

# CS 4072 - Topics in CS Process Mining

Lecture # 08

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FAST - NUCES, CFD Campus

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# Today's Topics

- ▶ Alpha algorithm
  - ▶ Activities ordering relation
  - ▶ Footprint matrix

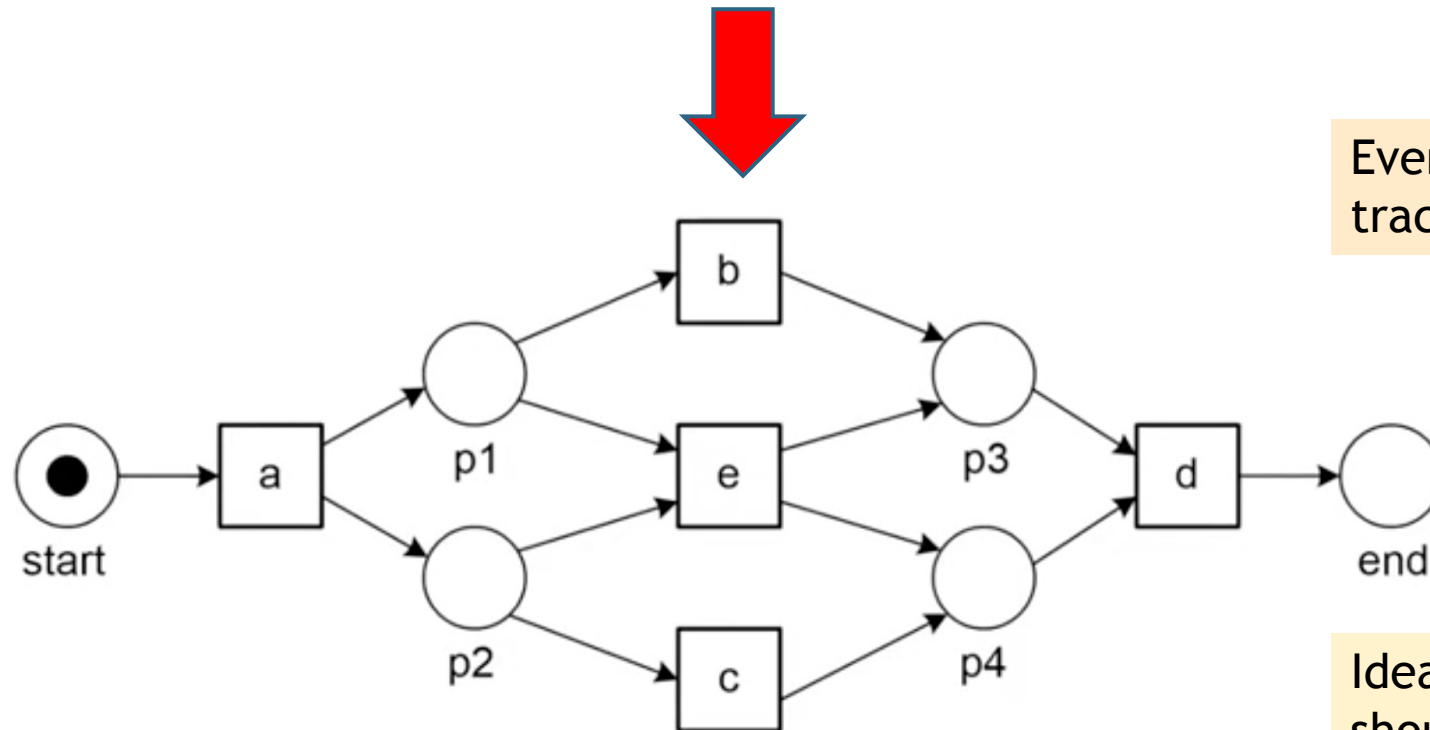
# Event log

$$L_1 = [<a, b, c, d>^3, <a, c, b, d>^2, <a, e, d>]$$

- ▶ An **event log** is a multiset of traces (same trace may appear multiple times).
- ▶ A **trace** is a sequence of **activity** names ordered by a **timestamp**.

