

21/03/2022

Box Plot: (Presentation)

↳ also called five points summary

Plot -  $\downarrow$ 

⇒ Measure of Central Tendency &amp; Dispersion

↳ 5 points are: (values)

①  $Q_1$ 

② Median

③  $Q_3$ ④  $Q_1 - 1.5 IQR$ ⑤  $Q_3 + 1.5 IQR$ 

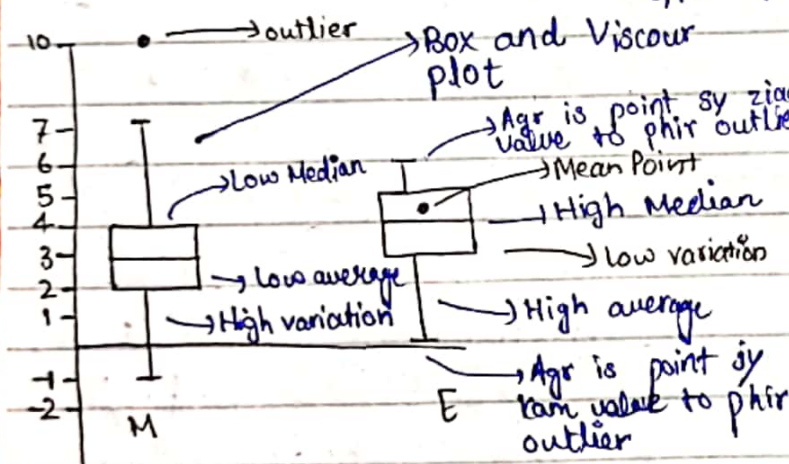
Suppose data is,

1  
2  
3  
4  
5

then,

①  $Q_1 = 2$ 

② Median = 3

③  $Q_3 = 4$ ④  $Q_1 - 1.5 IQR = -1$ ⑤  $Q_3 + 1.5 IQR = 7$ 

↳ It is for Quantitative Data

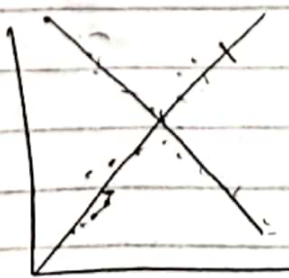
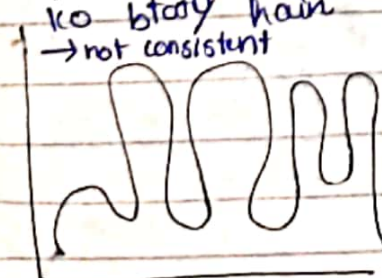
⇒ samajhna

↳ For Descriptive Data

↳ better for skewed-data plot

↳ asky 5 points summary statistics main aata hai isky 5 points ko present kr rahi hai jo data ki characteristics ko btati hai

→ not consistent



↳ If comparison is not possible then absolute - For example, marks in matric 1021 plus CGPA (3.1) can't be compared.

In order to compare we should convert them to % (Relative).

⇒ currency is relative

Weight:  $\bar{X} = 61\text{kg}$ ,  $SD = 2\text{kg}$ Height:  $\bar{X} = 5.6\text{ft}$ ,  $SD = 1.1\text{ft}$ 

→ STD is absolute measure (not comparable)

→ Agr inky units same bhi krain phir bhi comparison possible nhi

Coefficient of Variation: (For STD)

$$(normalized) C.V = \frac{S.D (kg)}{\bar{X} (kg)} \times 100 \text{ (Unitless)}$$

↳ ku k data k centres change

normalizing w.r.t mean, its effect ending

$$C.V = 3.278\% \text{ (Weight)}$$

→ In terms of percentage

Variation ↑ then consistency ↓

up player ko team main rkha jo consistent hoo

$$C.V = 19.64\% \text{ (Height)}$$

→ Height ki variation ziada hai

normalizing GPA

$$\frac{3.1}{4} \times 100$$

normalizing matric marks with total 100 eradicate its effect

$$\frac{1021}{1100} \times 100$$

↳ Relative measurement for STD is called Coefficient of Variation (C.V).

