CS 4072 - Topics in CS
Process Mining

Lecture # 05

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FAST - NUCES, CFD Campus

Dr. Rabia Maqsood

rabia.maqsood@nu.edu.pk

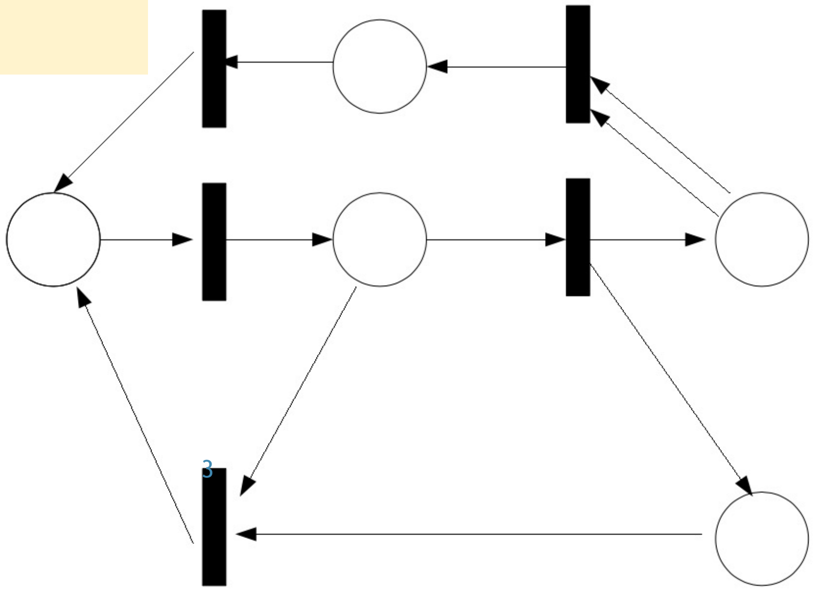
Today's Topics

- Petri-net
  - Markings and transition firing
  - Reachability graph
  - Properties: boundedness, k-bounded, safe

Petri Net

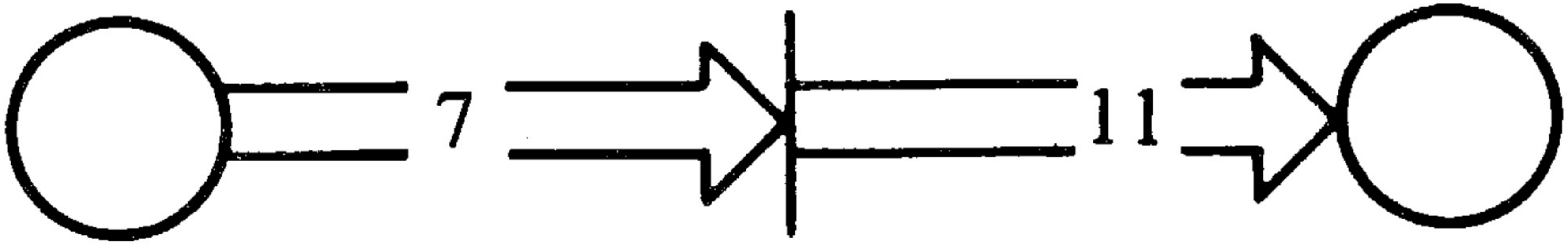
Definition 3.2 (Petri net) A *Petri net* is a triplet N = (P,T,F) where P is a finite set of *places*, T is a finite set of *transitions* such that $P \cap T = \emptyset$, and $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs, called the *flow relation*.

A *marked* Petri net is a pair (N,M), where N = (P,T,F) is a Petri net and where $M \in \mathbb{B}(P)$ is a *multi-set* over P denoting the *marking* of the net. The set of all marked Petri nets is denoted $\mathcal{N}$.



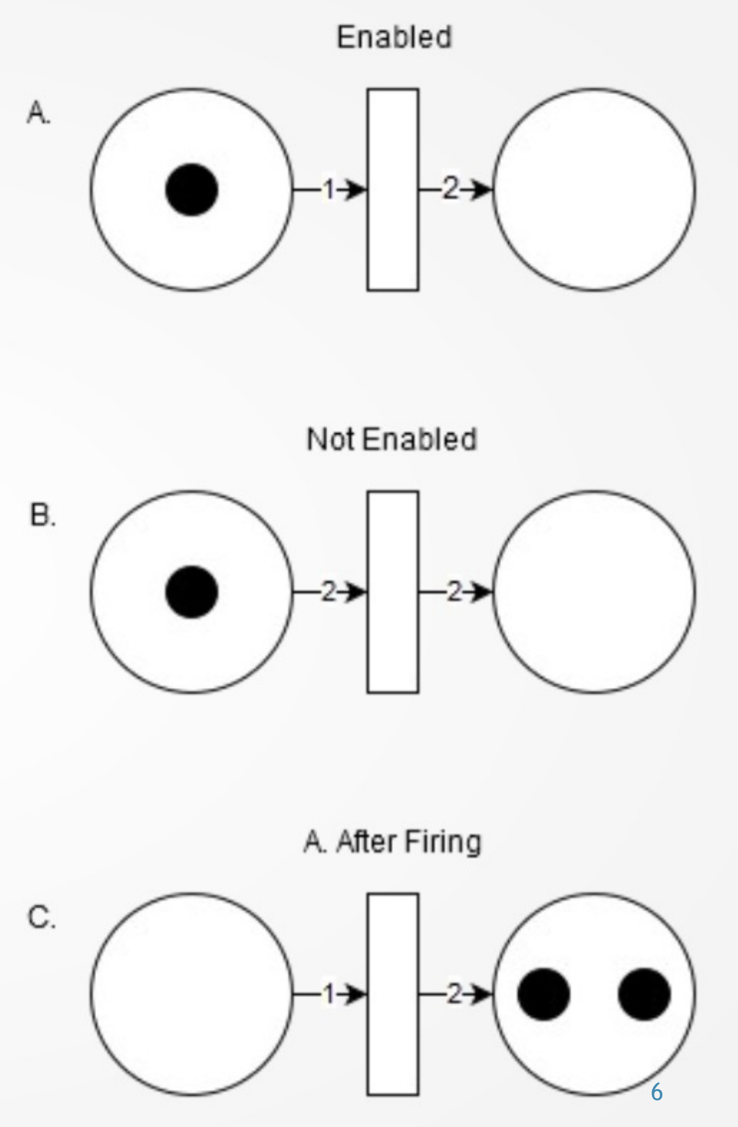
Petri Net

- Petri net graphs are multigraphs because a place may be a multiple input or output of a transition.
- This results in a graph with several arcs between the place and the transition.
- For convenience, integers are used to show high multiplicity of arcs.



