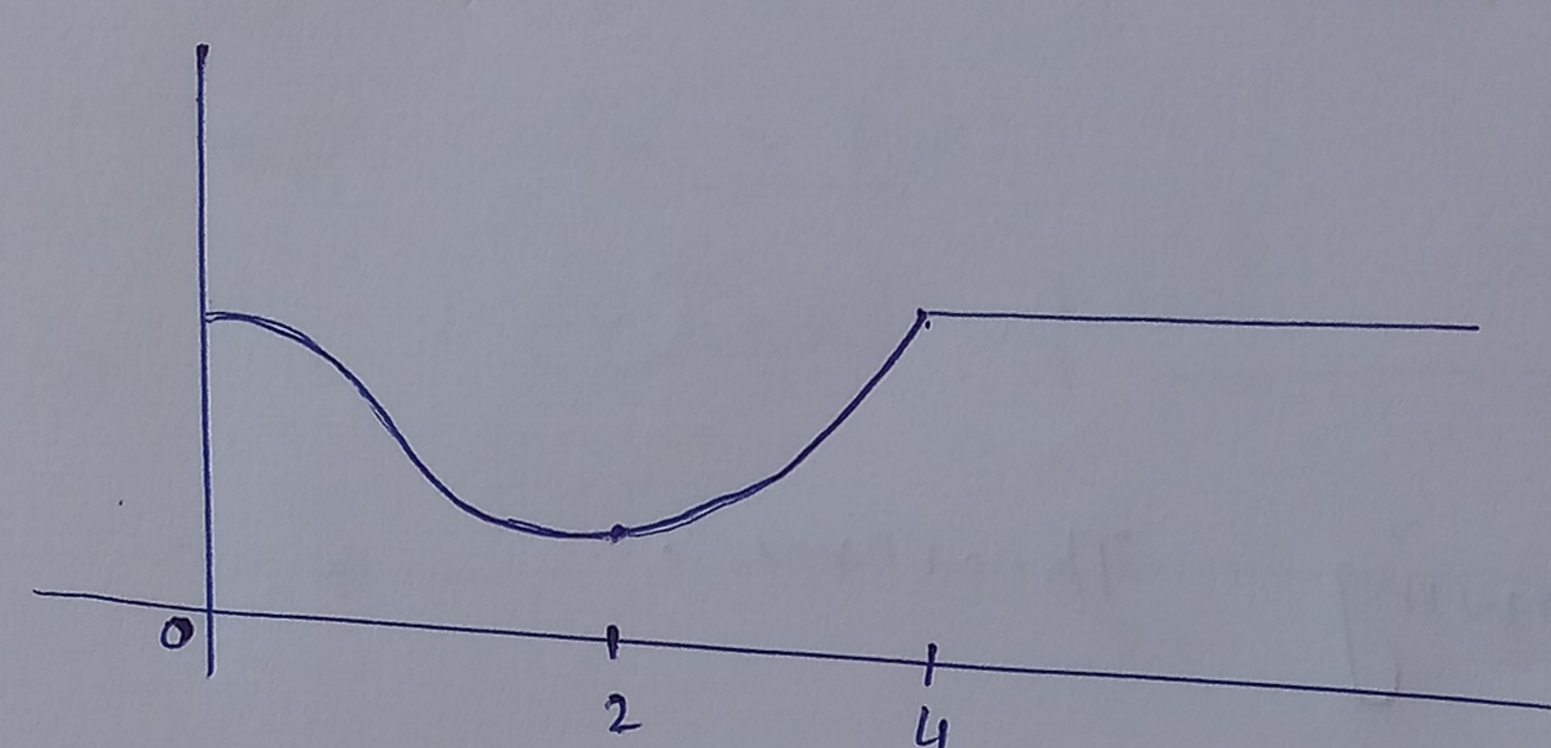
"Increasing & Decreasing Functions"

The term increasing, decreasing & constant are used to describe the behaviour of function as we travell from left to right along its graph.

