

# **AI 2002**

# **Artificial Intelligence**

Dr. Hashim Yasin

# Terminologies

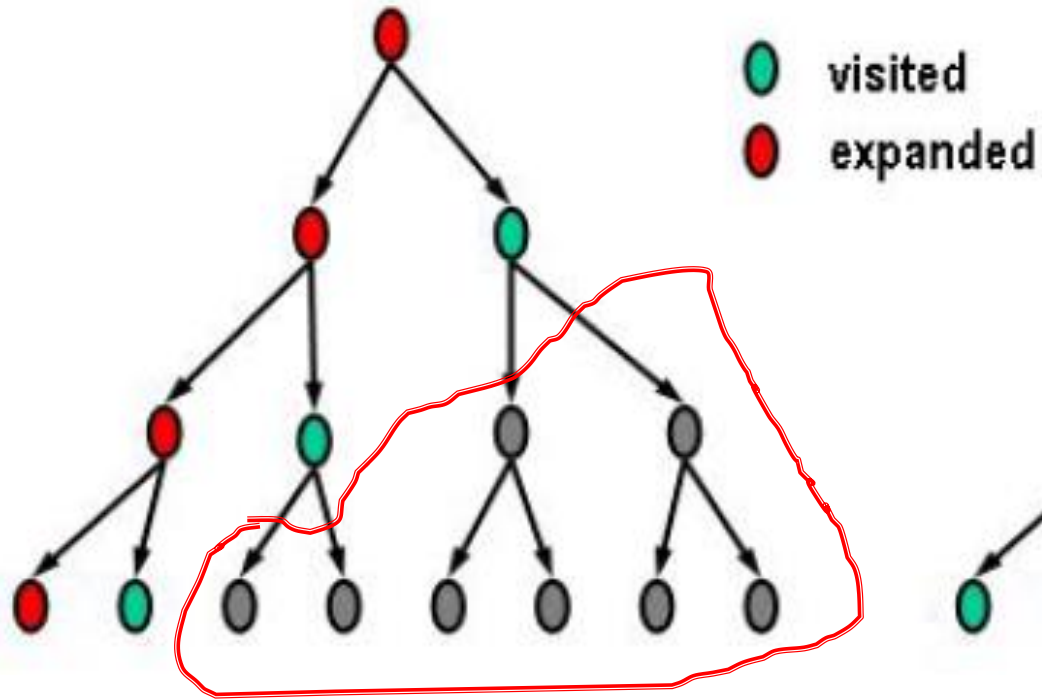
## Visited (Open) List:

- ▶ The *set of all leaf nodes available for expansion* at any given point is called the open list, (may be referred as **frontier**).
- ▶ In general, a state is said to be visited if it has ever shown up in search a node.
- ▶ The intuition is that *we have visited* them, but *we have not generated its descendants*.

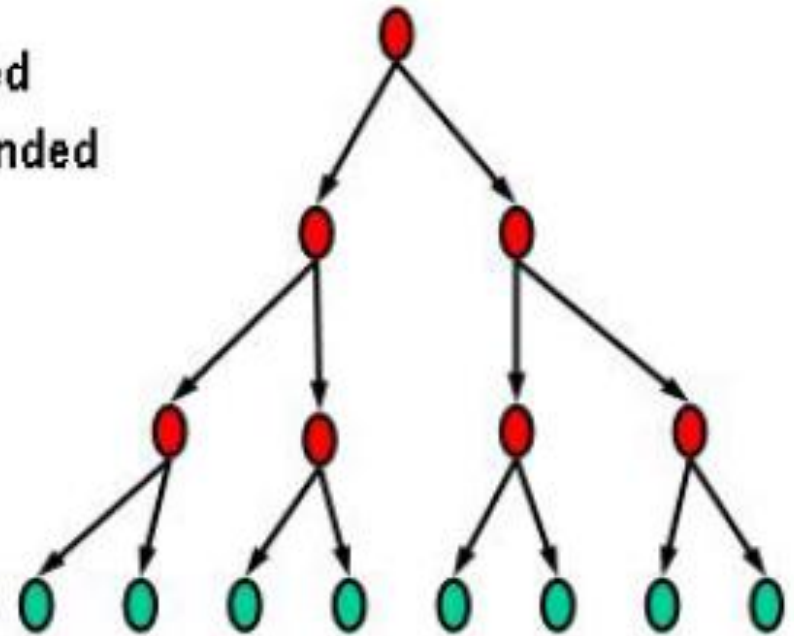
## Expanded (Closed, Explored) List:

- ▶ *Algorithms that forget their history are doomed to repeat it.*
- ▶ The way to avoid exploring redundant paths is to remember where one has been.
- ▶ To do this, we design **explored set** (also known as the **closed list**), which remembers every expanded node.

# Terminologies



# Depth First Search



## Breadth First Search

# Terminologies

Partial search trees for finding a route from Arad to Bucharest.

- ❑ Nodes that have been **visited but not yet expanded** are outlined in **bold**;
- ❑ Nodes that have been **expanded are shaded**;
- ❑ Nodes that have **not been visited** are shown in faint dashed lines.

