Date: __/__/2020

19F-0228

Q1.

$$x^2y + x\sin y + \frac{y}{x} = 1$$

$$x^2y + x\sin y + \frac{y}{x} = 1$$

Taking derivative

$$x^2\frac{d}{dx}y + y\frac{d}{dx}x^2 + x\frac{d}{dx}(\sin y) +$$

$$\sin y\frac{d}{dx}x + x\frac{d}{dx}y - y\frac{d}{dx}$$

$$x^2$$

$$\frac{x^2\frac{dy}{dx} + 2xy + x\cos y + \sin y + x\frac{dy}{dx} -}{x^2}$$

$$\frac{x^4\frac{dy}{dx} + 2x^3y + x^3\cos y + x^2\sin y + x\frac{dy}{dx}}{x^2}$$

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$$x^4\frac{dy}{dx}+x\frac{dy}{dx}=-2x^3y-x^3\cos y-x^2\sin y+y$$

$$\frac{dy}{dx}(x^4+x)=-2x^3y-x^3\cos y-x^2\sin y+y$$

$$\frac{dy}{dx}=\frac{-(2x^3y+x^3\cos y+x^2\sin y-y)}{x^4+x}$$

ii)

$$y=x^2\ln x+e^{-x}$$

$$y=x^2\ln x+e^{-x}$$

$$\frac{dy}{dx}(y)=\frac{d}{dx}(x^2\ln x+xe^{-x})$$

$$\frac{dy}{dx}=\frac{d}{dx}(x^2\ln x)+\frac{d}{dx}(xe^{2})$$

$$\frac{dy}{dx}=x^2\frac{1}{x}+2x\ln x+e^{-x}x(-1)+e^{-x}(1)$$

$$\frac{dy}{dx}=x+2x\ln x-xe^{-x}+e^{-x}$$

Date: / /20

19F0228
Q1 (c)

$$y = \frac{\sqrt{a^2+x^2} - \sqrt{a^2-x^2}}{\sqrt{a^2+x^2} + \sqrt{a^2-x^2}}$$

$$y = \frac{\sqrt{a^2+x^2} - \sqrt{a^2-x^2}}{\sqrt{a^2+x^2} + \sqrt{a^2-x^2}}$$

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{\sqrt{a^2+x^2} - \sqrt{a^2-x^2}}{\sqrt{a^2+x^2} + \sqrt{a^2-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sqrt{a^2+x^2} - \sqrt{a^2-x^2}}{(a^2+x^2)-(a^2-x^2)}\right)$$

$$= \frac{d}{dx}\left(\frac{\sqrt{a^2+x^2}^{\,2} + (\sqrt{a^2+x^2})^2 - 2(\sqrt{a^2+x^2})(\sqrt{a^2-x^2})}{a^2+x^2-a^2+x^2}\right)$$

$$= \frac{d}{dx}\left(\frac{a^2+x^2+a^2-x^2-2\sqrt{a^2+x^2}}{2x^2}\right)$$

$$= \frac{d}{dx}\left(\frac{2a^2-2\left(\sqrt{a^2+x^2}\right)\left(\sqrt{a^2-x^2}\right)}{2x^2}\right)$$

$$= \frac{2x^2\frac{d}{dx}\left(2a^2-2\sqrt{a^2+x^2}\sqrt{a^2-x^2}\right) - \frac{2a^2-2\left(\sqrt{a^2+x^2}\right)}{\left(\sqrt{a^2-x^2}\right)}}{\left(2x^2\right)^2}$$

$$= 2x^2 \cdot 4a - 2\left[\sqrt{a^2+x^2}\,\frac{d}{dx}\,a^2-x^2 + \sqrt{a^2-x^2}\frac{d}{dx}\sqrt{a^2+x^2}\right.$$

$$\frac{d}{dx}\sqrt{a^2-x^2} \cdot \left(2a^2-2\left(\sqrt{a^2+x^4}\right)\left(a^2-x\right)\right)^4$$

$$\frac{8x^2 - \frac{2a\sqrt{a^2+x^2}}{\sqrt{a^2-x^2}} - \frac{2\sqrt{a^2+x^2}}{\sqrt{a^2-x^2}} +}{\frac{2a\sqrt{a^2-x^2}}{\sqrt{a^2+x^2}} + \frac{2x\sqrt{a^2-x^2}}{a^2+x^2} - 8a^2x -}$$

$$1.\ x^4$$

19F-0228 Q2.

