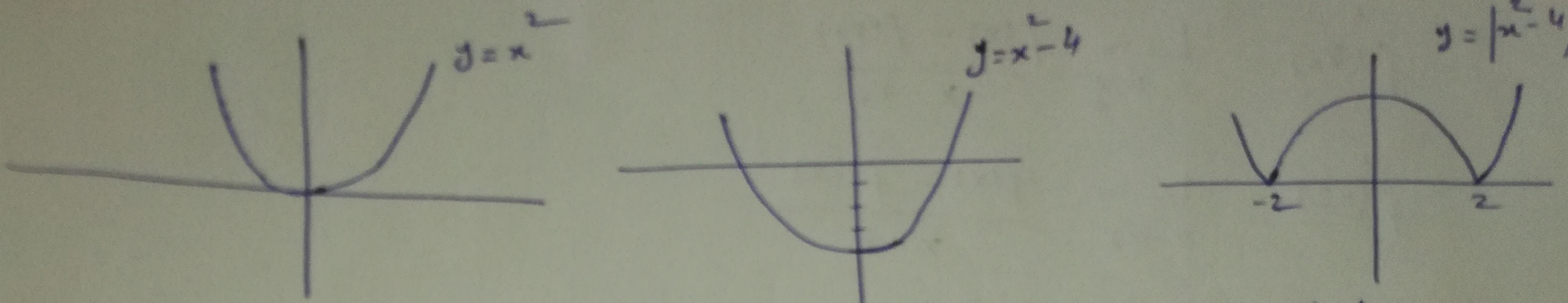
**Ex** Consider the ftn $f(x) = |x^2-4|$




As the graph of $f(x) = |x^2-4|$ has no hole in it so it is continuous in $(-\infty, \infty)$.
Although at corner points $f(x)$ is not diff.
let's check the differentiability of $f(x)$ at $x=2$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$Rf'(2) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^+} \frac{(x^2-4) - (0)}{x-2} = \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2} = 4$$

$$Lf'(2) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2^-} \frac{-(x^2-4) - 0}{x-2} = \lim_{x \to 2^-} \frac{-(x+2)(x-2)}{x-2} = -4$$

As $Rf'(2) \neq Lf'(2)$ so given ftn is not diff at $x=2$.

**Important Note**

$$f(x) = |x^2-4| = \begin{cases} (x^2-4) ; & x^2-4 \geq 0 \\ -(x^2-4) ; & x^2-4 < 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x^2-4 ; & x \geq 2 \text{ or } x \leq -2 \\ -(x^2-4) ; & -2 < x < 2 \end{cases}$$

let's check the differentiability at $x = -2$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'(-2) = \lim_{x \to -2} \frac{f(x) - f(-2)}{x + 2}$$

$$Rf'(-2) = \lim_{x \to -2^+} \frac{-(x^2 - 4) - 0}{x + 2}$$

$$= \lim_{x \to -2^+} \frac{-\cancel{(x+2)}(x-2)}{\cancel{x+2}}$$

$$= 4$$

$$Lf'(-2) = \lim_{x \to -2^-} \frac{(x^2 - 4) - 0}{x + 2}$$

$$= \lim_{x \to -2^-} \frac{(x+2)(x-2)}{x+2}$$

$$= -4$$

As $Rf'(-2) \neq Lf'(-2)$

so given ftw is ~~dis~~ not diff. at $x = -2$.

$\Rightarrow$ At corner points given ftw is not diff.

