Extreme Values of Function

A function "f" has an absolute maximum at c

if $\quad f(c) \geq f(x) \quad \forall\ x$ in D

where D is the domain of "f".

The value f(c) is called maximum value of f on

Similarly "f" has absolute minimum at d

if $\quad f(d) \leq f(x) \quad \forall\ x$ in D.

where D is the domain of f.

The value f(d) is called absolute minimum Va

of f on D.

Def:

The maximum & Minimum values of "f" are called extreme values of "f".

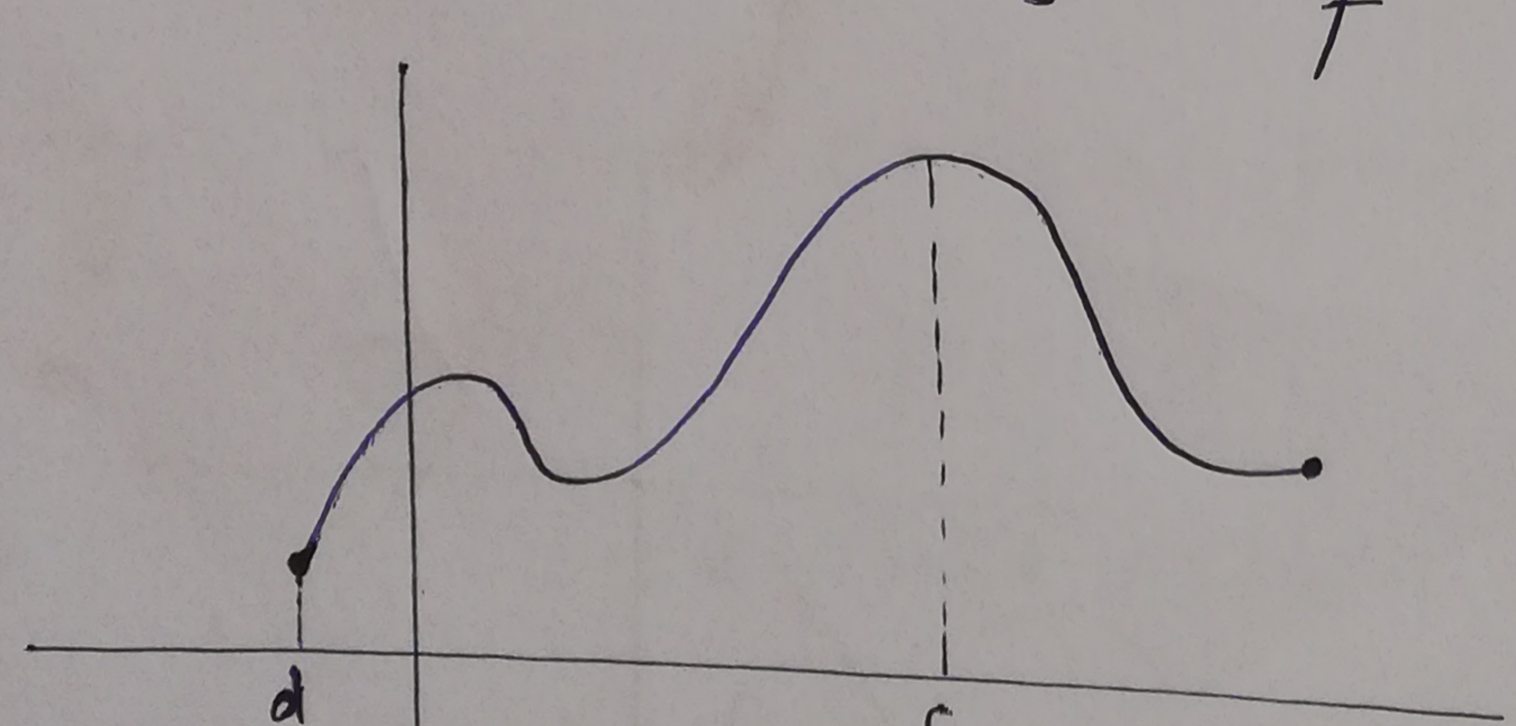


fig 1

fig (1) shows the graph of f with absolute maximum at "c" & absolute minimum at d.