For example, from the following graph



$f(x)$ is decreasing from $x = 0$ to $x = 2$

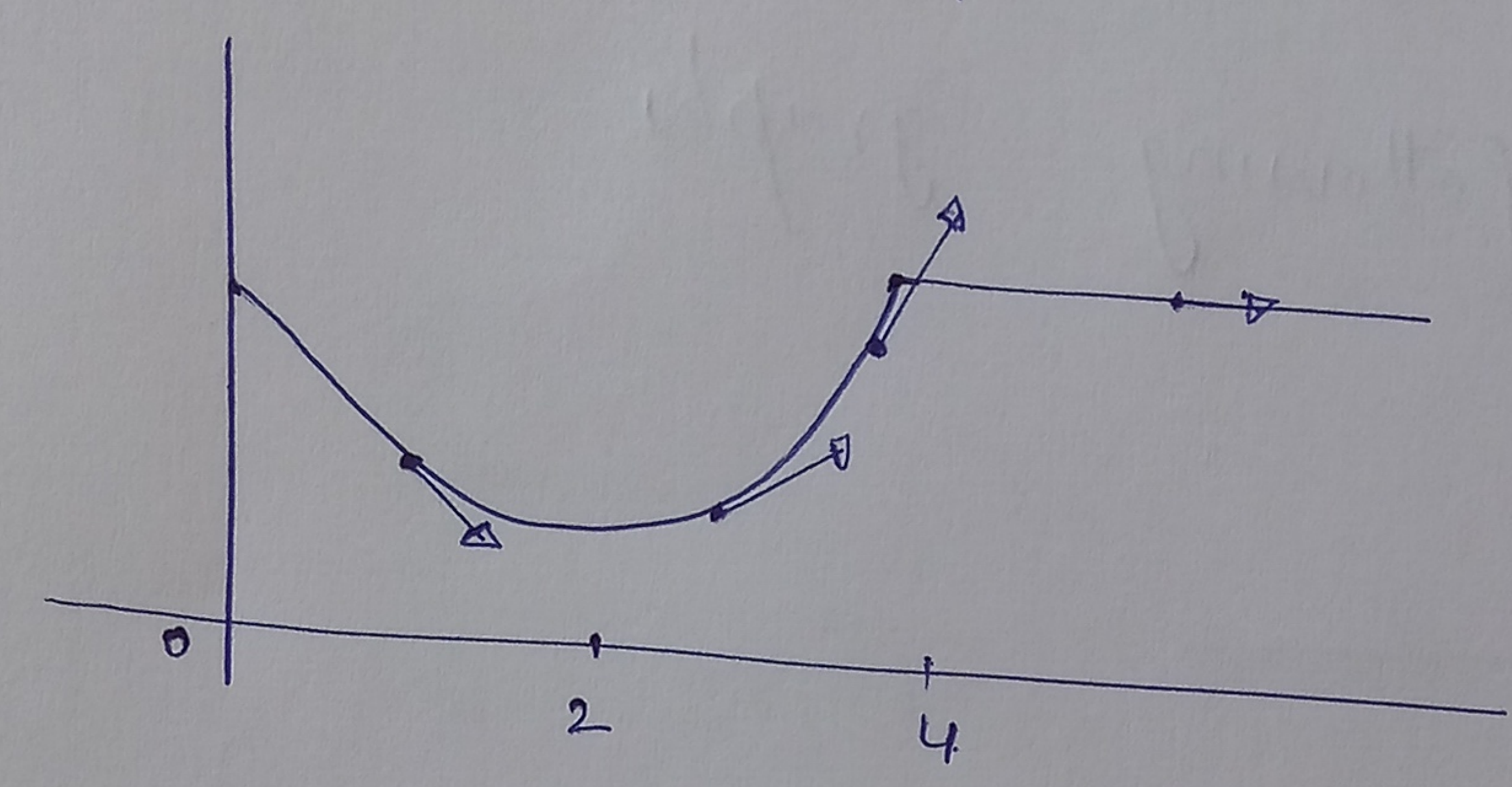
$f(x)$ is increasing from $x = 2$ to $x = 4$

& $f(x)$ is constant to the right of $x = 4$.

Also note that if we draw tangents in the interval from $x = 0$ to $x = 2$, All tangent are pointing downward so all tangents have -ve slope in from $x = 0$ to $x = 2$. Means $f'(x) < 0$ in $(0, 2)$.

Also if we draw all tangents in the interval from $x = 2$ to $x = 4$, All tangents are pointing upward so all tangents have +ve slope in $(2, 4)$. Means $f'(x) > 0$ in $(2, 4)$.

If we draw all tangents from $x = 4$ to $x \to \infty$, Then all tangents are parallel to X-axis, Means slope of all tangents are zero. i.e $f'(x) = 0$.