Transition Firing Rules

1. A transition *t* is *enabled* if each input place *p* of *t* is marked with at least w((p,t)) tokens (where w((p,t)) is the weight of the arc from p to t).
2. An enabled transition *t* may or may not fire.
3. The *firing* of an enabled transition *t* removes w((p,t)) tokens from each input place *p* of *t* and adds w((t,p')) tokens to each output place *p'* of *t*.



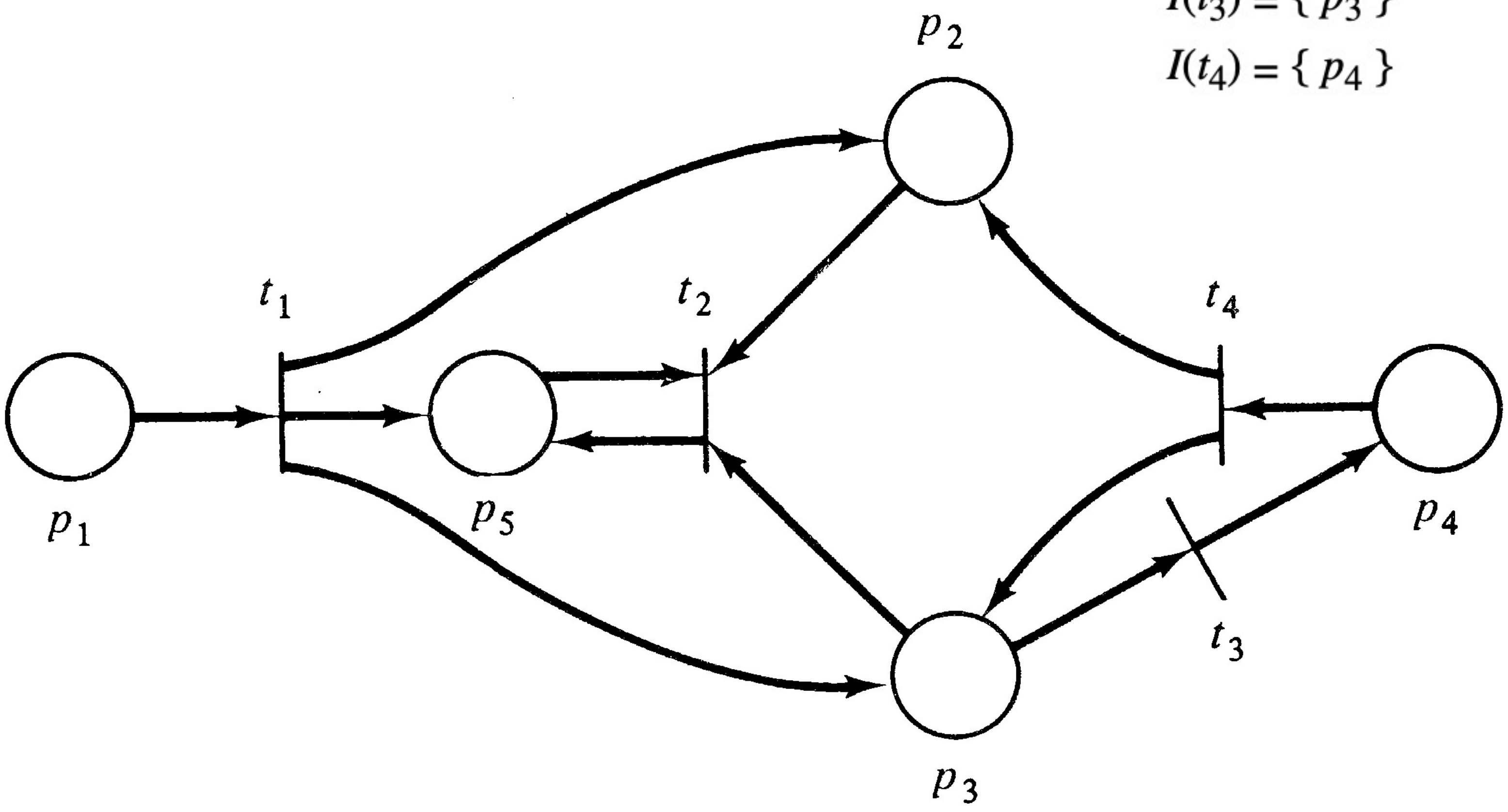
Solution – 3

$$C = (P, T, I, O)$$

$$P = \{ p_1, p_2, p_3, p_4, p_5 \}$$

$$T = \{ t_1, t_2, t_3, t_4 \}$$

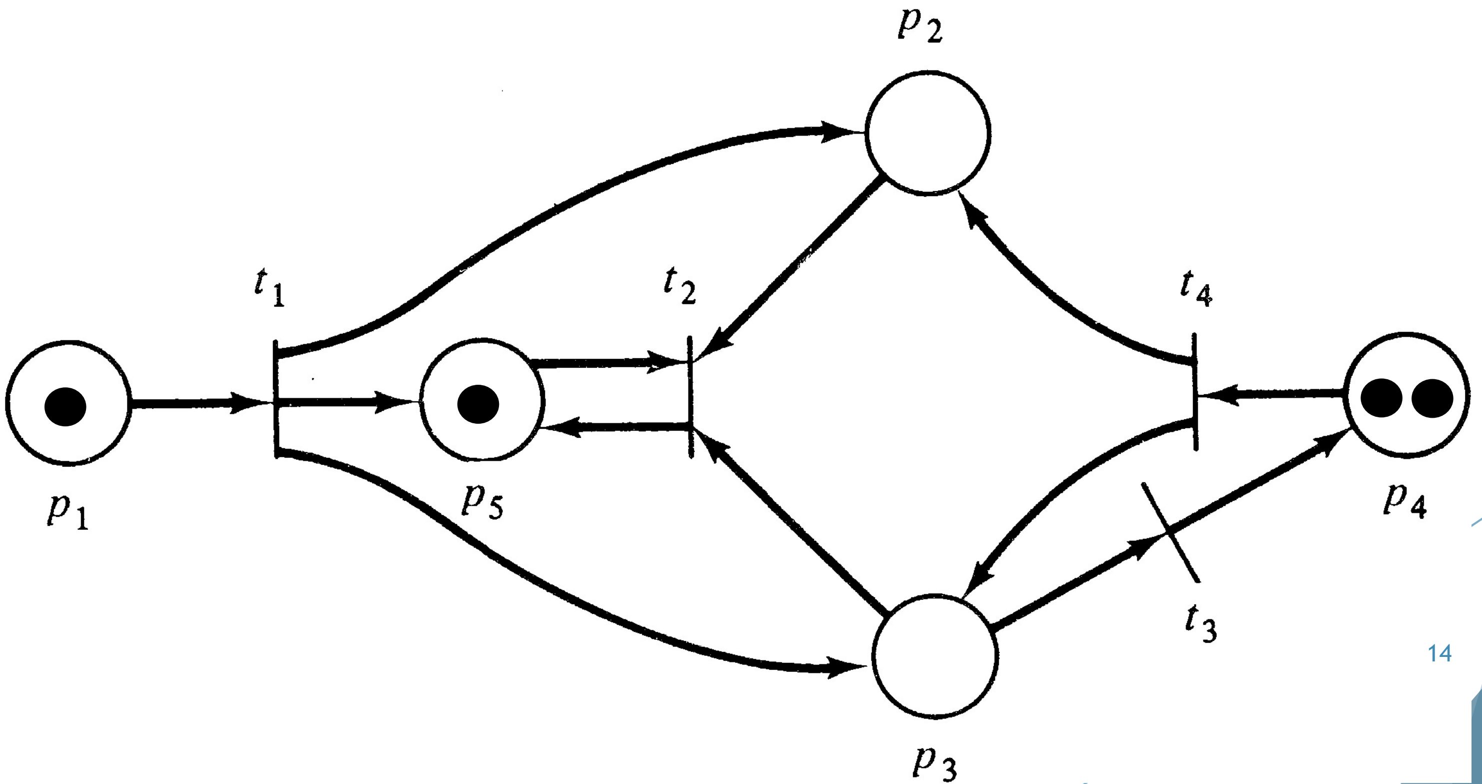
$I(t_1) = \{ p_1 \}$	$O(t_1) = \{ p_2, p_3, p_5 \}$
$I(t_2) = \{ p_2, p_3, p_5 \}$	$O(t_2) = \{ p_5 \}$
$I(t_3) = \{ p_3 \}$	$O(t_3) = \{ p_4 \}$
