$\boxed{Ex}$ Consider the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$, to find

[a] Find critical Points of $f(x)$.

[b] Find the interval on which given ftn is increasing & decreasing.

[c] Find Points of inflection.

[d] Find the interval on which Curve is Concave up & Concave down.

[e] Find local extreme values of given ftn.

[Sol]

[a] To find critical point, subsitute

$$f'(x) = 0$$

$$\Rightarrow 12x^3 + 12x^2 - 24x = 0$$

$$\Rightarrow 12x(x^2 + x - 2) = 0$$

$$\Rightarrow 12x(x^2 + 2x - x - 2) = 0$$

$$\Rightarrow 12x(x(x+2) - 1(x+2)) = 0$$

$$\Rightarrow 12x(x+2)(x-1) = 0$$

$$\Rightarrow x = 0, \quad x = 1, \quad x = -2$$

So $x = -2$, $x = 0$ & $x = 1$ are critical points of $f(x)$.

These are the points where 1st derivative is zero.

These are the points where slope of tangent line is zero. or we have horizontal tangent.

[b]

$$\begin{array}{ccccc} \hline & -2 & 0 & 1 & \end{array}$$

Interval	sign of $f'(x)$	Behaviour of $f(x)$.
$(-\infty, -2)$	$-ve$	$f(x)$ is decreasing
$(-2, 0)$	$+ve$	$f(x)$ is increasing
$(0, 1)$	$-ve$	$f(x)$ is decreasing
$(1, \infty)$	$+ve$	$f(x)$ is increasing

c To find Points of inflections, subsitute

$f''(x) = 0$

$36x^2 + 24x - 24 = 0$

$$x = \frac{-24 \pm \sqrt{(24)^2 - 4(36)(-24)}}{2(36)} = \frac{-24 \pm 24\sqrt{7}}{72} = 0.54, -1.21$$

so points of inflections are $x \simeq -1.21$ & $x \simeq 0.54$

These are the possible points of inflections.

d

Interval	sign of f''	Behaviour of $f(x)$
$(-\infty, -1.2)$	+ve	Concave up
$(-1.2, 0.5)$	-ve	Concave down
$(0.5, \infty)$	+ve	Concave up.

so in $(-\infty, -1.2)$ Curve is Concave up.

$(-1.2, 0.5)$ Curve is Concave down.

$(0.5, \infty)$ Curve is Concave up.

So at $x \simeq -1.2$ & at $x = -0.5$, Concavity changes

so $x \simeq -1.2$ & $x = -0.5$ are points of inflections.

e local extreme values exist only at critical points.

so by 1st derivative Test

f' changes sign from -ve to +ve, while crossing $x = -2$

so at $x = -2$ ftn has local minima.

f' changes sign from +ve to -ve while crossing $x = 0$

so ftn has local maximum at $x = 0$

f' changes sign from -ve to +ve while crossing $x = 1$

so ftn has local minimum at $x = 1$.

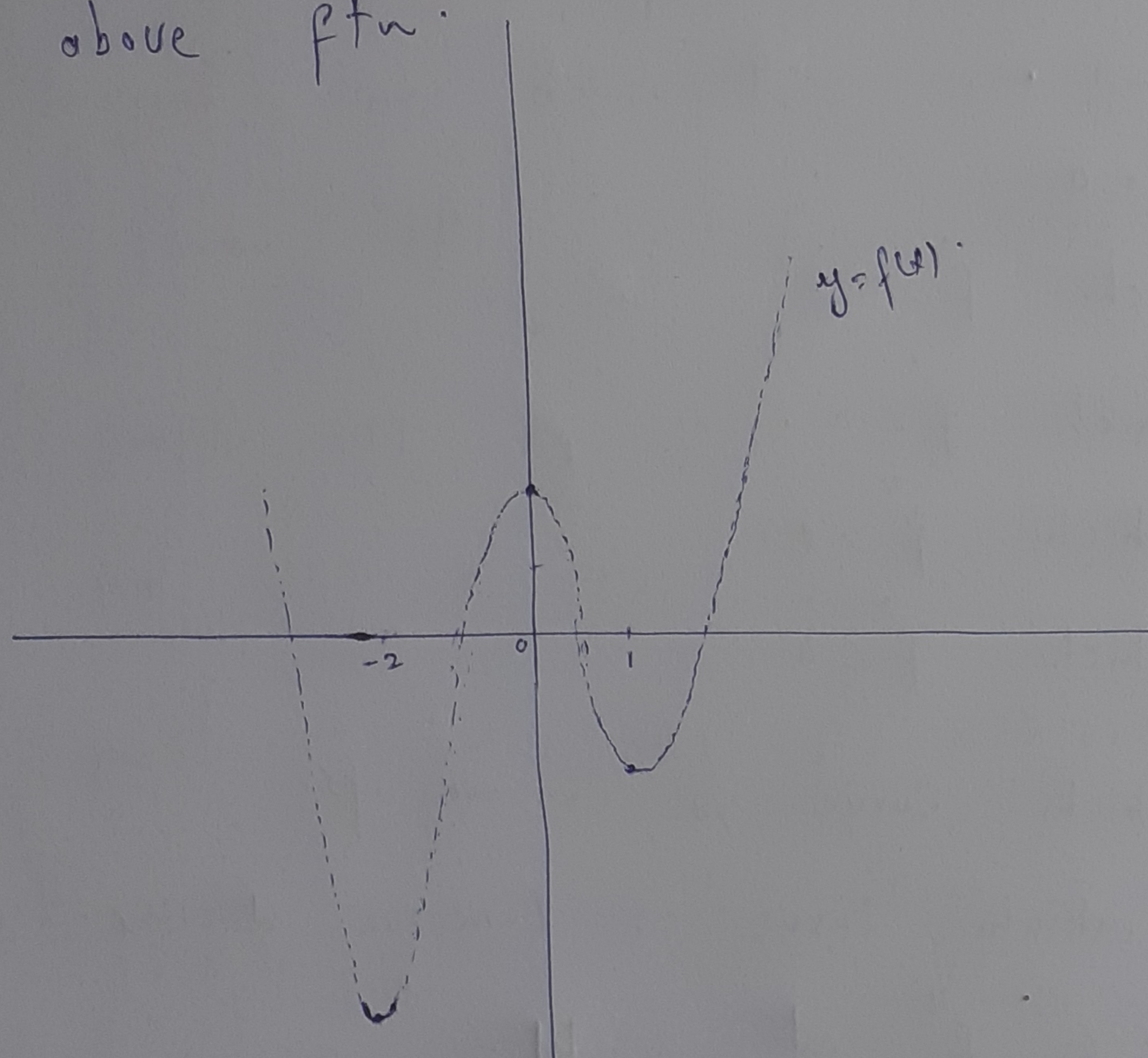
& These local extreme values are.