Coefficient of 4QR: (For IQR)

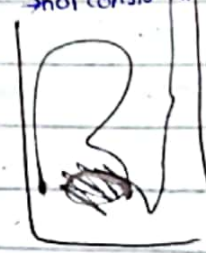
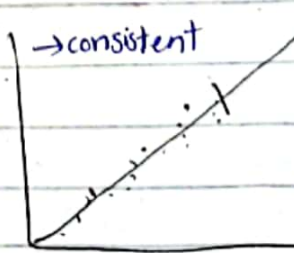
↳ just like coefficient of variation

$$\text{Coefficient of 4QR} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

→ total sy divide normalized krny k Jiye

Coefficient of Range: (For Range)

$$\text{Coefficient of Range} = \frac{\text{Max} - \text{Min}}{\text{Max} + \text{Min}}$$



→ normal data main mean aur median same ata hai issy Jiye hum unky variation ko is k through study kr skty



Question: Agr coin toss kia aur Head aya to phir ghr main khelna aur Tail aya to bahir khelny jana aur agr stand ho gaya to phir pharhai krha -  
 Ans: Probability of studying = 0 (Because coin standing does not exist in sample space)

$$y = \text{runit}(100, 10, 100)$$

hist(y)

summary(y)

mean(y)

04/02/2022

⇒ Probability

↳ Death (certain)

↳ Death kab aye gi (uncertain)

↳ Kal kia hoo skta (possibility)

↳ Kal kia hoo ga (no one knows)

↳ Uncertainty is the measure of chances of uncertainty or Quantification of uncertainty

↳ Probability started in 17<sup>th</sup> when rich people wanted to know their chances of winning gambling

So they hired intelligent mathematicians

↳ 1 bar ek rocket launch hooty hy tabch ho gaya uski jab probability nikali aye to phir wo

0.001% thi tab sy har field main almost 40% probability use hooi

↳ Patient chances of recovery (probability)

↳ Rain Forecasting (probability of rain)

Algorithm → Probability

→ Agr clothes wash kiye to phir kia chances k barish hoo

Experiment: (Pre-planned)

↳ Any planned activity is called activity experiment

↳ Observe krna k kon sun raha lecture aur kon nhi (pre-planned)

Random Experiments

In order to convert Experiment to Random Experiment the

① Outcomes are known

② Occurrence are unknown

↳ Dice Throwing ↳ Card nikalna ↳ Bowling

↳ Probability hamesha random experiment ki nikalti hai

↳ Random experiment hoony k liye experiment hona zroori hai ↳ coin tossing

Sample Space: ↳ Jahan per sb kuch available hoo

Gather all possible results of random experiment in a set

$S = \{H, T\}$  → Toss

$S = \{1, 2, 3, 4, 5, 6\}$  → Dice

→ Mathematical Probability

→ Mathematical Concept

Outcome:

Each element of the sample space is called an outcome

↳ Result of Random Experiment is called an outcome

Event: ↳ Agr batting team ki taraf to 6 main interested hoongy hum

↳ Outcomes in which we are interested is called an event

↳ Yeh humara event hai

↳ Head aya to main jeeta to phir Head mera event hai

2 Approaches to measure Probability:

Job data available  
 nhi to phir

Subjective Objective

↳ Biased expert opinion

↳ Neutral expert measuring chances of ATK vs IND (40%, 60%, 20%)

⇒ Dono main sy koi bhi best nhi ye scenario per depend krta

Algorithm

Probability

new factors

2020

Events Probability:

$0 \leq P(A) \leq 1$

↳ Probability of any event must be between 0 and 1. Agr isky ilawa to phir galat question solve

