CS 461
Artificial Intelligence

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First Order Logic

Propositional Logic

- Propositional logic is a **declarative language.**
  - its *semantics* is based on a ***truth relation*** ***between sentences and possible worlds***.
- Propositional logic **allows partial information using disjunction & negation** (unlike most data structures and databases).
- Propositional logic has a third property that is **compositionality**. Propositional logic is compositional, i.e., *the meaning of a sentence is a function of the meaning of its parts*.
  - For example: The meaning of $\boldsymbol{B_{1,1}} \wedge \boldsymbol{P_{1,2}}$ is derived from the meaning of $\boldsymbol{B_{1,1}}$ and of $\boldsymbol{P_{1,2}}$

Propositional Logic

- Meaning in propositional logic is **context-independent** (unlike natural language, where meaning depends on context).
- Propositional logic has very **limited expressive power** (unlike natural language)

For example: cannot say

"Pits cause Breezes in adjacent squares".

we have to write a separate rule about breezes and pits for **EACH** square.

First-order Logic

- The propositional logic assumes world contains ***facts*** while,
- The **first-order logic** (like natural language) assumes the world contains, ***objects***, ***relations*** and ***functions***.

Objects:

- The nouns and noun phrases refer to **objects**
  - *In wumpus-world*, the object examples are (squares, pits, wumpuses)
  - The people, houses, numbers, theories, baseball games, wars, centuries, etc.

First-order Logic

Relations:

- The verbs and verb phrases refer to **relations**
  - *In wumpus-world*, the relation examples are (is breezy, is adjacent to, shoots)
  - red, round, bogus, prime, brother of, bigger than, inside, part of, has colour, occurred after, owns, etc.

Functions:

- Some of these relations are **functions**—*relations in which there is only **ONE** "value" for a given "input"*.
  - father of, best friend, third inning of, one more than, end of

FOL Motivation

- The statements that cannot be made in propositional logics but can be expressed with FOL.
- First-order logic can also express facts about ***some* or *all*** of the objects in the universe.

1. When you paint a block with green paint, it becomes green.
   - In proposition logic, one would need a statement about **every single block** … for every single aspect of the situation, ***"if this block is black and I paint it, it becomes green"*** and ***"if block # 5 is red and I paint it, it becomes green"***
2. When you sterilize the jar, all the bacteria are dead.
   - In FOL, we can talk about all the bacteria without naming them explicitly.

Logics

Logics

Ontological commitment

- what exists in world

- what it assumes about the ***nature of reality (facts)***.

- Mathematically, this commitment is expressed through the ***nature of the formal models*** with respect to which the *truth of sentences is defined*.
  - For example, propositional logic assumes that there are facts that either hold or do not hold in the world.

Logics

Epistemological commitment

- the ***possible states of knowledge*** that it allows with respect to each fact.

- In both propositional and first order logic, a sentence represents a fact and the agent either believes the sentence to be true or false, or has no opinion.

- Thus the possible values are:

true/false/unknown

Types of Logic

Temporal logic

- assumes that **facts hold at particular *times*** and
- those times (which may be points or intervals) are ordered.

Probability theory

- Systems using **probability theory** can have any ***degree of belief***, ranging from 0 (total disbelief) to 1 (total belief).
  - For example, a probabilistic wumpus-world agent might believe that the wumpus is in [1,3] with probability 0.75

Types of Logic

Fuzzy logic

Fuzzy logic have a **degree of truth** between 0 and 1.

For example, the sentence **"Vienna is a large city"** might be true in our world only to degree 0.6 in fuzzy logic.