$I(t_4) = \{ p_4 \}$	$O(t_4) = \{ p_2, p_3 \}$



Solution – 4

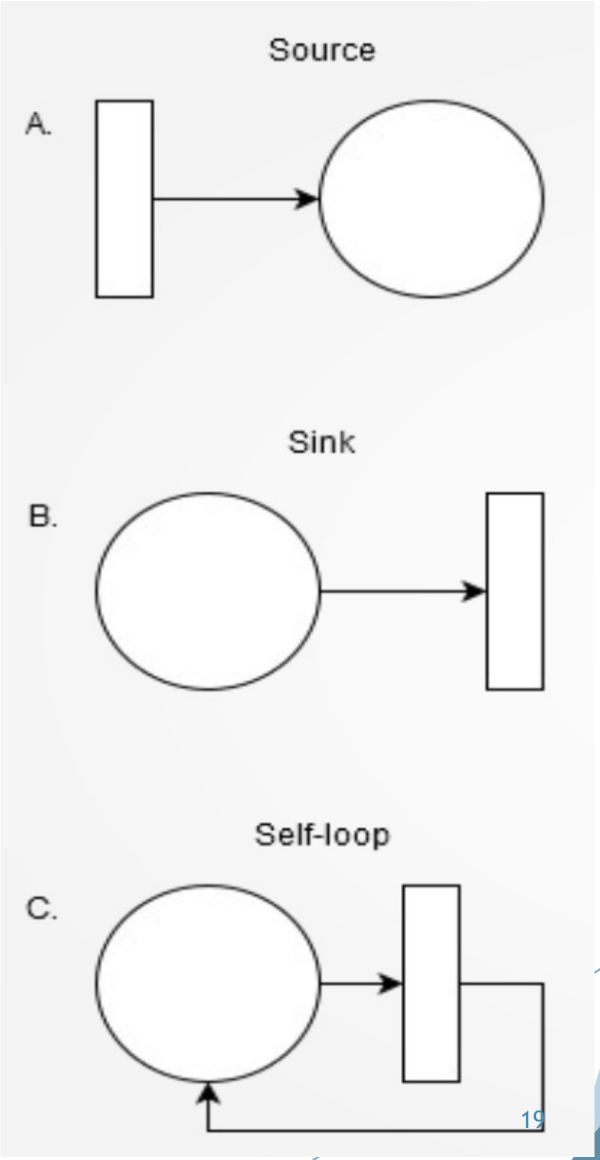
- Let M represents marking of a Petri net. Show the following marking on the previous Petri net.

$$M = (1,0,0,2,1)$$



Few more terminologies

- A transition does not need to have both input and output.
  - A transition with no input places is referred to as a source transition and a transition with no output places is called a sink transition. A source transition is always enabled.
- A tuple (p,t) is called a self-loop if p is both an input place and output place of t. A petri net which contains no self loops is called *pure*.
- A petri net whose arcs all have weight of 1 is called *ordinary*.



