CS 461
Artificial Intelligence

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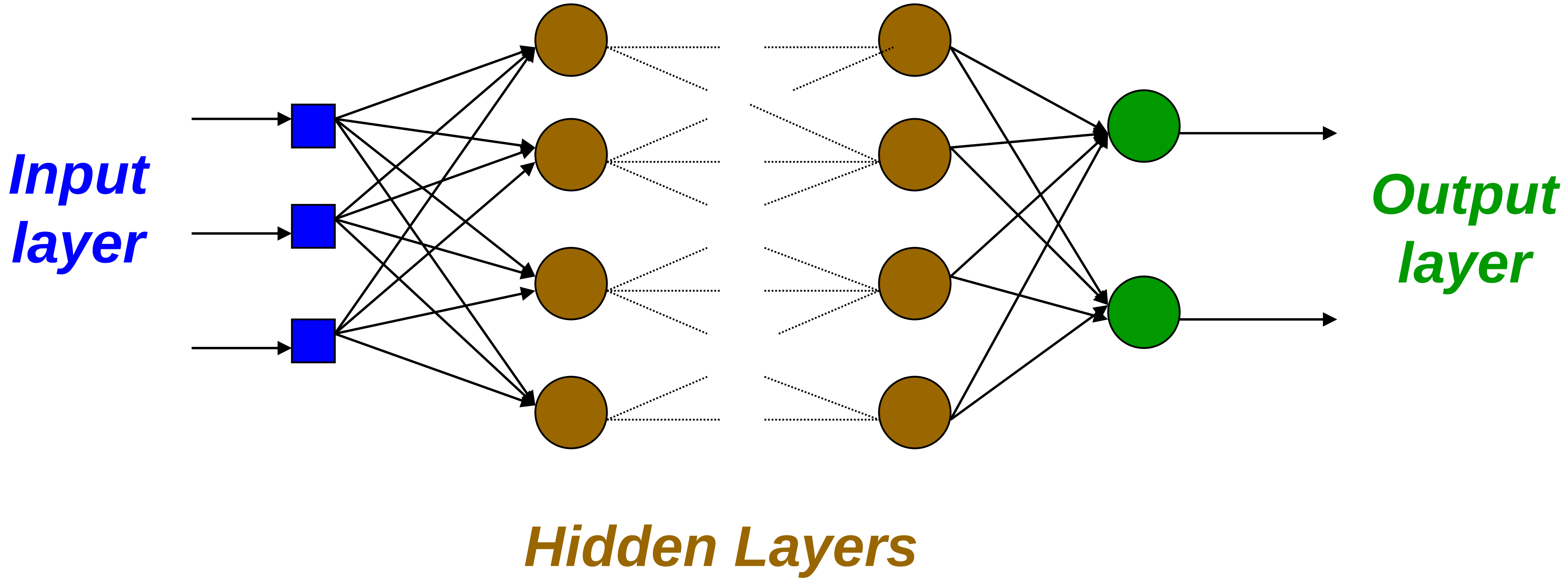
Multilayer Networks

Multilayer Perceptron Architecture

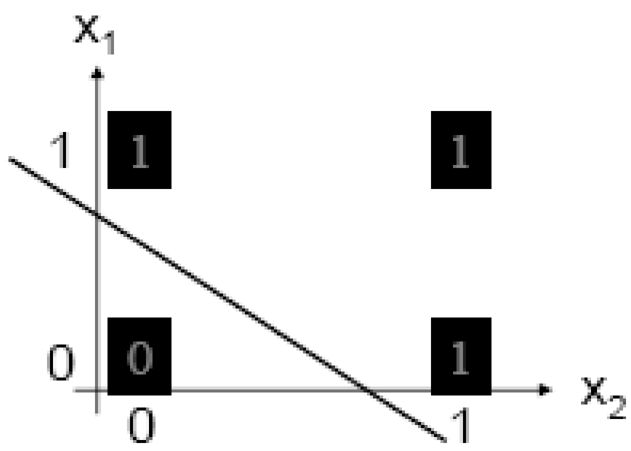
MLP used to describe any general feedforward (no recurrent connections) network



