02-09-2020 Wednesday

→ Easier than cal 1

eq including derivative ↑

3-4 weeks cal 2 then diffrential eq

Infinite Sequence (list of numbers).

$a_1, a_2, \ldots a_n$ in a given order

a_1 is the first term.

⋮

a_n is the n^{th} term.

→ the integer n is called index of a_n.

ie; (1) $\{1, 2, \ldots n\} \Rightarrow a_n = n$ or $\{n\}_{n=1}^{\infty}$

(2) $\{2, 4, 6, \ldots 2n\}$; $a_n = 2n$ (or) $\{2n\}_{n=1}^{\infty}$

(3) $\{12, 14, 16 \cdots 2n\}$; $a_n = 2n+10$ $\{2n\}_{n=6}^{\infty}$

(4) $\{1, \sqrt{2}, \sqrt{3}, \ldots \sqrt{n}\}$; $a_n = \sqrt{n}$; $\{\sqrt{n}\}_{1}^{\infty}$

(i) (ii) (iii)

forms of writing.

(5) $\{0, \frac{1}{2}, \frac{2}{3}, \ldots, 1 - \frac{1}{n}\}$; $a = 1 - \frac{1}{n}$; $\{1 - \frac{1}{n}\}_{n=1}^{\infty}$

where $\{n\}_{n=1}^{\infty} = \{2n\}_{1}^{\infty}$ is domain.

• Infinite Sequence:

An I.S of numbers is a function whose domain is set of +ive integers (Natural Numbers)

(EX 11.1 1-10)

→ Convergence & Divergence of Seq:-

→ Convergent:

A seq $\{a_n\}$ is said to be convergent seq if the number in a seq approach a single (finite) value as index n increases

$(n \to \infty)$

⇒ if approaches to some one value then convergent seq.

eg $a_n = 1 - 1/n$

$\lim_{n\to\infty} a_n = 1 - 0 = 1$, app to 1

eg $a_n = 1/n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1/n = 1/\infty = 0$ app to ze

→ Divergent :

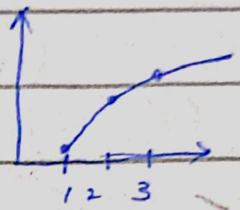
A seq $\{a_n\}$ is a convergent seq if $\lim_{n\to\infty} a_n$ exists otherwise seq will be divergent.

eg:

$a_n = n-1$

$\lim_{n\to\infty} a_n = 1 - 0 = 1$

convergent.

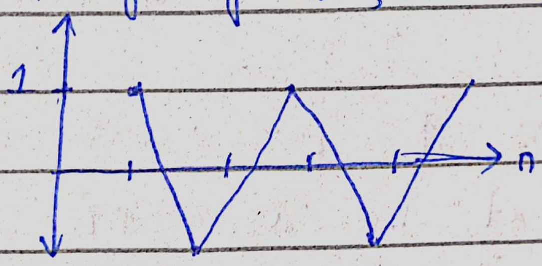


$a_n = (-1)^{n+1}$

$a_n = \lim_{n\to\infty} (-1)^{n+1} = \pm 1$

$\boxed{a_n \text{ is divergent}}$

It is giving even, odd value.



Theorem: Let $\{a_n\}$ & $\{b_n\}$ be seq of real numbers & if

$\lim_{n\to\infty} a_n = A$ (real).

(it means it is convergent)

$\lim_{n\to\infty} b_n = B$ (real)

then following rules hold

(1&2) Sum Rule : Diff Rule

$$\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$$

$$= A \pm B \quad (con)$$

(3) $\lim_{n\to\infty} (a_n, b_n) = AB$

(4) Constant Multiple

$\lim_{n\to\infty} (K a_n) = KA$

Any nonzero n multiple constant of con seq is a convergent

(5) Sum of two divergent seq may or may not be divergent

$a_n = -n = \{-1, -2, \ldots\ldots -n\}$

$b_n = n = \{1, 2, 3, \ldots\ldots n\}$

both are div

$a_n + b_n = n - n = 0$

it is convergent.

