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Probability & Statistics
Assignment # 02

{Laws of Probability}

{1- Addition Law}

{* Not-Mutually Exclusive Events}

{1}

From a Pack of 52 Playing Cards, a Card is drawn randomly. Find the Probability of getting it red or face card.

Solution :=

Let, $n(S)$ = exhaustive cases
$n(A)$ = no. of events of red card
$n(B)$ = no. of events of face cards

$n(S) = 52$, $n(A) = 26$, $n(B) = 12$

→ since there are 6 face cards which are red (3 face cards of heart + 3 face cards of diamond)

→ $n(A \cap B) = 6$

Now, by using law of addition of Probabilities of non-mutually exclusive events.

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$P(A \cup B) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$$

$$P(A \cup B) = \frac{26+12-6}{52}$$

$$P(A \cup B) = \frac{32}{52} \Rightarrow \frac{8}{13} \Rightarrow \boxed{0.61}$$

$$\boxed{P(A \cup B) = 61\%}$$ **Ans.**

* So, there is 61% chance that the card may be red or face card, if a card is drawn randomly from a Pack of 52 playing cards.

Q(2)

Two unbiased dice are rolled. Find the Probability that the sum of the numbers of two faces is either divisible by 3 or divisible by 4.

Solution :=

1st Dice \ 2nd Dice	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$n(S) = 36$

$A = \{((1,2), (2,1), (1,5), (2,4), (3,3), (4,2), (5,11),$
$(3,6), (4,5), (5,4), (6,3), (6,6))\}$

$n(A) = 12$

$B = \{((1,3), (2,2), (3,1), (2,6); (3,5), (4,4), (5,3)$
$(6,2), (6,6))\}$

$n(B) = 9$

∵ (6,6) both in A and B

$n(A \cap B) = 1$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$= \frac{12}{36} + \frac{9}{36} - \frac{1}{36}$$

$$P(A \cup B) = \frac{20}{36} \Rightarrow \frac{5}{9} \Rightarrow \boxed{0.55}$$

$\boxed{P(A \cup B) = 55\%}$ **Ans.**

* So, there is 55% chance that the dice is either divisible by 3 or 4, if two unbiased dices are rolled.

Q(3)

If a card is drawn from a deck, use the addition rule to find the probability of obtaining an ace or a heart.

Solution :=

Let, A be the event that the card is an ace, and H the event that it is a heart.

→ Since there are 4 aces, and 13 hearts in the deck. $n(S) = 52$, $n(A) = 4$, $n(H) = 13$

$$P(A) = \frac{4}{52} \text{ and } P(H) = \frac{13}{52}$$

→ Intersection of two events consists of only one card, the ace of heart:

$$P(A \cap H) = \frac{1}{52}$$

Now, find the probability:

$$P(A \cup H) = P(A) + P(H) - P(A \cap H)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} \Rightarrow \boxed{0.3076}$$

$$\boxed{P(A \cup H) = 30.76\%}$$ **Ans.**

* So, there is 30.76% Probability that a card drawn from a deck either be an ace or heart.

(* Mutually Exclusive Events)

Q(4)

One card is drawn at random from the numbered cards, numbered from 10 to 21. Find the probability that the card may be prime number or even numbered card.

Solutions:=

Let, $n(S)$ = exhaustive cases

$n(A)$ = no. of events of prime number cards

$n(B)$ = no. of events of even number cards

$S = \{10, 11, 12, \ldots, 21\}$

$n(S) = 12$

$A = \{11, 13, 17, 19\}$

$n(A) = 4$

$B = \{10, 12, 14, 16, 18, 20\}$

$n(B) = 6$

Now

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{12} \Rightarrow \frac{1}{3} \Rightarrow \boxed{0.33}$

$\boxed{P(A) = 33\%}$

$P(B) = \frac{n(B)}{n(S)} = \frac{6}{12} \Rightarrow \frac{1}{2} \Rightarrow \boxed{0.5}$

$\boxed{P(B) = 50\%}$

Now, using law of addition of Probability for mutually exclusive Events:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{3} + \frac{1}{2} - 0$$

$$= \frac{5}{6} = \boxed{0.83}$$

$\boxed{P(A \cup B) = 83\%}$ **Ans.**

* So, there is 83% chance that when one card is draw at random from the numbered cards from 10 to 21 then the card may be Prime or even number.