$\sum_{i=1}^n P(A_i) = 1$

Null Event →  $P(\cdot) = 0$

Sure Event →  $P(\cdot) = 1$

→ Tamam events ki probability ka sum equal to 1 hai

(certain)  $P(S) = 1$

(uncertain)  $P(\bar{S}) = 0$

↳ Agr head aya ya Tail aya to main jeetunga warna main haarunga

↳ Agr sample space main sb daqyo to phir ( $\bar{S}$ ) main us k ilawa sb zero



# Relative Frequency Definition of Probability

06/04/2022

$$P(A) = \frac{\text{no. of events}}{\text{total no. of outcomes}} = \frac{n(A)}{n(S)} \rightarrow \text{Formula (1)}$$

↳ sample space

→ A coin is tossed

$$S = \{H, T\}$$

$$P(H) = \frac{1}{2} = 0.5 \quad \text{or} \quad P(T) = \frac{1}{2} = 0.5$$

Chances of H = 50% or Chances of Getting T =  $\frac{0.5 \times 100}{100} = 50\%$

Diff. between Probability & Chances:

- ↳ Probability is always btw 0 & 1
- ↳ Chances are in terms of percent (%)

↳ A dice is rolled

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(1) = \frac{1}{6}$$

$$P(2) = \frac{1}{6}$$

$$P(6) = \frac{1}{6}$$

Limit Theory lehti hai k

agr 2 baar toss kro

to phir 1 baar Head

aur 1 baar Tail aye

ga lekin ye practically

possible nhi lekin agr

1000 baar toss kro aur

500 baar Head aye aur 500

baar Tail to phir ye cheez

possible hai hence

Limit Theory

Limit: Rain chances

ye tab valid hogi jab n

ziada hojaye

→ Dice is thrown

6: {1, 2, 3, 4, 5, 6}

except 6: {1, 2, 3, 4, 5}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

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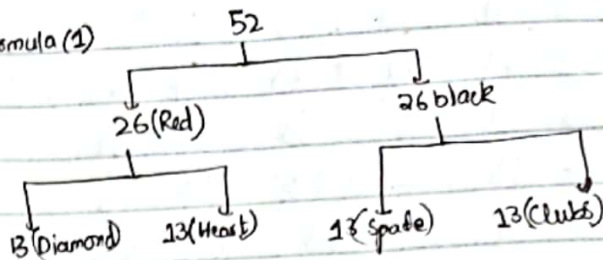
6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}

6: {1, 2, 3, 4, 5, 6}



2  
3  
4  
5  
6  
7  
8  
9  
10  
J  
Q  
K  
A

Face cards

$$\rightarrow \text{Probability of getting Red} = \frac{26}{52}$$

$$" " " 2 = \frac{4}{52}$$

$$" " \text{Face cards} = \frac{12}{52}$$

↳ Not face cards

$$P(F) = \frac{12}{52}$$

$$P(\bar{F}) = 1 - \frac{12}{52} = \frac{40}{52}$$

↳ Agr 80% chances pass hoongi to phir agr 100 baar exam dia to phir 80 baar hum pass hoongi (agr wohi conditions hoongi un dino main)

Limit: Rain chances

ye tab valid hogi jab n ziada hojaye

→ Dice is thrown

6: {1, 2, 3, 4, 5, 6}

$$P(S) = P(A \cup \bar{A}) = P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = (1 - P(A))$$

$$1 - P(\bar{A}) = 1 - P(A) \rightarrow \text{Formula (2)}$$

Future Events:

Basic LOGIC GATES:

① AND

② OR

→ NOT is actually negation

→ Agr dono courses main pass hoona to probability kam

Event A, B

$$P(A \text{ and } B) = P(A \cap B) \rightarrow \times$$

$$P(A \text{ or } B) = P(A \cup B) \rightarrow +$$

→ Simulation example of tossing by using size of n (changing) → R Studio

→ we have to increase our sample space for experiments

→ Stock Market Prediction

1 over main 1 size

$$0.2 \times 0.3 = 0.06$$

$$0.2 + 0.3 = 0.5$$

→ Agr dono courses main sy 1 course main pass hoona to probability ziada



⇒ Sample Space of 2 dices thrown together

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

Probability of getting 1 time 6 =  $\frac{11}{36}$

" " " 6 on both =  $\frac{1}{36}$

### Properties of Events:

If Event A & B

Mutually Exclusive | Not Mutually Exclusive  
↳ Mutually inclusive

#### Independent:

No effect on occurrence of one event on another. Humari degree pass aur England ka President change

#### Dependent:

Affects on occurrence of one event on another. Agr 2 students late hoon aur dono ki convince same.

	H	T
H	HH	HT
T	TH	TT

Atleast 1 head =  $\frac{3}{4}$

Exactly 1 head =  $\frac{2}{4}$

(0 or 1 Head) Atmost 1 head

↳ Probability & possibility are 2 different terms

→ Possibility are ways of happening  
→ Agr possibility pta to phir linked divide kr ky probability bati

→ Possibility (Combination): agr class main sy 2 buchon ko select krna to our possible combinations

Mutual → common

#### ① Mutually Exclusive:

Two events A & B are said to be mutually exclusive if they don't occur together. A coin is tossed - A dice is thrown - A card is extracted - Coin toss krta to phir Head ayega ya Tail lekin dono nhi aa skty

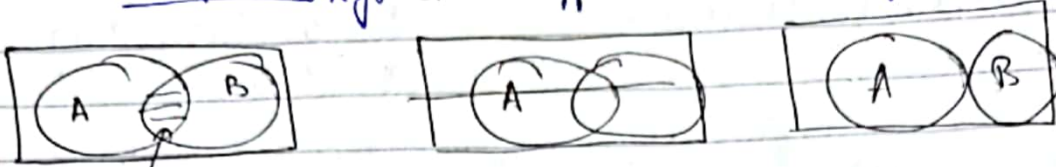
#### ② Non-mutually Exclusive:

Two events A & B are said to be non-mutually exclusive if they can occur together. Balso called mutually inclusive

↳ ~~Bad~~ aur bacarish  
Dhoop  
↳ Family functions  
↳ May be dependent or independent



M.E: Agr 2 students class main late aye aur 1 B section ka aur 1 C section ka (not occurring together)  
 N.M.E: Agr 2 students A section main late aye then 4'm interested in dependent or independent  
 8/04/2022  
 Dependent: Agr dono 1 convince per aye  
 Independent: Agr dono different crties sy aye



$A \cap B = \emptyset$  then mutually exclusive event

↳ Agr (non ME) to phir Dependent ya Independent Agr ~~non~~ ME to phir Dependent ya Independent ka nhi bta skty

M.E: (not occurring together)

$$P(A \cup B) = P(A) + P(B) \quad (M.E)$$

Example:

A: Red Card

B: Face Card

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} \\ &= 0.615 \end{aligned}$$



⇒ no common parts Example: (

A: 6 on 1st dice

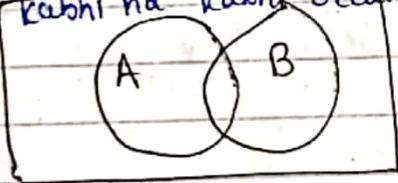
B: 6 on second dice

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} \end{aligned}$$

Non-M.E: Kissi bhi situation main a akhty aany main interested ya occur ho skty (possible)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

↳ Ho skta us situation main akhty na hoorn jekin kabhi na kabhi occur ho skty hain



→ common part 2 baar aata to balance krny k liye 1 baar

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} \end{aligned}$$

↳ Question main M.E or N.M.E subtract kr identity krna aur us hisab Independent: sy formula lagana

$$P(A \cap B) = P(A) \times P(B)$$

Dependent:

Conditional Probability & Bayes Theorem

Example:

S = {1, 2, 3, 4, 5, 6}

A: 6 on dice (Event)

B: 1 on dice (Event)

M.E:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_k) = \sum_{i=1}^k P(A_i)$$

Independent: (Non-M.E)

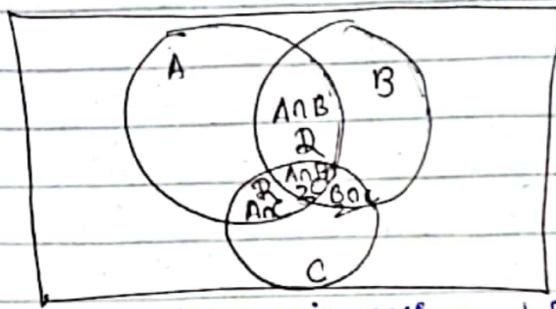
$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$P(A_1 \cap A_2 \cap \dots \cap A_k) = \prod_{i=1}^k P(A_i)$$

$$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$



↳ kitni missile fire hoone k target ko fire krny ki probability 100 percent or maximum considering current factors

↳ Security Purposes

↳ Decision making in target firing

$$P(A_1 \cup A_2 \cup \dots \cup A_k) = \sum_{i=1}^k P(A_i) - \sum \sum P(A_i \cap A_j) + \sum \sum \sum P(A_i \cap A_j \cap A_k) - \dots + \sum \sum \sum \dots \sum P(A_i \cap A_j \cap \dots \cap A_k)$$

Counting Techniques:

Example 2a:

$$m \times n$$

$$n_1 \times n_2 \times n_3 \times n_4 \times \dots \times n_k$$

$$10 \times 3 = 30$$

Example 2b:

$$3 \times 4 \times 5 \times 2$$

Dice throwing: (3 times)

$$6 \times 6 \times 6 = 6^3 \rightarrow \text{not actual formula}$$

Coin tossing five times:

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

Debit Card:

Password:

$$10 \times 10 \times 10 \times 10 = 10,000$$

$$0-9 = 10$$

$$a-z = 26$$

$$A-Z = 26$$

Special Character: 5

Total: 67

$$67 \times 67 \times 67 \times 67 \times 67 \times 67 \times 67 \times 67 = 67^8$$

$$67^8$$

$$10,000 \times 60 \times 60 \times 24 \times 30 \times 12 = 1305 \text{ years}$$

↳ 1 second minutes  
main computer 10K  
loop chikla

shows  
↓  
days months



11/04/2022

## Combination:

class main sy a bundo ko select krna hai

## Permutation:

A & B select krne aur pehla president aur dusra vice president

$m \times n$

$n_1 \times n_2 \times n_3 \dots \times n_n$

$10 \times 3$

$$2 \times 2 \times 2 = 2^3$$

$$6 \times 6 \times 6 \times 6 = 6^5$$

$$10 \times 10 \times 10 \times 10 = 10000 \quad 0-9 = 10$$

obj-3  
arrangement = 2

AB BA  
AC CA  
BC CB

$${}^3P_2 = 6 \rightarrow \text{order matters}$$

## Combination

obj-3 A B C

arrangements = 2

$$AB = BA$$

$$AC = CA$$

$$BC = CB$$

$${}^3C_2 = 3$$

$\rightarrow$  order does not matter

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$= \frac{10!}{6!}$$

$${}^{10}P_4 = \frac{10!}{(10-4)!} = \frac{n!}{(n-r)!}$$

$${}^nP_r = \frac{n!}{(n-r)!}$$

$${}^nC_r = \frac{n!}{(n-r)!} \rightarrow \text{For Duplicates removal}$$

01111  $\rightarrow$  Different passwords

$$72 \times 72 \times 72 \times 72 \times 72 \times 72 \times 72 \times 72 = 72^8$$

$$10 = 0-9$$

$$26 = A-Z$$

$$26 = A-Z$$

10 = Special Characters

72

$$10,000 \times 60 \times 60 \times 24 \times 30 \times 12$$

$\rightarrow$  If computer can run 10k

loops  $\rightarrow$  sequences in one second

$$= 2321.9 \text{ years}$$

(Password changes after every 10 seconds)  $\rightarrow$  Google

$$0300 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^7$$

$$10^7 \times 10 = 10^8$$

(10<sup>8</sup>)