(a) The initial state



(b) After expanding Arad



(c) After expanding Sibiu



# Uninformed Search

# Uninformed Search

- ▶ **Uninformed search** also called **blind search**
- ▶ The strategies have **no additional information** about the states beyond that provided in the problem definition.
- ▶ ***Use the information only provided*** in the problem definition.
  - ▶ Breadth-first search
  - ▶ Depth-first search
    - Depth-limited search
    - Iterative deepening search
  - ▶ Uniform cost search
  - ▶ Bidirectional search



# Breadth-First Search

# Breadth-First Search

## Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

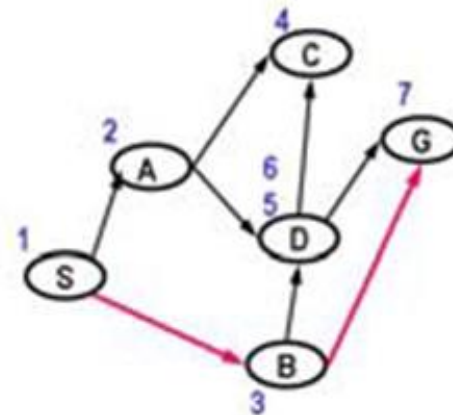
	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	(C A S) (D A S) (D B S) (G B S)*
5	(D A S) (D B S) (G B S)
6	(D B S) (G B S) (C D A S) (G D A S)
7	(G B S) (C D A S) (G D A S) (C D B S) (G D B S)

Added paths in blue

We show the paths in **reversed** order; the node's state is the first entry.

\* We could have stopped here, when the first path to the goal was visited

When we need the final path, we have to continue till the goal state is expanded.



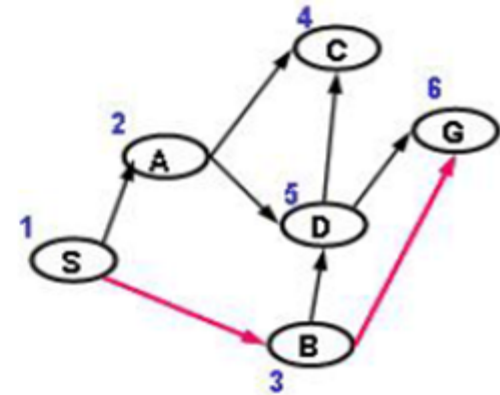


# Breadth-First Search

## Breadth-First

Pick first element of Q; Add path extensions to end of Q

	Q	Visited	Expanded
1	(S)	S	S
2	(A S) (B S)	A,B,S	A,S
3	(B S) (C A S) (D A S)	C,D,B,A,S	B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S	C,B,A,S
5	(D A S) (G B S)	G,C,D,B,A,S	D,C,B,A,S
6	(G B S)	G,C,D,B,A,S	G,D,C,B,A,S



Added paths in blue

We show the paths in **reversed** order; the node's state is the first entry.

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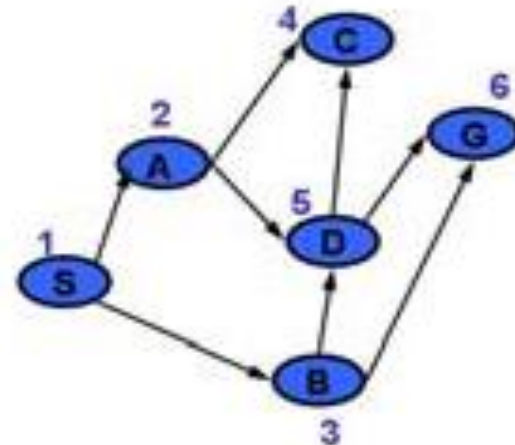
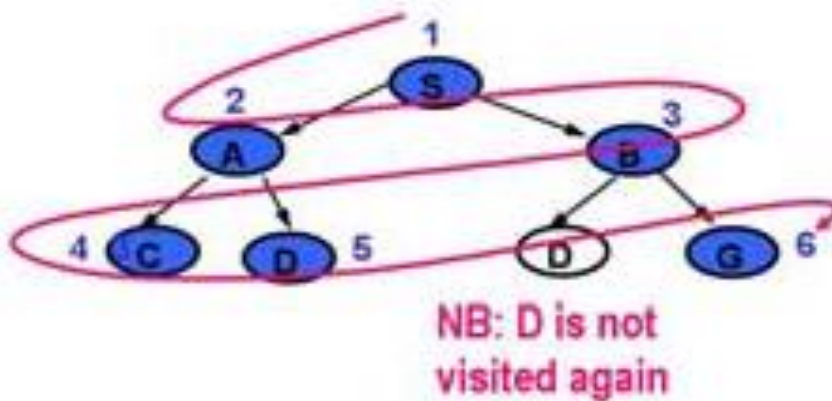
Tree Traversal = S, A, B, C, D, G

The Final path = S, B, G

# Breadth-First Search

## Breadth-First

Another (easier?) way to see it



Numbers indicate order pulled off of Q (expanded)

Dark blue fill = Visited & Expanded

Light gray fill = Visited

# Breadth-First Search

**Completeness:** Yes, (if  $b$  is finite)

**Time complexity:** Imagine *searching a uniform tree* where every state has  $b$  successors.

- The root of the search tree generates  $b$  nodes at the first level, each of which generates  $b$  more nodes,
- for a total of  $b^2$  at the second level, yielding  $b^3$  nodes at the third level, and so on. Now suppose that the solution is at depth  $d$

$$b + b^2 + b^3 + \dots + b^d = O(b^d)$$

- ▶ If the algorithm were to apply the goal test to nodes when selected for expansion, rather than when visited,
  - the whole layer of nodes at depth  $d$  would be expanded before the goal was detected and
  - the time complexity would be  $O(b^{d+1})$ .