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

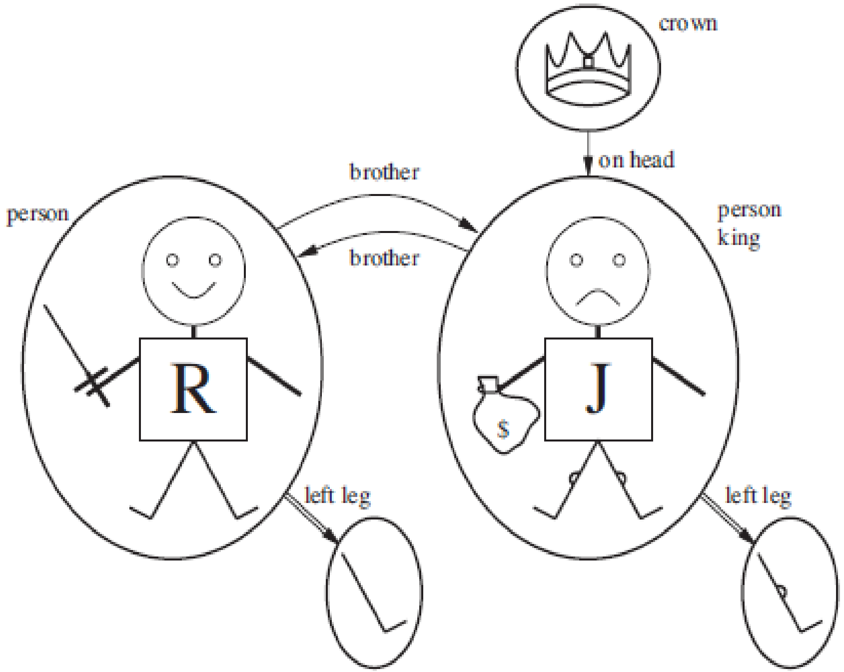
First-order Logic Models

Models for First-order Logic

- Models for first-order logic have **objects** in them.

Domain

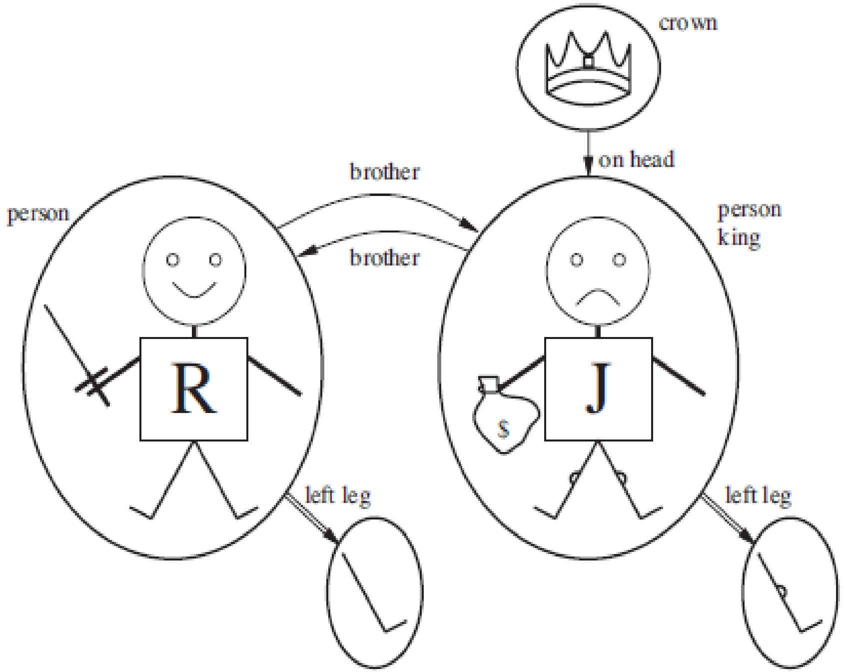
- The **domain** of a model is the ***set of objects*** or ***domain elements*** it contains.
- The domain is required to be ***nonempty***—*every possible world must contain at least one object.*



The first-order logic assumes the world contains, ***objects***, ***relations*** and ***functions***.