Multilayer Networks

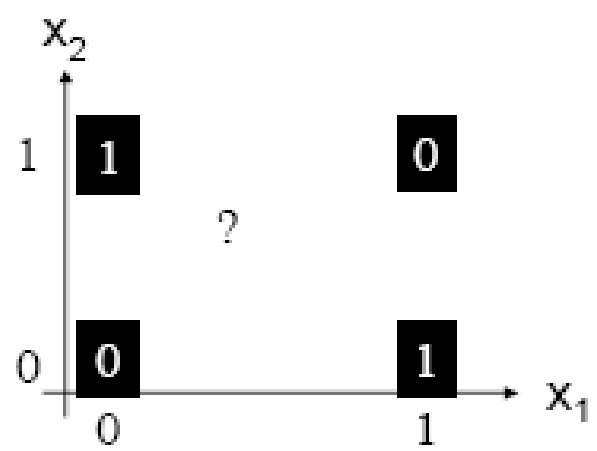
OR function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

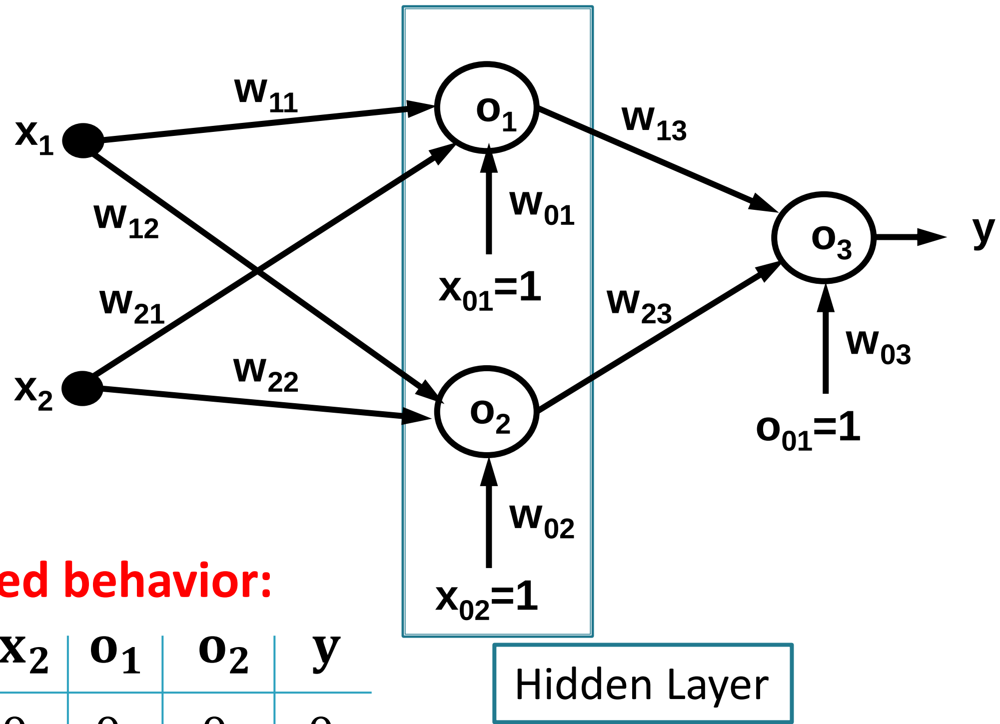


XOR function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



Multilayer Networks



Network Topology:
2 hidden nodes
1 output

Weights:

$$w_{11} = w_{12} = 1$$
$$w_{21} = w_{22} = 1$$
$$w_{01} = -1.5$$
$$w_{02} = -0.5$$
$$w_{13} = -1$$
$$w_{23} = 1$$
$$w_{03} = -0.5$$

Desired behavior:

x_1	x_2	o_1	o_2	y
0	0	0	0	0
1	0	0	1	1
0	1	0	1	1
1	1	1	1	0

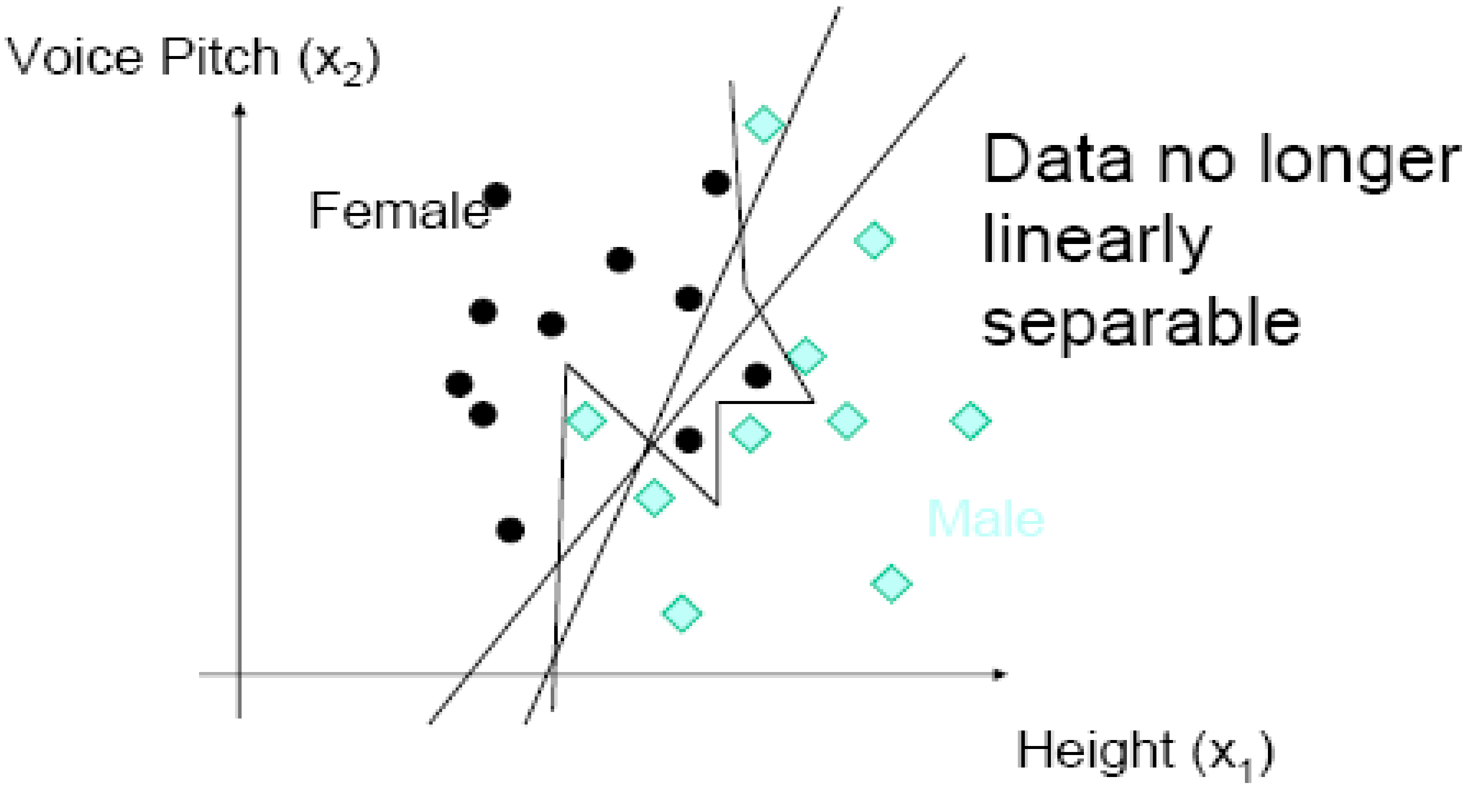
Piecewise linear classification using an MLP with threshold (perceptron) units

Multilayer Networks

- The single perceptron can only express **linear decision surfaces**.

- The kind of **multilayer networks** learned by the **back propagation** algorithm are capable of expressing a rich variety of **nonlinear decision surfaces**.