Def:-

A function f has local maximum at c

if $f(c) \geq f(x)$

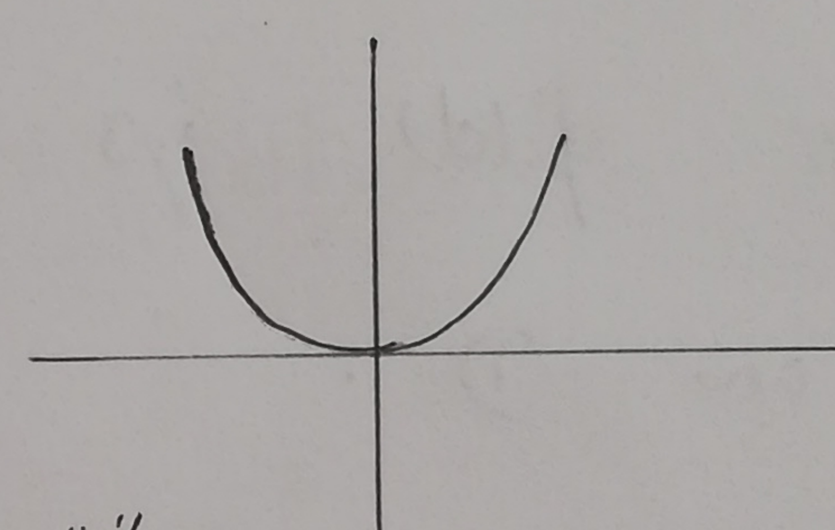
$\forall$ x in some open interval containing "c".

Similarly f has local minimum at c

if $f(c) \leq f(x)$

$\forall$ x in some open interval containing "c".

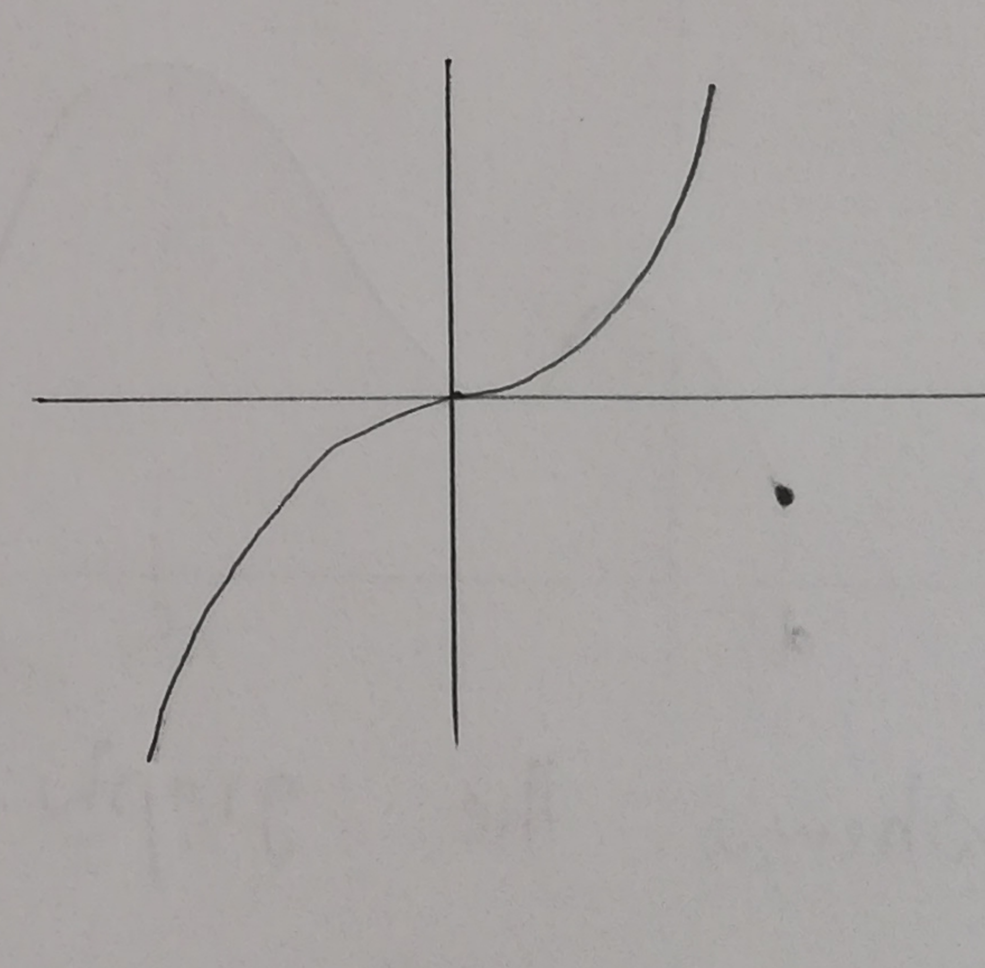
$\boxed{\text{Ex}}$ $f(x) = x^2$



$f(0) = 0$ is the absolute & local minimum value of "f".