Labeled Petri Net

Definition 3.4 (Labeled Petri net) A *labeled Petri net* is a tuple N = (P,T,F,A,l) where (P,T,F) is a Petri net, $A \subseteq \mathcal{A}$ is a set of *activity labels*, and $l \in T \rightarrow A$ is a *labeling function*.

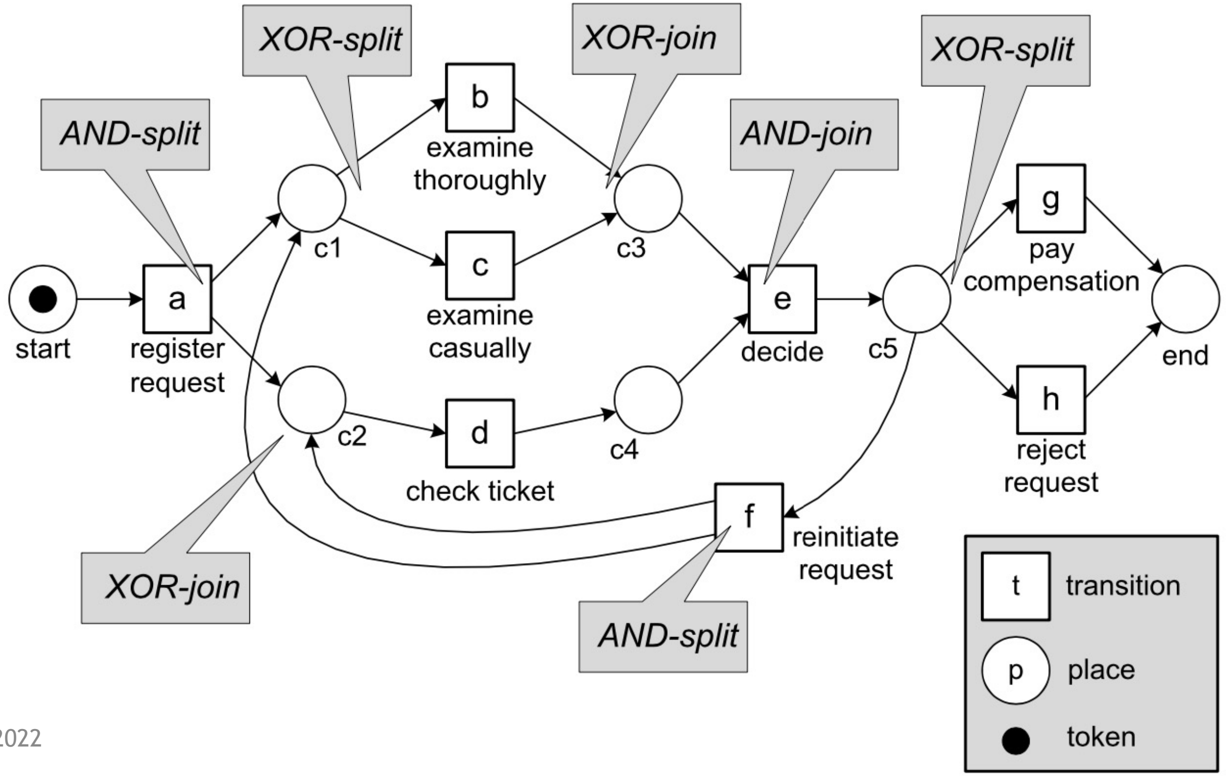


Fig. 3.2 A marked Petri net

Labeled Petri Net

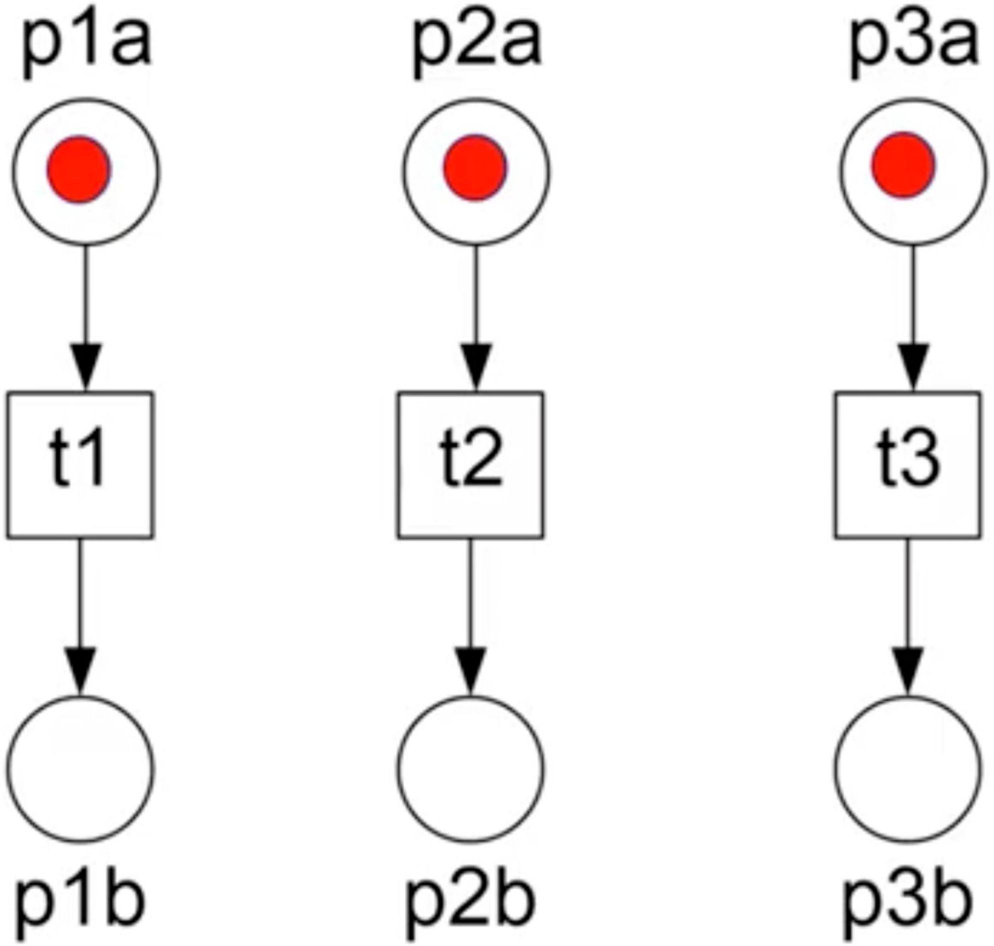
Definition 3.4 (Labeled Petri net) A *labeled Petri net* is a tuple $N = (P,T,F,A,l)$ where (P,T,F) is a Petri net, $A \subseteq \mathcal{A}$ is a set of *activity labels*, and $l \in T \to A$ is a *labeling function*.

