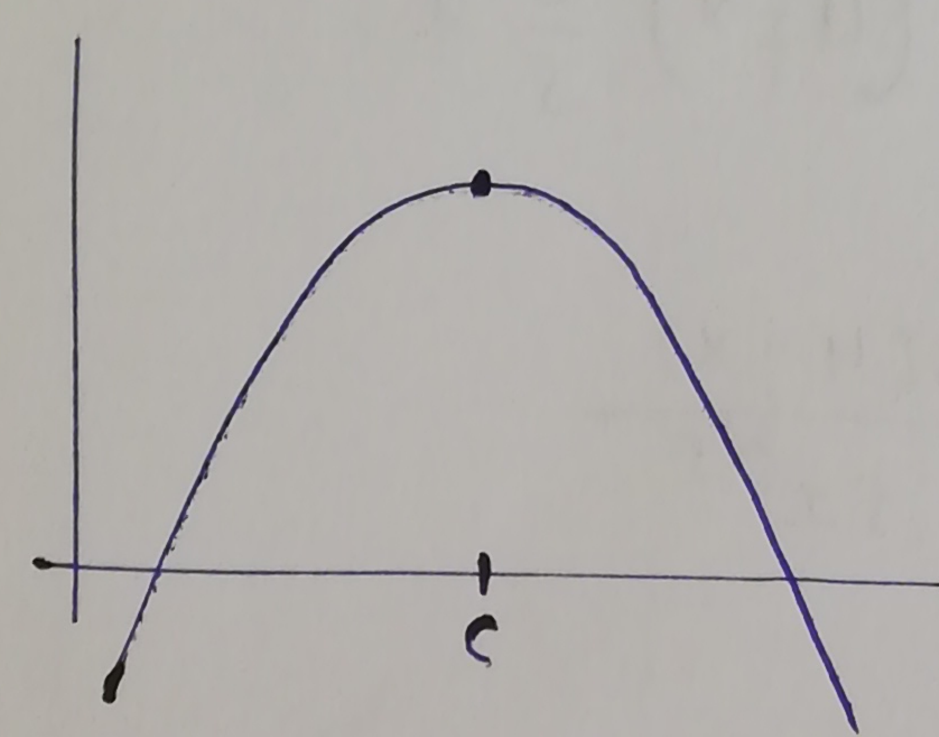
As if

$\boxed{a}$ $f' < 0$, Then f is decreasing ftn.

$\boxed{b}$ $f' > 0$, Then f is increasing ftn.