Models for First-order Logic

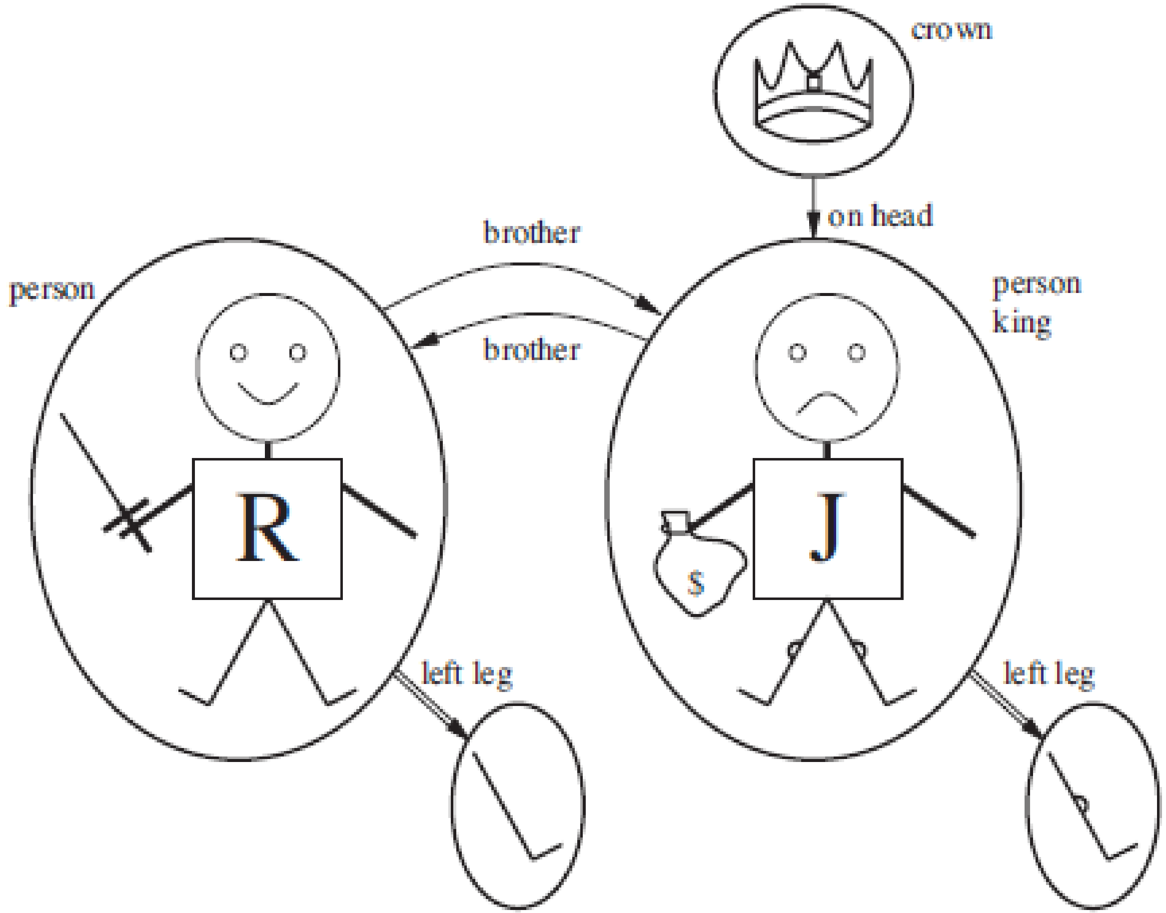
- Mathematically speaking, it doesn't matter ***what*** *these objects are,*
- ***all that matters is <u>how many</u> there are in each particular model.***



Example

<u>A model contains</u>

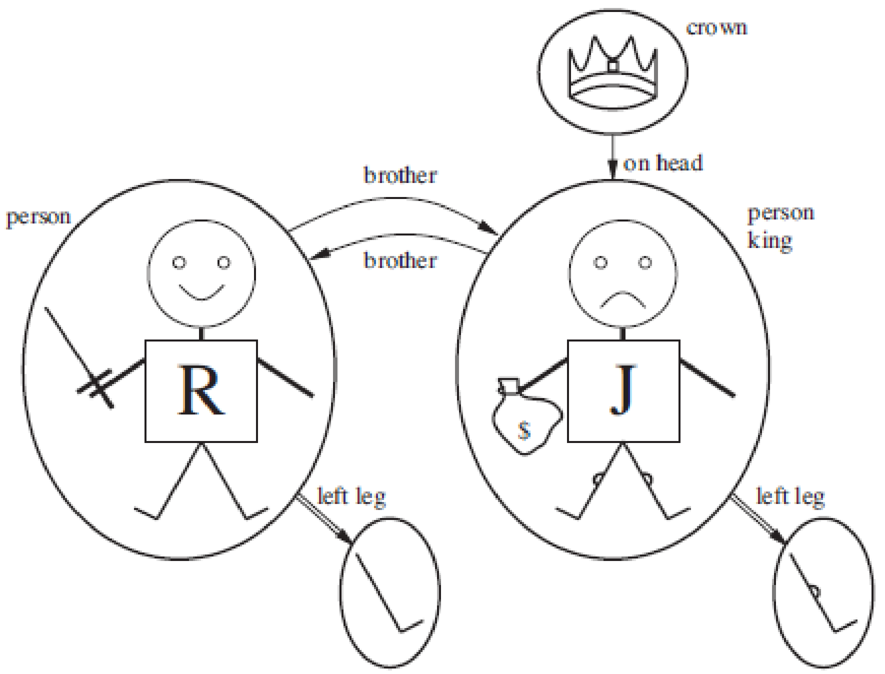
- Five objects,
- Two binary relations,
  - "brother"
  - "on head"
- Three unary relations,
  - "person"
  - "king"
  - "crown"
- One unary function,
  - "left leg"