So we have the following Theorem.

<u>Theorem</u>

let "f" be a Continuous ftn on a closed interval $[a,b]$ & differentiable on the open interval (a,b).

~~[d] if $f'(x) > 0$ for every x in (a,b), Then f(x) is increasing on $[a,b]$~~

[a] If $f'(x) > 0$ on an interval, Then f is increasing on that interval.

[b] If $f'(x) < 0$ on an interval, Then f is decreasing on that interval.

[c] If $f'(x) = 0$ on an interval, Then f is constant on that interval.

$\boxed{\text{Ex}}$ Find the interval on which $f(x) = x^2 - 4x + 3$ is increasing or decreasing.

$\boxed{\text{Sol}}$

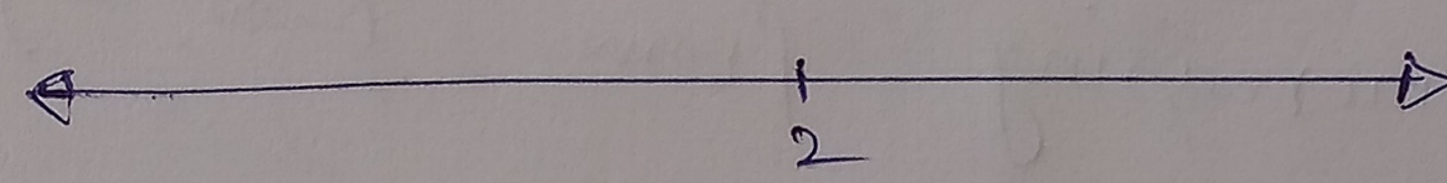
$$f'(x) = 2x - 4$$

To find critical points, substitute $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 2x - 4 = 0$$

$$\Rightarrow \quad \boxed{x = 2}$$

$x = 2$ is a critical point.



Interval	Sign of f'	Behaviour of f
$(-\infty, 2)$	$-ve$	f is decreasing.
$(2, \infty)$	$+ve$	f is increasing.