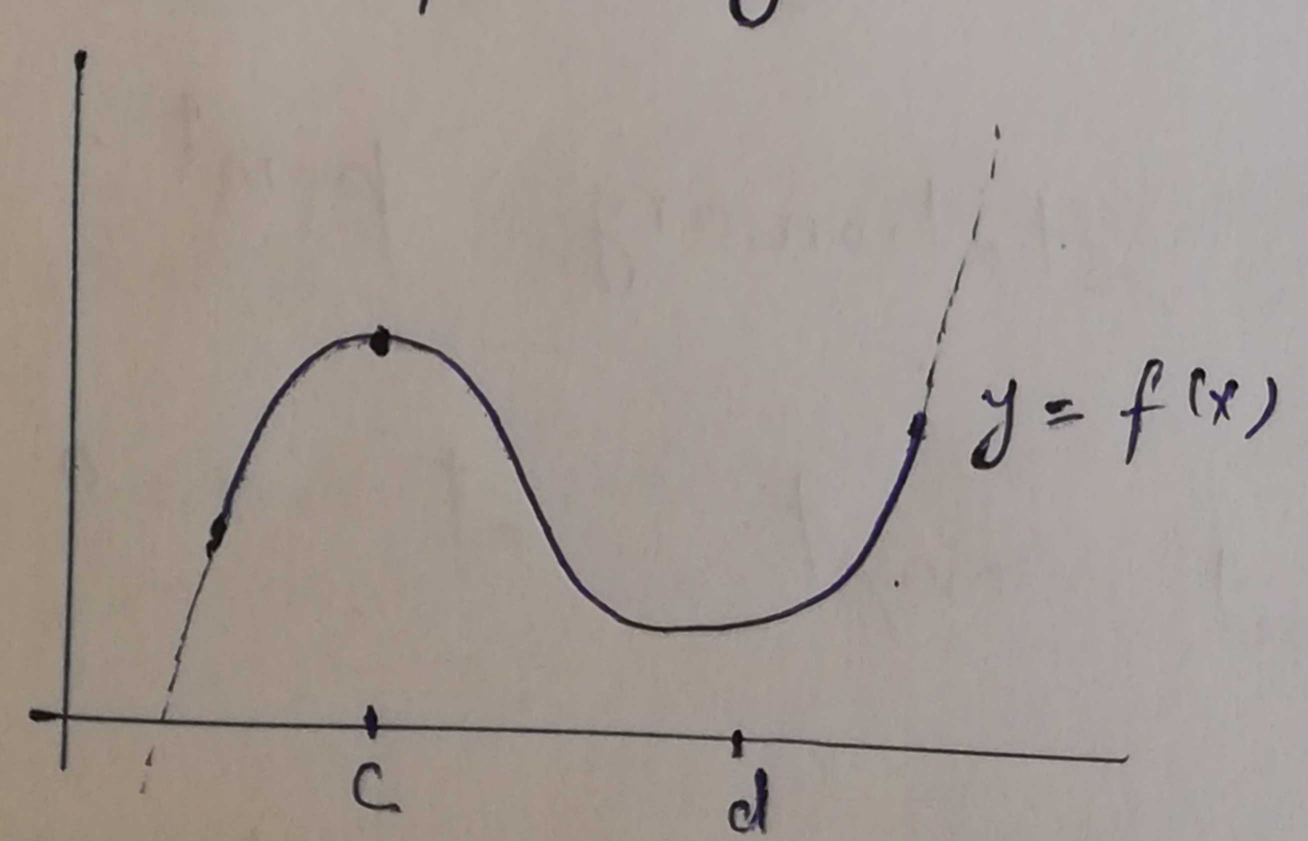
from the above graph,

$f'(c) = 0$

$f'(x) > 0$ in $(-\infty, c)$.

$f'(x) < 0$ in (c, ∞).

Now Consider the following Curve



Here "c" & "d" are critical points

1st derivative is zero at c & d.

i.e

$f'(c) = 0$

$f'(d) = 0$

$\Rightarrow$ "c" & "d" are critical points (stationary pt)

Also $f'(x) > 0$; for all "x" in $(-\infty, c)$.

$f'(x) < 0$; for all x in (c, d).

and again $f'(x) > 0$, for all x in (d, ∞).