Models for First-order Logic

Tuple:

- A tuple is a **collection of objects** arranged in a fixed order and is written with angle brackets surrounding the objects.

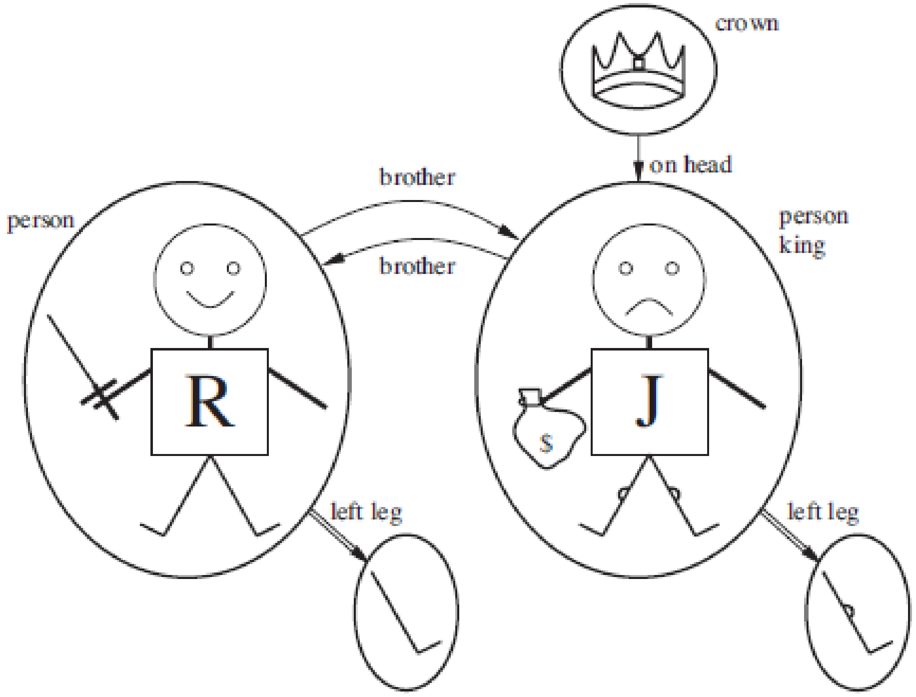


Models for First-order Logic

Tuple Example:

- The ***"brotherhood"*** relation in the model" is the set: **{<Richard the Lionheart, King John>, <King John, Richard the Lionheart>}.**
- The crown is on King John's head, so the ***"on head"*** relation contains just one tuple,