# Breadth-First Search

**Space complexity:** For breadth-first graph search in particular, every node generated remains in memory.

- ▶ There will be  $O(b^{d-1})$  nodes in the explored set
- ▶  $O(b^d)$  nodes in the frontier,

**Optimality:** Yes, if the cost = 1 per step

*The memory requirement is a bigger problem for breadth-first search than the execution time.*

# Breadth-First Search

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	$10^6$	1.1 seconds	1 gigabyte
8	$10^8$	2 minutes	103 gigabytes
10	$10^{10}$	3 hours	10 terabytes
12	$10^{12}$	13 days	1 petabyte
14	$10^{14}$	3.5 years	99 petabytes
16	$10^{16}$	350 years	10 exabytes

Time and memory requirements for breadth-first search. The numbers shown assume branching factor  $b = 10$ ; 1 million nodes/second; 1000 bytes/node.

# Breadth-First Search

- ▶ The root node is expanded by **first-in-first-out (FIFO), Queue data structure.**
- ▶ **Complete:** find the solution eventually
- ▶ **Optimal:** if the step cost is 1

## Disadvantages:

- ▶ The branching factor of a node is large,
- ▶ The **space complexity and time complexity** are enormous for even small instances (e.g., chess)

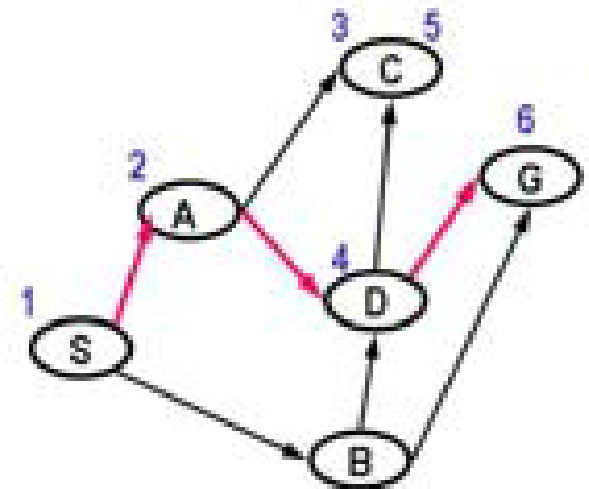


# Depth-First Search

# Depth-First Search (without-visited)

- ❑ Pick first element of Q
- ❑ Add path extensions **in front** to Q

	Q
1	(S)
2	<b>(A S) (B S)</b>
3	<b>(C A S) (D A S)</b> (B S)
4	(D A S) (B S)
5	<b>(C D A S) (G D A S)</b> (B S)
6	<b>(G D A S)</b> (B S)

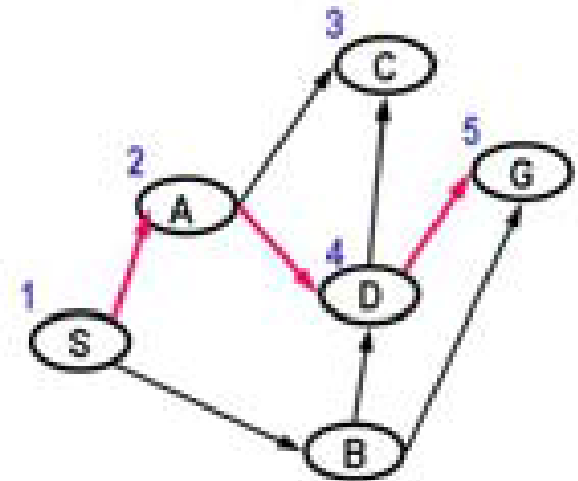


- ❑ **Blue Color represents added paths**
- ❑ Path has been shown in **reversed order**, node state is the first entry.
  - ❑ Tree Traversal = S, A, C, D, C, G
  - ❑ The Final path = S, A, D, G