# Goal of Alpha algorithm

$$L_1 = [<a, b, c, d>^3, <a, c, b, d>^2, <a, e, d>]$$

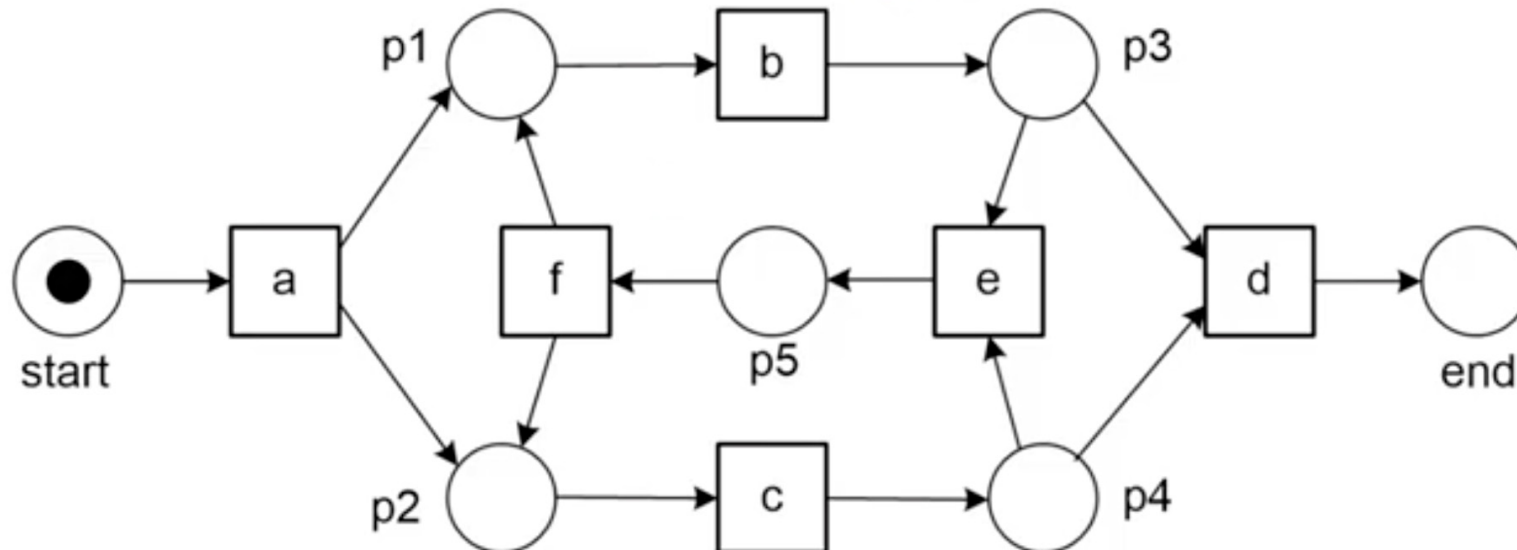


Event log contains all possible traces of model and vice versa.

Ideally, the discovered Petri net should be a sound WF-net.

# Another Example

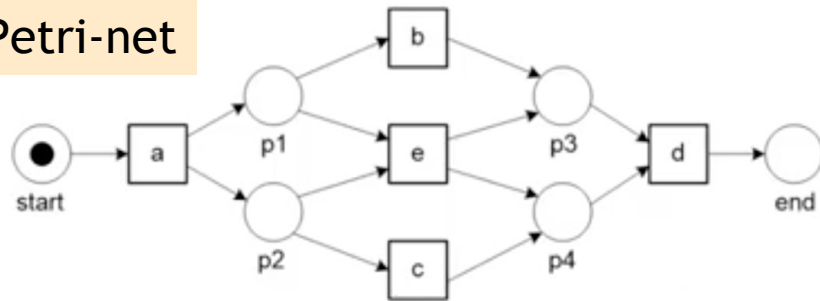
$L_2 = [<a, b, c, d>^3, <a, c, b, d>^4, <a, b, c, e, f, b, c, d>^2, <a, b, c, e, f, c, b, d>, <a, c, b, e, f, b, c, d>^2, <a, c, b, e, f, b, c, e, f, c, b, d>]$



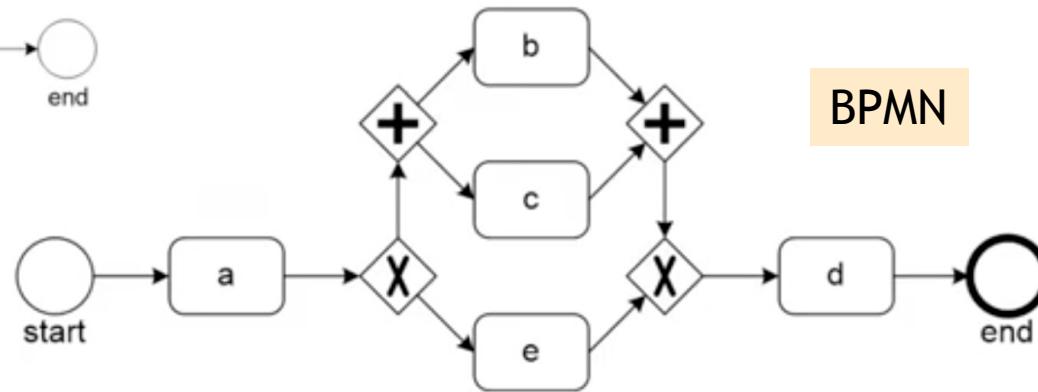
Generalization: event log contains only subset of all possible traces of model.