$f(-2) = -30$

$f(0) = 2$

$f(1) = -3$

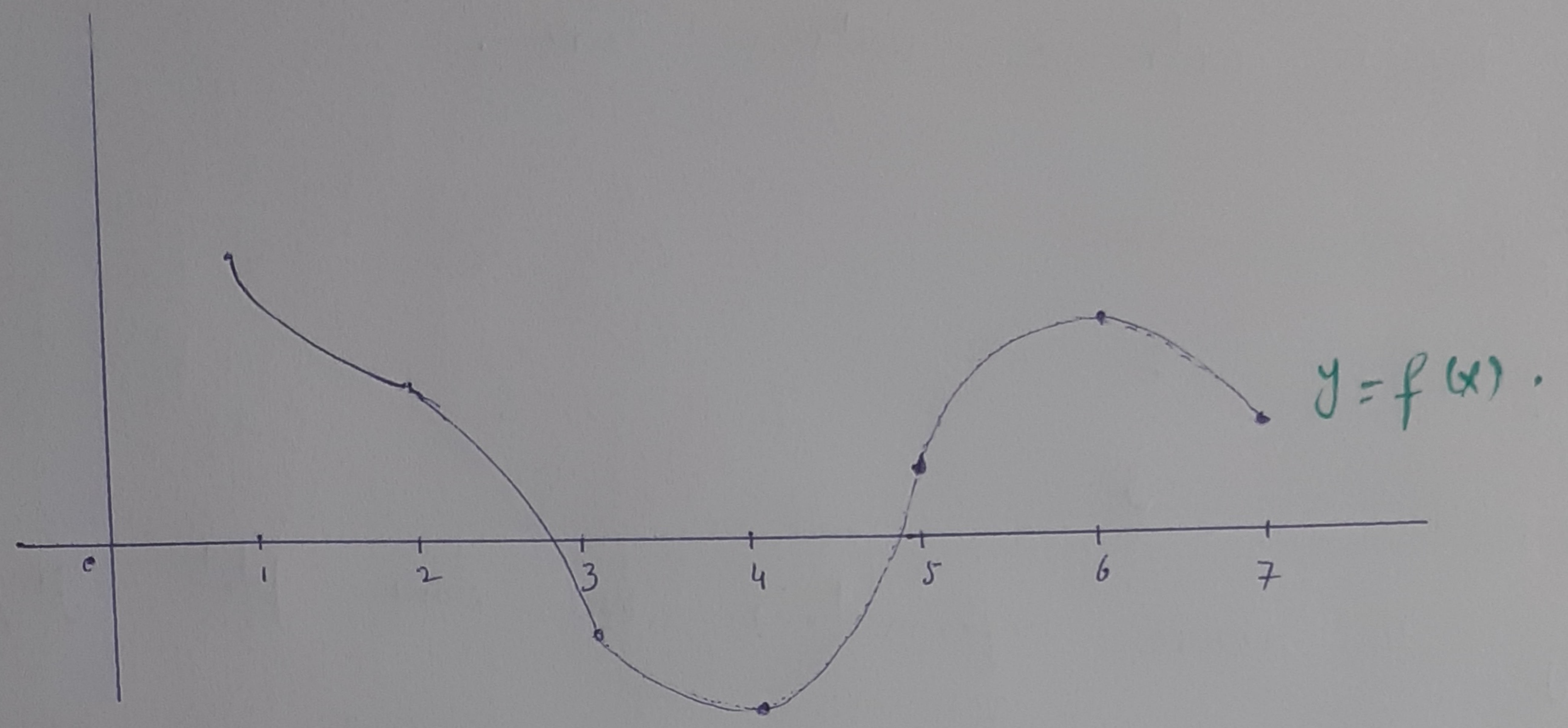
By using all above information we can plot graph of above ftn.