Multilayer Networks… Example



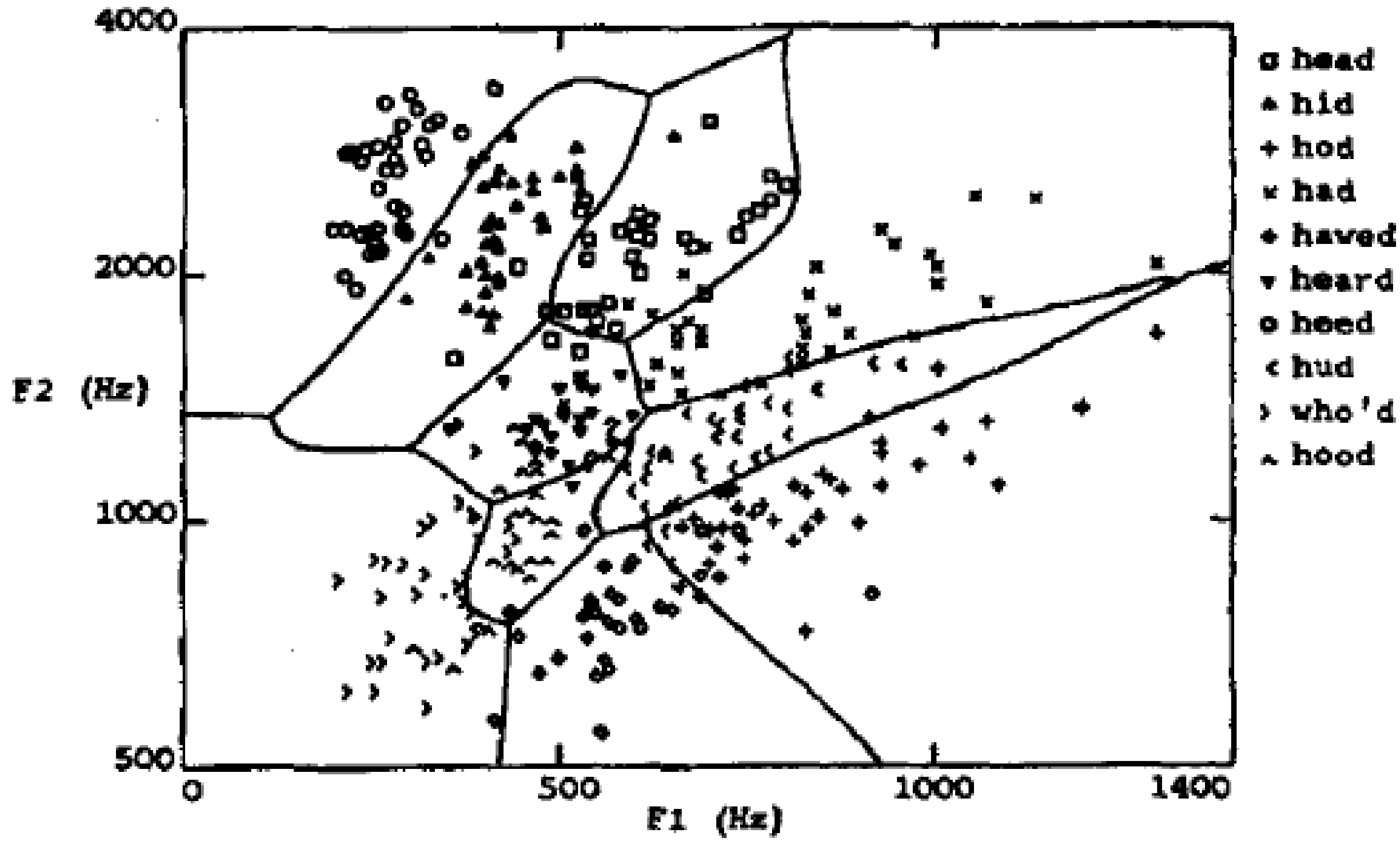
What is a good decision boundary ?

