10.018: Modelling Space and Systems
Term 2, 2024

Course Overview

This course provides the knowledge on how to model real life problems by casting them against a rigorous modelling framework on the topics of multivariable calculus and linear algebra. This course builds upon the Term 1 course, Modelling and Analysis, and will cover the following topics: differentiation and integration in multiple dimensions, optimisation, line integrals, linear maps, eigenvalues and eigenvectors. By working in group projects, students will appreciate the various topics and connections between mathematics and physics, computer science, probability, statistics and other topics.

The second half of the course will cover linear algebra, which includes linear maps, idea of a basis, subspaces, eigenvalues, and eigenvectors. Students will see how these topics tie in with modern day applications, and will also see some pitfalls for "standard algorithm" if they are applied blindly out of the box. Students will appreciate the various topics and connections between mathematics and physics, computer science, probability, statistics, and other topics.

Course Instructors

Course Lead: NG Wei Khim (Email: weikhim_ng@sutd.edu.sg)
Instructors: CAI Kui, CHING Chee Leong, Bernard EE, Justin FU, Sumbul KHAN, Omar ORTIZ, WONG Wei Pin, XUE Hansong, ZHANG Yining

Communications

Email: Announcements will be made via email. Do check your email on regular basis.
Office Hours: Instructors will discuss with their respective classes on the office hours.

Course Assessments

Component	**Weightage**
Exam 1: Multivariable Calculus	25%
Exam 2: Linear Algebra	25%
Homework	12%
Design 1D Project	25%
Math Modelling Activities	8%
Class Participation	5%
Course Survey Completion (Bonus)	2%
Total	**102%**

In addition to the overall weightage, students must demonstrate certain level of competencies in exams and 1D project to pass.

Exams

Format: Closed book exam and no calculator is allowed. An A4 double-sided **handwritten** help sheet is allowed.

	Scope	Date and Time
Exam 1	Week 1 to 6	Week 8: **15 March (Friday), 2.30pm to 4.30pm**
Exam 2	Week 8 to 13	Week 14: **25 April (Thursday), 9.00am to 11.00am**

Homework

Each homework submission carries 3%. That is $4 \times 3\% = 12\%$. Homework submission will be via MS TEAMS. Each cohort will have their own TEAMS page "MSS FXX Homework Submission 2024" for the submissions.

Homework	Scope	Due Date
HW 1	Week 1 and 2	Week 4: **15 Feb (Thursday), 7 pm**
HW 2	Week 3 and 4	Week 6: **29 Feb (Thursday), 7 pm**
HW 3	Week 5 and 6	No submission required
HW 4	Week 8 and 9	Week 11: **04 Apr (Thursday), 7 pm**
HW 5	Week 10 and 11	Week 13: **18 Apr (Thursday), 7 pm**
HW 6	Week 12 and 13	No submission required

Design 1D Project

Design 1D project uses concepts from week 1 to 4 and the math modelling (covered in week 1 to 3). You are encouraged to do the project continually over the weeks. The project is due **Week 6: 27 February (Tuesday), 6pm**. The 1D project groupings can be found in eDimension. Refer the file "Design 1D_Questions.pdf" for more details including the submission process.

Math Modelling Activities

In the second classes in week 1 to 3, you will work in your 1D groupings on math modelling activities.

MM Activity	Due Date	Weightage
1	11.59pm on the day of week 1 class 2	2%(attempt) + 1%(decent effort)
2	Week 3: 8 February (Thursday), 7.00pm	2%(attempt) + 1%(decent effort)
3	Week 4: 14 February (Wednesday), 7.00pm	1%(attempt) + 1%(decent effort)

The submission will be via the Discussion Forum in eDimension. Each cohort will have their own discussion thread named "SCXX - MM forum". Refer to the class slides for more details.

Class Participation

Class participation includes **regular attendance, punctuality, classroom decorum and active participation during class activities.**

Progressive Learning Session

Timing: Friday 2.30pm to 4.30pm (Starting week 1) Venue: Lecture Theatre 3

This is a slow-paced session that will cover the materials from the previous and current weeks. It is meant to help students with a weak foundation in math. This session focuses more on being able to do the computations by hand, and understand what the answer means. You are encouraged to attend this session if you have queries **especially those who have received an invitation to join**.

Bootcamp

Timing: 20 May 2024 to 31 May 2024 **(to be confirmed)

Modelling Space and Systems is a required subject that students must pass in order to proceed to Term 3. Failures will have to attend the bootcamp. The attendance to the bootcamp is compulsory, failing to attend the bootcamp will result in an immediate failure of bootcamp. **DO NOT plan** for any overseas trip during bootcamp period if you are in the danger zone of failing the course based on your performance during the term.