4 hackers PC run 10k sequences (loops) in 1 second then  $\frac{10000}{10000} = 1 \text{ sec. to crack password}$

LICENSE PLATE:

$$① \underline{F} \times \underline{S} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10}$$

$$② \underline{F} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10}$$

$$③ \underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \quad \begin{matrix} A \\ B \end{matrix}$$

$${}^nP_n = \frac{n!}{(n-n)!} = n!$$

$\rightarrow$  Factorial is special case of Permutation

$${}_5P_5 = 5!$$

## Factorial:

$\rightarrow$  cheezin jahan per one by one decrease hoo ohi hooli hai

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

object = 5  
arrangements = 5

$\rightarrow$  Agr bus main banda aya aur seats class  
 $\rightarrow$  students & per behtina start

$$10 \times 9 \times 8 \times 7$$

object = 10  
arrangement = 4

$$2! = 2 \times 1$$

$\rightarrow$  Agr kuch nhi to phir arrangement ki khat hum whi karain gay - Agr exist krta to phir 1 arrangement hogi hy

Deck of cards:

$\rightarrow$  Digit Arrangements

$$\frac{1}{52} \text{ then } \frac{1}{51} \text{ then } \frac{1}{50}$$

$\rightarrow$  Factorial of -ve number is not possible

1!: Arrangement can't be zero. There must be one arrangement itself

$n_1 \times n_2 \times \dots \times n_k \rightarrow$  not included

Ross 8th Edition

$${}^{10}C_3 = \frac{10!}{(3)(10-3)!} = \frac{10 \times 9 \times 8 \times \dots \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)} = 120$$

start up  
way  
number that  
pr divide

$$\frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

Example 3b

$$6! \times 4!$$

Q#5

(a)  $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

(b) (1, 1, 0, 0, 0)

(1, 1, 1, 1, 1)

~~(1, 0, 0, 0, 1)~~

(0, 0, 0, 0, 0)

(0, 0, 0, 0, 1)

(0, 0, 0, 1, 0)

(0, 0, 0, 1, 1)

(0, 0, 1, 0, 0)

(0, 0, 1, 0, 1)

(0, 0, 1, 1, 0)

(0, 0, 1, 1, 1)

(0, 1, 0, 0, 0)

(0, 1, 0, 0, 1)

(0, 1, 0, 1, 0)

(0, 1, 0, 1, 1)

0, 1, 1, 0, 0  
0, 1, 1, 0, 1  
0, 1, 1, 1, 0  
0, 1, 1, 1, 1  
1, 0, 0, 0, 0  
1, 0, 0, 0, 1  
1, 0, 0, 1, 0  
1, 0, 0, 1, 1  
1, 1, 0, 0, 0  
1, 1, 0, 0, 1  
1, 1, 0, 1, 0  
1, 1, 0, 1, 1  
1, 1, 1, 0, 0  
1, 1, 1, 0, 1  
1, 1, 1, 1, 0  
1, 1, 1, 1, 1