# Depth-First Search

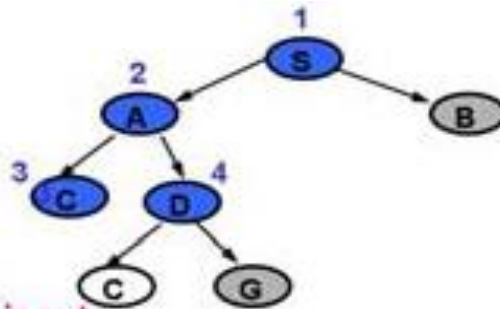
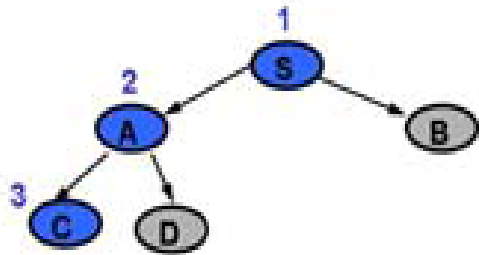
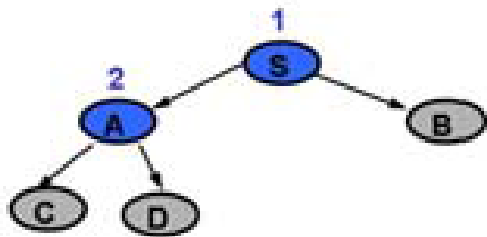
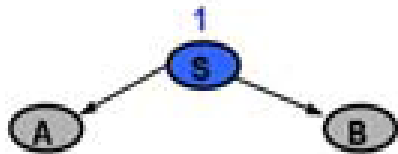
- ❑ Pick first element of Q
- ❑ Add path extensions **in front** to Q

	Q	Visited	Expanded
1	(S)	S	S
2	<b>(A S) (B S)</b>	A, B, S	A, S
3	<b>(C A S) (D A S)</b> (B S)	C, D, B, A, S	C, A, S
4	(D A S) (B S)	C, D, B, A, S	D, C, A, S
5	<b>(G D A S)</b> (B S)	G, C, D, B, A, S	G, D, C, A, S

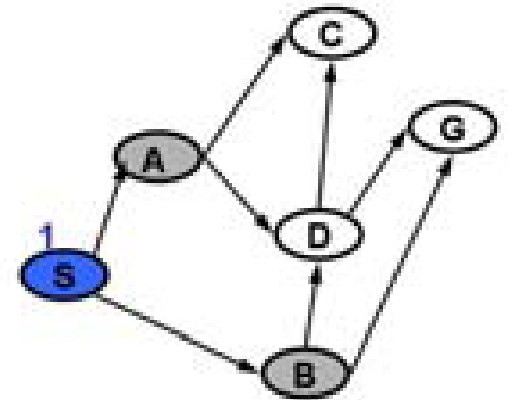
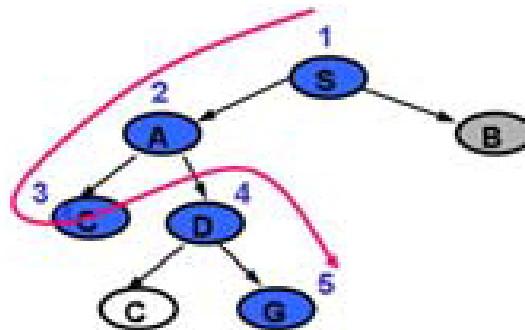


- ❑ **Blue Color represents added paths**
- ❑ Path has been shown in **reversed order**, node state is the first entry.
- ❑ **Tree Traversal = S, A, C, D, G**
- ❑ **The Final path = S, A, D, G**