(6) Quotient Rule

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{A}{B}$$

Q.

$$a_n = (-1)^{n+1} + 1/n$$

$$\lim_{n\to\infty} a_n = \pm 1 + 0/\infty$$

$$= \pm 1$$

Doesnot exists, $\{a_n\}$ is div.

Theorem: Following six sequences converge to the limits listed below.

① $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

② $\lim_{n\to\infty} (n)^{1/n} = 1$

③ $\lim_{n\to\infty} (x)^{1/n} = 1$ (for $x > 0$)

④ $\lim_{n\to\infty} x^n = \begin{cases} 0 & -1 < x < 1 \\ 1 & x = 1 \end{cases}$

$$\left[\left(1+\frac{x}{n}\right)^{n/x}\right]^x = e$$

⑤ $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$ (Binomial theorem)

⑥ $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for any x

$$\boxed{\lim_{n\to\infty} (-1)^n = \pm 1}$$

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eg:

$$\lim_{n \to \infty} \frac{100^n}{n!} = 0 \quad ; \quad \lim_{n \to \infty} \left(1 + \frac{2}{n}\right)^n = e^2$$

$$\lim_{n \to \infty} (2)^{1/n} = 1 \quad ; \quad \lim_{n \to \infty} (1)^n = 1$$

$\boxed{\text{final answer} = \text{limit of convergence}}$

→ When limit approaches ∞ limit of convergence is thet final answer.

E.X 10.1 → (27-60)

Lec # 3 9-9-2020

"Series"

Series is sum of sequence.

→ Sequence of partial sum is called series.

$\{a_n\}_{n=1}^{\infty}$; $\sum_{n=1}^{n=\infty} a_n$

Sequence — series

$a_1, a_2, a_3, \dots$ — $a_1 + a_2 + a_3 \dots$

↓

denoted by { }

→ The convergence of series is dependent on the convergence of partial sum of sequence.

Partial Sum:-

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots \qquad \vdots$$
$$S_n = a_1 + a_2 + a_3 + \dots + a_n.$$

(P.Q) (10.2 1-6)

Example

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$S_1 = 1 = 2 - \frac{1}{2^0}$$
$$S_2 = 1 + \frac{1}{2} = \frac{3}{2} = 2 - \frac{1}{2^1}$$
$$S_3 = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4} = 2 - \frac{1}{2^2}$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$S_n = 2 - \frac{1}{2^{n-1}}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 2 - \frac{1}{2^{(n-1)}}$$

$$\boxed{\lim_{n\to\infty} S_n = 2}$$

The sum of infinite series is 2.

$(r) = a_n / a_{n-1}$

→ **Geometric Series** (same common ratio)

Series of form

$$a + ar + ar^2 + ar^3 \cdots + ar^n$$

$$= \sum_{n=1}^{\infty} ar^{n-1}$$

in which a & r are real numbers &

$a \neq 0$.

Example

$1 + 1/2 + 1/4 + 1/8 + \cdots 1/2^{n-1}$

Theorem

$$\boxed{\begin{array}{l}\text{if } |r| < 1 \\ \text{The G.S converges.}\end{array}}$$

and it's sum is $\dfrac{a_1}{1-r}$

$$\boxed{\begin{array}{l}\text{if } |r| \geq 1 \\ \text{then G.S diverges.}\end{array}}$$

Example

$1/9 + 1/27 + 1/81 + \cdots$

$r = 1/27 / 1/9 = 1/3$

$|r| < 1$

$\Rightarrow$ series converges.

(10.2) P.Q (15-22)

$S_n = a/1-r$

$= 1/9 / 1-1/3$

$\boxed{S_n = 1/6}$

$\boxed{S_n = \dfrac{a(1-r^n)}{1-r}}$

n^{th} partial term.

Example:

$5 - 5/4 + 5/16 - 5/64 + \cdots$

$r = \frac{\cancel{5}/4}{\cancel{8}} = \cancel{28}/\cancel{4} - 1/4$

$|r| < 1$

$\Rightarrow$ series converges

$$S_n = \frac{a_n}{1-r} = \frac{5}{1+1/4} = \frac{5}{\frac{4+1}{4}}$$

$$S_n = \frac{20}{5}$$

$$\boxed{S_n = 4}$$

Express the repeating decimals 5.232323— as ratio of two integers.