As $y = x^2$ has no highest point so function $y = x^2$ has no maximum value.

$\boxed{\text{Ex}}$ $f(x) = x^3$



$f(x) = x^3$ has neither an absolute maximum value nor an absolute minimum value.

Infact it has no local extreme values either.

How to find absolute maximum & absolute minimum values on a closed interval $[a,b]$.

Step I Find the values of f at the critical points of f in (a,b).

Step II Find the values of f at the end points of interval.

Step III The largest of the values in step 1 & step II is absolute maximum & lowest value in step I & step II is absolute minimum.

Ex Consider $f(x) = 3x^4 - 16x^3 + 18x^2$ $\quad -1 \le x \le 4$

find absolute extreme values of $f(x)$.

Sol

Step I To find critical points

subsitute $f'(x) = 0$

$$12x^3 - 48x^2 + 36x = 0$$

$$6x\left(2x^2 - 8x + 6\right) = 0$$

$$12x\left(x^2 - 4x + 3\right) = 0$$

$$12x\left[x^2 - 3x - x + 3\right] = 0$$

$$12x\left[x(x-3) - 1(x-3)\right] = 0$$

$$12x(x-3)(x-1) = 0$$

So critical points are

$x = 0$, $x = 1$ & $x = 3$

$f(0) = 0$

$f(1) = 5$

$f(3) = -27$

Step II

$f(-1) = 37$

$f(4) = 32$

Step III

$f(-1) = 37$

$f(3) = -27$.

So given ftn $y = f(x)$ has absolute maximum at $x = -1$ & absolute minimum at $x = 3$

& absolute Maximum value $= 37$.

absolute Minimum value $= -27$.

Criterion for finding Local Extreme Values

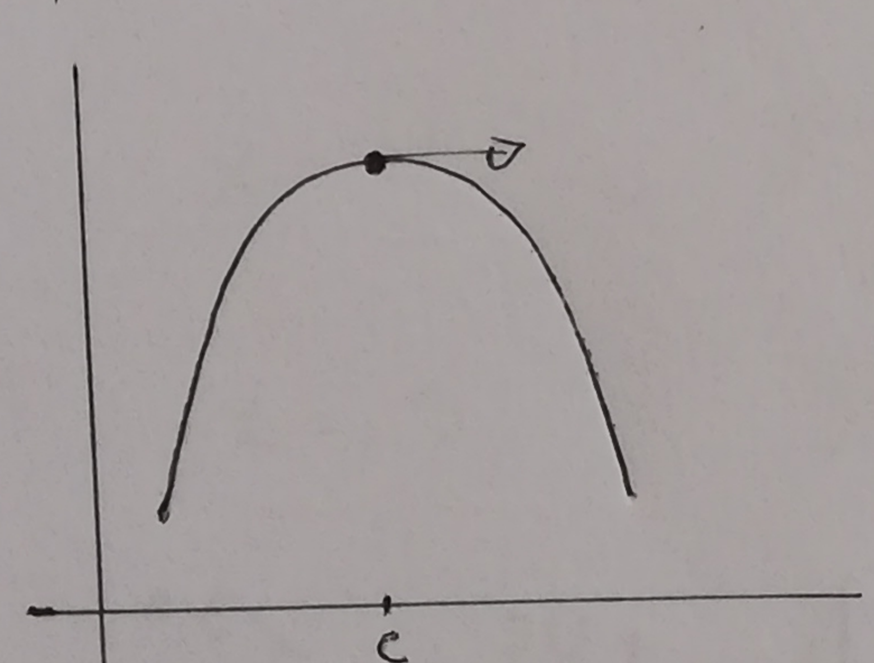
1st Derivative Test :-

Suppose that "c" is a critical point of a continuous ftn "f"