Multilayer Networks… Example

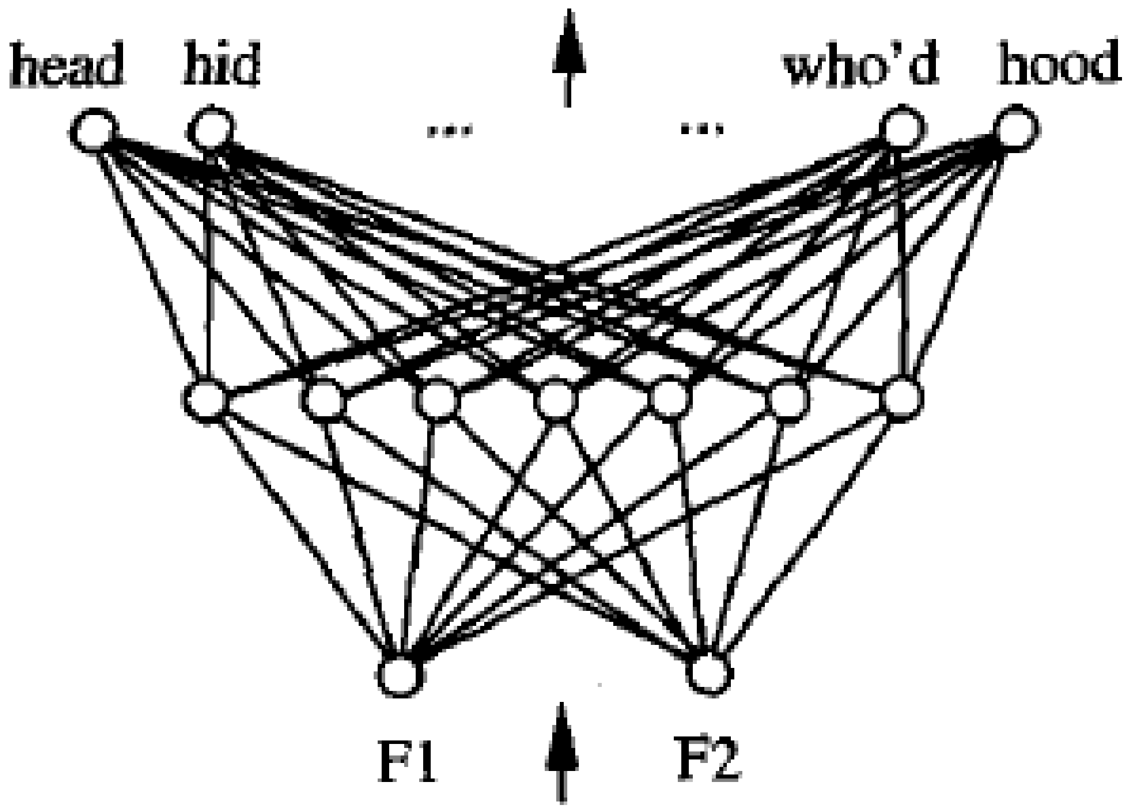
Example:

- The speech recognition task involves distinguishing among 10 possible vowels, all spoken in the context of "h-d" (i.e., "hid," "had," "head," "hood," etc.).

Multilayer Networks… Example



Multilayer Networks… Example



Multilayer Networks

- **What type of unit shall we use as the basis for constructing multilayer networks?**

- Multiple layers of cascaded linear units still produce only linear functions, and we prefer networks capable of representing highly nonlinear functions

- The perceptron unit is another possible choice, is it?
  - its discontinuous threshold makes it undifferentiable and hence unsuitable for gradient descent.

Multilayer Networks

Solution:

- One solution is the **sigmoid unit:**
  - a unit very much like a perceptron, but based on a **smoothed**, **differentiable threshold function**.

- Like the perceptron, the sigmoid unit
  - first computes a linear combination of its inputs,
  - then applies a threshold to the result.

Multilayer Networks

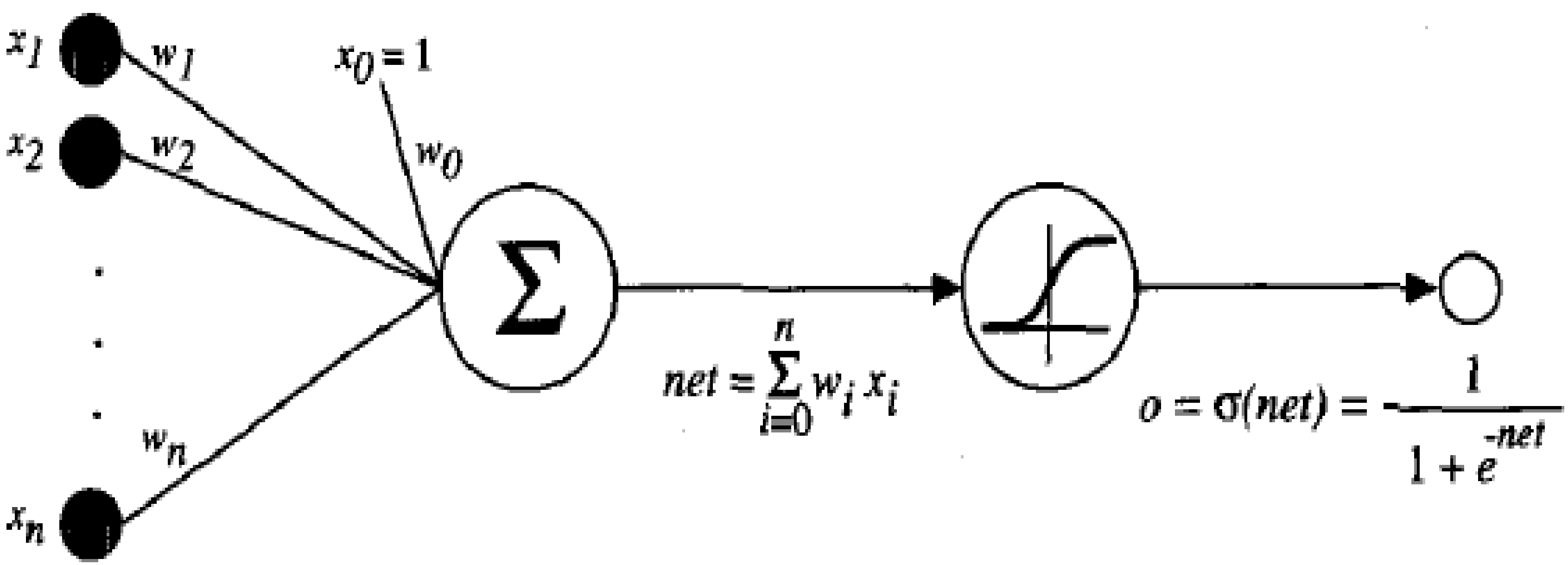
- In case of **sigmoid unit**, however, the **threshold output is a continuous function** of its input.
- More precisely, the sigmoid unit computes its output ***o*** as,