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

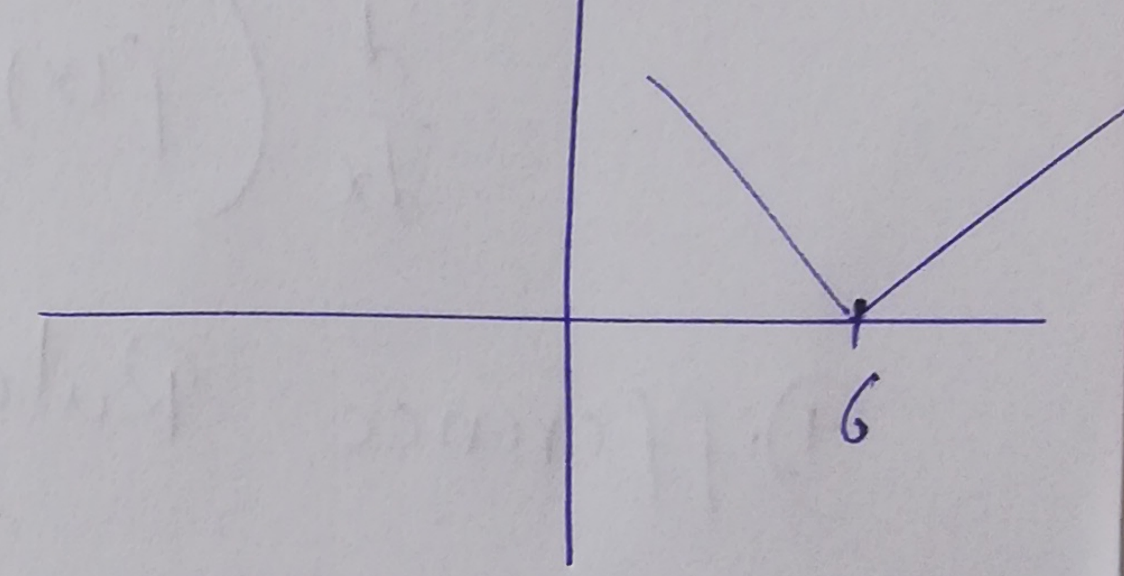
<u>Theorem</u>

Every differentiable ftn is continuous, but every Continuous ftn need not to be differentiable.

$\boxed{Ex}$ Consider the ftn

$$f(x) = |x - 6|$$

The graph of above ftn is




let's check the Continuity at $x = 6$.

$$f(6) = 0$$

$$\lim_{x \to 6^+} f(x) = \lim_{x \to 6^+} (x - 6) = 0$$

$$\lim_{x \to 6^-} f(x) = \lim_{x \to 6^-} -(x - 6) = 0$$

As $f(6) = \lim_{x \to 6^+} f(x) = \lim_{x \to 6^-} f(x)$

so given ftn is Continuous at $x = 6$.

lets check the differentiability at $x = 6$.

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6}$$

$$R f'(6) = \lim_{x \to 6^+} \frac{(x - 6) - (0)}{x - 6} = 1$$

$$L f'(6) = \lim_{x \to 6^-} \frac{-(x - 6) - 0}{x - 6} = -1$$

As $Rf'(6) \neq Lf'(6)$

So given ftw is not diff. at $x=6$.

Power Rule for differentiation :-

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Sum Rule :-

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x)).$$

Difference Rule :-

$$\frac{d}{dx}(f(x)-g(x)) = \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x)).$$

Product Rule :-

$$\frac{d}{dx}(f\cdot g) = fg' + gf'$$

Quotient Rule :-

$$\frac{d}{dx}(f/g) = \frac{gf' - fg'}{g^2}$$

**Ex** Find the points on the curve
$y = x^4 - 6x^2 + 4$
where tangent line is horizontal.

**Sol** Horizontal tangents occur, when derivative is zero. So substitute,

$$\frac{dy}{dx} = 0$$

$$\Rightarrow 4x^3 - 12x = 0$$

$$\Rightarrow 4x(x^2 - 3) = 0$$

$$\Rightarrow x = 0, \quad x = +\sqrt{3}, \quad x = -\sqrt{3}$$

when $x = 0$, $y = 4$

when $x = +\sqrt{3}$, $y = -5$

when $x = -\sqrt{3}$, $y = -5$

So at $(0, 4)$, $(+\sqrt{3}, -5)$, $(-\sqrt{3}, -5)$ are the points where tangent line is horizontal.