- Multiple transitions may bear the same label.
- One can think of the transition label as the *observable action*.

Petri Net State Space

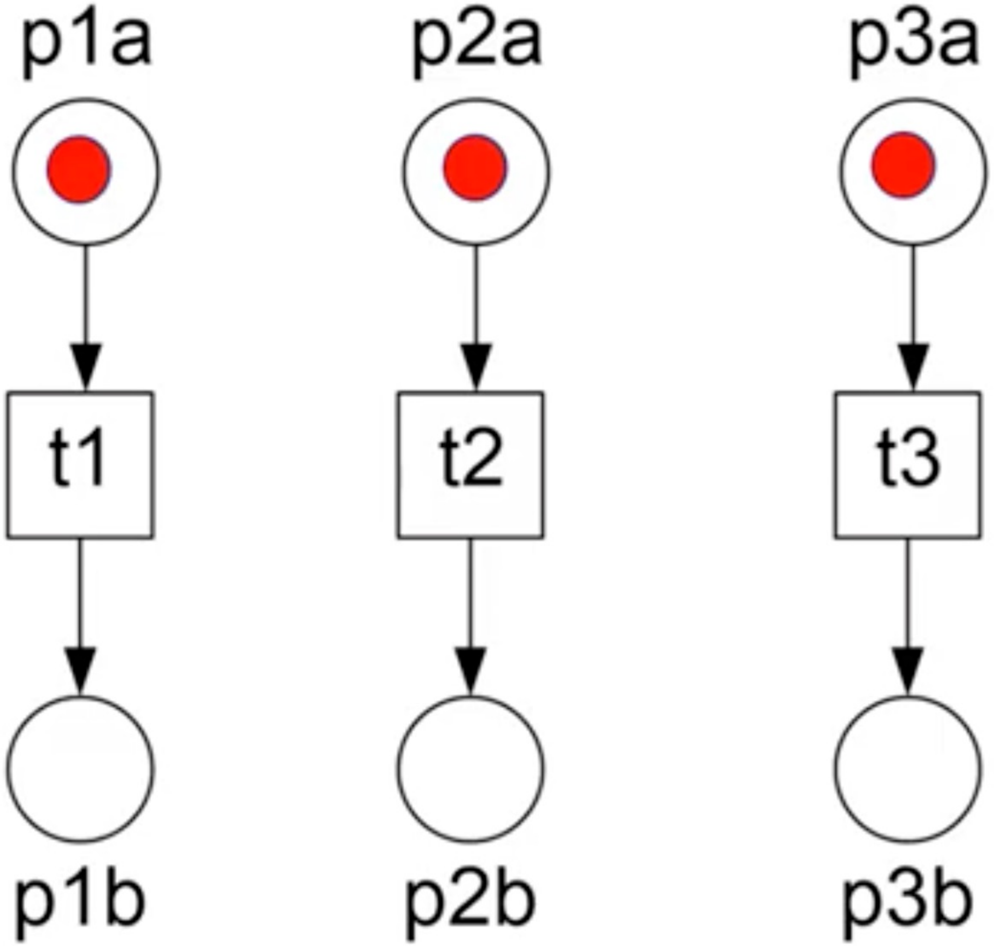
- Firing of a transition represents a change in the state of the Petri net by a change in the marking of the net.
- The ***state space*** of a Petri net with *n* places is the set of all markings, that is, M^n.

Concurrent Transitions



All three transitions are enabled and can fire in any order or even concurrently.

Concurrent Transitions



2 x 2 x 2 = 8 reachable states

Reachability Graph

Definition 3.5 (Reachability graph) Let (N,M_0) with $N = (P,T,F,A,l)$ be a marked labeled Petri net. (N, M_0) defines a transition system $TS = (S, A', T')$ with $S = [N, M_0\rangle$, $S^{start} = \{M_0\}$, $A' = A$, and $T' = \{(M, l(t), M') \in S \times A \times S \mid \exists_{t \in T} (N, M)\ [t\rangle\ (N, M')\}$.

TS is often referred to as the ***reachability graph*** of (N,M_0).

A marking M is *reachable* from the initial marking M_0 if and only if there exists a sequence of enabled transitions whose firing leads from M_0 to M'.

The set of reachable markings of (N, M_0) is denoted $[N, M_0\rangle$.

