18F-0137
18F-0326

Q:1÷

Consider the following sentence

$[(Food \Rightarrow Party) \lor (Drinks \Rightarrow Party)] \Rightarrow [(Food \land Drinks) \Rightarrow Party]$

a) Determine, using enumeration whether this sentence is valid, satisfiable or unsatisfiable.

Food: F   Party: P   Drink: D

Sol÷

F	P	D	$F \Rightarrow P$	$D \Rightarrow P$	$(F \Rightarrow P) \lor (D \Rightarrow P)$	$(F \land D) \Rightarrow P$	$[(F \Rightarrow P) \lor (D \Rightarrow P)] \Rightarrow [(F \land D) \Rightarrow P]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	T	T	T	T

Result÷

As, with enumeration of sentence we found = Tautology

So, the given sentence is valid.

b)

Sol: Convert main Implication into CNF:

$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$

$[(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \Rightarrow [(\neg(Food \wedge Drinks)) \vee Party]$

$[(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \Rightarrow [(\neg Food \vee \neg Drinks) \vee Party]$

$[(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \Rightarrow [(\neg Food \vee Party) \vee (\neg Drinks \vee Party)]$

Result:
As, the side before implication and the side after implication are equal. So, the given sentence is valid, which contains answer of Part (a).

c) Prove (a) using Resolution:

CNF

$[F \vee (\sim F \vee \sim D \vee P)] \wedge [P \vee (\sim F \vee \sim D \vee P)] \wedge [D \vee (\sim F \vee \sim D \vee P)]$

Proof:

As, $F \rightarrow T, D \rightarrow T, P \rightarrow T, \sim F \rightarrow F, \sim D \rightarrow F, \sim P \rightarrow F$

$\Rightarrow [F \vee (\sim F \vee \sim D \vee P)] \wedge [P \vee (\sim F \vee \sim D \vee P)] \wedge [D \vee (\sim F \vee \sim D \vee P)]$

$\Rightarrow [T \vee (F \vee F \vee T)] \wedge [T \vee (F \vee F \vee F)] \wedge [T \vee (F \vee F \vee T)]$

$\Rightarrow [T \vee T] \wedge [T \vee F] \wedge [T \vee T]$

$\Rightarrow T \wedge T \wedge T$

$\Rightarrow T$

Hence, given prediction is valid.

Scanned with CamScanner

18F-0326
18F-0137

Q.2:-

Decide if following sentences are valid, unsatisfiable or neither

a) Smoke ⇒ Smoke

Elimination of Implication

Smoke ⇒ Smoke = ¬Smoke ∨ Smoke

Smoke	¬Smoke	Smoke ⇒ Smoke	¬Smoke ∨ Smoke
T	F	T	T
F	T	T	T

Smoke ⇒ Smoke is "Valid" preposition.

b) Smoke ⇒ Fire

Smoke ⇒ Fire = ¬Smoke ∨ Fire

Smoke	Fire	¬Smoke	Smoke ⇒ Fire	¬Smoke ∨ Fire
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Smoke ⇒ Fire is "Neither" valid nor unsatisfiable.

c) (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

Smoke	Fire	¬Smoke	¬Fire	Smoke ⇒ Fire	¬Smoke ⇒ ¬Fire	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Given preposition is "Neither valid nor unsatifiable."

d) Smoke ∨ Fire ∨ ¬Fire

∵ Smoke = S
Heat = H
Fire = F

S	F	¬F	S∨F∨¬F
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	T

So, as Tautology exists then the preposition is valid.

e) ((Smoke ∧ Heat) => Fire) <=> ((Smoke => Fire) ∨ (Heat => Fire))

S	H	F	S∧H	(S∧H)=>F	S=>F	H=>F	(S=>F) ∨ (H=>F)	[((S∧H)=>F)] <=> [(S=>F) ∨ (H=>F)]
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Result: As, we found Tautology on Last coloumn. So, preposition is "valid".