$5.\overline{23}$

$$5.232323 = 5 + 0.23 + 0.0023 + 0.000023$$

$$= 5 + \frac{23}{100} + \frac{23}{(100)^2} + \frac{23}{(100)^3} + \cdots$$

10.2

P.Q

(23-30)

$$= 5 + \frac{23}{100}\left(1 + \frac{1}{100} + \frac{1}{(100)^2} + \cdots\right)$$

$a = 1$

$r = 1/100 < 1$

$$= 5 + \frac{23}{100}\left(\frac{1}{1-\frac{1}{100}}\right)$$

$$= 5 + \frac{23}{100}\left(\frac{1/99}{100}\right) = 5 + \frac{23}{100}\left(\frac{100}{99}\right)$$

$$= 5 + \frac{23}{99} \Rightarrow \boxed{\frac{518}{99}}$$

only for divergence ec#3

11-09-20

may or may not be convergent

(Theorem)

The n^{th} Term test for a Divergent Series $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

$$\lim_{n\to\infty} a_n = \begin{cases} \infty \\ \neq 0 \end{cases} \text{ Diverge}$$

(Test fails)

(Move to next test)

PQ (31–38)

Example:

$$\sum_{n=1}^{\infty} n^2 = \lim_{n\to\infty} n^2 = \infty \quad \text{(diverge)}$$

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right) = \lim_{n\to\infty} \left(1+\frac{1}{n}\right) = 1 \quad \text{(diverge)}$$

Partial sum will outgrow

Note:

If $\sum a_n$ converges & $\sum b_n$ diverges then $\sum(a_n+b_n)$ & $\sum(a_n-b_n)$ both diverge.

(H. Work)

$$\sum_{n=1}^{n=\infty} a_n = \frac{1}{2^{n-1}} \text{ is convergent } = (S_{a_n}=2)$$

$$\sum_{n=1}^{n=\infty} b_n = n^2 \text{ is divergent } = (\infty) \quad S_{b_n}=\infty$$

$$S_{a_n} \pm S_{b_n} = 2+\infty=\infty \;/\; 2-\infty=\infty$$

which shows sum & diff diverges.

* Remember that:

$\sum(a_n+b_n)$ can converge even if both $\sum a_n$ & $\sum b_n$ diverge.

→ <u>For example</u>:

$$\sum a_n = 1+1+1+\cdots$$

$$\sum b_n = (-1)+(-1)+(-1)+\cdots$$

where as:

$$\sum(a_n+b_n) = 0+0+0+\cdots$$

Converges to 0.

Telescoping Series:-

A telescoping series is any series where nearly every term cancels with a preceeding or following terms.

<u>Example</u>:

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$S_k = \left(1-\tfrac{1}{2}\right)+\left(\tfrac{1}{2}-\tfrac{1}{3}\right)+\left(\tfrac{1}{3}-\tfrac{1}{4}\right)+ \cdots - \left(\tfrac{1}{k}-\tfrac{1}{k+1}\right)$$

$$S_K = 1 - \frac{1}{K+1}$$

$$\lim_{K\to\infty} S_K = 1 - \lim_{K\to\infty} \frac{1}{K+1}$$

$$\boxed{\lim_{K\to\infty} S_K = 1}$$

$\Rightarrow$ The sum is. $\underline{1}$.

(P-Q
39-44)
(45-52)

(23-09-2020)

→ Integral test for ~~sequence~~ :-

let $\{a_n\}$ be a sequence of positive term. Suppose that $a_n = f(n)$, where f is continuous, positive & decreasing function for all x. then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)\,dx$ both converge or diverge.

(10.3) practice

• Example $\sum_{n=1}^{\infty} 1/n^2$

It is valid for positive seq. alternative sign is not applied. on integral test for seq.