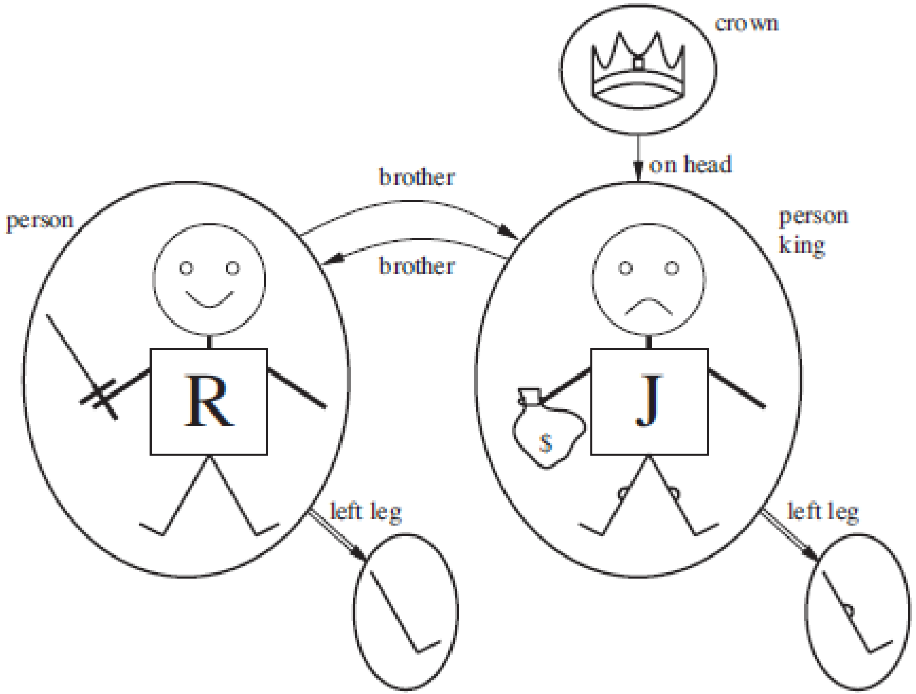
  <the crown, King John>.



Models for First-order Logic

Example:

- The **"brother"** and **"on head"** relations are ***binary relations***.
- The model also contains ***unary relations***, or ***properties***:
  - The **"person"** property is true of both Richard and John;
  - The **"king"** property is true only of John,
  - The **"crown"** property is true only of the crown



FOL Symbols and Interpretations

FOL Symbols and Interpretations

Symbol:

- The basic syntactic elements of first-order logic are the symbols that *stand for objects, relations, and functions*
- The symbols will begin with UPPERCASE letters.
- The symbols come in three kinds:

1. **constant symbols** --- which stand for objects, like Richard and John
2. **predicate symbols** --- which stand for relations, like Brother , OnHead, Person, King, and Crown
3. **function symbols** --- which stand for functions, like LeftLeg

Syntax of FOL: Basic Elements

Constants	$KingJohn,\ 2,$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\wedge\ \ \vee\ \ \neg\ \ \Rightarrow\ \ \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

FOL Symbols and Interpretations

- **Interpretation** specifies exactly which objects, relations and functions are referred to by the constant, predicate, and function symbols.

Arity:

- Each predicate and function symbol comes with an arity that ***fixes the number of arguments***.

FOL Symbols and Interpretations

Term:

- A term is a logical expression that refers to an object.
- *A term may contain:*
  1. **Constant symbols**: Fred, Japan, Becterium 39
  2. **Variables:** a,b, x
  3. **Functional symbol** applied to one or more terms. F(x), Mother-of(John)

- A term with no variables is called a **ground term**.

FOL Sentences

Sentence

1. A **predicate symbol** may be applied to terms. **On(a, b), Sister(Jane, John), Sister(Mother-of(Jane), Jen)**
2. $\boldsymbol{term_1 = term_2}$
3. A **functional symbol** may be applied to one or more terms. **F(x)**, **Mother-of(John)**.
4. If $\boldsymbol{v}$ is a variable and $\boldsymbol{S}$ is a sentence, then
   - $(\forall \boldsymbol{v}\ \boldsymbol{S})$ and $(\exists \boldsymbol{v}\ \boldsymbol{S})$ are sentences too.

FOL Sentences

Atomic sentence

- (or atom for short) is formed from a predicate symbol optionally followed by a parenthesized list of terms, such as

Brother (Richard , John)

- Atomic sentences can have **complex terms** as arguments.

Married(Father (Richard), Mother (John))

FOL Sentences

Complex sentence

- We can use **logical connectives** to construct more complex sentences, with the same syntax and semantics as in propositional calculus.