Q(5)

From a Pack of 52 cards, a card is drawn at random. What is the Probability of getting a card with face or an ace?

Solutions:-

Let, $n(S)$ = exhaustive cases

$n(A)$ = no. of events of face card

$n(B)$ = no. of events of ace

$n(S) = 52, \quad n(A) = 12, \quad n(B) = 4$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - 0$$

$$P(A \cup B) = \frac{12}{52} + \frac{4}{52}$$

$$P(A \cup B) = \frac{16}{52} \Rightarrow \frac{4}{13} \Rightarrow \boxed{0.3076}$$

$$\boxed{P(A \cup B) = 30.76\,\%}$$ **Ans.**

* So, there is 30.76% chance that when a card is drawn at random from a Pack of 52 cards, then the cards may be with face or an ace.

(2. Complement Law)

(1)

Find the probability that when roll a dice we get a number different than 1 ~~or~~ and 6.

Solution:-

- First find $P(A)$ being A getting the number 1 and 6.

$$P(A) = \frac{2}{6}$$

$\boxed{P(A) = 0.33} \Rightarrow \boxed{P(A) = 33\%}$

now find $P(A')$

$P(A') = 1 - P(A)$

$= 1 - 0.33$

$P(A') = \boxed{0.67}$

$\boxed{P(A') = 67\%}$ **Ans.**

* So, there is 67% probability of getting a number different from 1 and 6 when a dice is rolled.

Q(2)

A young man goes to buy a new phone model which has a 10 different color collection, but he does not like two of those colors, if when he buys the phone they give him a random color. What are the chances of him getting a color that he actually like?

Solution :=

- First we find $P(C)$ being C getting one of the colors he didn't like

$P(C) = \frac{2}{10}$

$\boxed{P(C) = 0.2} \Rightarrow \boxed{P(C) = 20\%}$

Then, find $P(C')$

$P(C') = 1 - P(C)$

$= 1 - 0.2$

$\boxed{P(C') = 0.8}$

$\boxed{P(C') = 80\%}$ • **Ans.**

* So, there is 80% chance that he will get a color he like when they give him a random ~~pho~~ color phone model.

(3)

A teacher is asking to some students to read for the class the lesson of the day, if the teacher is going to put to read to 7 of his 40 students, but most of the students are very shy to read for the class. What ~~is~~ are the chances for each student to not read out loud?

Solution :-

• First we find $P(R)$ where R is reading for the class.

$P(R) = \frac{7}{40}$

$\boxed{P(R) = 0.175} \Rightarrow \boxed{P(R) = 17.5\%}$

Now, we find $P(R')$

$P(R') = 1 - P(R)$

$= 1 - 0.175$

$\boxed{P(R') = 0.825}$

$\boxed{P(R') = 82.5\%}$ **Ans.**

* So, there is 82.5% chances that each of the 7 student will not read out loud the lesson given by the teacher.

(3. Conditional Law)

(1)

Two dies are thrown simultaneously and the sum of the numbers obtained is found to be 7. what is the probability that the number 3 has appeared at least once?

Solution :=

Sample Space $= n(S) = 6 \times 6 = 36$ events

Event A indicates the combinations in which 3 has appeared atleast once.

Event B indicates the combinations of numbers which sum upto 7.

$A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (1,3), (2,3), (5,3), (6,3), (4,3)\}$

$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$P(A) = \frac{11}{36} \Rightarrow \boxed{0.305} \Rightarrow \boxed{P(A) = 30.5\%}$

$P(B) = \frac{6}{36} \Rightarrow \boxed{0.166} \Rightarrow \boxed{P(B) = 16.6\%}$

$A \cap B = 2$

$\boxed{P(A \cap B) = \frac{2}{36}}$

Now, Apply Conditional Probability Formula:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2/36}{6/36} = \frac{1}{3} \Rightarrow \boxed{0.33}$$

$\boxed{P(A/B) = 33.3\%}$ **Ans.**

* So, there is 33.3% chance that the number 3 will appeared atleast once when two dies are throw simultaneously.