→ check inc/dec. if decreasing then apply test otherwise not applicable.

a_n $\boxed{f'(x) < 0}$

→ Check continuity (Not undef on any point)

Ans is unique (convergent)

for infinity (divergence)

why fail for inc fun?

$$\sum_{n=1}^{\infty} 1/n^2$$

suppose $f(x) = 1/x^2$

continuous at boundry (in limit)

$f'(x) = -2/x < 0$ dereasing.

and +ive integer so this law is applicable.

$$f(x) = \int_1^{\infty} 1/x^2 \, dx$$

$$= \lim_{b \to \infty} \int_1^{b} 1/x^2 \, dx$$

$$= -\lim_{b \to \infty} \left[\frac{x^{-1}}{1} \right]_1^b$$

$$= -\lim_{b \to \infty} \left(1/b - 1/1 \right)$$

$$= -(0-1)$$

$$\boxed{\sum_{n=1}^{\infty} 1/n^2 = 1 \quad = \text{convergent}}$$

→ Graph pe end behaviour

→ $f'(x) = 0$ mein whole domain check.

(eg) $\sum_{n=1}^{\infty} n e^{-n^2}$

$$\int_1^{\infty} x e^{-x^2} = \lim_{b\to\infty} \int_1^b x e^{-x^2}$$

$$= \lim_{b\to\infty} \int_1^b \frac{1}{2}\left(e^{-x^2} \cdot x\right) dx$$

$$= \lim_{b\to\infty} \int_1^b -\frac{1}{2}\left(e^{-x^2} - 2x\right) dx$$

$$= \lim_{b\to\infty} -\left[e^{-x^2}\right]_1^b$$

$$= -\lim_{b\to\infty} e^{-b^2} - e^{-1}$$

$$= -\lim_{b\to\infty} \frac{1}{e^{b^2}} - \frac{1}{e}$$

$$= 0 + \frac{1}{e}$$

$$\sum_{n=1}^{\infty} n e^{-n^2} = \frac{1}{e} = \text{conv.}$$

⇒ P series test : always be $1/n^p$

$\frac{1}{(n-1)^p}$ → not possible

$$\sum_{n=1}^{\infty} 1/n^p = 1/1^p + 1/2^p + 1/3^p + \dots$$

conditions for p-series test. $\boxed{p \text{ is a num}}$

if $p > 1$ (convergent)

if $p \leq 1$ (divergent)

→ proof of p-series is most imp.

proof :

case #1 : where $p > 1$

$$\sum_{n=1}^{\infty} 1/n^p \quad , \quad a_n = f(x)$$

$$f(x) = 1/x^p$$

$$\int_1^{\infty} 1/x^p \, dx$$

$$\lim_{b \to \infty} \int_1^b x^{-p} \, dx$$

$$\lim_{b \to \infty} \left[\frac{x^{-p+1}}{1-p} \right]_1^b$$

$$= \frac{1}{1-p} \lim_{b \to \infty} \left(x^{-(1-p)} \right)_1^b$$

$$= \frac{1}{1-p} \lim_{b \to \infty} \left(\frac{1}{x^{(1-p)}} \right)_1^b$$

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$$= \frac{1}{1-p} \lim_{b\to\infty} \left[\frac{1}{x^{b-1}} - \frac{1}{x^{1-1}} \right]$$

$$= \frac{1}{1-p} \left(0 - \frac{1}{1} \right)$$

$$= \frac{1}{p-1} \quad \text{where } p>1$$

This series converges by integral test. We emphasize that the sum of the p-series is not $1/(p-1)$. The series converges but we donot know the value it converges to.

for $p \leq 1$

$$= \frac{1}{1-p} \lim_{b\to\infty} \left[x^{-p+1} \right]_1^b$$

$$= \frac{1}{1-p} \lim_{b\to\infty} \left[x^{1-p} \right]_1^b$$

$$= \frac{1}{1-p} \lim_{b\to\infty} \left[x^{1-b} - x^{1-1} \right]$$

$$= \frac{1}{1-p} \left[x^{1-\infty} - 1 \right]$$

$$= \frac{1}{1-p} \left[\infty - 1 \right]$$

$$= \infty = \text{diverges}$$

case 3:

for $p=1$

$$\frac{1}{1-p} / \cancel{1}$$

$$\sum_{n=1}^{\infty} 1/n \rightarrow \text{Divergent Harmonic Series.}$$

dirvege harmonic series for $p=1$.

Note:

The slowness with which partial sums of harmonic series approaches infinity is impressive. for example it takes more than 178 million terms of harmonic series to move the partial sums beyond 20.