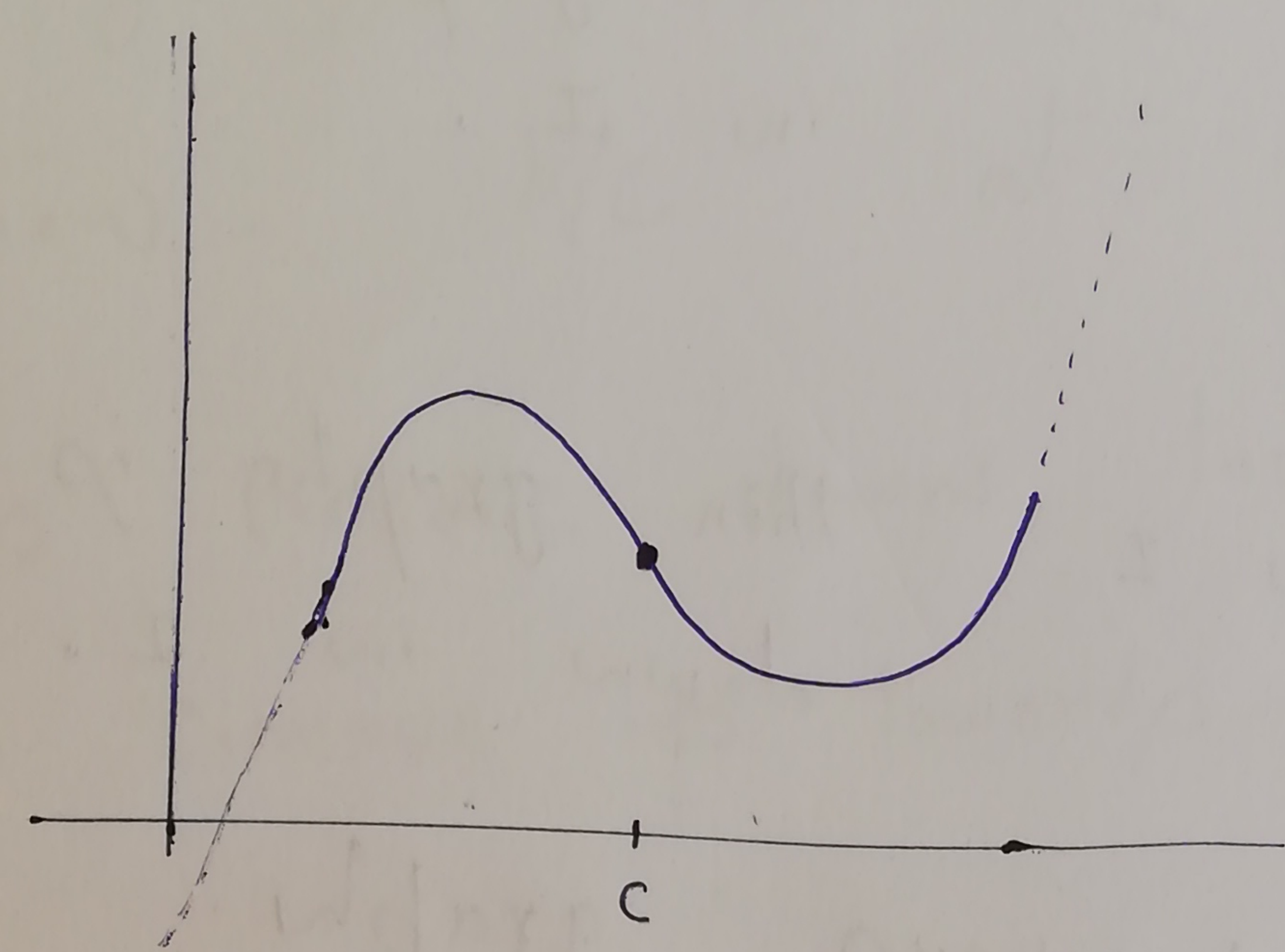


fig (a)

from the above graph

f' is decreasing in $(-\infty, c)$

f' is increasing in (c, ∞).

So we have the following definitions.

if f is decreasing, then $f' < 0$.
if f is increasing, then $f' > 0$.

if f' is decreasing, then $f'' < 0$.
if f' is increasing, then $f'' > 0$.

Concavity :-

let $y = f(x)$ is twice differentiable on an interval I.

1. If $f'' > 0$ on I, then graph is Concave up in I.

2. If $f'' < 0$, on I, then graph is Concave down in I.

So from the previous graph.