18F-0326
18F-0137

f)

S	F	H	$S \Rightarrow F$	$S \wedge H$	$(S \wedge H) \Rightarrow F$	$[(S \Rightarrow F)] \Rightarrow [(S \wedge H) \Rightarrow F]$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Result:-
Thus, this preposition is "Valid" due to Tautology.

g) Big ∨ Dumb ∨ (Big ⟹ Dumb)

Big	Dumb	Big ⟹ Dumb	Big ∨ Dumb ∨ (Big ⟹ Dumb)
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Result:-
Thus, the preposition is "Valid" due to Tautology.

Scanned with CamScanner

18F-0326
18F-0137

Q.3:

(a) A, B, C, D

A	B	C	D	B∨C	
T	T	T	T	T	✓
T	T	T	F	T	✓
T	T	F	T	T	✓
T	T	F	F	T	✓
T	F	T	T	T	✓
T	F	T	F	T	✓
T	F	F	T	F	
T	F	F	F	F	
F	T	T	T	T	✓
F	T	T	F	T	✓
F	T	F	T	T	✓
F	T	F	F	T	✓
F	F	T	T	T	✓
F	F	T	F	T	✓
F	F	F	T	F	
F	F	F	F	F	

(b) $\sim A \vee \sim B \vee \sim C \vee \sim D$

A	B	C	D	$\sim A$	$\sim B$	$\sim C$	$\sim D$	$\sim A \vee \sim B \vee \sim C \vee \sim D$
T	T	T	T	F	F	F	F	F
T	T	T	F	F	F	F	T	T
T	T	F	T	F	F	T	F	T
T	T	F	F	F	F	T	T	T
T	F	T	T	F	T	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	T	F	T	T	F	T
T	F	F	F	F	T	T	T	T
F	T	T	T	T	F	F	F	T
F	T	T	F	T	F	F	T	T
F	T	F	T	T	F	T	F	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	T	F	F	T
F	F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	F	T
F	F	F	F	T	T	T	T	T

18F-0137

18F-0328

(c)

A	B	C	D	~B	A→B	(A→B) ∧ A ∧ ~B ∧ C ∧ D
T	T	T	T	F	T	F
T	T	T	F	F	T	F
T	T	F	T	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	F	F
T	F	T	F	T	F	F
T	F	F	T	T	F	F
T	F	F	F	T	F	F
F	T	T	T	F	F	F
F	T	T	F	F	F	F
F	T	F	T	F	F	F
F	T	F	F	F	F	F
F	F	T	T	T	T	F
F	F	T	F	T	T	F
F	F	F	T	T	T	F
F	F	F	F	T	T	F

No model which gives tr

18F-0187
18F-0328

QNO4:

Q_1 = "There is no pit in [2,2]"

Q_2 = "There is a wampus in [1,3]"

concerned block [1,3], [2,2], [3,1]

following are the Possible in concerned block.

P → Pit

W → wampus.

UB ⇒ stench in [1,2] → W in [1,3] ∨ [2,2]

Breez in [2,1] → P in [2,2], [3,1] or both.

		[1,3]	[2,2]	[1,1]
1	P	1	1	1
	W	0	0	1
2	P	1	1	1
	W	0	1	0
3	P	1	1	1
	W	1	0	0
4	P	0	1	0
	W	0	0	1
5	P	0	0	0
	W	0	1	0
6	P	0	0	0
	W	1	0	0
7	P	0	0	1
	W	0	0	1
8	P	0	0	1
	W	0	1	0
9	P	0	0	1
	W	1	0	0
10	P	0	1	0
	W	0	0	1
11	P	0	1	0
	W	0	1	0
12	P	0	1	0
	W	1	0	0
13	P	1	0	0
	W	0	0	1
14	P	1	0	0
	W	0	1	0
15	P	1	0	0
	W	1	0	0
16	P	1	1	0
	W	0	0	1
17	P	1	1	0
	W	0	1	0
18	P	1	1	0
	W	1	0	0
19	P	0	1	1
	W	0	0	1
20	P	0	1	1
	W	0	1	0
21	P	0	1	1
	W	1	0	0
22	P	1	0	1
	W	0	0	1