# Notation is less relevant

Petri-net



BPMN



$$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$$

# Process Discovery

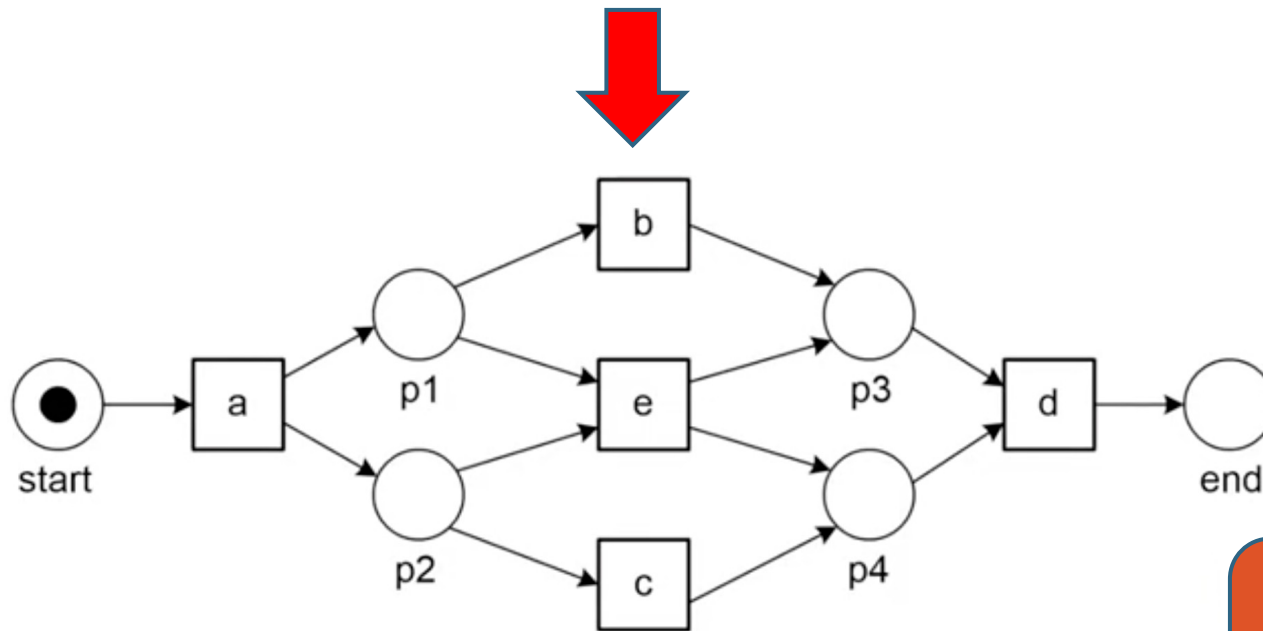
**Definition 6.2** (Specific process discovery problem) A *process discovery algorithm* is a function  $\gamma$  that maps a log  $L \in \mathbb{B}(\mathcal{A}^*)$  onto a marked Petri net  $\gamma(L) = (N, M)$ .

Ideally,  $N$  is a *sound WF-net* and all traces in  $L$  correspond to possible firing sequences of  $(N, M)$ .

Note that it is not necessary that an event log contains all firing sequences of  $(N, M)$ .

# Challenge for Process Discovery

$L_1 = [<a, b, c, d>^3, <a, c, b, d>^2, <a, e, d>]$



**Fitness**

(ability to explain observed behavior)

**Precision**

(avoiding underfitting)

**Generalization**

(avoiding overfitting)

**Simplicity**

(Occam's razor principle)