Example

- There are four sentences,

$$\neg Brother(LeftLeg(Richard), John)$$
$$Brother(Richard, John) \wedge Brother(John, Richard)$$
$$King(Richard) \vee King(John)$$
$$\neg King(Richard) \Rightarrow King(John) .$$

Quantifiers

Quantifiers - Universal Quantifier (∀)

- **"All kings are persons"** is written in first-order logic as,

$$\forall x \;\; King(x) \Rightarrow Person(x)$$

- **∀** is usually pronounced **"For all . . ."**
- *Intuitively, the sentence* **∀x P**, *where* **P** *is any logical expression, says that* **P** *is true for every object* **x**.
- More precisely, **∀x P** *is true in a given model if* **P** *is true in* ***ALL*** *possible* ***extended interpretations*** *constructed from the interpretation given in the model,*
  - where each extended interpretation specifies a domain element to which x refers.

Quantifiers - Universal Quantifier (∀)

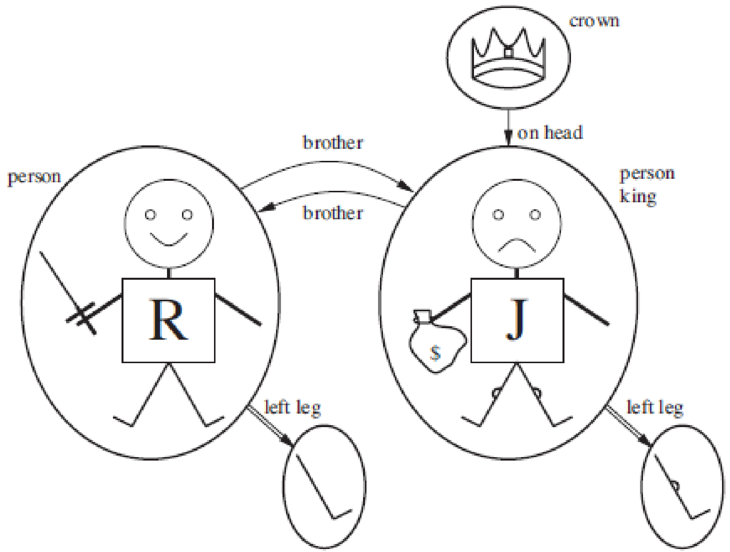
- **"All kings are persons"** is written in first-order logic as,

$$\forall x \;\; King(x) \Rightarrow Person(x)$$

- **∀** is usually pronounced **"For all . . ."**
- **Example:** **"For all x, if x is a king, then x is a person."**

We can extend the interpretation in **five** ways:

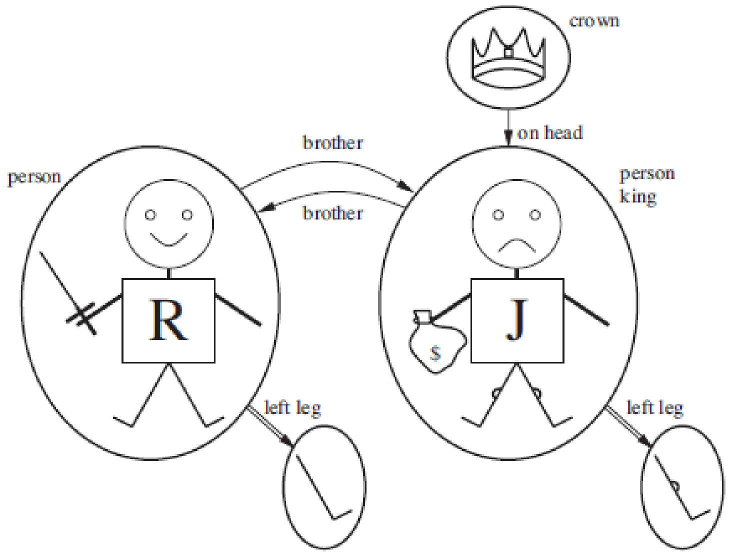
$$x \to \text{Richard the Lionheart},$$
$$x \to \text{King John},$$
$$x \to \text{Richard's left leg},$$
$$x \to \text{John's left leg},$$
$$x \to \text{the crown}.$$



Quantifiers - Universal Quantifier (∀)

- The **universally quantified sentence** is equivalent to asserting the following five sentences:

Richard the Lionheart is a king $\Rightarrow$ Richard the Lionheart is a person.
King John is a king $\Rightarrow$ King John is a person.
Richard's left leg is a king $\Rightarrow$ Richard's left leg is a person.
John's left leg is a king $\Rightarrow$ John's left leg is a person.
The crown is a king $\Rightarrow$ the crown is a person.