<u>Exercise</u> Consider $f(x) = xe^{-x}$

find all terms which we have calculated above.

In each part use the graph of $y = f(x)$ in the accompanying fig. to find the requested information. (44)



[a] Find points where $f' = 0$.

[b] Find points where $f'' = 0$

[c] Find intervals on which $f' > 0$.

[d] Find intervals on which $f' < 0$.

[e] Find intervals on which $f'' > 0$.

[f] Find intervals on which $f'' < 0$.

[g] Find intervals on which Curve is Concave up.

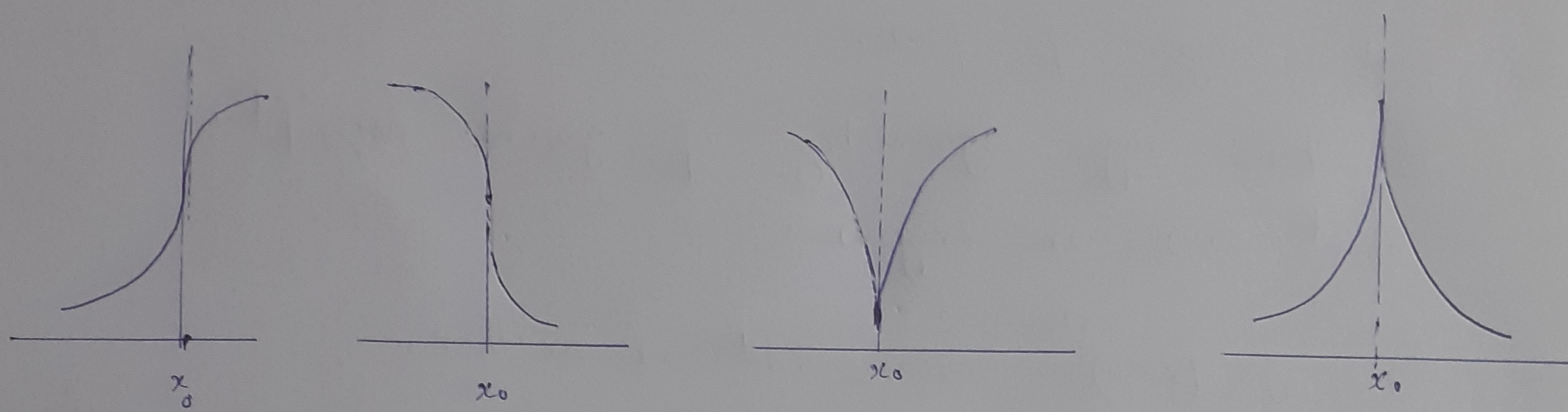
[h] Find intervals on which Curve is Concave down.

[i] what is the sign of f'' at $x = 4$.

[j] what is the sign of f' at $x = 6.7$.

Def "Cusp"

Consider the following graphs



In all four cases, ftn is not differentiable at x_0.
b/c tangent line approaches a vertical position when $x \to x_0$ either from left or from right.

In graph (a) & (b), x_0 is point of inflection but in (c) & (d), x_0 is not a point of inflection.

In part (c) & (d) f' approaches to $+\infty$ from one side $-\infty$ from other, we say that graph has a cusp at $x = x_0$.

in graph (a)

$$\lim_{x \to x_0^+} f(x) = +\infty$$

$$\lim_{x \to x_0^-} f(x) = +\infty$$

in graph (b)

$$\lim_{x \to x_0^+} f(x) = -\infty$$

$$\lim_{x \to x_0^-} f(x) = -\infty$$

in graph (c)

$$\lim_{x \to x_0^+} f(x) = +\infty$$

$$\lim_{x \to x_0^-} f(x) = -\infty$$

in graph (d)

$$\lim_{x \to x_0^+} f(x) = -\infty$$

$$\lim_{x \to x_0^-} f(x) = +\infty$$

Exercise Consider $y = (x-4)^{2/3}$ (46)

(a) find domain of $f(x)$.

(b) find range of $f(x)$.

[c] Is there any symmetry of given ftn about axis or origin.

[d] Is $y = (x-4)^{2/3}$, even, odd or neither.

[e] what are the x-intercept & y-intercept.

[f] what is the end behaviour of ftn.

[g] find horizontal & Vertical asymptotes. if exist.

[h] find critical points.

[i] find the interval on which given ftn is increasing. i.e $f' > 0$.

[j] find the interval on which given ftn is decreasing i.e $f' < 0$.

[k] find point of inflection.

[l] find interval on which curve is Concave up & Concave down.

[m] find Cusp. if exist.

[n] find local extreme values of given ftn.

[o] Use all above information to plot graph.