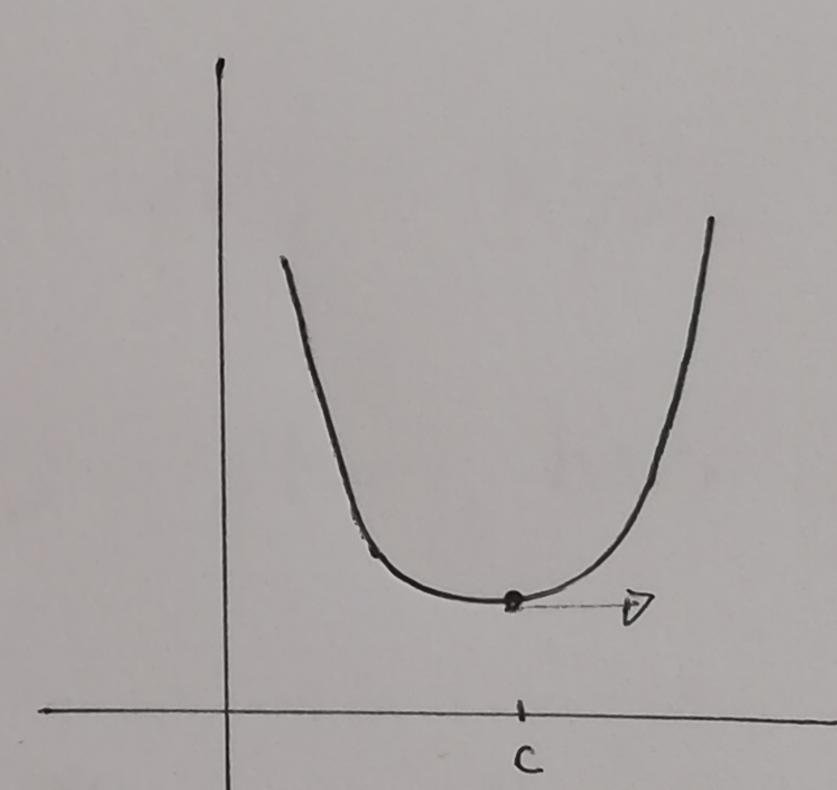
[a] if f' changes sign from +ve to -ve while crossing "c", then f has local maximum at c.

[b] if f' changes sign from -ve to +ve while crossing c, then f has local minimum at c.

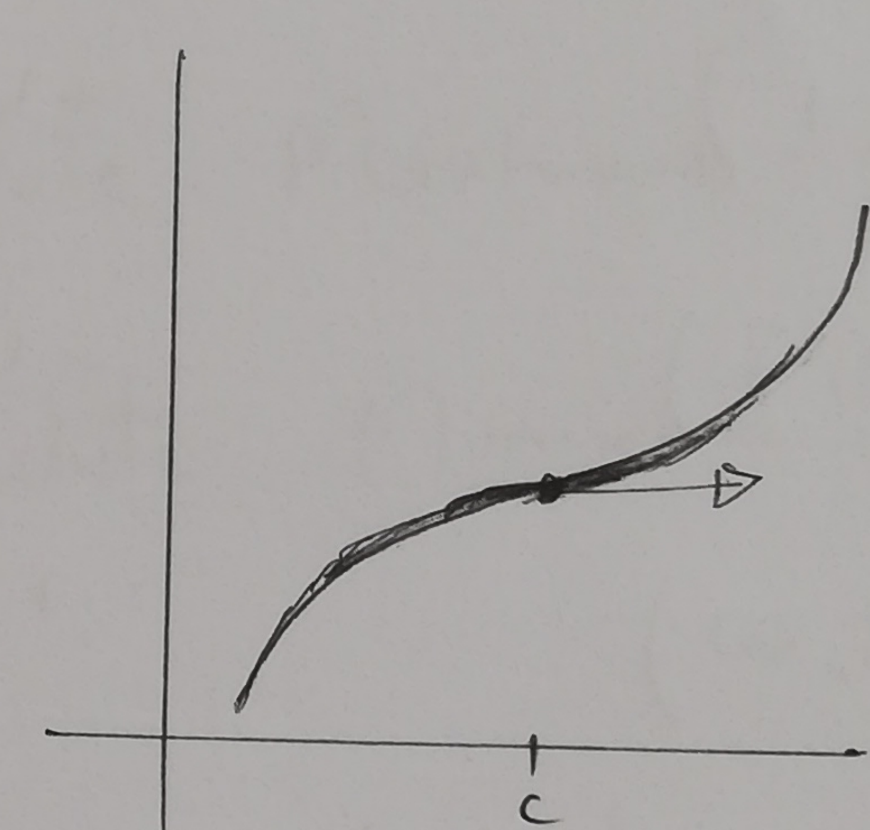
[c] if f' does not changes sign at c, then f has no local maximum or Minimum at c.



local Max. at $x = c$



local Min. at $x = c$



No Extreme value.

$\boxed{Ex}$ As from previous exp

$$f(x) = 3x^4 - 16x^3 + 18x^2 \qquad (-\infty, \infty)$$

find local Extreme values of $f(x)$.