Q(2)

In a group of kids, if one is selected at random the probability that he/she like ~~orgames~~ oranges is 0.6, the Probability that he/she like oranges AND apples is 0.3. If a kid, who likes oranges is selected at random, what is the Probability that he/she also like apples?

Solution:

Let,

Event O = kids likes oranges,

Event A = kids likes apples

Given,

$P(O) = 0.6$

$P(A \text{ and } O) = 0.3$

$$P(A/O) = \frac{P(A \text{ and } O)}{P(O)} = \frac{0.3}{0.6} = \boxed{0.5}$$

$\boxed{P(A/O) = 50\%}$ **Ans.**

* So, there is 50% chance that the kid likes apples if selected at random in a group of kids.

(3)

The results of a Survey of a group of 100 People who bought either a mobile phone or a tablet from any of two Brands A and B is shown in table below:

	Mobile	Tablet	Total
A	20	10	30
B	30	40	70
Total	50	50	100

If a Person is selected at random from the group, what is the probability that he/she

a) bought brand B?

b) bought a mobile phone from brand B?

c) bought a mobile phone given that he/she bought brand B?

Solutions:=

Let,

Event M = bought a mobile phone

Event T = bought a tablet

Event A = bought brand A

Event B = bought brand B

(a)

70 people bought brand B out of the total 100:

$$P(B) = \frac{70}{100} = \boxed{0.7} \Rightarrow \boxed{P(B) = 70\%}$$

Ans.

(b)

30 people bought a mobile phone from brand B out of the total 100.

$$P(M \text{ and } B) = \frac{30}{100} = \boxed{0.3}$$

$$\boxed{P(M \cap B) = 30\%}$$ **Ans.**

(c)

There are 30 mobile phones out of total of 70 bought from brand B.

$$P(M/B) = \frac{P(M \text{ and } B)}{P(B)} = \frac{0.3}{0.7} = \frac{3}{7}$$

$$P(M/B) = \boxed{0.4285} \Rightarrow P(M/B) = \boxed{42.85\%}$$

Ans.

* So, there is 70% chance that the person selected random from a group bought brand B, 30% chance that he/she bought a mobile Phone from brand B and

42.85% chance that he/she bought Mobile phone given that he/she bought brand B.

(4)

The results of Survey of a group of 100 people having insurances with a certain Company are as follows. 40% have both home and car insurances with the Company. The Probability that person selected at random from this group, has a car insurances is 0.7. What is the probability that a person selected at random has a home insurance knowing that he has a car insurance?

Solution :=

Let,

Event H = people with home insurance

Event C = People with car insurance

We are given:

$P(C) = 0.7$, $P(H \text{ and } C) = 0.4$

Now, apply conditional probability law:

$$P(H/c) = \frac{P(H \text{ and } c)}{P(c)} = \frac{0.4}{0.7} = \boxed{0.5714}$$

$$\boxed{P(H/c) = 57.14\,\%}$$ **Ans.**

*So, there is 57.14 % chance that the person Selected at random has a home insurance Knowing That he has car insurance.

« (5) »

You toss a fair coin three times :

a) what is the Probability of three heads, HHH?

b) what is the probability that you observe exactly on heads?

c) Given that you have observed at least one heads, what is the Probability that you observe at least two heads?

Solutions :=

Let assume that the coin tosses are independent

« (a) »

$$P(HHH) = P(H) \cdot P(H) \cdot P(H) = (0.5)^3 = \frac{1}{8}$$

$P(HHH) = \boxed{0.125}$

$\boxed{P(HHH) = 12.5\%}$ **Ans.**

(b)

To find the probability of exactly one head:

$P(\text{exactly one heads}) = P(HTT \cup THT \cup TTH)$

$= P(HTT) + P(THT) + P(TTH)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8} = \boxed{0.375}$

$\boxed{P(\text{exactly one heads}) = 37.5\%}$

(c)

Let, A_1 be the event that you observe at least one heads, and A_2 be the event that you observe atleast two heads.

then,

$A_1 = S - \{TTT\}$, and $P(A_1) = \frac{7}{8}$

$A_2 = \{HHT, HTH, THH, HHH\}$

and $P(A_2) = \frac{4}{8}$

Now, we can write,

$$P(A_2|A_1) = \frac{P(A_2 \cap A_1)}{P(A_1)}$$

$$= \frac{P(A_2)}{P(A_1)} = \frac{4}{8} \cdot \frac{8}{7} = \frac{4}{7} = \boxed{0.5714}$$

$\boxed{P(A_2/A_1) = 57.14\ \%}$ **Ans.**

So, there is 12.5% chance that three heads ~~if~~ will come up if a toss is happen three times, 37.5% chance that you obseve ~~ort~~ exactly one heads, and 57.14% chance that you observe at least two heads.