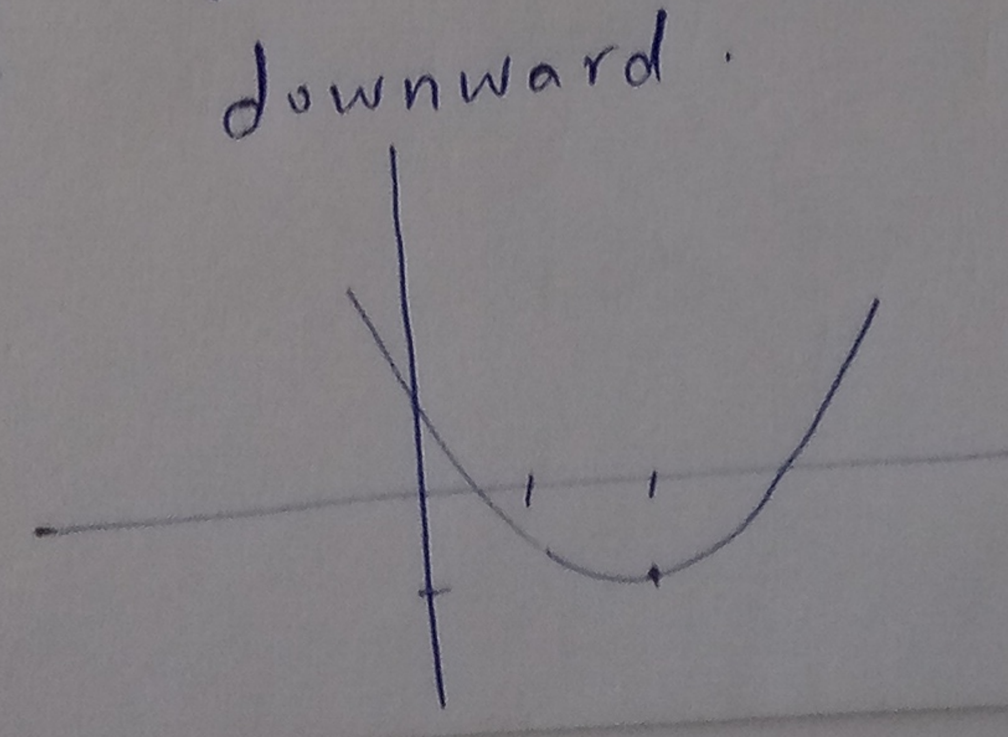
We can also plot the graph of $f(x)$ as

$$f(x) = x^2 - 4x + 3 = (x)^2 - 2(x)(2) + (2)^2 + 3 - (2)^2$$

$$= (x-2)^2 - 1.$$

So $f(x) = x^2 - 4x + 3 = (x-2)^2 - 1$

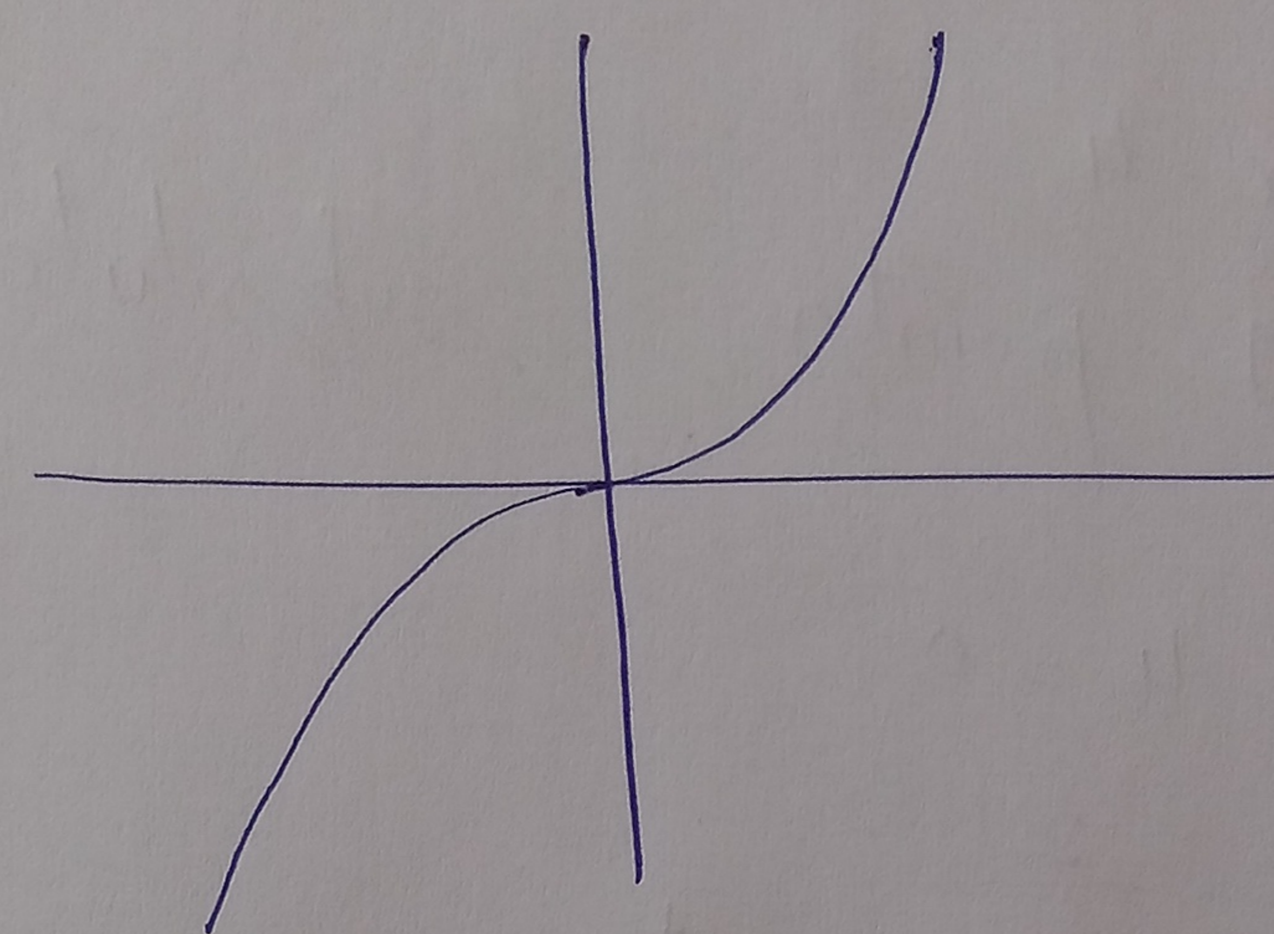
We can plot the graph of $f(x)$ by shifting the graph of $y = x^2$, two unit towards right & one unit downward.



