**15**	**12**	**20**	**30**	**15**	**6**	**26**	**140**

Question 1 [4 +12 marks][20 min]

A. Consider the following functions and sort them in the increasing order of growth.

10^n , $n^{1/3}$, 2^{2n}, n^{20}, lgn, $n!$, 2^{2^n}, $\sqrt{n}$

B. For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ milliseconds.

	1 sec	1 min	Number of minutes represented by the last 4 digits of your Roll No.	Number of hours represented by the last 4 digits of your Roll No.	Describe the process used for calculating last 2 columns
10^n					
2^{2n}					
n^{20}					
lgn					
$n!$					
2^{2^n}					

Question 2 [7.5+7.5marks][15 min]

Calculate the upper bound (O) of the following recurrence using a recursion tree.

A. $T(n) = 3T(n^{1/3}) + lgn$

B. $T(n) = T(n/2) + T(n/4) + n^2$

Question 3 [5+5+2 marks][15 min]

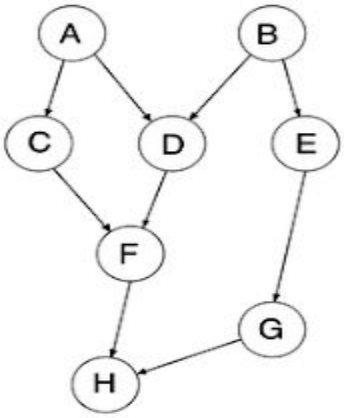
You are provided with two algorithms based on divide and conquer technique. You have to suggest the best algorithm to your programming team. Suppose you are choosing between the following two algorithms:

A. Algorithm X solves problems by dividing them into eight subproblems of half the size, recursively solving each subproblem, and then combining the solutions in constant time.
B. Algorithm Y solves problems of size n by dividing them into seven subproblems of size n/3, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time.

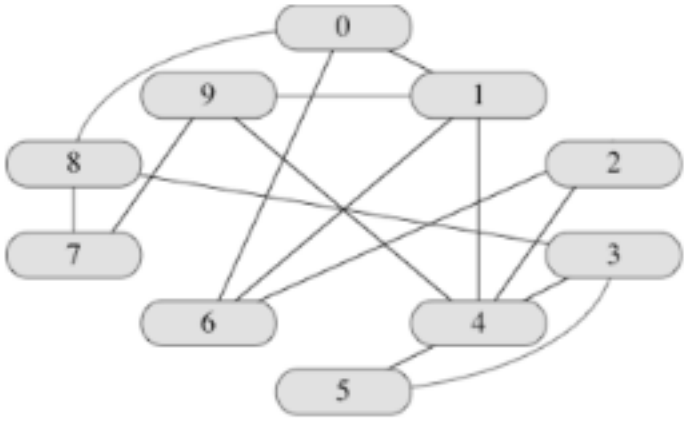
Write a recursive relation of each of the above algorithms. What are the running times of each of these algorithms (in asymptotic notation), and which would you choose? [Note: Solution with direct answer will not be acceptable]

Question 4 [10+10 marks][15 min]

A. Provide all topological sort solutions for the following graph. Mention the selected algorithm and show each step.

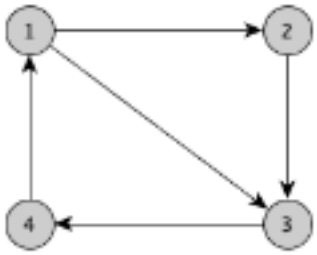


B. Provide all BFS traversal trees for the following graph starting with node number represented by the last digit of your roll number.

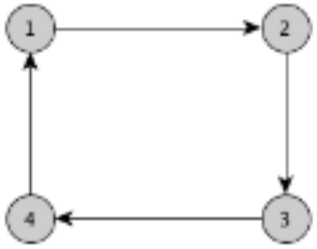


Question 5 [10+5+10+5 marks][30 min]

A. Provide an algorithm for printing number of cycles with frequency. The following graph has 1 cycle of length 4 and 1 cycle of length 3
B. Provide the dry run using the given directed graph with 5 edges.



C. Provide an algorithm to find all MSTs of a given graph.
D. Provide the dry run using the given directed graph with 4 edges



Question 6[15 marks][20 min]

We would like to make a telco tower deployment plan for Murree expressway. The government has identified the possible sites for the towers given by $s_1,s_2,s_3....s_n$. Also, it is required that two towers cannot be deployed within 25 KM of each other.

The length of the expressway is assumed to be K kilometers. So, all the towers are to be deployed in the interval [0, K]. Depending on the location of the tower, the telco can earn the revenue x_i for the tower is where $x_i>0$. We would like to deploy the towers at a subset of the sites while maximizing the revenue.

Example K=100 and n=4

$[s_1, x_1] = [16, 500]$

$[s_2, x_2] = [37, 600]$

$[s_3, x_3] = [62, 500]$

$[s_4, x_4] = [84, 100]$

The optimal placement would be to use sites s_1 and s_3 giving the revenue of 1000.

Question 7 [3+3marks][10 min]

For each of the following scenarios, you have to provide any suitable sorting algorithm that can reduce the expected running time. Provide detailed reasoning of your proposed solution .

A. You are running an Instagram account for your super popular business. You have many Instagram followers of your business. Now you want to give a special discount to your

early followers. For this purpose you decide to sort your followers by their ids, which are 32-bit integers.

B. You are organizing a product repository. You know that the products in your repository are almost in sorted descending order by product ids, with the exception of one product which is in the wrong place. You want the repository to be completely sorted in descending order.

Question 8 [3+6+3+7+2+5 marks][25 min]

Daraz.pk is an online shopping store. It has warehouses in almost all the major cities of Pakistan. As the trend of online shopping increases day by day in Pakistan, new cities are being added frequently in the list of warehouses. Daily thousands of products shipped from one city to another due to the large number of transactions. Product shipment management team decides the best shortest path to deliver their product from one city to another. Cities are connected through roads. They also have to check the conditions of the road before deciding the path for shipment. Some roads can damage their vehicles and the cost to repair the damage can reduce their profit. It is very hard to find the best shortest path daily considering all parameters manually. So now, we would like to suggest an appropriate algorithm that can provide a suitable shortest path among all the cities in their network. List of cities along with connections between the cities are provided to you. The cost of road connection can be positive or negative. Positive cost can increase the profit of the company while the negative cost can do the opposite.

You are required to provide a solution based on dynamic programming techniques. You are required to answer the following questions.

a. Identify optimal substructure property and explain it by example [3]
b. Identify overlapping subproblems property and explain it by example [3+2+1]
    1. Identify number of subproblems
    2. Identify time to solve each problem
c. Identify the recurrence relation derived from your solution. It must contain base case and inductive step → [3]
d. Provide the pseudocode of the proposed solution. [7]
e. Provide the running time of the proposed algorithm [2]
f. Provide one example to show the correctness of the proposed algorithm. You are only required to show the computation of the shortest path from any one city to another city. Don't provide examples for showing best solutions for all cities in the network. [5]