$(N,M)[t\rangle(N,M')$ denotes that firing the enabled transition results in marking M'

For a petri net N, $(N,M)[t\rangle$ denotes that t is enabled at marking M

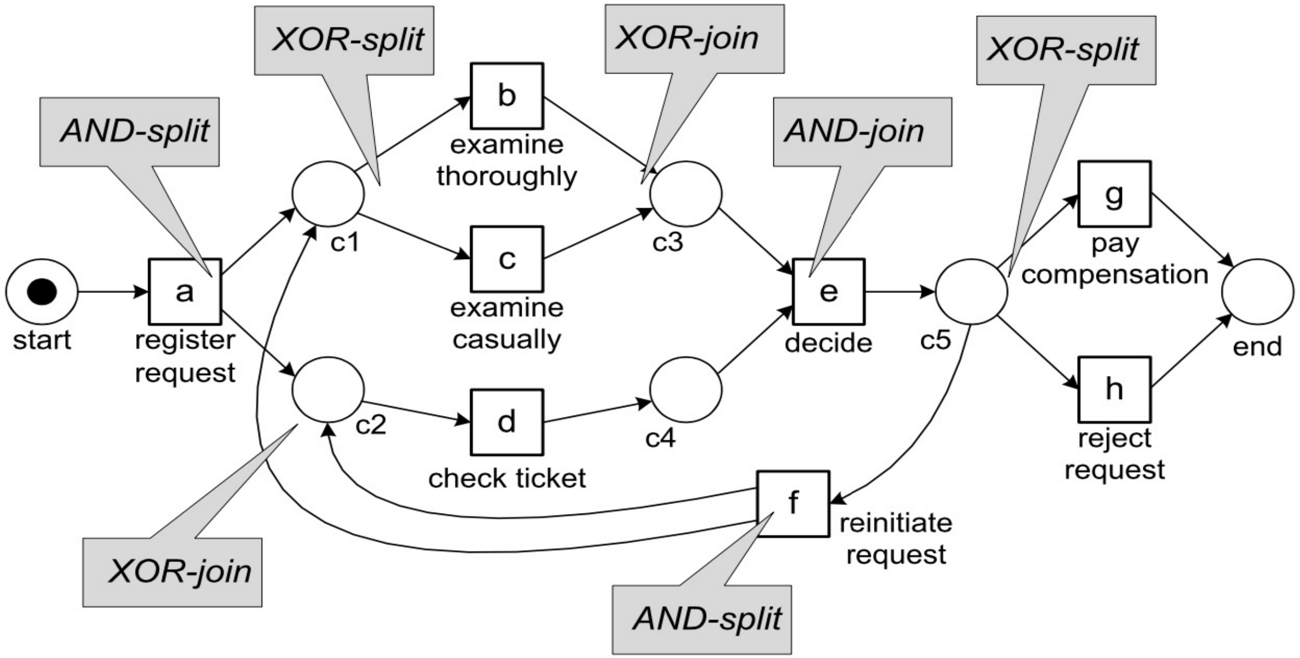


Fig. 3.2 A marked Petri net

<u>Note:</u> a marked Petri net may have infinitely many reachable states.

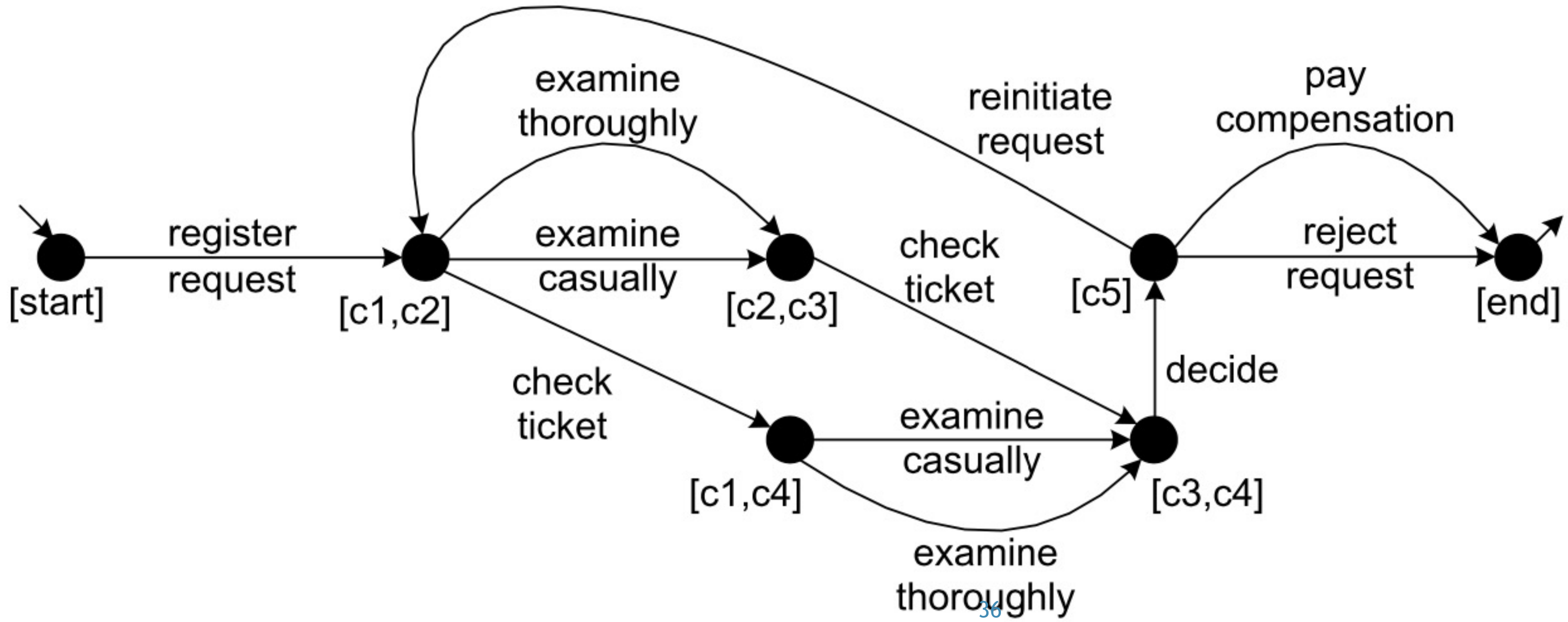
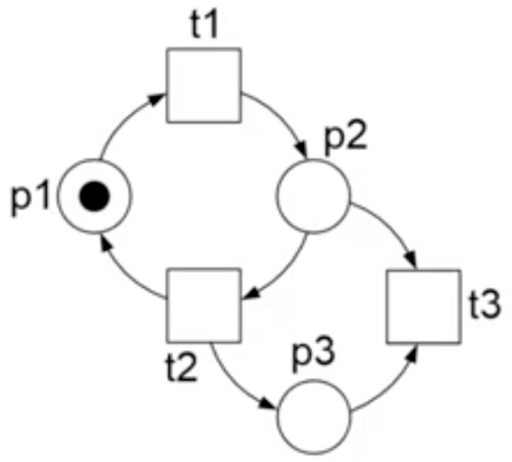
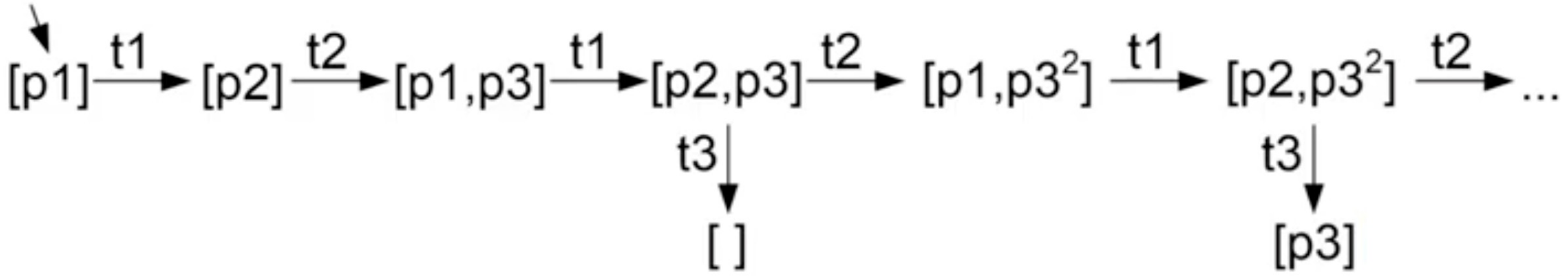


