$\boxed{Ex}$ At what point on the hyperbola $xy = 12$ is the tangent line parallel to the line $3x + y = 0$. $\boxed{6}$

$\boxed{Sol}$

slope of given st. line is -3.

As $xy = 12$

$y = 12x^{-1}$

$\frac{dy}{dx} = -12x^{-2} = \frac{-12}{x^2}$

To find required result.

$-\frac{12}{x^2} = -3$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

So $(2, 6), (-2, -6)$ are the point where the slope of hyperbola $xy = 12$ is the same as slope of $3x + y = 0$.

Verification

$y = 12x^{-1}$

$\frac{dy}{dx} = -12x^{-2} = -12/x^2$

$\left.\frac{dy}{dx}\right|_{x=2} = -3$; $\left.\frac{dy}{dx}\right|_{x=-2} = -3$

Ex let $y = \dfrac{x^2 + x - 2}{x^3 + 6}$, find $\dfrac{dy}{dx}$.

Sol

$$\frac{dy}{dx} = \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}$$

Ex Find the equation of tangent line to the Curve $y = \dfrac{\sqrt{x}}{1+x^2}$ at $(1, 1/2)$

Sol

$$\frac{dy}{dx} = \frac{(1+x^2)\left(\frac{1}{2}x^{-1/2}\right) - (\sqrt{x})(2x)}{(1+x^2)^2}$$

$$= \frac{\frac{1+x^2}{2\sqrt{x}} - 2x\cdot\sqrt{x}}{(1+x^2)^2}$$

$$= \frac{1+x^2 - 4x^2}{2\sqrt{x}\cdot(1+x^2)^2}$$

$$= \frac{1-3x^2}{2\sqrt{x}\cdot(1+x^2)^2}$$

Ex

The position of a particle is given by

$$s = f(t) = t^3 - 6t^2 + 9t$$

a) Find velocity at time "t".

b) Find velocity after 2s.

c) when is the particle at rest?

d) when the particle is moving forward?

e) find the total distance travelled by particle during 1st five sec.

Sol

a) $v(t) = \frac{ds}{dt} = 3t^2 - 12t + 9$

b) $v(2) = 3(2)^2 - 12(2) + 9 = -3$ m/s.

c) Particle is at rest when $v(t) = 0$

$\Rightarrow 3t^2 - 12t + 9 = 0$

$\Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16 - 12}}{2}$

$\Rightarrow t = \frac{4 \pm 2}{2}$

$t = 1, 3$

so particle is at rest, when $t = 1$ sec & $t = 3$ sec.

d

The particle moves forward (+ve direction) if $v(t) > 0$

$\Rightarrow \quad 3t^2 - 12t + 9 > 0$

$\Rightarrow \quad 3(t-1)(t-3) > 0$

which is possible only if

$t-1 > 0$ & $t-3 > 0$ or $t-1 < 0$ & $t-3 < 0$

$t > 1$ & $t > 3$ $\quad$ $t < 1$ & $t < 3$

$\Rightarrow t > 3$ or $\Rightarrow t < 1$

So particle moves forward if

$t < 1$ or $t > 3$

& moves backward if

$1 < t < 3$

e

Distance travelled in 1st sec is

$|f(1) - f(0)| = |4 - 0| = 4\text{ m}$

Distance travelled in $1 < t < 3$

$|f(3) - f(1)| = |0 - 4| = 4\text{ m}$

Distance travelled in $t = 3$ to $t = 5$

$|f(5) - f(3)| = |20 - 0| = 20$

Total distance in 1st five sec $= 4 + 4 + 20$

$= 28\text{ m}.$

Derivative of Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\csc x) = -\csc x \cdot \cot x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$.

Ex) Differentiate $f(x) = \frac{\sec x}{1+\tan x}$. For what values of x does the graph of f have horizontal tangent?

$\boxed{\text{S.l}}$

$$f(x) = \frac{\sec x}{1+\tan x}$$

$$f'(x) = \frac{(1+\tan x)(\sec x \cdot \tan x) - \sec x(\sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x \cdot \tan x + \sec x \cdot \tan^2 x - \sec^3 x}{(1+\tan x)^2}$$

$$= \frac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x(\tan x - 1)}{(1+\tan x)^2}$$

To find horizontal tangents. subsitute

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{\sec x(\tan x - 1)}{(1+\tan x)^2} = 0$$

$$\Rightarrow \sec x(\tan x - 1) = 0$$

As $\sec x \neq 0$

so $\tan x - 1 = 0$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow x = n\pi + \frac{\pi}{4}$$

where n is an integer.