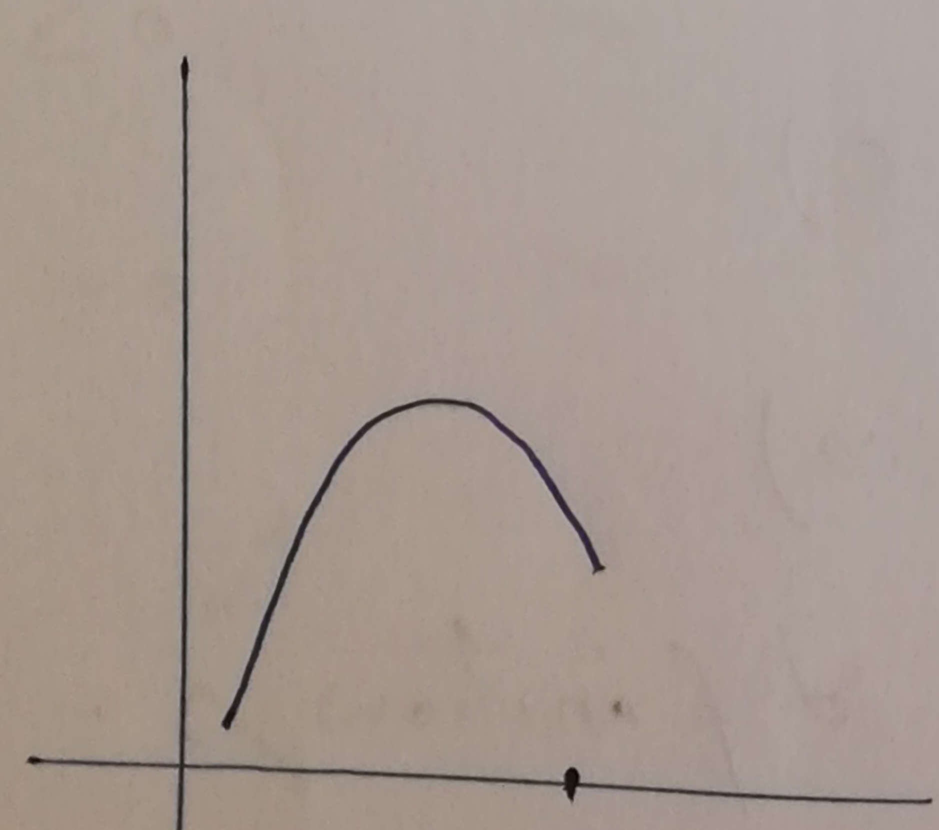


Fig 1

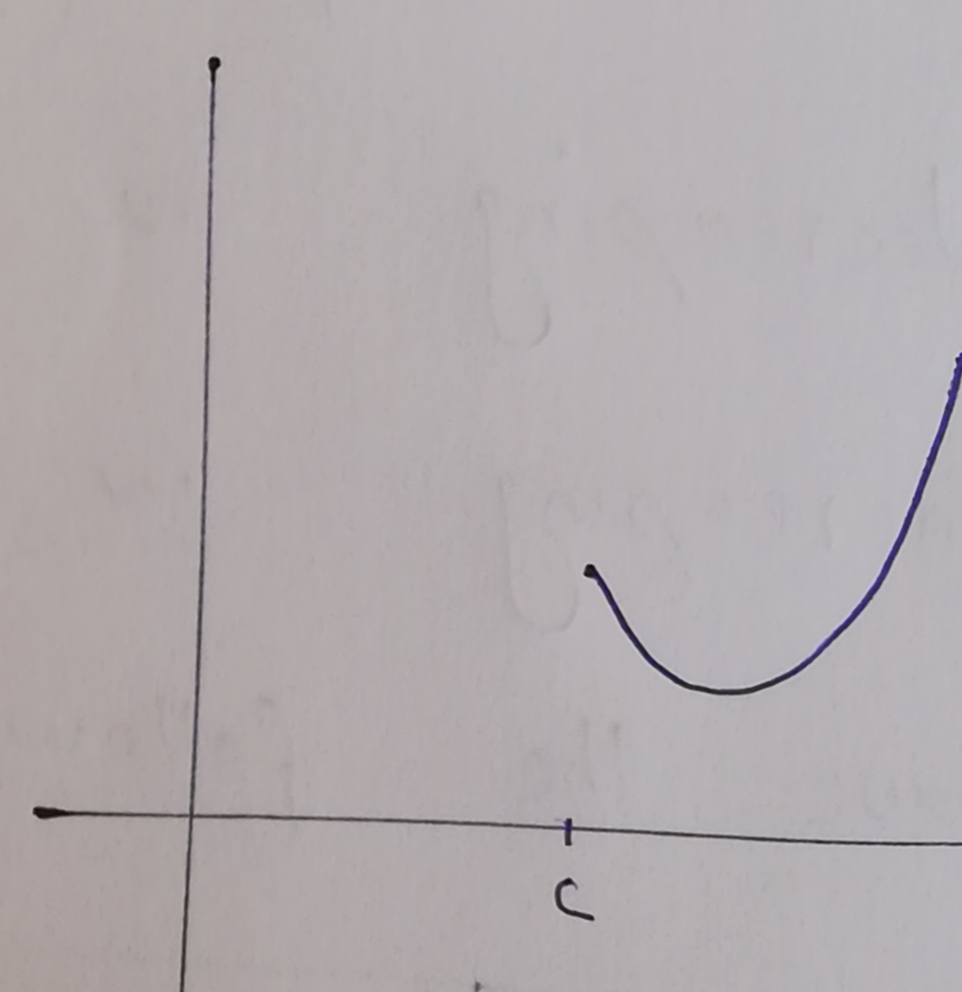


fig 2

In fig 1, f' is decreasing, so $f'' < 0$

So Curve is Concave down.

In fig 2, f' is increasing, so $f'' > 0$

So Curve is Concave up.

Point of inflection :-

The point where Concavity changes is called point of inflection.

In fig (a) "c" is the point of inflection. b/c at "c" Concavity changes

Curve is Concave down towards the left of c.

Curve is Concave up towards the right of c.

i.e

$f'' < 0$ in $(-\infty, c)$.