Students must sit for bootcamp exam(s) at the end of the bootcamp. Students who fail the bootcamp exam(s) will have to take a self-study exam in the following term.

Resources

Visualisation

GeoGebra will be used to visualise many examples and exercises. You may use the web version or download the GeoGebra Calculator Suite or GeoGebra 3D Calculator from the App Store or Google Play. These apps allow for augmented reality (AR) experiences to improve the visualisations.

Discussion Forum

Discussion forums are set up (in eDimension) for the general content, homework and design 1D. Feel free to post any queries. You are also encouraged to reply to your classmates queries if you know the answers.

Personalised AI Tutor[1]

You will received the login information for access to the Personalised AI Tutor. The course materials have been loaded to the AI platform. The replies from this AI will be Socratic in nature and relevant to the course. In addition, all in-class activities are also preloaded to the platform (to be released weekly). You can use this AI tutor to test yourself by re-doing these activities and ask the AI on certain steps.

Similar to other AI platform, the replies for this platform are not 100% accurate. Do not rely purely on this platform. Consult your instructors if you have doubts. If you can spot the errors in the replies, this implies that you have achieved certain level of knowledge competency.

Skills-Tree[2]

For those who have signed up for the Skills-Tree initiative in around November 2023, there will be a gamification section ("quest") in the app for our course Modelling Space and System. There are some MCQs for each lessons where you can "level up" by answering them correctly.

The MCQs are generated by AI and have been vetted (could have missed some). For those who did not sign up, you can still sign up by referring the invitation email sent out a few months ago. You are encouraged use this platform to test your concepts.

References

- G.F. Simmons, *Calculus with Analytic Geometry*, 2E, McGraw-Hill, 1996.
- D. Hughes-Hallett, *Calculus: Single and Multivariable*, 7E, John Wiley, 2019.
- J. Stewart, *Multivariable Calculus*, 7E, Brooks/Cole., 2012.
- D. Poole, *Linear Algebra: A Modern Introduction*, 3E, Brooks/Cole/Cengage Learning, 2011.

Relevant sections will be listed in each lesson and you may attempt the exercises therein.

Some online resources:

- MIT Open Courseware on Multivariable Calculus
- Calculus by Gilbert Strang
- Khan Academy Multivariable Calculus (with many interesting analogies)
- 3Blue1Brown essence of calculus and linear algebra playlists – very good animations

[1] This is an initiative by campusX. The url is `https://campusxpat.com`. It will go live by early week 2.
[2] Initiative by campusX.

Schedule of Topics

Week	Topics
Week 1 22 Jan – 26 Jan	Class 1: Vectors and multivariable functions Class 2: Level curves and partial differentiation
Week 2 29 Jan – 02 Feb	Class 1: Tangent plane approximation, directional derivative and the gradient Class 2: Chain rule, implicit differentiation and critical points
Week 3 05 Feb – 09 Feb	Class 1: Optimisation I – Extreme Value Theorem Class 2: Optimisation II – Unbounded regions
Week 4 12 Feb – 16 Feb	Class 1: Optimisation III – Lagrange Multipliers Class 2: Introduction to double integrals
Week 5 19 Feb – 23 Feb	Class 1: Double integrals, polar coordinates and triple integrals Class 2: Change of variables and vector fields
Week 6 26 Feb – 01 Mar	Class 1: Line integrals Class 2: Conservative vector fields, divergence and curl
Week 7	Mid-term break
Week 8 11 Mar – 15 Mar	Class 1: Systems of linear equations, elementary row operations and Gaussian elimination Class 2: Gauss Jordan elimination **Exam 1: 15 March (Friday), 2.30pm to 4.30pm**
Week 9 18 Mar – 22 Mar	Class 1: Introduction to matrices, matrix operations and inverses Class 2: Matrix inverses
Week 10 25 Mar – 29 Mar	Class 1: Matrix properties – trace and determinant Class 2: Span, linear independence and subspaces
Week 11 01 Apr – 05 Apr	Class 1: Basis, orthogonal basis and change of basis Class 2: Subspaces associated with matrices and rank
Week 12 08 Apr – 12 Apr	Class 1: Eigenvalues and eigenvectors Class 2: Diagonalisation
Week 13 15 Apr – 19 Apr	Class 1: Linear transformations I – geometric transformation Class 2: Linear transformations II – projection and regression
Week 14 22 Apr – 26 Apr	**Exam 2: 25 April (Thursday), 9.00am to 11.00am**

Updated: January 19, 2024