$$y = \frac{5x}{2x^2+4}$$

Quotient rule

$$= \frac{5(2x^2+4) - 4x(5x)}{(2x^2+4)^2}$$

$$= \frac{10x^2+20-20x^2}{(2x^2+4)^2}$$

$$= \frac{-10x^2+20}{(2x^2+4)^2}$$

Putting $\sqrt{3}$

$$m = \frac{-10(\sqrt{3})^2+20}{(2(\sqrt{3})^2+4)^2}$$

$$m = \frac{-10(3)+20}{(2\times3+4)^2}$$

$$m = \frac{-30+20}{(6+4)^2} = \frac{-10}{(10)^2} = -\frac{1}{10}$$

Date: __/__/2020 19F-0228 M T W T F S

$$2y - \sqrt{3} = -\frac{1}{10}x + \frac{\sqrt{3}}{10}$$

$$2y = -\frac{1}{10}x + \frac{\sqrt{3}}{10} + \sqrt{3}$$

$$y = \left(-\frac{1}{10}x + \frac{\sqrt{3}}{10} + \sqrt{3}\right)\frac{1}{2}$$

19F -0228

Q3

$$y = 8x^3 + 9x^2 + 3x - 4$$

a)

$$f'(y) = 24x^2 + 18x + 3$$

$$f''(y) = 48x + 18$$

$48x + 18 = 0$

$16x + 6 = 0$

$8x + 3 = 0$

$8x = -3$

$$\boxed{x = -\frac{3}{8}}$$

$-\infty \quad -3/8 \quad \infty$

b)

$\cancel{f(-3/8) = 8(-\tfrac{3}{8})^3} + 9(-\tfrac{3}{8})^2 + 3(-\tfrac{3}{8}) - 4$

$\cancel{= (-3)^3 +}$

$(-\infty \quad -\frac{3}{8}]$	-2	$8(-2)+3 = -13$ $= 13$
$(-\frac{3}{8}, \infty]$	2	

Date: ___/___/2020

$(-\infty, -3/8) < 0$ concave down

$\left(-\frac{3}{8}, \infty\right) > 0$ concave up

$-\frac{3}{8}$ point of inflextion

c)

$f'(x) = 0$

$24x^2 + 18x + 3 = 0$

$8x^2 + 6x + 1 = 0$

$8x^2 + 4x + 2x + 1 = 0$

$(4x+1)(2x+1) = 0$

$x = -\frac{1}{4}, \quad x = -\frac{1}{2}$

$f''(x) = 48x + 18$

$= 48\left(-\frac{1}{4}\right) + 18$

$= -12 + 18$

$= 6 > 0$

$f(x)$ has relative min value at $x = \frac{-1}{4}$

&

$f''(x) = -6 < 0$ at $x = \frac{-1}{2}$

so $f(x) = -6 < 0$
at

at $x = \frac{-1}{2}$

so $f(x)$ has relative at $x = \frac{-1}{2}$

and relative max value at

$x = \frac{-1}{2}$

$f\left(-\frac{1}{2}\right) = 8\left(-\frac{1}{2}\right)^3 + 9\left(-\frac{1}{2}\right)^2$

$+ 3\left(-\frac{1}{2}\right) - 4 = \frac{-17}{4}$

Date:___/___/2020 M T W T F S

and relative min value at $x = -\frac{1}{4}$

$$f\left(-\frac{1}{4}\right) = 8\left(-\frac{1}{4}\right)^3 + 9\left(-\frac{1}{4}\right)^2 + 3\left(-\frac{1}{4}\right)$$

$$= 8\left(-1/64\right) + 9/16 - \frac{3}{4} - 4$$

$$= -\frac{1}{8} + \frac{9}{16} - \frac{3}{4} - 4$$

$$= \frac{-69}{16}$$

Q4

This function satisfies all ~~values~~ conditions of mean value theorem

$f(x) = \ln(2x+1) \quad [0, 2]$

$f(0) = 0$

$f(2) = \ln 5$

$f'(x) = \frac{2}{2x+1} = \frac{2}{2c+1}$

$f'(c) = \frac{2}{2c+1}$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$f'(c) = \frac{\ln 5 - 0}{2 - 0} = \frac{\ln 5}{2}$

$\frac{2}{2c+1} = \frac{\ln 5}{2} \Rightarrow \frac{4}{\ln 5} - 1$

$= 2c \Rightarrow \frac{4 - \ln 5}{2\ln 5} = c$