$\boxed{Sol}$ local extreme values exists only at critical points.

critical points of $f(x)$ are

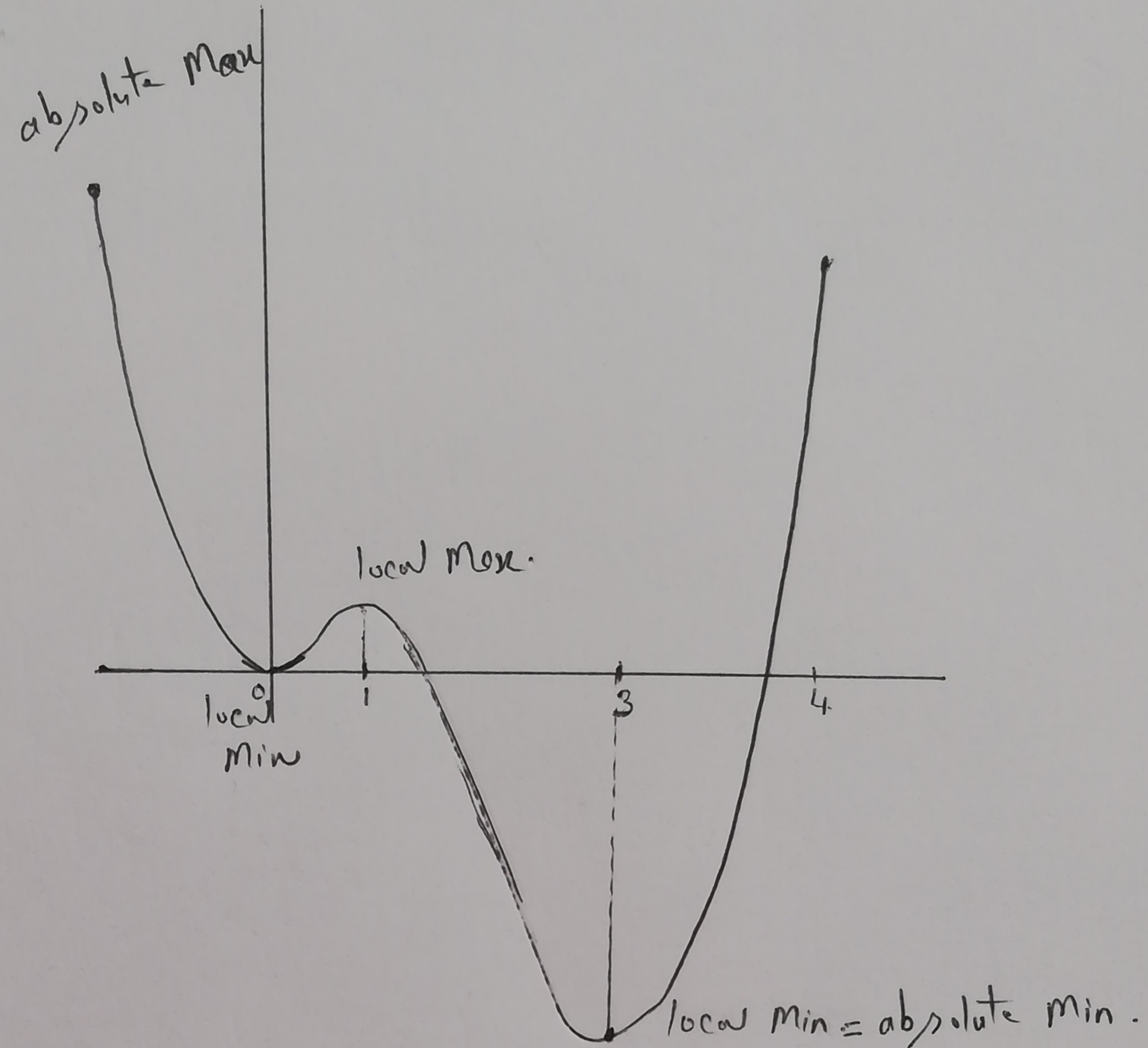
$x = 0$, $x = 1$, $x = 3$.

Interval	Sign of f'
$(-\infty, 0)$	$-$ve.
$(0, 1)$	$+$ve
$(1, 3)$	$-$ve
$(3, \infty)$	$+$ve.

So f' changes sign from $-$ve to $+$ve, so given ftn has local minimum at $x = 0$.

As f' changes sign from $+$ve to $-$ve while crossing $x = 1$, so f has local max. at $x = 1$.

As f' changes sign from $-$ve to $+$ve while crossing $x = 3$, so f has local min. at $x = 3$.



$-1 \leq x \leq 4.$

At $x = 0$, given ftn has local minimum.

at $x = 1$, given ftn has local maximum.

at $x = 3$, given ftn has local minimum.

At $x = -1$, given ftn has absolute Maximum.

At $x = 3$, given ftn has absolute Minimum.