Basic Comparison Test / Direct Comp. Test

Let $\sum a_n$ & $\sum b_n$ be two series:

Greater + conv.

(i) $\sum a_n \leq \sum b_n \Rightarrow$ Must be conv.

↳ if conv. then greater else lessert div honi chaheye.

(ii) $\sum a_n \geq \sum b_n \Rightarrow$ Must be div

↳ choti honi chaheye an se.

اس سے یہ
بنانا ہے

c.x $\quad a_n = \dfrac{n+2}{n^2-n}$, $b_n = \dfrac{n}{n^2} = 1/n$

$a_n \rightarrow \dfrac{2}{1}, \dfrac{5}{6}, \dfrac{6}{12}$

$b_n \rightarrow \dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}$

$a_n \geq b_n$

b_n is div by P-Series

a_n is div by B.C.T.

Making b_n :

$$a_n = \frac{\cos^2 n}{n^{3/2}}$$

$$b_n = 1/n^{3/2},\ n/n^{3/2},\ \frac{n^2}{n^{3/2}}$$

check & see which goes best according to rules.

1.

- The Comparison Test -

Let $\sum a_n$, $\sum c_n$ & $\sum d_n$ be series with nonnegative terms, suppose for some integer N.

$$d_n \leq a_n \leq c_n \quad \text{for All} \quad n > N$$

(a) If $\sum c_n$ converges then $\sum a_n$ also converges.
(b) if $\sum d_n$ diverges the $\sum a_n$ also diverges.

$$c_n \leq a_n \leq b_n$$

Small + div. ↲ ↳ Greater + Conv

(2)

(E.X 10.4) 30-09-2020

Limit Comparison test:

→ Suppose that $a_n > 0$ & $b_n > 0$ for all $n > N$ (+ve term series.)

(i) $\lim_{n\to\infty} a_n/b_n = c > 0$ then $\sum a_n$ & $\sum b_n$ both con or div. (depend on b_n.)

(ii) $\lim_{n\to\infty} a_n/b_n = 0$ & $\sum b_n$ con then $\sum a_n$ conv. (must)

(iii) $\lim_{n\to\infty} a_n/b_n = \infty$ & $\sum b_n$ div then $\sum a_n$ div. (must)

(Eg)

$$a_n = \frac{2n+1}{n^2+2n+1}$$

$$b_n = 1/n$$

$$a_n/b_n = \frac{2n+1}{n^2+2n+1} / 1/n = \frac{2n^2+n}{n^2+2n+1}$$

$$\lim_{n\to\infty} \frac{2n^2+n}{n^2+2n+1}$$

Apply L·Hopital rule

$$\frac{4n+1}{2n+2} \quad \left(\frac{\infty}{\infty}\right)$$

again

$$\lim_{n\to\infty} 4/2 = 2$$

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$$\lim_{n \to \infty} \frac{a_n}{b_n} = 2 > 0$$

Now check $\sum b_n$

using p-seri—

$$\sum_{n=1}^{\infty} b_n = 1/n$$

$\Rightarrow$ diverges by P serie.

$\Rightarrow \sum_{n=1}^{\infty} a_n$ diverges by Limit Comp Test.

$\longleftrightarrow$

→ Ratio Test:

absolut conv
we can do any type of test.

Let $\sum a_n$ be any series, then

mostly applicable to factorial seri— $\frac{2n!}{n!}$

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = P$$

(i) Converge absolutely if $P < 1$. → whole domain

(ii) Diverge if $P > 1$

(iii) inconclusive if $P = 1$ (fail)

(eg) $a_n = \frac{2^n + 5}{3^n}$

$a_n \to$ given
$a_{n+1} \Rightarrow n = n+1$
→ drive

$$|a_n| = \left| \frac{2^n + 5}{3^n} \right|$$

(eg) an =

$$|a_{n+1}| = \left|\frac{2^{n+1}+5}{3^{n+1}}\right|$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{2^{n+1}+5}{3^{n+1}} \times \frac{3^n}{2^n+5}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2^n \cdot 2+5}{\cancel{3^n}\times 3} \times \frac{\cancel{3^n}}{2^n+5}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2^n \cdot 2+5}{3(2^n+5)}\right| = \lim_{n\to\infty}\left|\frac{2+5/2^n}{3(1+5/2^n)}\right|$$

Apply limit

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 2/3 < 1$$

$\Rightarrow$ converge Absolutely by Ratio test.