Fig. 3.3 The reachability graph of the marked Petri net shown in Fig. 3.2

Reachability Graph

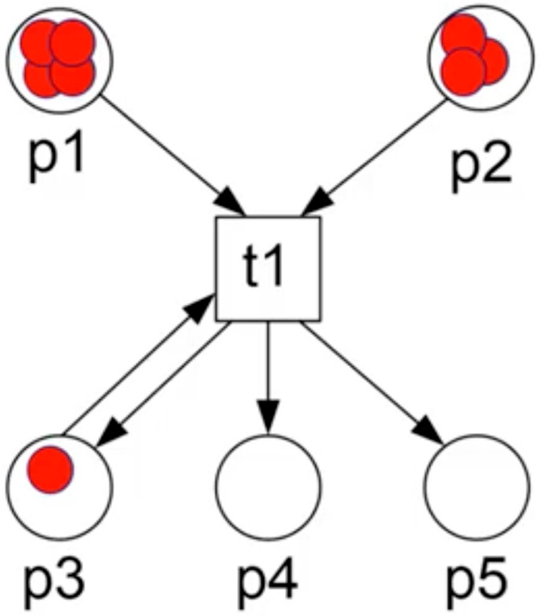


Reachability graph may be infinite …

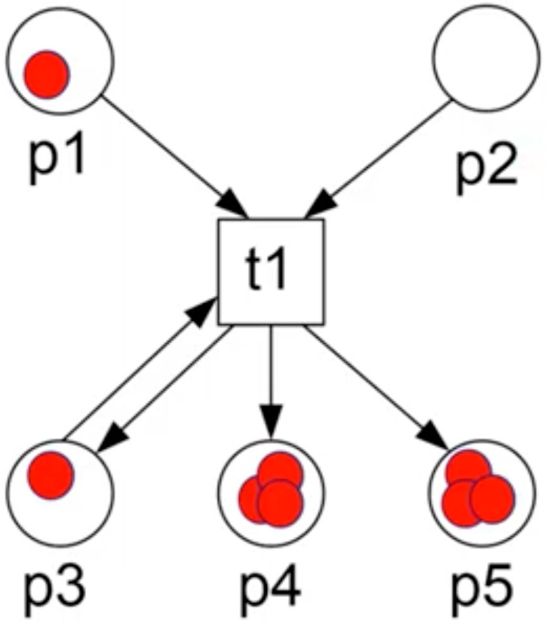


Practice Work – 6

- Draw the reachability graph of the following Petri net.