Ex

Find the interval on which $f(x) = x^3$ is increasing & decreasing.

Sol



As we can see from the graph of $f(x) = x^3$ given ftn is increasing from $(-\infty, \infty)$

__Mathematically__

$$f'(x) = 3x^2$$

Interval	Sign of f'	Behaviour of f''
$(-\infty, 0)$	+ve	f is increasing
$(0, \infty)$	+ve	f is increasing.

Ex Find the interval on which $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ is increasing & decreasing.

Sol

$$f'(x) = 12x^3 + 12x^2 - 24x$$

To find critical points, subsitute $f'(x) = 0$

$$\Rightarrow 12x^3 + 12x^2 - 24x = 0$$

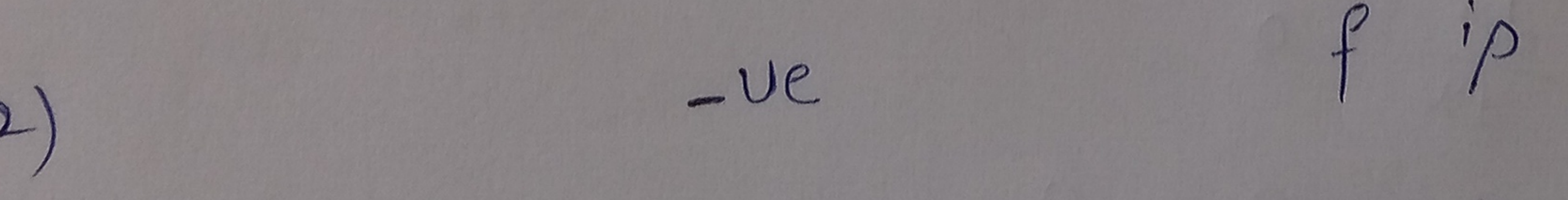
$$\Rightarrow x(x^2 + x - 2) = 0$$

$$\Rightarrow x(x^2 + 2x - x - 2) = 0$$

$$\Rightarrow x(x+2)(x-1) = 0$$

So critical points are $x = 0$, $x = -2$, $x = 1$

$x = -2$, $x = 0$, $x = 1$ are the points where $\frac{dy}{dx} = 0$.



Interval	Sign of f'	Behaviour of f''.
$(-\infty, -2)$	$-ve$	f is decreasing
$(-2, 0)$	$+ve$	f is increasing
$(0, 1)$	$-ve$	f is decreasing
$(1, \infty)$	$+ve$	f is increasing.

<u>Def</u> :-

In general we define the critical point of a function $y = f(x)$ to be a point where f'' has horizontal tangent (i.e $dy/dx = 0$) or f' does not exist.

$\boxed{Ex}$ Find the critical points of $f(x) = x^3 - 3x + 1$

$\boxed{Sol}$ The given function being the polynomial, is differentiable everywhere so its critical points are all stationary points.

To find critical points, substitute

$$\frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 - 3 = 0$$

$$\Rightarrow 3(x^2 - 1) = 0$$

$$\Rightarrow 3(x+1)(x-1) = 0$$

So $x = -1$ & $x = 1$ are critical points

These are the points where given ftn has horizontal tangents.

$\boxed{\text{Ex}}$ Find the critical points of

$$f(x) = x^{3/5}(4-x).$$

$\boxed{\text{Sol}}$

$$f'(x) = x^{3/5}(-1) + (4-x)\frac{3}{5}x^{-2/5}$$

$$= -x^{3/5} + \frac{3(4-x)}{5x^{2/5}}$$

$$= \frac{-5x^{3/5} \cdot x^{2/5} + 3(4-x)}{5x^{2/5}}$$

$$= \frac{-5x + 12 - 3x}{5x^{2/5}}$$

$$f'(x) = \frac{12 - 8x}{5x^{2/5}}$$

Here $f'(x) = 0$, when $12 - 8x = 0$

$\Rightarrow x = 3/2$

So $x = 3/2$ is a stationary point.

Also $f'(x)$ does not exist at $x = 0$

So $x = 0$ is also a critical point.

So given ftn has two critical points

$x = 0$ & $x = 3/2$.