{4. Multiplication Law}

{Independent Events}

{1}

Suppose you take out two cards from a standard Pack of cards one after another, without replacing the first card. What is Probability that the first card is the ace of spades, and the second card is a heart?

Solution :=

The two events are dependent events because the first card is not replaced.

→ There is only one ace of spades in a deck of 52 cards: so:

$$p(\text{1st card is ace of spades}) = \frac{1}{52}$$

→ If the ace of spades is drawn first, then there are 51 cards left in the deck of which 13 are hearts:

$$p(\text{2nd card is a heart} \mid \text{1st card is ace of spade})$$
$$= \frac{13}{51}$$

So, applying multiplication law, we get:

$$p(\text{ace of spade, then a heart}) = \frac{1}{52} \cdot \frac{13}{51}$$
$$= \frac{13}{4 \cdot 13 \cdot 51} = \frac{1}{204}$$
$$\boxed{= 0.0049}$$

$$p(\text{ace of spade, then a heart}) = \boxed{0.49\%}$$

* So, there is 0.49 % chance that the two Cards taken out from Pack of cards are ace of Spade and Second card is a heart.

Q(2)B

Shareen has to select two students from a class of 23 girls and 25 boys. what is the probability that both students chosen are boys?

Solution :=

Total no. of students = 23+25 = 48

- $P(B1) = \frac{25}{48}$
- $P(B2) = \frac{24}{47}$

Now,

$P(B1 \text{ and } B2) = P(B1) \text{ and } P(B2/B1)$

$= \frac{25}{48} \cdot \frac{24}{47}$

$= \frac{600}{2256} \Rightarrow \boxed{0.2659}$

$\boxed{P(B1 \text{ and } B2) = 26.59\%}$ **Ans.**

* So, there is 26.59% chance that the two student selected randomly will be boys.

(3)

In a Pack of 52 cards, a card is drawn at random without replacement. Find the probability of drawing a queen followed by a jack.

Solutions:-

P(drawing queen in the first place) $= \frac{4}{52}$

P(drawing jack in second place given that queen is in first place) $= \frac{4}{51}$

P(drawing queen followed by a jack) =

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} \Rightarrow \frac{4}{663}$$

$$\boxed{\Rightarrow 0.0060}$$

P(drawing queen followed by jack) = $\boxed{0.60\%}$

Ans.

* So, there is 0.60 % chance that a queen followed by a jack will come up when a 52 pack of cards are randomly drawn without replacement.

« Independent Events »

«4»

A coin is tossed and a die is rolled. What is the probability of getting a head and a 4?

Solution:-

We have two independent events:

Event A = toss a coin to get heads.

Event B = Roll a die and get a 4.

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{6}$$

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

$P(A \cap B) = \boxed{0.0833}$

$\boxed{P(A \cap B) = 8.33\%}$ **Ans.**

* So, there is 8.33% chance that we will get a head and a 4 when a coin is tossed and a die is rolled.

Q(5)

A card is drawn from a deck of 52 cards and then replaced and a second card is drawn. Find the Probability of getting a 2 and then a King.

Solution:-

Event A = draw a card and get a 2.

Event B = draw a card and get a King

∵ Because the cards are replaced, so the two events are independent.

$$P(A) = \frac{4}{52} = \frac{1}{13} = \boxed{0.0769} \Rightarrow \boxed{7.69\%}$$

$$P(B) = \frac{4}{52} = \frac{1}{13} = \boxed{0.0769} \Rightarrow \boxed{7.69\%}$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169} \simeq \boxed{0.0059}$$

$\boxed{P(A \text{ and } B) = 0.59\%}$ **Ans.**

* so, there is 0.59% chance of getting a 2 and then a King when a card is drawn from a deck 52 cards and then replaced and a second card is drawn.