Quantifiers - Universal Quantifier (∀)

- Asserting the universally quantified sentence is **equivalent** to asserting a *whole list of individual implications*.
- The **implication is true whenever its premise is false**—*regardless of the truth of the conclusion.*

Richard the Lionheart is a king $\Rightarrow$ Richard the Lionheart is a person.
King John is a king $\Rightarrow$ King John is a person.
Richard's left leg is a king $\Rightarrow$ Richard's left leg is a person.
John's left leg is a king $\Rightarrow$ John's left leg is a person.
The crown is a king $\Rightarrow$ the crown is a person.

Quantifiers – Existential Quantifiers (∃)

- **"King John has a crown on his head"**, we write

$$\exists x \;\; Crown(x) \land OnHead(x, John)$$

- **∃x** is pronounced **"There exists an x such that . . ."** or **"For some x . . .".**
- *Intuitively, the sentence* **∃x P** *says that* **P** *is true for at least one object* **x**.
- More precisely, **∃x P** *is true in a given model if* **P** *is true in* ***at least one extended interpretation*** *that assigns* **x** *to a domain element*.

Quantifiers – Existential Quantifiers (∃)

- That is, at least one of the following is true:

Richard the Lionheart is a crown ∧ Richard the Lionheart is on John's head;
King John is a crown ∧ King John is on John's head;
Richard's left leg is a crown ∧ Richard's left leg is on John's head;
John's left leg is a crown ∧ John's left leg is on John's head;
The crown is a crown ∧ the crown is on John's head.

- The **fifth assertion is true in the model**, so the original existentially quantified sentence is true in the model.

Quantifiers – Existential Quantifiers (∃)

- Notice that, by the definition, the sentence would also be true in a model in which **King John was wearing two crowns**.

- There is a variant of the existential quantifier, usually written **$\exists^1$** or **∃!**, that means

"There exists exactly one."

- Typically, **$\wedge$** is the main connective with **∃.**

Quantifiers - Universal Quantifier (∀)

- Typically → is the main connective with ∀.

Common Mistake:

- Using ∧ as the main connective with ∀.

Everyone at Berkeley is smart:

$$\forall x \;\; At(x, Berkeley) \;\Rightarrow\; Smart(x)$$

$$\forall x \;\; At(x, Berkeley) \land Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Quantifiers – Existential Quantifiers (∃)

Common Mistake:

- Using → as the main connective with ∃

Someone at Stanford is smart:

$\exists x \;\; At(x, Stanford) \wedge Smart(x)$

$$\exists x \;\; At(x, Stanford) \;\Rightarrow\; Smart(x)$$

is true if there is anyone who is not at Stanford!

The **implication is true whenever its premise is false**—*regardless of the truth of the conclusion.*

Nested Quantifiers

- For example, **"Brothers are siblings"** can be written as

$$\boxed{\forall x \ \ \forall y} \; Brother(x, y) \ \Rightarrow \ Sibling(x, y) \, .$$

- <u>**Consecutive quantifiers**</u> of the same type can be written as one quantifier with several variables.
- For example, to say that siblinghood is a symmetric relationship, we can write,

$$\boxed{\forall x, y} \; Sibling(x, y) \ \Leftrightarrow \ Sibling(y, x)$$

Nested Quantifiers

- The **order of quantification** **is very important**. For example: **"Everybody loves somebody"** means that for every person, there is someone that person loves:

$$\forall x \ \exists y \ \ Loves(x, y)$$

- On the other hand, to say **"There is someone who is loved by everyone"** we write

$$\exists y \ \forall x \ \ Loves(x, y)$$

Nested Quantifiers

- *Some confusion may arise when two quantifiers are used with the same variable name.*
- Consider the sentence

$$\forall x \ \ (Crown(x) \lor (\exists x \ \ Brother(Richard, x)))$$

- Here the **x** in **Brother (Richard, x)** is *existentially quantified.*
- ***The <u>rule</u> is that the variable belongs to the innermost quantifier that mentions it; then it will not be subject to any other quantification.***

$$\exists z \ Brother(Richard, z).$$

Reading Material

- **Artificial Intelligence,** A Modern Approach
  Stuart J. Russell and Peter Norvig
  - **Chapter 8.**