		[1,3]	[2,2]	[1,1]
23	P	1	0	0
	W	0	1	0
24	P	1	0	0
	W	1	0	0
25	P	1	1	1
	W	0	0	0
26	P	0	0	0
	W	0	0	0
27	P	1	0	0
	W	0	0	0
28	P	0	1	0
	W	0	0	0
29	P	0	1	1
	W	0	0	0
30	P	1	0	1
	W	0	0	0
31	P	1	1	0
	W	0	0	1
32	P	0	0	0
	W	0	0	0

world in which a_1 is ~~true~~ true : 4, 5, 6, 7, 8, 9, 13, 14, 15, 22, 23, 24, 26, 27, 30, 32

world in which a_2 is true : 3, 6, 9, 12, 15, 18, 21, 24

world in which KB is true : 8, 9, 12, 21

Proof

18F-0386
19F-0137

Wampus World.

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3 : B_{2,1} \leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

→ Apply the inference rule and derive the proof step by step that $\neg P_{1,2}$.

$\neg P_{1,2}$ (there is no Pit in [1,2])

Proof:

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ (as R_2 has $P_{1,2}$)

$R_6 : B_{1,1} \longrightarrow (P_{1,2} \vee (P_{2,1})) \wedge (P_{1,2} \vee P_{2,1})$ ⟹ {Bi, Bidirectional Elimination}

$R_7 : (P_{1,2} \vee P_{2,1}) \longrightarrow B_{1,1}$ (and elimenation)

$R_8 : \neg B_{1,1} \longrightarrow \neg(P_{1,2} \vee P_{2,1})$ {logical equalence of contrapositive)

$R_9 : \neg(P_{1,2} \vee P_{2,1})$ (Percep by R_4)

$R_{10} : (\neg P_{1,2} \wedge \neg P_{2,1})$ (Demorgens law)

Hence

$\neg P_{1,2}$ is Proved Neither [1,2] nor [2,1] contain or is a pit.

Q) Apply resolution theorem & Prove $\neg P_{1,2}$

Proof:

R_2: $B_{1,1} \iff (P_{1,2} \vee P_{2,1})$ (R_2 has $P_{1,2}$)

Convert to CNF:

$(B_{1,1} \rightarrow (P_{1,2} \vee P_{2,1}) \wedge ((P_{1,2} \vee P_{2,1}) \rightarrow B_{1,1})$ (Biconditional)

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee (B_{1,1}))$ (implication)

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$ (Demorgans)

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$ Distribution

Knowledge Base:

$KB = R_2 \wedge P_1$

$KB = (\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{1,1} \vee B_{1,1}) \wedge (\neg B_{1,1})$

$\alpha = \neg P_{1,2}$

$\neg \alpha = P_{1,1}$

$KB \wedge \neg \alpha = (\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1}) \wedge \neg B_{1,1} \wedge P_{1,2}$

$(\neg B_{1,1} \vee P_{1,2} \vee P_{1,1})$ $(\neg P_{1,2} \vee B_{1,1})$ $(\neg P_{2,1} \vee B_{1,1})$ $(\neg B_{1,1})$ $(P_{1,2})$

$(\neg P_{2,1})$ $(\neg P_{1,2})$ $(P_{1,2} \vee P_{2,1} \vee \neg P_{2,1})$ $(P_{1,2} \vee P_{2,1} \vee \neg P_{2,1})$ $(\neg B_{1,1} \vee P_{1,2} \vee B_{1,1})$ $(\neg B_{1,1} \vee P_{2,1} \vee B_{1,1})$

Hence $P \vee \neg P$ is found which is unsatisfiable nither valid nor satisfying) query is entailed.

Scanned with CamScanner