$$o = \sigma(\vec{w} \cdot \vec{x})$$

$$\sigma(y) = \frac{1}{1 + e^{-y}}$$

- σ is often called the sigmoid function or, alternatively, the logistic function.

Sigmoid Threshold Unit



Sigmoid Function

- Sigmoid function maps a very large input domain to a small range of outputs, it is often referred to as the ***squashing function*** of the unit.
- The sigmoid function has the **useful property** that **its derivative is easily expressed in terms of its output**.

$$\sigma(y) = \frac{1}{1+e^{-y}}$$

$$\frac{d\sigma(y)}{dy} = \sigma(y) \cdot (1 - \sigma(y))]$$

Sigmoid Function

- The term $\boldsymbol{e^{-y}}$ in the sigmoid function definition is sometimes replaced by $\boldsymbol{e^{-k.y}}$
  - where k is some positive constant that determines the steepness.
- The function ***tanh*** is also sometimes used in place of the sigmoid function.

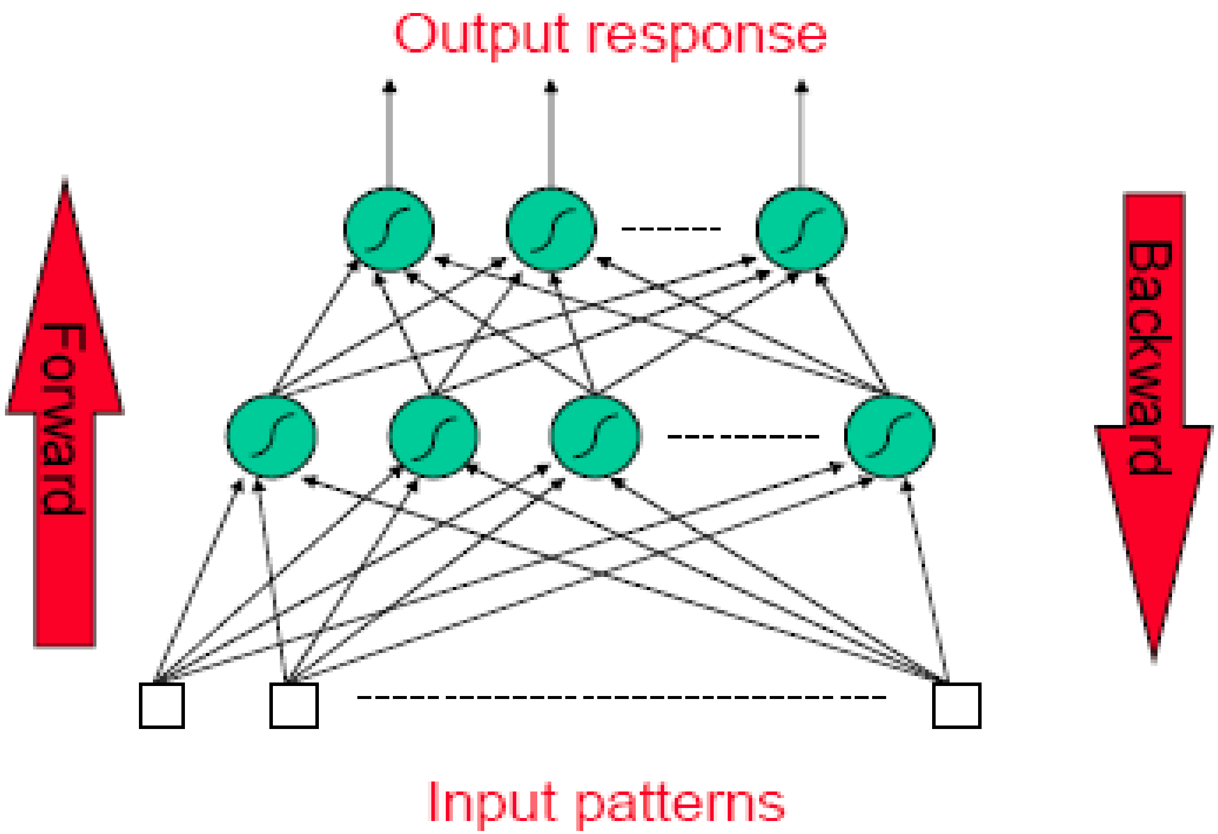
Back Propagation Algorithm

The Back Propagation Algorithm

The Back Propagation algorithm has two phases:

- **Forward pass phase:** computes 'functional signal', feed forward propagation of input pattern signals through network
- **Backward pass phase:** computes 'error signal', *propagates* the error *backwards* through network starting at output units
  - (where the error is the difference between actual and desired output values)

The Back Propagation Algorithm



Conceptually: Forward Activity - Backward Error

The Back Propagation Algorithm

- The back propagation algorithm learns the weights for a multilayer network,
  - given a network with a fixed set of units and interconnections.
- It **employs gradient descent** to attempt to minimize the squared error between the network output values and the target values for these outputs.
- As we are considering networks with multiple output units, we begin by redefining **E** to sum the errors over all of the network output units.

The Back Propagation Algorithm