# Alpha Algorithm

Process discovery from event log

# Log-based ordering relations ( $\succ$ , $\rightarrow$ , $\parallel$ , $\#$ )

- ▶ Let  $L$  be an event log over  $\mathcal{A}$ , i.e.,  $L \in \mathbb{B}(\mathcal{A}^*)$ . Let  $a, b \in \mathcal{A}$ 
  - ▶  $a \succ_L b$  if and only if there is a trace  $\sigma = \langle t_1, t_2, t_3, \dots, t_n \rangle$  and  $i \in \{1, \dots, n-1\}$  such that  $\sigma \in L$  and  $t_i = a$  and  $t_{i+1} = b$
  - ▶  $a \rightarrow_L b$  if and only if  $a \succ_L b$  and  $b \succ_L a$
  - ▶  $a \#_L b$  if and only if  $a \not\succ_L b$  and  $b \not\succ_L a$
  - ▶  $a \parallel_L b$  if and only if  $a \succ_L b$  and  $b \succ_L a$

# Log-based ordering relations: ( $>$ , $\rightarrow$ , $||$ , $\#$ )

$L_1 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^2, \langle a, e, d \rangle]$


- ▶ **Direct succession:**  $x > y$ 
  - ▶ Iff for some case  $x$  is directly followed by  $y$
- ▶ **Causality:**  $x \rightarrow y$ 
  - ▶ Iff  $x > y$  and not  $y > x$
- ▶ **Parallel:**  $x || y$ 
  - ▶ Iff  $x > y$  and  $y > x$
- ▶ **Choice:**  $x \# y$ 
  - ▶ Iff not  $x > y$  and not  $y > x$

$a > b$   
 $a > c$   
 $a > e$   
 $b > c$   
 $b > d$   
 $c > b$   
 $c > d$   
 $e > d$

$a \rightarrow b$   
 $a \rightarrow c$   
 $a \rightarrow e$   
 $b \rightarrow d$   
 $c \rightarrow d$   
 $e \rightarrow d$

$b || c$   
 $c || b$

$b \# e$   
 $e \# b$   
 $c \# e$   
 $a \# d$   
...



$abcd$   
 $acbd$   
 $aed$

# Footprint of a log

- ▶ A matrix capturing relations between all the activities

**Table 6.1** Footprint of  $L_1$ :  
 $a \#_{L_1} a$ ,  $a \rightarrow_{L_1} b$ ,  $a \rightarrow_{L_1} c$ ,  
etc.

	$a$	$b$	$c$	$d$	$e$
$a$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$
$b$	$\leftarrow_{L_1}$	$\#_{L_1}$	$\parallel_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
$c$	$\leftarrow_{L_1}$	$\parallel_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$
$d$	$\#_{L_1}$	$\leftarrow_{L_1}$	$\leftarrow_{L_1}$	$\#_{L_1}$	$\leftarrow_{L_1}$
$e$	$\leftarrow_{L_1}$	$\#_{L_1}$	$\#_{L_1}$	$\rightarrow_{L_1}$	$\#_{L_1}$

# Answer

- Create a footprint matrix for this log.

$L_2 = [\langle a, b, c, d \rangle^3, \langle a, c, b, d \rangle^4, \langle a, b, c, e, f, b, c, d \rangle^2, \langle a, b, c, e, f, c, b, d \rangle, \langle a, c, b, e, f, b, c, d \rangle^2, \langle a, c, b, e, f, b, c, e, f, c, b, d \rangle]$

**Table 6.2** Footprint of  
 $L_2 = [\langle a, b, c, d \rangle^3,$   
 $\langle a, c, b, d \rangle^4,$   
 $\langle a, b, c, e, f, b, c, d \rangle^2,$   
 $\langle a, b, c, e, f, c, b, d \rangle,$   
 $\langle a, c, b, e, f, b, c, d \rangle^2,$   
 $\langle a, c, b, e, f, b, c, e, f, c, b, d \rangle]$

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
<i>a</i>	#	→	→	#	#	#
<i>b</i>	←	#		→	→	←
<i>c</i>	←		#	→	→	←
<i>d</i>	#	←	←	#	#	#
<i>e</i>	#	←	←	#	#	→
<i>f</i>	#	→	→	#	←	#

# Answer

- Create a footprint matrix for this log.

$L_3 = \langle a, b, c, d, e, f, b, d, c, e, g \rangle, \langle a, b, d, c, e, g \rangle^2, \langle a, b, c, d, e, f, b, c, d, e, f, b, d, c, e, g \rangle$

**Table 6.3** Footprint of  $L_3$

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
<i>a</i>	#	→	#	#	#	#	#
<i>b</i>	←	#	→	→	#	←	#
<i>c</i>	#	←	#		→	#	#
<i>d</i>	#	←		#	→	#	#
<i>e</i>	#	#	←	←	#	→	→
<i>f</i>	#	→	#	#	←	#	#
<i>g</i>	#	#	#	#	←	#	#

# Reading Material

- ▶ Chapter 6: Aalst