# Depth-First Search



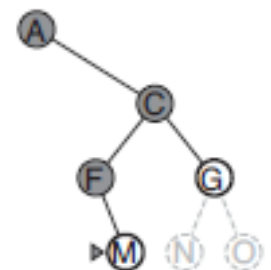
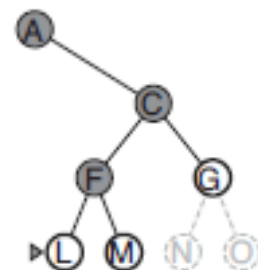
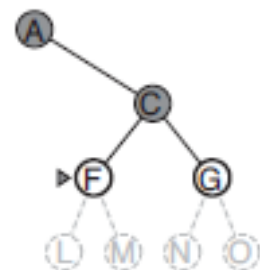
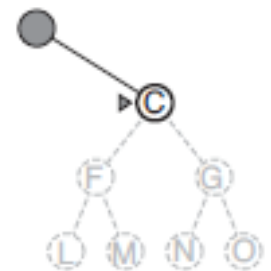
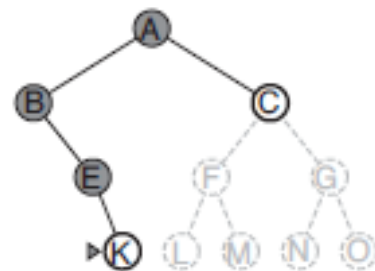
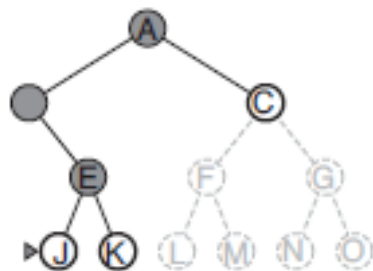
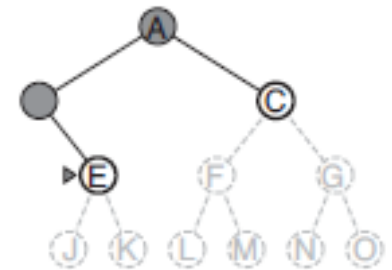
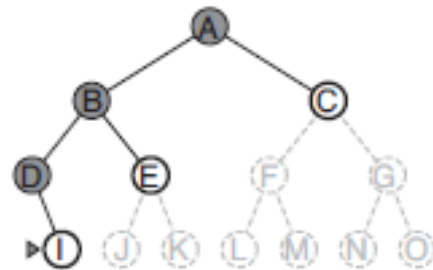
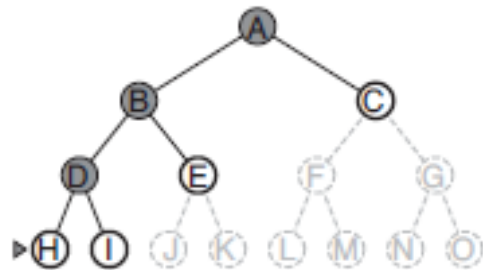
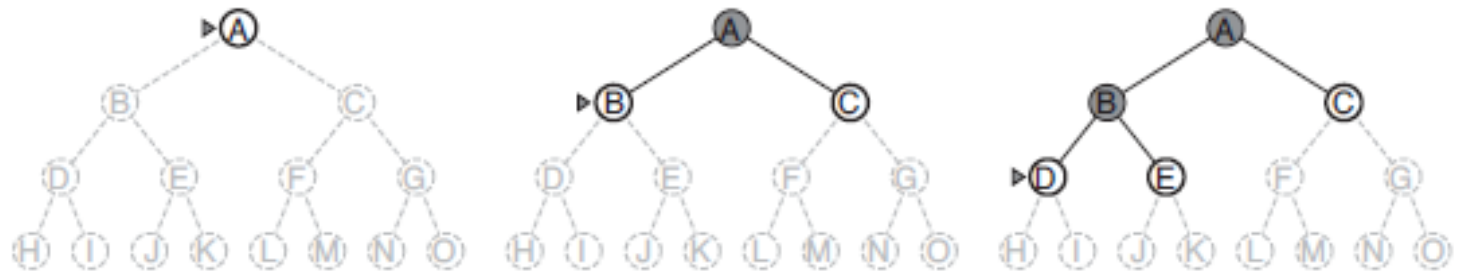
NB: C is not  
visited again



Numbers indicate order pulled off of Q (expanded)

Dark blue fill = Visited and Expanded

Light gray fill = Visited



## DFS Example 2

# Depth-First Search

- ▶ Depth-first search always expands the ***deepest node***
- ▶ Implemented with a **last-in-first-out (LIFO)** strategy, also known as a **stack**.
- ▶ As an alternative to the ***Graph-Search-style*** implementation, depth-first search is implemented with a **recursive function** that calls itself on each of its children in turn.



# Depth-First Search

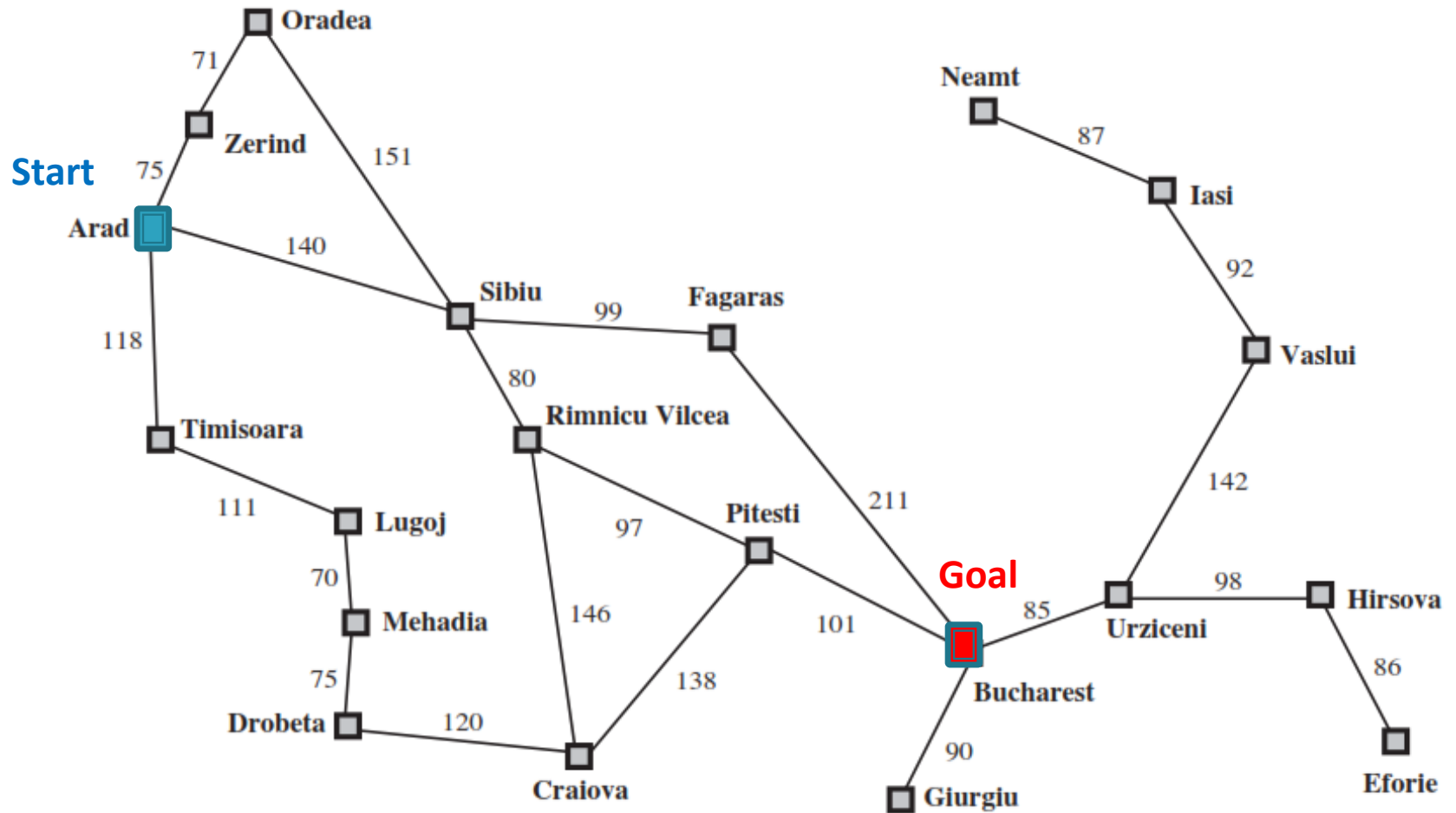
## Completeness:

- ▶ The graph-search version, which *avoids repeated states and redundant paths*, is **complete** in finite state spaces because it will eventually expand every node.
- ▶ The tree-search version, on the other hand, it is **not complete**.

## Optimality:

- ▶ It doesn't guarantee the best solution.

# Depth-First Search



A simplified road map of part of Romania.

(a) The initial state



(b) After expanding Arad



(c) After expanding Sibiu



# Depth-First Search

It overcomes **time and space** complexities.

## Time:

- ▶  $O(b^m)$ : terrible if  $m$  (maximum length of the depth) is much larger than the size of the state space.
- ▶ *If solutions are dense, may be much faster than breadth-first.*

## Space:

- ▶  $O(bm)$ , i.e., linear space!
- ▶ A variant of depth-first search with **backtracking search** uses still less memory  $O(m)$ .

# Comparison with BFS

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	$10^6$	1.1 seconds	1 gigabyte
8	$10^8$	2 minutes	103 gigabytes
10	$10^{10}$	3 hours	10 terabytes
12	$10^{12}$	13 days	1 petabyte
14	$10^{14}$	3.5 years	99 petabytes
16	$10^{16}$	350 years	10 exabytes

Time and memory requirements for breadth-first search. The numbers shown assume branching factor  $b = 10$ ; 1 million nodes/second; 1000 bytes/node.

The depth-first search would require **156 kilobytes** instead of 10 Exabyte at depth  $d = 16$ , a factor of **7 trillion times less space**.

# Depth-limited Search



# Depth-limited Search

- ▶ The problem of depth first search can be alleviated by supplying depth-first search with a **pre-determined depth limit  $l$** .

$$l < d$$

- ▶ Nodes at depth  $l$  are treated as if they have no successors.

## Completeness:

- ▶ It may be incomplete if we choose  $l < d$ , that is, the shallowest **goal is beyond the depth limit  $l$** .

## Optimality:

- ▶ The depth-limited search may be non-optimal if we choose  $l > d$ .

# Depth-limited Search

Time complexity:  $O(b^l)$

Space complexity:  $O(bl)$

- ▶ *Depth-first search can be viewed as a special case of depth-limited search with*

$$l = \infty$$

- ▶ Depth limited search can be based on knowledge of the problem.
  - For the most problems, however, **we will not know a good depth limit until** we have solved the problem.

# Iterative Deepening Search

# Iterative Deepening Search

- ▶ No selection of the best depth limit
- ▶ It tries all possible depth limits:
  - first 0, then 1, 2, and so on
- ▶ Combines the benefits of depth-first and breadth-first search,