$$E(\vec{w}) \equiv \frac{1}{2}\sum_{d\in D}\sum_{k\in outputs}(t_{kd} - o_{kd})^2$$

- where ***outputs*** is the set of output units in the network, and t_{kd} and O_{kd} are the target and output values associated with the *k*th output unit and training example **d**.

The Back Propagation Algorithm

BACKPROPAGATION($training\_examples, \eta, n_{in}, n_{out}, n_{hidden}$)

Each training example is a pair of the form $\langle \vec{x}, \vec{t} \rangle$, where $\vec{x}$ is the vector of network input values, and $\vec{t}$ is the vector of target network output values.

η is the learning rate (e.g., .05). n_{in} is the number of network inputs, n_{hidden} the number of units in the hidden layer, and n_{out} the number of output units.

The input from unit i into unit j is denoted x_{ji}, and the weight from unit i to unit j is denoted w_{ji}.

- Create a feed-forward network with n_{in} inputs, n_{hidden} hidden units, and n_{out} output units.
- Initialize all network weights to small random numbers (e.g., between −.05 and .05).
- Until the termination condition is met, Do

The Back Propagation Algorithm

For each $\langle \vec{x}, \vec{t}\, \rangle$ in *training_examples*, Do

Propagate the input forward through the network:

1. Input the instance $\vec{x}$ to the network and compute the output o_u of every unit u in the network.

Propagate the errors backward through the network:

2. For each network output unit k, calculate its error term δ_k

$$\delta_k \leftarrow o_k(1 - o_k)(t_k - o_k)$$

3. For each hidden unit h, calculate its error term δ_h

$$\delta_h \leftarrow o_h(1 - o_h) \sum_{k \in outputs} w_{kh} \delta_k$$

4. Update each network weight w_{ji}

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

where

$$\Delta w_{ji} = \eta\, \delta_j\, x_{ji}$$

Reading Material

- **Artificial Intelligence,** A Modern Approach

  Stuart J. Russell and Peter Norvig
  - **Chapter 18.**
- **Machine Learning**

  Tom M. Mitchell
  - **Chapter 4.**