Initial marking



Final marking

Example

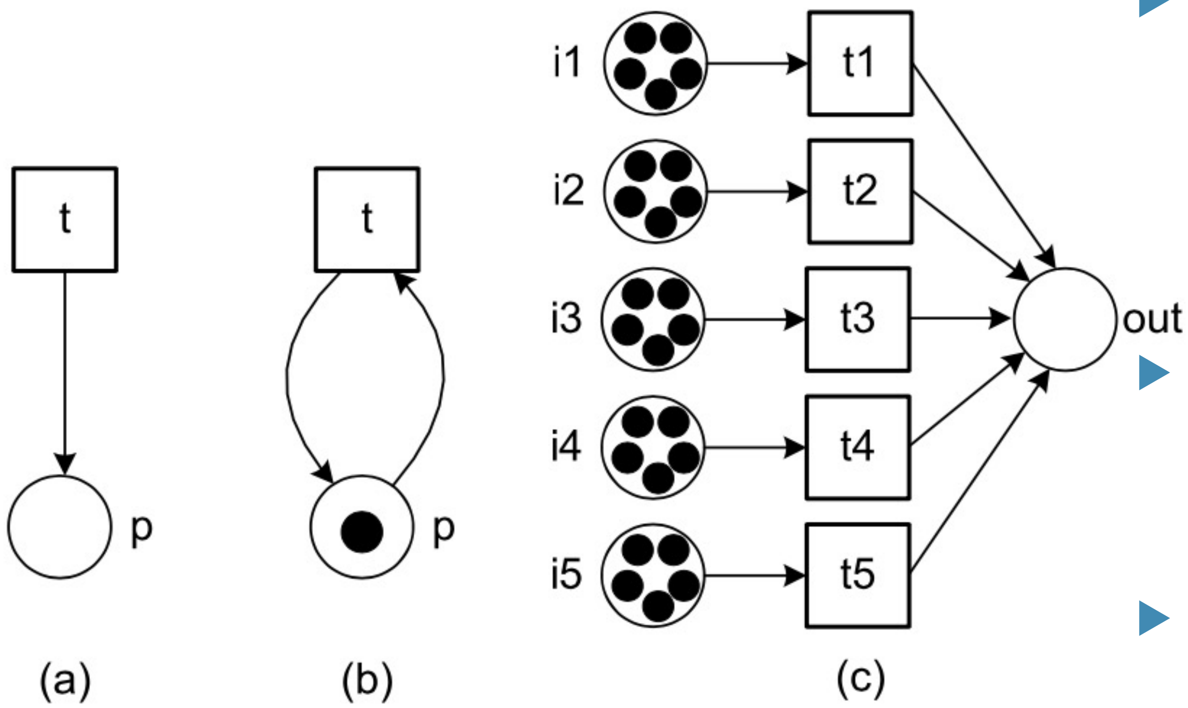


Fig. 3.4 Three Petri nets: (**a**) a Petri net with an infinite state space, (**b**) a Petri net with only one reachable marking, (**c**) a Petri net with 7776 reachable markings

- In (a), transition t is continuously enabled because it has no input place. Therefore, it can put any number of tokens in p.
- In (b), the only reachable state is [p].
- In (c), shows the effect of concurrency. The corresponding transition system has 6^5 = 7776 states and 32,400 transitions.

Petri Net Properties

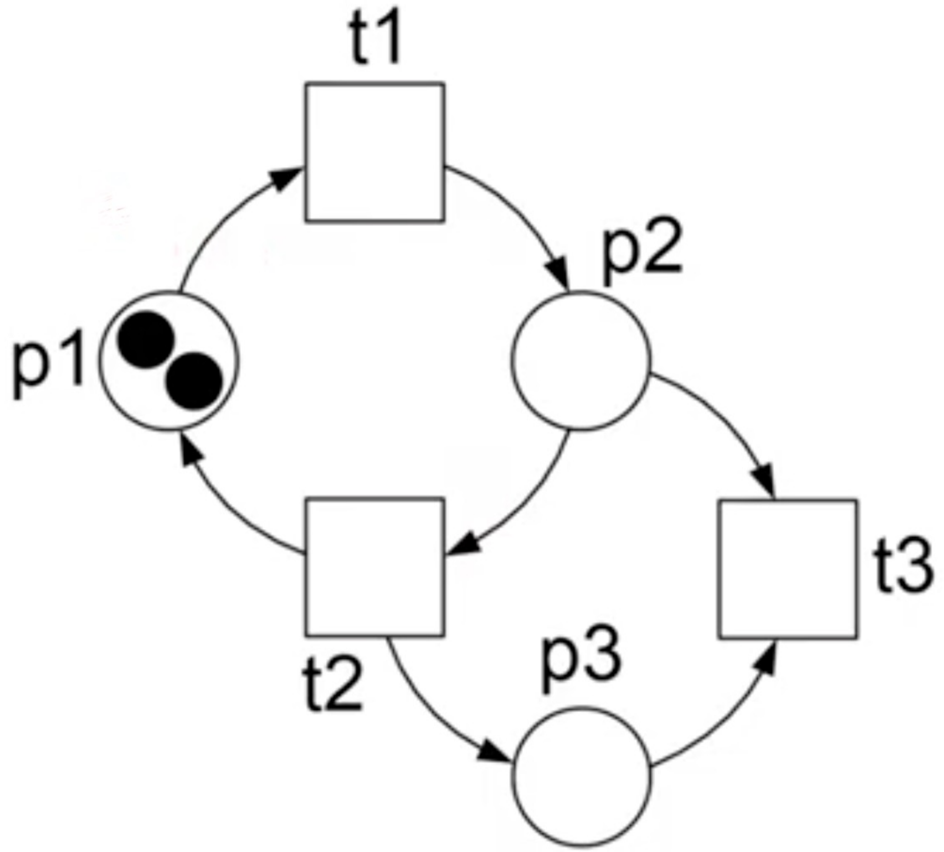
- A marked Petri net (N,M0) is ***k-bounded*** if no place ever holds more than *k* tokens.

  Formally, for any $p \in P$ and any $M \in [N,M0\rangle$: $M(p) \leq k$

- A marked Petri net is ***safe*** if and only if it is 1-bounded.

- A marked Petri net is ***bounded*** if and only if there exists a $k \in N$ such that it is k-bounded.

Boundedness



- P1 is 2-bounded
- P2 is 2-bounded
- P3 is unbounded
- Hence, the whole Petri net is unbounded

Reading Material

- Chapter 3: Aalst