A+B+C

A:  $1 \times 1 \times 2 \times 2 \times 2 = 2^3$

B:  $2 \times 2 \times 1 \times 1 \times 2 = 2^3$

C:  $1 \times 2 \times 1 \times 2 \times 1 = 2^2$

$$= A+B+C - AB - AC - BC + ABC = 16$$

Ch#02

Q#6  $\rightarrow 14$



⇒ Discrete & continuous random variable and their probability distribution

13/04/2022

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10$$

$$26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7$$

$$26P_3 \times 10^4$$

$$\frac{n!}{n!(n-1)! \dots 1!} \Rightarrow \text{not included in paper}$$

Q#06

$$\frac{2 \times 3}{6} = \frac{1}{3}$$

$$\begin{array}{cc} 04 & 14 \\ 09 & 19 \\ 03 & 13 \end{array} = 2$$

$$(C) 1 \times 3 = 3$$

$$B^c \cup A = 5$$

Q#07

$$(a) 6 \times 6 \times 6 \times \dots \times 6 = 6^{15} \xrightarrow{\text{Person}} \text{Repetition}$$

7(b)

$$6^{15} - 3^{15} =$$

Event

$$P(\overline{\text{FUGUS}})$$

$$= 1 - (P(\text{FUGUS}))$$

(Conditional Probability)

$$P(A|B)$$

Q#08

$$(a) P(A) = 0.3$$

$$P(B) = 0.5$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$(b) P(A \cap \bar{B}) = P(A) \Rightarrow \text{not common } P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$(c) 0$$

Q#09

A: American  
V: Visa

$$P(A) = 0.24$$

$$P(V) = 0.61$$

$$P(A \cap V) = 0.11$$

$$P(A \cup V) = P(A) + P(V) - P(A \cap V)$$

If A &amp; B are independent

$$P(A|B) = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

If A &amp; B are dependent:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$P(A \cap B) = P(B|A) \cdot P(A)$$



18/07/2022

Atleast 2:

2,3,4,...

Atmost 30:

..., 28, 29, 30

More upto dated  
Posterior Probability

$$P(Pw) = 0.90$$

$$P(Pw/India) = 0.6$$

$$P(Pw/Zimbabwe) = 0.99$$

Information from data  
current information

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Agar budal Gaojay  
to barish ki  
probability

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

prior info  
normalization  
constant

Probability of A such that  
B has occurred

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

India ke khilaf  
match khela ho aur  
match jeeta hoo

Pakistan ke  
team too per jeeta  
ke chances

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Independent:

Naive Bayes Classifier

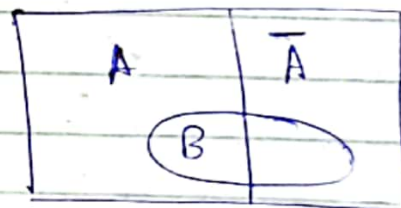
$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$P(Pw)$  1st over

Given that sign  
if events are Dependent:



$$B = (A \cap B) \cup (\bar{A} \cap B)$$

Total Probability Theorem

$$P(B) = \frac{P(A \cap B) + P(\bar{A} \cap B)}{P(A \cap B) + P(\bar{A} \cap B)}$$

Prob marks

Except Prob total marks

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B) \cdot P(B) \rightarrow \text{I}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \rightarrow P(A \cap B) = P(B|A) \cdot P(A) \rightarrow \text{II}$$

$$P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Bayes Theorem

Bayes Theorem  
with prior probability

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

$$P(A \cap B) + P(\bar{A} \cap B)$$

$$P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})$$

Example:

Machine Learning (Words Prediction)

This is  
was

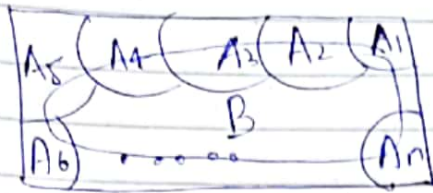
$$P(was|This)$$

$$P(is|This)$$

$$P(A \cap B) = P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$$

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|\bar{B}) \cdot P(\bar{B})}$$





$$P(B) = P(A_1 \cap B) + \dots + P(A_n \cap B)$$

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \dots + P(B|A_n) \cdot P(A_n)$$

$$= \sum_{i=1}^n P(B|A_i) \cdot P(A_i)$$

$$P(A_i|B) = \frac{P(B|A_i) \cdot P(A_i)}{\sum_{i=1}^n P(B|A_i) \cdot P(A_i)}$$

Example 2a:

F: French

C: Chemistry

A: A grade

$$P(A \cap C) =$$

$$P(A \cap C) = P(A|C) \cdot P(C)$$

$$= \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

$$P(A \cap F) = P(A|F) \cdot P(F)$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(A|C) = \frac{2}{3}$$

$$P(A|F) = \frac{1}{2}$$

$$P(C) = \frac{1}{2}$$

$$P(F) = \frac{1}{2}$$

Example 3a (Part I)

↳ Homework

Example 2a:

less than  $x$  hours =  $Lx$

Full hours =  $F$

$$P(Lx) = \frac{x}{2}$$

$$P(L_{0.75}) = \frac{0.75}{2}$$

$$P(\overline{F} | \overline{L_{0.75}}) = \frac{P(\overline{F} \cap \overline{L_{0.75}})}{P(\overline{L_{0.75}})} = \frac{1/2}{0.625} = 0.8$$

$$P(\overline{L_{0.75}}) = 1 - P(L_{0.75})$$

$$= 1 - \frac{0.75}{2}$$

$$= 0.625$$

$$P(\overline{F} \cap \overline{L_{0.75}}) = P(\overline{F})$$

$$\begin{array}{c} x=0.75 \\ 1 \\ 0.75 \end{array} \quad \left( \frac{1}{2} \right)$$

$$x=1$$

$$P(\overline{F}) = P(\overline{L_1})$$

$$= 1 - 1/2 = 1/2$$



20/04/2022

Example: 3a (I):

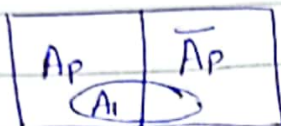
$A_p$ : Accidental Poome.

$A_1$ : Accident in 1 year

$$P(A_1 | A_p) = 0.4$$

$$P(A_1 | \bar{A}_p) = 0.2$$

$$P(A_p) = 0.3 \Rightarrow P(\bar{A}_p) = 0.7$$



$$(A_1 \cap A_p) + (A_1 \cap \bar{A}_p)$$

→ Total Probability Theorem

$$P(A_1) = P(A_1 | A_p) \cdot P(A_p) + P(A_1 | \bar{A}_p) \cdot P(\bar{A}_p)$$

$$P(A_1) = (0.4)(0.3) + (0.2)(0.7)$$

$$P(A_1) = 0.26$$

Part II:

$$\begin{aligned} P(A_p | A_1) &= \frac{P(A_1 | A_p) \cdot P(A_p)}{P(A_1)} \\ &= \frac{(0.4)(0.3)}{(0.26)} \\ &= 0.46 \end{aligned}$$

Example: 3c:

K: knowing

C: correctly

$$P(K | C) = P(C | K) \cdot P(K)$$

$$P(C | K) \cdot P(K) + P(C | \bar{K}) \cdot P(\bar{K})$$

$$1 \times 0.7$$

$$(1 \times 0.7) + \left(\frac{1}{4} \times 0.3\right)$$

$$P(K) = 0.7$$

$$P(\bar{K}) = 0.3$$

$$P(C | \bar{K}) = \frac{1}{m} = \frac{1}{4}$$



Examples 3d:

Y : Has Disease

N : Don't have disease

+

-

$$P(+|Y) = 0.95 \Rightarrow P(-|Y) = 0.05 \checkmark$$

$$P(+|N) = 0.01 \Rightarrow P(-|N) = 0.99$$

$$P(Y) = 0.005$$

$$P(Y|+) = \frac{P(+|Y) \cdot P(Y)}{P(+|Y) \cdot P(Y) + P(+|N) \cdot P(N)}$$
$$= (0.05)(0.$$

$$= 0.32$$