$\leftrightarrow$

→ Root Test:

Let Σa_n be any series, then

Apply where whole power is "n"

$$\lim_{n\to\infty} |a_n|^{1/n} = P$$

(i) converge absolutely if $P<1$

(ii) Diverge if $P>1$

(iii) inconclusive if $P=1$ (fail)

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(eg)

$$a_n = (-1)^n \left(\frac{2n^2+4}{n^3+1} \right)^n$$

$$|a_n| = \left| \frac{2n^2+4}{n^3+1} \right|^n$$

$$|a_n|^{1/n} = \left| \frac{2n^2+4}{n^3+1} \right|$$

$$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \left| \frac{2n^2+4}{n^3+1} \right| \qquad \infty/\infty$$

$$= \lim_{n\to\infty} \left| \frac{4n}{3n^2} \right| \qquad \text{L.H.L}$$

$$= \lim_{n\to\infty} \left| \frac{4}{3n} \right|$$

$$= 0 < 1$$

$\Rightarrow$ converge Absolutely by Root test.

E.K b.5

3-10-2020.

Not applicable to +ve or -ve series.

→ only applicable to $(-1)^n$ sign series.

→ Alternating Series Test

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 + \cdots \quad b_n > 0$$

satisfies (i.i) $(-1)^n$, must have alternate test.

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim_{n\to\infty} b_n = 0$ (nth term)

then the series is convergent.

if $\lim_{n\to\infty} b_n \neq 0$ divergence

↓

By div test.

⇒ div by nth term test.

- Seq must be decreasing. (i) next term must be less then previous term.

(eg) $(-1)^n \, 1/n$

(i) $(-1)^n$ ✓

(ii) $1/n \geq 1/n+1$ ✓

$1/n$ is div but $(-1)^n \, 1/n$ is conv

(iii) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} 1/n = 0$

converge by alternating series test.

- $1/n$ is abs of $(-1)^n (1/n)$

$$\boxed{\begin{aligned} \cos n\pi &= (-1)^n \\ \sin(n+1/2)\pi &= (-1)^n \end{aligned}}$$

Abs lenay ke bad Abs test apply nahi hoga baki sb apply hongay.

→ A series $\sum a_n$ is called absolutely convergent if the series of absolute values $\sum |a_n|$ is convergent.

→ A series $\sum a_n$ is called conditionally conv if it is convergent but not abs convergent.

→ Check if abs con, div or cond conv

$$a_n = \frac{(-2)^n}{n^2}$$

$$a_n = \frac{(-1)^n (2)^n}{n^2}$$

$$|a_n| = \left|\frac{2^n}{n^2}\right|$$

$$|a_{n+1}| = \left|\frac{2^{n+1}}{(n+1)^2}\right|$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{2^{n+1}}{(n+1)^2} \times \frac{n^2}{2^n}\right|$$

$$= \left|\frac{\cancel{2^n}\cdot 2}{(n+1)^2} \times \frac{n^2}{\cancel{2^n}}\right|$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{2n^2}{(n+1)^2}\right| = \left|\frac{4n}{n^2+2n+1}\right|$$

$a_n = (-1)^n$?
C.A ?
C.C ?
div ?

↓ (by any test)

$|a_n| \to C$
$|a_n| \to D$

if(C)
(conv abs)
(end)

else if(D)
$a_n \Rightarrow$ A.S.T

↙ C → (C.C) (Result)
↘ D → (div) (result)

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 2 > 1$$

$\Rightarrow$ Div by ratio test.

Now A.S.T

$$a_n = \frac{(-1)^n \; 2^n}{n^2}$$

All cond are satisfied.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2^n}{n^2}$$

$$= \lim_{n\to\infty} \frac{2^n \ln(2)}{2n} \quad (\infty/\infty)$$

$$= \lim_{n\to\infty} \frac{2^n (\ln 2)^2}{2}$$

$$\lim_{n\to\infty} a_n = \infty$$

$\Rightarrow$ A.S.T implies that a_n is, divergent series