$f'' > 0$ in (c, ∞)

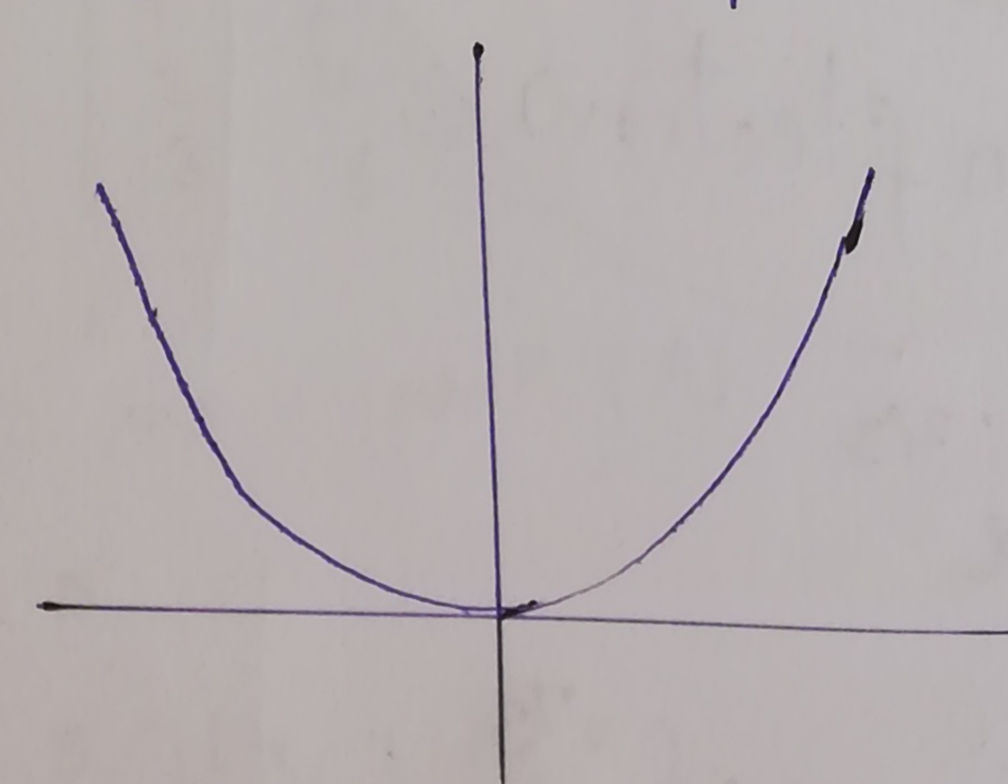
So at "c", $f'' = 0$.

Ex $f(x) = x^2$

This ftn has no point of inflection b/c we Cannot subsitute $f'' = 0$.

$f'' = 2 > 0 \quad \forall x$, so Curve is Concave up in $(-\infty, \infty)$.

At all points on the graph of $y = x^2$, $f'' > 0$.

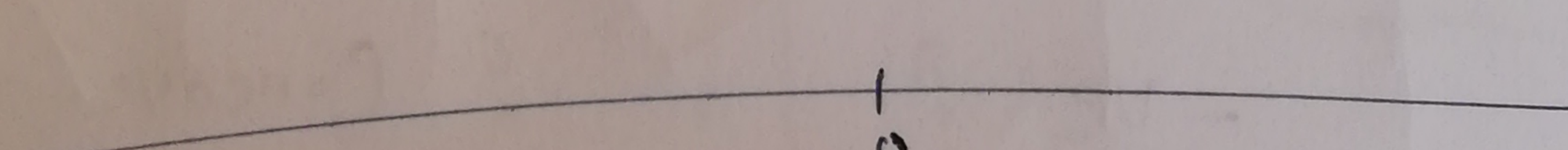


Ex $f(x) = x^3$

find the interval on which Curve is Concave up & Concave down.

S-l $f''(x) = 6x$, subsitute $f'' = 0$

$\Rightarrow \boxed{x = 0}$



Interval	sign of f''	Behaviour of f.
$(-\infty, 0)$	-ve	f is Concave down
$(0, \infty)$	+ve	f is Concave up.

$\Rightarrow x = 0$ is a point of inflection because at $x = 0$ Concavity changes.

Def :

At the point of inflection $(c, f(c))$ either $f''(c) = 0$ or $f''(c)$ does not exist.

$\boxed{\text{Ex}}$ let $f(x) = x^{5/3}$

find point of inflection.

$\boxed{\text{Sol}}$

$$f'(x) = \frac{5}{3} x^{2/3}$$

$$f''(x) = \left(\frac{5}{3}\right)\left(\frac{2}{3}\right) x^{-1/3}$$

$$f''(x) = \frac{10}{9x^{1/3}}$$

2nd derivative does not exist at $x = 0$

Interval	sign of f''	Behaviour of f.
$(-\infty, 0)$	$-ve$	Concave down
$(0, \infty)$	$+ve$	Concave up.

So Concavity changes at $x = 0$

So $x = 0$ is a point of inflection