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution
  inputs: problem, a problem
  for depth ← 0 to  $\infty$  do
    result ← DEPTH-LIMITED-SEARCH(problem, depth)
    if result  $\neq$  cutoff then return result
  end
```

The cutoff value indicates that there is no any solution within the depth limit.

# Iterative Deepening Search

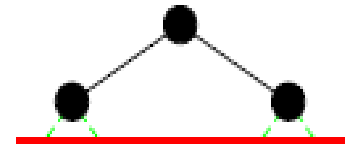
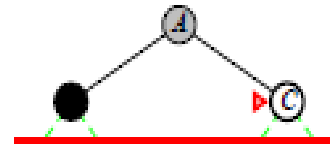
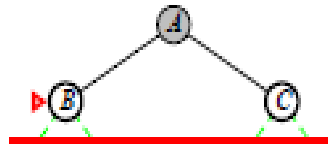
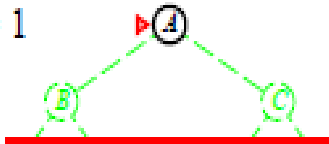
►  $\ell = 0$ :

Limit = 0



►  $\ell = 1$ :

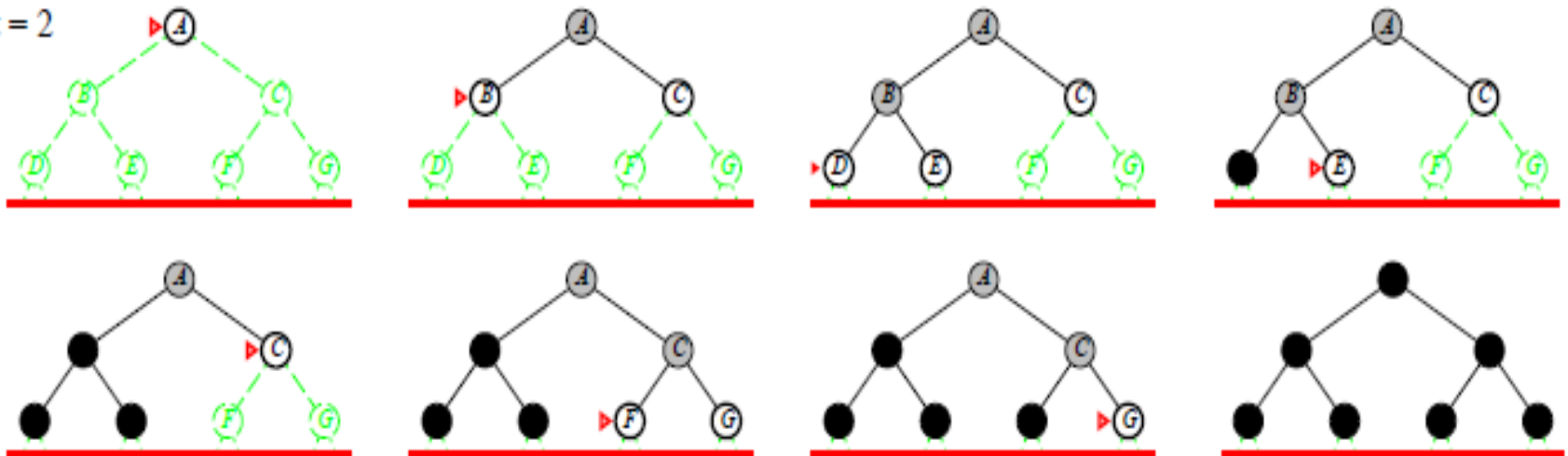
Limit = 1



# Iterative Deepening Search

►  $\ell = 2$ :

Limit = 2

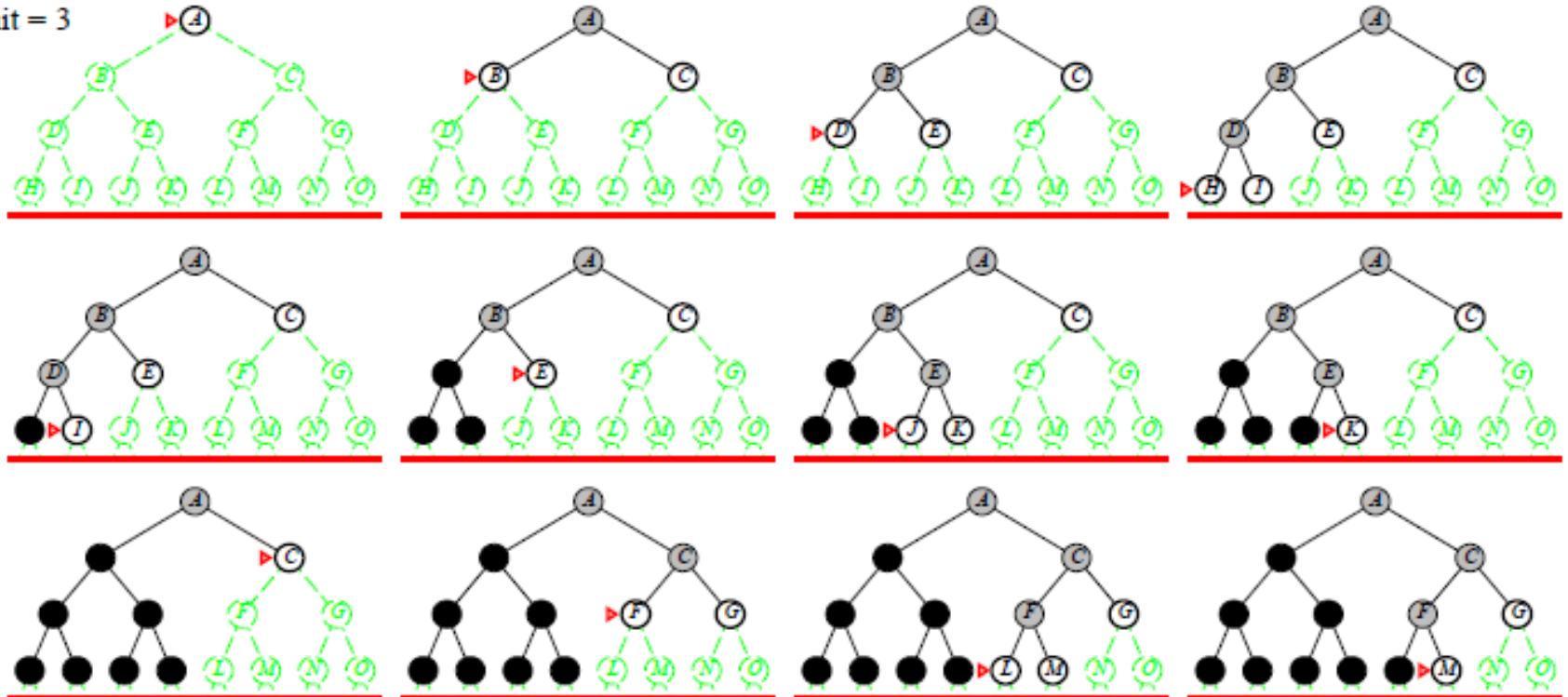




# Iterative Deepening Search

►  $\ell = 3$ :

Limit = 3



# Iterative Deepening Search

Complete?? Yes if ***b*** is finite

Time??  $(d)b + (d-1)b^2 + \dots + (1)b^d = O(b^d)$

Space??  $O(bd)$

Optimal?? Yes, if step cost = 1

Can be modified to explore uniform-cost tree

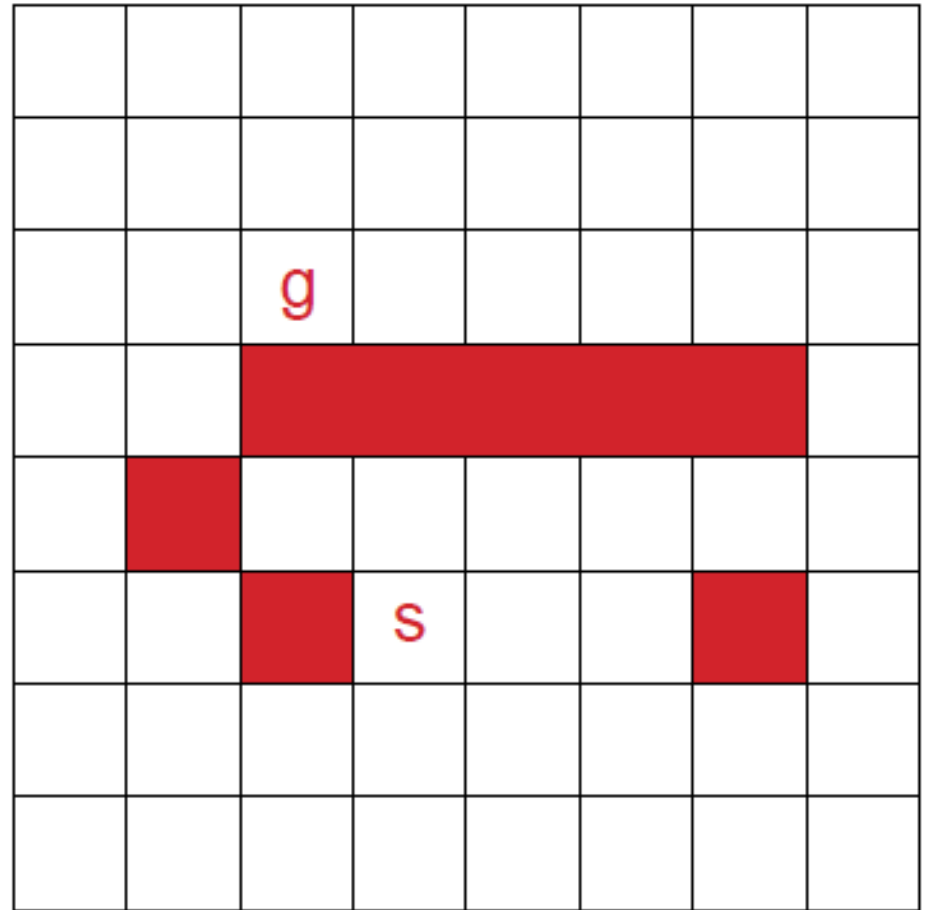
## IDS suitable for the problem

- having a large search space
- and the *depth of the solution is not known*

# Real Life Example

# Example ... DFS

- ❑ Finding a path in the given grid using DFS.
- ❑ The order of the actions are **up, left, right, then down**.
- ❑ Maintain the visited list to avoid looping.
- ❑ Number the square according to the traversal.



# Reading Material

- ▶ **Artificial Intelligence, A Modern Approach**  
**Stuart J. Russell and Peter Norvig**
  - Chapter 3.